PHILOSOPHICAL

TRANSACTIONS

OF THE

ROYAL SOCIETY

OF

LONDON.

FOR THE YEAR MDCCCLXV.

VOL. 155.

LONDON:
PRINTED BY TAYLOR AND FRANCIS, RED LION COURT, FLEET STREET.
MDCCCLXV.

CONTENTS

OF VOL. 155.

I. *On the Spectra of Ignited Gases and Vapours, with especial regard to the different Spectra of the same elementary gaseous substance. By Dr.* J. PLÜCKER, *of Bonn, For. Memb. R.S., and Dr.* J. W. HITTORF, *of Münster* . . . . page 1

II. *On the Osteology of the genus* Glyptodon. *By* THOMAS H. HUXLEY, *F.R.S.* . . . 31

III. *Investigations of the Specific Heat of Solid Bodies. By* HERMANN KOPP. *Communicated by* T. GRAHAM, *Esq., F.R.S.* 71

IV. *On the Composition of Sea-water in the different parts of the Ocean. By* GEORG FORCHHAMMER, *Professor at the University, and Director of the Polytechnic Institution at Copenhagen. Communicated by the President* 203

V. *On the Magnetic Character of the Armour-plated Ships of the Royal Navy, and on the Effect on the Compass of particular arrangements of Iron in a Ship. By* FREDERICK JOHN EVANS, *Esq., Staff Commander R.N., F.R.S., Superintendent of the Compass Department of Her Majesty's Navy; and* ARCHIBALD SMITH, *Esq., M.A., F.R.S., late Fellow of Trinity College, Cambridge, Corresponding Member of the Scientific Committee of the Imperial Russian Navy* 263

VI. *On some Foraminifera from the North Atlantic and Arctic Oceans, including Davis Straits and Baffin's Bay. By* W. KITCHEN PARKER, *F.Z.S., and Professor* T. RUPERT JONES, *F.G.S. Communicated by Professor* HUXLEY, *F.R.S.* . . . 325

VII. *New Observations upon the Minute Anatomy of the Papillæ of the Frog's Tongue. By* LIONEL S. BEALE, *M.B., F.R.S., Fellow of the Royal College of Physicians, Professor of Physiology and of General and Morbid Anatomy in King's College, London; Physician to King's College Hospital, &c.* 443

VIII. *A Dynamical Theory of the Electromagnetic Field. By* J. CLERK MAXWELL, *F.R.S.* 459

IX. *On the Embryogeny of* Antedon rosaceus, *Linck* (Comatula rosacea *of Lamarck*). *By Professor* WYVILLE THOMSON, *LL.D., F.R.S.E., M.R.I.A., F.G.S., &c. Communicated by* THOMAS HENRY HUXLEY, *F.R.S.* . . . . page 513

X. *On the Sextactic Points of a Plane Curve. By* A. CAYLEY, *F.R.S.* 545

XI. *A Description of some Fossil Plants, showing Structure, found in the Lower Coal-seams of Lancashire and Yorkshire. By* E. W. BINNEY, *F.R.S.* 579

XII. THE BAKERIAN LECTURE.—*On a Method of Meteorological Registration of the Chemical Action of Total Daylight. By* HENRY ENFIELD ROSCOE, *B.A., F.R.S., Professor of Chemistry in Owens College, Manchester* 605

XIII. *On the Commissures of the Cerebral Hemispheres of the Marsupialia and Monotremata as compared with those of the Placental Mammals. By* WILLIAM HENRY FLOWER, *F.R.S., F.R.C.S., Conservator of the Museum of the Royal College of Surgeons of England* 633

XIV. *On the Sextactic Points of a Plane Curve. By* WILLIAM SPOTTISWOODE, *M.A., F.R.S., &c.* 653

XV. *On the Marsupial Pouches, Mammary Glands, and Mammary Fœtus of the* Echidna Hystrix. *By Professor* OWEN, *F.R.S., &c.* 671

XVI. *On the Influence of Physical and Chemical Agents upon Blood; with special reference to the mutual action of the Blood and the Respiratory Gases. By* GEORGE HARLEY, *M.D., Fellow of the Royal College of Physicians, Professor of Medical Jurisprudence in University College, London. Communicated by Professor* SHARPEY, *M.D., Sec. R.S.* 687

XVII. *On a New Geometry of Space. By* J. PLÜCKER, *of Bonn, For. Memb. R.S.* 725

Index 793

APPENDIX.

Presents . . . . [1]

LIST OF ILLUSTRATIONS.

Plates I. to III.—Drs. J. Plücker and J. W. Hittorf on the Spectra of Ignited Gases and Vapours.

Plates IV. to IX.—Professor Huxley on the Osteology of the genus *Glyptodon*.

Plates X. & XI.—Staff-Commander Evans and Mr. A. Smith on the Magnetic Character of the Armour-plated Ships of the Royal Navy.

Plates XII. to XIX.—Mr. W. K. Parker and Professor T. R. Jones on some Foraminifera from the North Atlantic and Arctic Oceans.

Plate XX.—Professor Kopp on the Specific Heat of Solid Bodies.

Plates XXI. and XXII.—Professor Beale's New Observations upon the Minute Anatomy of the Papillæ of the Frog's Tongue.

Plates XXIII. to XXVII.—Professor W. Thomson on the Embryogeny of *Antedon rosaceus*, Linck (*Comatula rosacea* of Lamarck).

Plates XXVIII. & XXIX.—Professor Roscoe on a Method of Meteorological Registration of the Chemical Action of Total Daylight.

Plates XXX. to XXXV.—Mr. E. W. Binney on some Lower-coal-seam Fossil Plants.

Plates XXXVI. to XXXVIII.—Mr. W. H. Flower on the Cerebral Commissures of the Marsupialia and Monotremata.

Plates XXXIX. to XLI.—Professor Owen on the Marsupial Pouches, Mammary Glands, and Mammary Fœtus of the *Echidna Hystrix*.

PHILOSOPHICAL
TRANSACTIONS

OF THE

ROYAL SOCIETY

OF

LONDON.

FOR THE YEAR MDCCCLXV.

VOL. 155.—PART I.

LONDON:
PRINTED BY TAYLOR AND FRANCIS, RED LION COURT, FLEET STREET.
MDCCCLXV.

ADVERTISEMENT.

THE Committee appointed by the *Royal Society* to direct the publication of the *Philosophical Transactions*, take this opportunity to acquaint the Public, that it fully appears, as well from the Council-books and Journals of the Society, as from repeated declarations which have been made in several former *Transactions*, that the printing of them was always, from time to time, the single act of the respective Secretaries till the Forty-seventh Volume; the Society, as a Body, never interesting themselves any further in their publication, than by occasionally recommending the revival of them to some of their Secretaries, when, from the particular circumstances of their affairs, the *Transactions* had happened for any length of time to be intermitted. And this seems principally to have been done with a view to satisfy the Public, that their usual meetings were then continued, for the improvement of knowledge, and benefit of mankind, the great ends of their first institution by the Royal Charters, and which they have ever since steadily pursued.

But the Society being of late years greatly enlarged, and their communications more numerous, it was thought advisable that a Committee of their members should be appointed, to reconsider the papers read before them, and select out of them such as they should judge most proper for publication in the future *Transactions*; which was accordingly done upon the 26th of March 1752. And the grounds of their choice are, and will continue to be, the importance and singularity of the subjects, or the advantageous manner of treating them; without pretending to answer for the certainty of the facts, or propriety of the reasonings, contained in the several papers so published, which must still rest on the credit or judgement of their respective authors.

It is likewise necessary on this occasion to remark, that it is an established rule of the Society, to which they will always adhere, never to give their opinion, as a Body, upon any subject, either of Nature or Art, that comes before them. And therefore the thanks, which are frequently proposed from the Chair, to be given to the authors of such papers as are read at their accustomed meetings, or to the persons through whose hands they received them, are to be considered in no other light than as a matter of civility, in return for the respect shown to the Society by those communications. The like also is to be said with regard to the several projects, inventions, and curiosities of various kinds, which are often exhibited to the Society; the authors whereof, or those who exhibit them, frequently take the liberty to report and even to certify in the public newspapers, that they have met with the highest applause and approbation. And therefore it is hoped that no regard will hereafter be paid to such reports and public notices; which in some instances have been too lightly credited, to the dishonour of the Society.

The Meteorological Journal hitherto kept by the Assistant Secretary at the Apartments of the Royal Society, by order of the President and Council, and published in the Philosophical Transactions, has been discontinued. The Government, on the recommendation of the President and Council, has established at the Royal Observatory at Greenwich, under the superintendence of the Astronomer Royal, a Magnetical and Meteorological Observatory, where observations are made on an extended scale, which are regularly published. These, which correspond with the grand scheme of observations now carrying out in different parts of the globe, supersede the necessity of a continuance of the observations made at the Apartments of the Royal Society, which could not be rendered so perfect as was desirable, on account of the imperfections of the locality and the multiplied duties of the observer.

A List of Public Institutions and Individuals, entitled to receive a Copy of the Philosophical Transactions of each year, on making application for the same directly or through their respective agents, within five years of the date of publication.

Observatories.

Armagh.
Cape of Good Hope.
Dublin.
Edinburgh.
Greenwich.
Kew.
Liverpool.
Madras.
Oxford (Radcliffe).

Institutions.

Barbadoes........Library and Museum.
Calcutta.........Asiatic Society.
Geological Museum.
Cambridge......Philosophical Society.
Cape Town......South African Library.
Dublin..........Royal Dublin Society.
Royal Irish Academy.
Edinburgh.......Royal Society.
London.........Admiralty Library.
Chemical Society.
College of Surgeons.
Entomological Society.
Geological Society.
Geological Survey of Great Britain.
Horticultural Society.
Institute of British Architects.
Institution of Civil Engineers.
Linnean Society.
London Institution.
Royal Asiatic Society.
Royal Astronomical Society.
Royal College of Physicians.
Royal Geographical Society.
Royal Institution of Great Britain.
Royal Medical and Chirurgical Society.
Royal Society of Literature.
Society of Antiquaries.
Society of Arts.
The Queen's Library.
The Treasury Library.
United Service Museum.
Zoological Society.
Malta...........Public Library.
Manchester......Literary and Philosophical Society.
Melbourne......University Library.
Montreal........McGill College.
Oxford..........Ashmolean Society.
Radcliffe Library.
Swansea.........Royal Institution.
Sydney..........University Library.
Woolwich........Royal Artillery Library.

Belgium.

Brussels.........Académie Royale de Médecine.
Royal Academy of Sciences.

Denmark.

Copenhagen......Royal Society of Sciences.

France.

Montpellier......Academy of Sciences.
Faculté de Médecine.
Paris...........Academy of Sciences.
Dépot de la Marine.
Ecole des Mines.
Geographical Society.
Geological Society.
Jardin des Plantes.
Société d'Encouragement pour l'Industrie Nationale.
Toulouse........Academy of Sciences.

Germany.

Altona..........Observatory.
Berlin..........Royal Academy of Sciences.
Society of Experimental Philosophy.
Brünn..........Naturforschender Verein.
Dresden........Cæsarean Acad. of Naturalists.
Frankfort........Natural History Society.
Giessen..........University.
Göttingen........University.
Hamburg........Naturwissenschaftlicher-Verein.
Konigsberg......Königlichen Physikalisch Ökonomischen Gesellschaft.
Leipzig..........Royal Saxon Society of Sciences.
Mannheim......Observatory.
Munich..........Royal Academy of Sciences.
Prague..........Bohemian Society of Sciences.
Vienna..........Imperial Academy of Sciences.
Geologische Reichsanstalt.
Würzburg........Physico-Medical Society.

A List of Public Institutions and Individuals, entitled to receive a Copy of the Philosophical Transactions of each year, on making application for the same directly or through their respective agents, within five years of the date of publication (*continued*).

Hungary.

Pesth	Hungarian Academy of Sciences.

Italy.

Bologna	Academy of Sciences.
Catanea	Accademia Gioenia di Scienze Naturali.
Florence	Royal Observatory.
Milan	Institute of Sciences, Letters, and Arts.
Modena	Italian Society of Sciences.
Naples	Institute of Sciences.
Palermo	Academy of Sciences and Letters.
Rome	Academy de' Nuovi Lincei. Collegio Romano.
Turin	Royal Academy of Sciences.
Venice	Institute of Sciences, Letters, and Arts.

Java.

Batavia	Batavian Society of Sciences.

Netherlands.

Amsterdam	Royal Institute.
Haarlem	Dutch Society of Sciences.
Rotterdam	Batavian Society of Experimental Philosophy.

Portugal.

Lisbon	Royal Academy of Sciences.

Russia.

Kazan	Imperial University.
Moscow	Imperial Society of Naturalists. Public Museum.
Pulkowa	Observatory.
St. Petersburg	Imperial Academy of Sciences.

Spain.

Cadiz	Observatory.
Madrid	Royal Academy of Sciences.

Sweden and Norway.

Christiania	Royal University.
Drontheim	Royal Society of Sciences.
Gottenburg	Kongl. Vetenskaps och Vitterhets Samhälle.
Stockholm	Royal Academy of Sciences.

Switzerland.

Bern	Allg. Schweizerischen Gesellschaft.
Geneva	Société de Phys. et d'Hist. Naturelle.

Transylvania.

Klausenburg	Society of the Transylvanian Museum.

United States.

Albany	New York State Library.
Boston	American Academy of Sciences.
Newhaven (Conn.)	The Editors of the American Journal.
Cambridge	Harvard University.
Philadelphia	Academy of Natural Sciences. American Philosophical Society.
Washington	Smithsonian Institution. Observatory.

The *fifty* Foreign Members of the Royal Society.

A List of Public Institutions and Individuals, entitled to receive a Copy of the Astronomical Observations (including Magnetism and Meteorology) made at the Royal Observatory at Greenwich, on making application for the same directly or through their respective agents, within two years of the date of publication.

Observatories.

Altona.
Armagh.
Berlin.
Breslau.
Brussels.
Cadiz.
Cambridge.
Cape of Good Hope.
Coïmbra.
Copenhagen.
Dorpat.
Dublin.
Edinburgh.
Helsingfors.
Königsberg.
Madras.
Mannheim.
Marseille.
Milan.
Munich.
Oxford.
Palermo.
Paris.
Seeberg.
Tübingen.
Turin.
Vienna.
Wilna.

Institutions.

Aberdeen	University.
Berlin	Academy of Sciences.
Bologna	Academy of Sciences.
Boston	American Academy of Sciences.
Brunswick, U.S.	Bowdoin College.
Cambridge	Trinity College Library.
Cambridge, U.S.	Harvard University.
Dublin	University.
Edinburgh	University.
	Royal Society.
Glasgow	University.
Göttingen	University.
Leyden	University.
London	Board of Ordnance.
	Royal Institution.
	Royal Society.
	The Queen's Library.
Oxford	Savilian Library.
Paris	Academy of Sciences.
	Board of Longitude.
	Dépôt de la Marine.
Pesth	Hungarian Academy of Sciences.
Philadelphia	American Philosophical Society.
St. Andrews	University.
St. Petersburg	Imperial Academy.
Stockholm	Royal Academy of Sciences.
Upsal	Royal Society.
Waterville, Maine (U.S.)	College.

Individuals.

Lowndes' Professor of Astronomy	Cambridge.
Plumian Professor of Astronomy	Cambridge.
President of the Royal Society	London.
South, Sir James	Kensington.
The Earl of Rosse	Parsonstown.

A List of Observatories, Institutions and Individuals, entitled to receive a Copy of the Magnetical and Meteorological Observations made at the Royal Observatory, Greenwich.

Observatories.

Bombay	Lieut. P. W. Mitcheson.
Cambridge, United States	Prof. J. Lovering.
Christiania	C. Hansteen.
Gotha	P. A. Hansen.
Heidelberg	M. Tiedemann.
Kew	B. Stewart.
Kremsmünster	P. A. Reslhuber.
Leipzig	Professor Möbius.
Lisbon	Senhor da Silveira.
Marburg	Professor Gerling.
Prague	K. Jelinek.
Stockholm	Professor H. Selander.
St. Petersburg	(Twelve copies for distribution to the Russian Mag. and Met. Obs.)
Toronto	Professor Kingston.
Upsal	Professor Svanberg.
Washington	

Institutions.

Bombay	Geographical Society.
Bonn	University.
Boston, U.S.	The Public Library (late Bowditch).
Cambridge	Philosophical Society.
Cherkow	University.
Falmouth	Royal Cornwall Polytechnic Society.
London	House of Lords, Library.
	House of Commons, Library.
	King's College.
	Royal Society.
	University College, Library.
Paris	Meteorological Society.
St. Bernard	Convent.
Washington	Smithsonian Institution.
Woolwich	Office of Mag. and Met. Publication.

Individuals.

Bache, Dr. A. D	Washington.
Buys Ballot, Dr.	Utrecht.
Dove, Prof. H. W.	Berlin.
Erman, Dr. Adolph	Berlin.
Fox, R. W., Esq.	Falmouth.
Harris, Sir W. Snow	Plymouth.
Hoskins, Dr. S. E.	Guernsey.
Kaemtz, Prof. L. F.	Dorpat.
Kreil, Prof. K.	Vienna.
Lloyd, Rev. Dr.	Dublin.
Loomis, Prof. E.	Yale College, Newhaven (Conn.).
Phillips, Prof. John	Oxford.
Quetelet, A.	Brussels.
Sabine, Major-General, R.A.	London.
Senhor da Souza	Coimbra.
Vernon, G. V., Esq.	Manchester.
Wartmann, Prof. Elie	Geneva.
Younghusband, Col., R.A.	Woolwich.

CONTENTS.

I. *On the Spectra of Ignited Gases and Vapours, with especial regard to the different Spectra of the same elementary gaseous substance. By Dr.* J. PLÜCKER, *of Bonn, For. Memb. R.S., and Dr.* J. W. HITTORF, *of Münster* . . . . . . page 1

II. *On the Osteology of the genus* Glyptodon. *By* THOMAS H. HUXLEY, *F.R.S.* . . . 31

III. *Investigations of the Specific Heat of Solid Bodies. By* HERMANN KOPP. *Communicated by* T. GRAHAM, *Esq., F.R.S.* 71

IV. *On the Composition of Sea-water in the different parts of the Ocean. By* GEORG FORCHHAMMER. *Professor at the University, and Director of the Polytechnic Institution at Copenhagen. Communicated by the President* 203

V. *On the Magnetic Character of the Armour-plated Ships of the Royal Navy, and on the Effect on the Compass of particular arrangements of Iron in a Ship. By* FREDERICK JOHN EVANS, *Esq., Staff Commander R.N., F.R.S., Superintendent of the Compass Department of Her Majesty's Navy; and* ARCHIBALD SMITH, *Esq., M.A., F.R.S., late Fellow of Trinity College, Cambridge, Corresponding Member of the Scientific Committee of the Imperial Russian Navy* 263

VI. *On some Foraminifera from the North Atlantic and Arctic Oceans, including Davis Straits and Baffin's Bay. By* W. KITCHEN PARKER, *F.Z.S., and Professor* T. RUPERT JONES, *F.G.S. Communicated by Professor* HUXLEY, *F.R.S.* . . . 325

VII. *New Observations upon the Minute Anatomy of the Papillæ of the Frog's Tongue. By* LIONEL S. BEALE, *M.B., F.R.S., Fellow of the Royal College of Physicians, Professor of Physiology and of General and Morbid Anatomy in King's College, London; Physician to King's College Hospital, &c.* 443

VIII. *A Dynamical Theory of the Electromagnetic Field. By* J. CLERK MAXWELL, *F.R.S.* 459

LIST OF ILLUSTRATIONS.

Plates I. to III.—Drs. J. Plücker and J. W. Hittorf on the Spectra of Ignited Gases and Vapours.

Plates IV. to IX.—Professor Huxley on the Osteology of the genus *Glyptodon*.

Plates X. and XI.—Staff-Commander Evans and Mr. A. Smith on the Magnetic Character of the Armour-plated Ships of the Royal Navy.

Plates XII. to XIX.—Mr. W. K. Parker and Professor T. R. Jones on some Foraminifera from the North Atlantic and Arctic Oceans.

Plate XX.—Professor Kopp on the Specific Heat of Solid Bodies.

Plates XXI. and XXII.—Professor Beale's New Observations upon the Minute Anatomy of the Papillæ of the Frog's Tongue.

PHILOSOPHICAL TRANSACTIONS.

I. *On the Spectra of Ignited Gases and Vapours, with especial regard to the different Spectra of the same elementary gaseous substance.* *By Dr.* J. Plücker, *of Bonn, For. Memb. R.S., and Dr.* J. W. Hittorf, *of Münster.*

Received February 23,—Read March 3, 1864.

1. In order to obtain the spectra of all the elementary bodies, you may make use either of flame or the electric current. For this purpose flame is preferable on account of its easy management, and therefore was immediately introduced into the laboratory of the chemist. But its use is rather limited, the metals of alkalies being nearly the only substances which, if introduced into flame, give spectra exhibiting well-defined bright lines. In the case of the greater number of elementary substances the temperature of flame, even if alimented by oxygen instead of air, is too low. Either these substances are not reduced into vapour by means of flame, or, if reduced, the vapour does not reach the temperature necessary to render it luminous in such a degree that by prismatic analysis we obtain its characteristic rays. The electric current, the heating-power of which may be indefinitely increased by increasing its intensity, is alone fitted to produce the peculiar spectra of all elementary bodies.

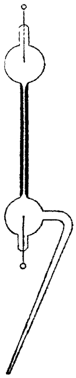
2. In applying the electric current we may proceed in two ways. In one mode of proceeding the substance to be examined by its spectrum is at the same time, by means of the current, transformed into vapour and rendered luminous. In the other mode the substance is either in the gaseous state, or, if not, has been converted into it by means of a lamp, and the electric current ignites the substance in passing through.

3. The first way of proceeding is the least perfect, but we are obliged to recur to it in the case of all such elementary bodies as neither by themselves nor combined with other substances can be vaporized without altering the least-fusible glass. If the substance to be examined be a metal, the extremities of the conducting-wires are made of it and placed at a short distance from one another. When the strong spark of a large Leyden jar, charged by Ruhmkorff's powerful induction-coil, is sent through the space between the two extremities of the conducting-wires, minute particles of the metal,

starting off from them, are volatilized: even in the gaseous state they conduct the electric current from point to point, and exhibit, while heated by it, the characteristic spectral lines of the metal. In all experiments made in this way, either air or another permanent gas occupied the space between the two extremities of the wires. The consequence of this is, the interposed gas partly conducting the electric current on its way through it, two spectra are obtained at the same time—the spectrum of the metal and the spectrum of the interposed gaseous medium. This inconvenience is the greater, as in most cases the number of bright lines constituting gas-spectra is a considerable one; it is least in the case of hydrogen, the spectrum of which, if appearing under these conditions, becomes nearly a continuous one (59). If the substance submitted to experiment be not a metal or charcoal, the extremities of the metallic wires are to be covered with it. Then we get with the spectrum of the non-conducting substance at the same time the spectrum of the metal covered by it.

4. The spectra are obtained the most beautifully and are the most suitable for examination in their minute details, if the substance be in the gaseous state before the electric discharge is sent through it. The spectral tubes for enclosing gas, first proposed and employed by one of us, were in most cases, with some modifications, adopted for our more recent researches. Our tubes, as represented by the diagram (fig. 1), generally consist of a capillary middle part 30–40 millims. long, and 1·5–2 millims. in diameter, forming a narrow channel, by which two larger spheres, with platinum electrodes traversing the glass, communicate with one another. The small tube starting from one of the spheres serves to establish the communication with the exhauster, to which it is either attached by means of a cement (sealing-wax for instance), or soldered by the blowpipe. The exhauster, made solely of glass, without any metal, is connected with an additional system of glass tubes and glass cocks, by means of which the spectral tube is most easily filled with the gas to be examined. If the gas be a permanent one, the apparatus by which it is developed, and its accessory parts, by which it is purified and dried, may, as well as the spectral tube, simultaneously and separately be evacuated. The gas arrives directly from the apparatus into the tube, which, *ad libitum*, may be alternately filled and exhausted again. Finally, the tension of the gas is regulated and measured by means of a manometer in connexion with the exhauster.

Fig. 1.



5. In order to compare with one another the spectra corresponding to different densities of the gas, or even to a mixture of different gases, the tube may be examined by the spectroscope while attached to the exhauster. But generally the spectral tube was blown off and hermetically sealed at the extremity of the narrow tube starting from one of the spheres. This tube equally serves to attach the spectral tube before the slit of the spectroscope.

6. If the substance submitted to examination were at the ordinary temperature in the liquid or solid condition, the tube destined to receive it was made of a glass diffi-

cultly fusible, and bent as shown by the diagram (fig. 2). After having introduced into it a small quantity of the substance, the last traces of air were expelled from the tube, which was finally blown off. Put before the slit of the spectroscope, the enclosed substance was, by means of a lamp, reduced into vapour and, if necessary, kept in the gaseous state (fig. 3), and the density of the vapour regulated. The glass of our spectral tubes of this description is fused with such difficulty, that these highly evacuated tubes, when becoming red-hot by the lamp, are not altered by the pressure of the surrounding air.

Fig. 2.

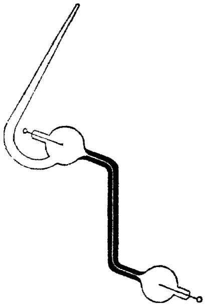
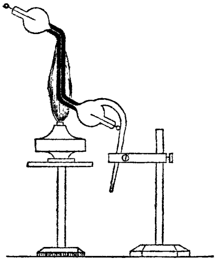


Fig. 3.



7. Before giving a general account of the results we have obtained, it seems necessary to enter into some preliminary discussions regarding the admirable working of GEISSLER's exhauster, and the phenomena shown by our tubes when highly evacuated by it. The essential part of GEISSLER's exhauster is a large glass ball, containing ten to twenty kilogrammes of mercury, which in its upper part communicates, by means of a doubly perforated stopcock of glass, either with the free air, or with the spectral tube to be evacuated. From the lower part of the ball, which is invariably fixed, descends a longer tube of glass communicating at its lower extremity with a moveable similar tube, the free end of which enters into a large open bottle. When this bottle with the moveable tube is lifted up, the mercury within the apparatus entirely fills the ball, if communicating with the air. This communication having been interrupted, a Torricellian vacuum is formed when the bottle descends. By establishing the communication with the spectral tube, the gas within it will be dilated. After the ascent and descent of mercury has thus been alternately produced often enough, no perceptible trace of air will remain within the spectral tube.

8. A tube evacuated in this way does not permit the induction current of RUHMKORFF's smaller apparatus (which in air gives a spark of about 15 millims.) to pass through. The current of his large apparatus forces a passage; but the spectrum we obtain in this case is very faint; it shows no traces of the bands of nitrogen, but solely the lines of hydrogen and the large fields of vaporized carbon (51). The hydrogen-lines take their origin from hygroscopic water covering the interior surface of the spectral tube, the carbon-bands probably from the minute traces of fatty matter hitherto employed in

greasing the stopcocks. (The oxygen simultaneously obtained by decomposition is not indicated.) The hydrogen-lines given by spectral tubes made of common glass are more brilliant than those of tubes made of less fusible glass, the hygroscopic state of the glass not being the same in both cases. Though within the interior of the exhauster the air is in contact with the surface of concentrated English sulphuric acid, or, what is preferable, with anhydrous phosphoric acid, we never succeeded in expelling the last traces of hygroscopic water, not even by strongly heating the spectral tube during evacuation.

If, in the usual way, a Leyden jar be intercalated into the current of RUHMKORFF'S large induction coil, we must conclude, from the powerful charge of the jar, as proved by flashes of light, that within the spectral tube the tension of electricity, before it effects its passage, is very high. In this case the electric light is more bright, and of a fine colour like that of blue steel. When analyzed by the prism, it shows the spectral lines of hydrogen and oxygen, mixed with other spectral lines, among which those of sodium and silicium are the brightest. At the same time the interior surface of the capillary part of the tube tarnishes. Hence we conclude that the decomposed glass partly conducts the current.

By means of our tubes, therefore, the theoretical conclusions of Dr. FARADAY, that electricity being merely a peculiar condition of ponderable matter cannot exist without it, and cannot move without being carried by it, are confirmed and supported in a striking way*.

9. As soon as the tube encloses perceptible traces of air, the spectral lines resulting from the ingredients of the glass entirely disappear. Though the temperature of the gas be raised by the passing current to an immense height, nevertheless, on account of its great tenuity and the short duration of the discharge, the gas is not able to heat the surface of the glass sufficiently to volatilize it. In this case also no spectral lines owing to particles starting from the platinum electrodes appear in the capillary part of the tube. Those lines are to be seen only near the electrodes, namely, in the aureola surrounding the negative pole.

10. The temperature of the particles of air seized by the weakest electric spark by far surpasses the temperature of the hottest obtainable flame. For no flame whatever shows the spectral lines of air, which are constantly seen in the spark. In order to raise the temperature of the discharge of RUHMKORFF'S induction coil, you may either increase the power of the inducing current, or diminish the duration of the induced one. The last plan may be found preferable in most cases. The heat excited in a given conductor by a current sent through it increases in the ratio of the square of intensity, but decreases in the ratio of the duration of the current. Admitting, therefore, that the conductibility is not altered by elevation of temperature, and that the quantity of induced electricity remains the same, we conclude that the heating-power of the induced current is in the inverse ratio of its duration. But the resistance opposed by gases to the passage of

---

* Mr. GASSIOT has already obtained vacua so nearly perfect as to present an obstacle to electric conduction. See Philosophical Transactions for 1859, p. 148.

electricity depends essentially upon their temperature. At the ordinary temperature it is rather too great to be measured, but, according to hitherto unknown laws, it rapidly decreases when the temperature rises beyond that of red heat. The law above mentioned is therefore not strictly applicable in the case of gaseous conduction.

11. Electricity can only be discharged through a given stratum of air, from one point to another, after a certain electric tension takes place in these points. This tension depends upon the chemical constitution of the gas, and, the gas being the same, it is nearly in the ratio of its density and the distance of the two points. The quantity of electricity required to produce that degree of tension which must precede the electric discharge through our spectral tubes, enclosing gas of a given density, may be indefinitely increased by interposing a Leyden jar. The less the distance between the coatings of the jar, and the larger their surface, the greater quantities of electricity will be accumulated on them, ready for discharge at the moment when the electric tension of the electrodes entering our tube reaches that intensity which alone allows the discharge to take place. Thus the Leyden jar is the most proper and most easy means for shortening the duration of the discharge, and consequently increasing the temperature of the gas.

In several cases, especially if a vapour like that of mercury be examined, which isolates less, it will be found more convenient, instead of replacing the Leyden jar by a larger one, to increase the charge of the same jar by intercalating into the circuit a spark micrometer, by means of which you may add to the resistance within the spectral tube the resistance of any stratum of air.

12. The leading idea by which one of us was guided when he first (1857) directed his attention to spectral analysis, was to concentrate the light in GEISSLER's tubes by confining the electric current within a capillary channel *. The construction of our tubes immediately follows from it. Accordingly we gave, for different purposes, a different diameter to their capillary part. The length of this part is of very little influence if the tubes are very highly exhausted; we had to shorten our recent tubes, intended to enclose gases and vapours of a greater density, rendered luminous by a powerful induction coil.

13. We employed in our researches the large spectral apparatus constructed by M. STEINHEIL. The refracting angle of one of the four flint prisms belonging to the apparatus is 60°, the angle of the three others 45°. Generally we made use of only two prisms (of 60° and 45°), and of a magnifying power of only 18.

It is well known that the slit of the apparatus, if illuminated by sodium-light (by the flame of alcohol containing common salt), is seen double. According to the width of the slit and the dispersive power of the prisms, the two well-defined images, having both

---

* PLÜCKER: "Spectra der elektrischen Lichtströmungen," 30 März 1858, POGGENDORFF's 'Annalen,' vol. civ.; "Ueber die Spectra der verschiedenen Gase, wenn durch dieselben bei starker Verdünnung die elektrische Entladung hindurchgeht," 25 Aug. 1858, Ibid. vol. cv.; "Ueber die Constitution der elektrischen Spectra von verschiedenen Gasen und Dämpfen," 5 Mai 1859, Ibid. vol. cvii.

the breadth of the slit as observed without the interposed prisms, are either superposed, or touch one another, or are separated by a black space. In making use of the two prisms, we generally regulated the aperture of the slit so that the two small sodium-bands appeared separated by a black space having nearly the breadth of these bands. In this case the angle at which the aperture of the slit is seen is equal to half the angular distance of the two middle lines of the bands, and therefore equal to half the angular distance of the two sodium-bands themselves after being reduced by narrowing the slit to mathematical lines.

If the images touch each other, the aperture of the slit and the two sodium-lines are seen at the same angle.

14. The first fact which we discovered in operating with our tubes, guided by the above explained principles, was the following one:—

There is a certain number of elementary substances, which, when differently heated, furnish two kinds of spectra of quite a different character, not having any line or any band in common.

The fact is important, as well with regard to theoretical conceptions as to practical applications—the more so as the passage from one kind of spectra to the other is by no means a continuous one, but takes place abruptly. By regulating the temperature you may repeat the two spectra in any succession *ad libitum*.

We will now treat more explicitly the case of *Nitrogen*, which first unfolded to us its different spectra. These spectra, obtained in the easiest and most striking way, have been examined by us in every point of view. The other cases of double spectra may hereafter be spoken of in a more summary manner.

15. We examined nitrogen prepared in different ways, even in the state of greatest purity; but we found that, in order to get pure spectra of it, it was not necessary to free the gas from all traces of air *. Therefore we may select the following preparation, imperfect as it is, in order to give an instance of constructing nitrogen-tubes. Three absorbing apparatus were connected with one another and, by means of a stopcock, with the exhauster, the first two being filled with a solution of pyrogallic acid in hydrate of potash, and the third with concentrated sulphuric acid. After having evacuated the interior of the exhauster and the spectral tube connected with it, by carefully turning the stopcock air was very slowly admitted, leaving its oxygen and carbonic acid to the first two, and its aqueous vapour to the third absorbing apparatus. Thus by and by the exhauster, with the tube, was filled with nitrogen, the manometer always indicating the tension of the gas. These operations being repeated several times by alternately evacuating and introducing new nitrogen, finally, the tension of the gas

---

* Whatever may be, under certain conditions, the practical importance of prismatic analysis in detecting certain substances converted into vapour, whatever may be its use in indicating traces of a single gas imperceptible by other means, mixtures of permanent gases are not fitted to be examined by the prism. A gas, if mixed in rather small proportion with another one, entirely escapes observation. The proportion necessary to render it visible depends upon the nature of the gas as well as upon the temperature of ignition.

(measured by means of the manometer) being from 40 millims. to 80 millims., the spectral tube was melted off and hermetically sealed.

16. When we send through our nitrogen-tube the direct discharge of Ruhmkorff's large induction coil, without making use of the Leyden jar, we observe a beautiful richly coloured spectrum. This spectrum is not a continuous one, but divided into bands, the character of which differs essentially at its two extremities; its middle part is in most cases less distinctly traced. Towards the more refracted part of the spectrum, the bands, illuminated by the purest blue or violet light, present a channeled appearance*. This effect is produced by a shading, the intensity of which decreases from the more to the less refracted part of each band. On applying four prisms instead of two, we perceive a small bright line, forming an interstice between two neighbouring channels, and the shading is, by the telescope of the spectral apparatus, resolved into dark lines. The number of such dark lines of one of the brightest bands (of the eighth band, we always count from the red to the violet) was found to be thirty-four, or nearly so. Their mutual distance is nearly the same, but their darkness decreases towards the least-refracted limit of each channeled band. Hence we concluded, the breadth of the band having been measured, that the angular distance of two contiguous shading-lines was nearly equal to the distance of the two sodium-lines. The breadth of the channeled bands varies, but the character of all is absolutely the same; only if foreign bright lines like those of hydrogen are simultaneously seen, it becomes slightly disturbed. We may distinguish seventeen bands of this description; the first three are smaller ones, the fourth is traversed by $H\beta$, the eleventh by $H\gamma$†. At the violet extremity the light is very faint.

17. The bands of the less refracted part of the spectrum are all of nearly the same breadth, but smaller than those just described, and of quite a different appearance. Making use of only a single prism, and of a small magnifying power, we count eighteen such bands, starting from the extreme red and extending to the greenish yellow, where they are bounded by a dark space. $H\alpha$ falls within the fourth, and the double sodium-line (Na) within the fourteenth of these bands. Under favourable circumstances, both extremities of the spectrum being equally developed, these bands extend to the channeled part, their number rising to thirty-five. All have the same general character, but not the same brightness. From the extreme red the intensity of light

---

* Under favourable conditions such a band appears furrowed semicircularly; but psychological effects of this description may be quite different: partly by our own will, partly by exterior circumstances, the bands may be seen convex as well as concave. Even the engraving of the bands (Plate I.) shows it. Let it be illuminated by daylight through a window, you will see the bands concave if their more refracted and shaded part be directed towards the window; if in the opposite direction, the bands will appear convex. The shade passes from one side to the other if really concave and convex bands are replaced by one another; so it does if the illuminating light pass to the opposite side. Accordingly, the stereoscopic appearance depending upon the direction from which the light comes, the mind passes judgment on it *unconsciously*.

† We denote by $H\alpha$, $H\beta$, and $H\gamma$ the three bright lines of the spectrum of hydrogen (the red, the bluish green, and the violet one). See 57.

increases to the eighth band; over the ninth, tenth, and eleventh, especially over the two last, a shadow is spread, which gives to the red a rather brownish tint. The next seven bands are of a fine orange and yellow colour. The nineteenth and twentieth bands are very dark, the twenty-first is less dark. The following bands have a green colour, varying in brightness. The darkest are the twenty-eighth and twenty-ninth, succeeding the lightest ones.

The cause producing these bands and their shading by dark transverse lines is evidently not the same as that which produces the shadow overspreading some of them. This may be concluded, for instance, from the fact that the shadow which darkens the nineteenth and twentieth bands, without entirely destroying their limits, spreads at the same time over the neighbouring third part of the preceding eighteenth band.

18. When the light sent out from the incandescent nitrogen within the capillary tube is dispersed by means of four prisms, the shading of the less refracted bands also is resolved into dark narrow lines; but these lines are smaller than the similar lines of the more refracted bands, and their distribution quite different. If the dispersion increase, in each band we at first perceive a new dark limit; but the design becoming gradually more defined, we observe in each band extremely delicate bright lines bounded by a shadow or by dark lines.

By closer examination of a band we distinguish first a least-refracted small part, occupying about the seventh part of the whole, formed by two bright lines including a somewhat larger dark space. The first of these two bright lines touches the dark extremity of the preceding band; the second is bounded by a subtle dark line, to which succeeds a third bright line, smaller than the two first. A fourth bright line divides the whole band into two parts, one less refracted, comprising the small one just described, the other more refracted and larger—the breadth of the two parts being about in the ratio of 4:5. Starting from the bright middle line, a feeble shading is produced by a number of most subtle dark lines, the darkness of which decreases towards the least-refracted part. Similar but darker lines produce the stronger shading of the larger more refracted part, decreasing in the same direction from the extremity of the whole band towards its bright middle line. The stereoscopic effect produced by the shading of the bands is represented by the diagram (Plate I.).

The configuration of all the bright orange and yellow bands is exactly the same; it is rather obscured in the case of the preceding bands by the shadow spreading over them, but becomes the same again in the bright red ones. Even in the dark bands 19 to 21, traces of the design are to be seen. The appearance of the green bands, though the general character be the same, slightly differs; the shading in the middle part of them being increased, they rather seem to be divided into two.

The accordance of these bands, even to the minute detail of their configuration, is a fact worthy of attention.

19. The character of the two systems of bands on the extremities of the spectrum is

entirely stereotype; all apparent changes result from the different intensity of light. The middle part of the spectrum, on the contrary, may much differ from that which we have described; you may even say that this part varies more or less essentially on replacing one spectral tube enclosing nitrogen by any other. Sometimes the traces of the less refracted bands are seen far beyond Hβ, spreading over the channeled part of the spectrum; in other cases the channeled appearance goes in the opposite direction as far as the sodium-line, disturbing the character of the bands.

20. Now, instead of the direct discharge of RUHMKORFF's large induction coil, let us send through the very same spectral tubes the discharge of the interposed Leyden jar. The spectrum then obtained (Plate II.) has not the least resemblance to the former one. The variously shaded bands which we have hitherto described are replaced by brilliant lines on a more or less dark ground. Neither the distribution of these new lines nor their relative brightness gives any indication whatever of a law. Nevertheless the place occupied by each of them remains under all circumstances invariably the same. If exactly determined, not only does each line undoubtedly announce the gas within the tube, but the gas may even, without measuring, be recognized at first sight by characteristic groups into which the lines are collected.

21. The new spectrum of nitrogen extends towards the red slightly beyond the hydrogen-line Hα, which if the gas be not dried with care will be seen simultaneously, enclosed by two red nitrogen-lines, the less refracted of which is twice as distant as the more refracted. There are in the spectrum five groups of brilliant lines especially remarkable. The orange group, slightly less refracted than Na, is formed by four lines, the second of which is the brightest; the third, not quite so bright, is closely followed by the fourth, which is very faint. The second (yellow) group contains seven lines, among which the fifth is brightest. The third (light-green) and the fourth (dark-green) group contain each nine lines. The third and sixth lines of the light-green group and the sixth and seventh (both near to each other) of the dark-green group are brightest. The fifth (light-blue) group (the distance of its middle part from Hβ and Hγ is about in the ratio of 3 : 4) is formed by six lines, the second of which is the brightest, the first slightly less bright; the last four lines, nearly equally distant from each other, are slightly less bright again. Two groups, of three fainter lines each, fall between the two green groups and between the dark-green and the blue. We may mention also two bright single lines, placed out of the groups—a green line preceded by an expanded one, and a light-violet line followed at a short distance by a bright band. Besides, there are in the spectrum more or less faint bands or expanded lines extending beyond Hγ nearly as far as the distance between this line and Hβ, *i. e.* about to FRAUNHOFER's line H.

22. We may denote the orange, yellow, light-green, dark-green, and blue groups by I, II, III, IV, and V, and the single lines of them by the arabic numbers, the place they occupy in each group being reckoned from the less to the more refracted. Thus by adding the chemical symbol of the gas we get a general method of denomination,

according to which N II 5, N IV 6, N IV 7, and N V 2, for instance, indicate the brightest lines of the groups of the nitrogen-spectrum.

23. Not only is the general character of the two kinds of spectra we obtained when nitrogen was heated in our tubes, either by the direct discharge or by the discharge of the interposed Leyden jar, quite different, but the difference is even so great that the bright lines of one of the spectra do not in the least fall within the brighter part of the bands constituting the other. Thus, for instance, the bright yellow line (N II 5) falls within the nineteenth band, the darkest of all the bands constituting the less refracted part of the spectrum; the bright blue line (N V 2) falls into the darker part of one of the channeled spaces. Accordingly it appears by no means probable that by increasing the temperature the shaded bands of one spectrum may be transformed gradually into the bright lines of the other: nevertheless it would be desirable to prove by experiment that the passage from one spectrum to another is a discontinuous and abrupt one.

24. For a given nitrogen-tube which without the Leyden jar gives the spectrum of bands, and by means of the commonly used jar the spectrum of bright lines, you may easily select a jar of smaller covering, which, if intercalated, exhibits the curious phenomenon of two rival spectra disputing existence with each other. Sometimes one of the spectra, sometimes the other appears; and for moments both are seen simultaneously. Especially the brighter lines of the second spectrum abruptly appear in the blue and violet channeled spaces of the first, and, according to the fluctuation of the induced current, either suddenly disappear again or subsist for some time, and constitute with the added fainter lines the second spectrum.

We obtain in an easier and a continuous way both spectra simultaneously by making use of a small Leyden jar, and increasing its charge by an intercalated stratum of air the thickness of which increases till the bright lines appear within the bands of the primitive spectrum.

25. By these and other experiments it is evidently proved that ignited nitrogen shows two quite distinct spectra. Each bright line of one of these spectra, each of the most subtle lines into which, by means of the telescope, the bands of the other are resolved, finally depends upon the molecular condition of the ignited gas, and the corresponding modification of the vibrating ether within it. Certainly, in the present state of science, we have not the least indication of the connexion of the molecular constitution of the gas with the kind of light emitted by it; but we may assert with confidence that, if one spectrum of a given gas be replaced by quite a different one, there must be an analogous change of the constitution of the ether, indicating a new arrangement of the gaseous molecules. Consequently we must admit either a chemical decomposition or an allotropic state of the gas. Conclusions derived from the whole series of our researches led us finally to reject the first alternative and to adopt the other.

26. The same spectral tube exhibits, in any succession whatever, as often as you like, each of the two spectra. You may show it in the most striking way by effecting the intercalation of the Leyden jar by means of a copper wire immersed in mercury. As

often as the wire is taken out of the mercury we shall have the spectrum of bands; as soon as the communication is restored, the spectrum of bright lines. Hence we conclude that the change of the molecular condition of nitrogen which takes place if the gas be heated beyond a certain temperature by a stronger current, does not permanently alter its chemical and physical properties, but that the gas, if cooled below the same limit of temperature, returns again to its former condition.

27. The essentially different character of the two extremities of the first spectrum of nitrogen, as described (16–19), and the indistinctness of its middle part, suggested to us the idea that, in reality, the observed spectrum might originate from the superposition of two single spectra. Accordingly one of these single spectra, the more refracted part of which is best developed, must be formed by channeled spaces; the other one, the less refracted part of which is best developed, must be a spectrum of shaded bands. In different cases, either the one or the other of the spectra may be predominant.

In order to confirm our conjecture it was necessary to get the two spectra separated.

28. The discharge of RUHMKORFF's coil through a spectral tube is changed the less by introducing the Leyden jar, the weaker is the resistance opposed to it by the tube. Accordingly the two different degrees of temperature to which the gas rises by the discharge when, the coil remaining the same, we either make use of the jar or not, may be regulated in such a way as to approach one another more and more. Let the tension of the gas of about 10 millims. remain the same, the temperature produced by the discharge will be diminished by increasing the interior diameter of the capillary part of the spectral tube. Thus we succeeded in constructing a tube which, when the direct discharge was sent through it, became incandescent with the most brilliant gold-coloured light, which might easily be confounded with the light of highly ignited vapours of sodium; but with the intercalated jar, the light of the incandescent gas within the same tube had a fine bluish-violet colour. The yellow light, when analyzed by the prism, gave a beautiful spectrum of shaded bands, extending with decreasing intensity to the blue, the channeled spaces being scarcely perceptible. The bluish light, when examined, was resolved by the prism into channeled spaces extending towards the red, while the former bands almost entirely disappeared. We may transform each colour and its corresponding spectrum into the other *ad libitum.*

Hence it follows that there is another allotropy of nitrogen, which, like the former, is not a stable and permanent one, but depends only upon temperature. The modification in which nitrogen becomes yellow corresponds to the lower, the modification in which it becomes blue to the higher temperature.

29. When we send the direct discharge of RUHMKORFF'S coil through one of GEISSLER'S wider tubes enclosing very rarefied nitrogen or air (the oxygen of air becomes not visible here), we see the negative pole surrounded by blue light, the light at the positive pole being reddish yellow. In such of GEISSLER'S tubes as are especially calculated to show how the light starting in all directions from the different points of the negative electrode is by the action of an electro-magnet concentrated along the magnetic curves

passing through these points, the blue light is most beautiful. It belongs generally to the nitrogen alone, which, on account of the greater resistance at the negative electrode opposed to the discharge, reaches a higher intensity of heat there than at the positive pole. When analyzed by the prism, the blue light gives the spectrum of channeled spaces, with traces only of the less refracted bands. The reddish-yellow light of the positive pole is more faint, and therefore not so easy to be submitted to spectral analysis.

30. When RUHMKORFF's large induction coil is discharged in common air between two points the distance of which does not exceed a few centimetres, we obtain, as is well known, a brilliant spark surrounded by an aureola, the colour of which is partly bluish violet, partly reddish yellow. In order to separate these colours more distinctly from each other, the aureola, moved by the slightest breath, may be extended into a large surface by blowing it sideways. But the separation may be best made when the discharge takes place between the two poles of an electro-magnet in the equatorial direction. While the straight spark is not acted upon by the electro-magnet to any sensible degree, the aureola is expanded into a fine surface, bounded by the spark starting from one to the other extremity of the electrodes, and by a semicircle passing through these extremities. At a certain rarefaction of air this surface appeared most beautifully bounded by a semicircular golden-coloured band, and divided by a similar band into two parts*. We may explain now in a satisfactory way the appearance, hitherto mysterious, of the golden light. Both the yellow and the blue light are owing to the nitrogen of the air, reduced by the heat of the current into the two allotropic states which exhibit the spectra of channeled spaces and of bands. The brilliant white light of the spark partly belongs to the oxygen, partly to the nitrogen of the air, both highly ignited, the nitrogen being in that allotropic state in which it exhibits the spectrum of bright lines.

31. In order to complete the history of the spectrum of nitrogen we add two remarks. First, by intercalating a Leyden jar and, in order to weaken the current, at the same time a stratum of water or a wet thread, we may also reduce the spectrum of bright lines to the spectrum of bands. Secondly, by increasing the density of the gas, or, if the gas be less dense, by intercalating at the same time a large jar and a stratum of air, the bright lines of the spectrum, at the highest obtainable temperature, will expand. Out of a great number of observations made in this direction we shall describe only one.

32. A short spectral tube enclosing nitrogen of a tension of about 250 millims. refused passage to the discharge of RUHMKORFF's large induction coil, when three of GROVE's elements were made use of and the jar intercalated. Without the jar the discharge passed through and produced a bright but rather undefined spectrum of bands. When the current continued to pass, the indistinctness of the spectrum increased, and after short intervals brilliant coloured lines appeared and disappeared again, like lightning-flashes. These lines, occupying always the same place, belonged to the second spectrum of nitrogen, the brightest yellow and green lines of which

---

* PLÜCKER, "Ueber die Einwirkung des Magnetes auf die elektrische Entladung," POGGENDORFF's 'Annalen,' vol. cxiii. p. 267.

(N II 5, N IV 6, N IV 7) were specially observed. When we made use of twelve of GROVE's elements ranged into three sets of four combined ones, the current even passed after we interposed the jar, and we got a most dazzling second spectrum of the gas. The bright lines of this spectrum, rising from a ground itself brighter than it usually is, ceased at an increased brilliancy to be well defined. The two brilliant green lines both expanded, and were united into a single broad line; the double yellow lines, though expanded, yet remained double. *The spectrum was progressing towards a continuous one.*

33. In recapitulating, we get the following results:—

Nitrogen in the state of greatest rarefaction, such as may be obtained by GEISSLER's exhauster, like other gases does not allow the induction current to pass through. But when its tension is only a small fraction of a millimetre, the current begins to pass and renders the gas luminous. Below a certain limit of temperature ignited nitrogen sends out a golden-coloured light, giving the spectrum of bands. Above this limit the colour of the light is replaced by a bluish violet, the spectrum of channeled spaces replacing simultaneously the spectrum of bands. When, by means of the intercalated jar for instance, the temperature rises to a second higher limit, the light of the gas, becoming white and most brilliant, gives, if analyzed by the prism, a spectrum of quite a different description: bright lines of different intensity, with the colour indicated by the place they occupy, rise from a dark ground. By increasing the power of the discharge these lines become more brilliant, but the brilliancy does not increase in the same ratio for them all. New bright lines appear, which formerly, on account of their extreme faintness, were not visible; but the number of such lines is not unlimited. By increasing the heat of the ignited nitrogen to the last extremity, the lines, especially the brighter ones, gradually expand, approaching thus to a continuous spectrum.

34. Those spectra which are composed of larger bands showing various appearances *according to their being differently shaded by subtle dark lines*, we generally call *spectra of the first order*. In the same spectrum the character of the bands is to a certain extent the same, the breadth of the bands varies in a more or less regular way. On the contrary, those spectra in which brilliant coloured lines rise from a more or less dark ground, we call *spectra of the second order*.

Ignited nitrogen therefore exhibits, if its temperature increase, successively two spectra of the first and one of the second order.

35. In the case of sulphur, which we may select as another instance, there are two different spectra, one of the first and one of the second order.

In common air the flame of sulphur gives a continuous spectrum; if fed with oxygen we get a spectrum of the first order, but it is faint and its bands are not well defined. In order to get the sulphur-spectrum most perfect, we must recur to our spectral tubes.

A doubly bent short tube (6), into which we introduced a small quantity of sulphur, was evacuated by means of GEISSLER's exhauster, and while attached to it heated by a lamp, in order to expel as much as possible the moisture it contained. Finally, the manometer showing no more tension of the remaining gas, the tube was hermetically sealed by a blowpipe. The direct charge of Ruhmkorff's large induction coil sent through it, generally indicates by their spectra traces of remaining foreign substances (8). But when the tube was heated by a small alcohol-lamp, at a certain moment a fine sulphur-spectrum of the first order appeared, undisturbed by any former spectrum. The beauty of the spectrum increased when we continued to heat moderately.

36. We counted thirty-seven well-defined bands, extending nearly from Hα to Hγ. Seven of these bands, the first of which was of a dark-red colour and visible only under favourable circumstances, preceded the sodium-line, eighteen fell between this line and Hβ, and eleven between Hβ and Hγ, the last of which being broader, appears sometimes divided into two. After a last band, traversed by Hγ, a larger and strongly shaded space extended towards the extreme violet. The breadth of the bands increased from the less to the more refracted part of the spectrum. In each band, contrary to what takes place in the case of nitrogen, namely, with regard to its channeled spectrum, the shading produced by fine dark lines decreases from the less to the more refracted extremity. The darkest part of the shadow is bounded by a small separate band of a varied appearance, generally formed by two small bright lines including a somewhat larger dark one. By these small bands the purely channeled character of the spectrum is disturbed.

37. If, while the discharge is passing, we continue to heat the tube by a lamp, the brightness of the spectrum always increases; but if we approached to a certain degree of temperature, in different parts of the spectrum we have described, bright-coloured lines belonging to the sulphur-spectrum of the second order appeared and disappeared again according to the fluctuating heat, till at last the second of the two rival spectra remained undisturbed. The colour of the light was changed. In cooling again after the lamp was taken off, the light within the tube changed its colour again, while the spectrum of the second order was replaced by the spectrum of the first order.

There is a certain elevation of temperature at which the increased density of the vapour does not permit the discharge to pass; the light within the tube is extinguished, but abruptly reappears after cooling.

38. Well-defined bright lines, constituting a fine sulphur-spectrum of the second order, are obtained if moderate discharges of Rhumkorff's large induction coil are sent through the tube, the tube being slightly heated by means of an alcohol-lamp, and a small Leyden jar being intercalated. At first the spectrum extends only from about the sodium-line to Hβ. One observes chiefly a characteristic group of sixteen lines, followed at some distance by two separate lines. The spectrum once developed persists even after taking off the lamp. When we continue to heat, the brightness of the group increases and its lines begin to expand, while at the same time the hitherto black ground is coloured. The brilliancy may be increased to such an extent as to be unbearable to the eye. Beyond the sodium-line, towards the red extremity, new distinct lines appear, among which we particularly distinguish a triple line, remarkable as well for its fine red colour as for its distinctness, and nearer to Hα a second such triple line, at first well

defined but soon merging into a single one. Like the less refracted part of the spectrum, the most refracted part is developed only at a higher ignition of the vapour of the sulphur. At its violet extremity (we do not give here a full description of the middle part) we observe at the same distance from one another five well-defined fainter bright lines. Then follows, after an expanded violet band, a group of four bright lines, the second of which is accompanied by a more refracted, the fourth by a less refracted faint line. The fourth line especially is distinct to a degree seldom observed at so high a refraction and so great a power of the discharge. After two bands of faint light, there is seen at the end of the spectrum a group of four slightly expanded bright lines, preceded by an expanded violet band.

39. Like sulphur, selenium has two spectra—one of the first, another of the second order.

40. Ignited carbon, even in a state of greatest division, gives a continuous spectrum.

41. We select, among the various compound gases which, if decomposed in flame, give the spectrum of carbon, in the first place *cyanogen*. The gas was procured by heating cyanide of mercury introduced into a retort of glass by means of a lamp. The flame of it may be fed either with oxygen or with air.

When a jet of cyanogen mixed with oxygen is kindled, in the interior part of the flame a most brilliant cone of a whitish-violet light is seen, the limit between the ignited and the cold part of the jet. This cone exhibiting the spectrum of vapour of carbon best developed, we conclude that the cyanogen must be decomposed into carbon and nitrogen, the carbon being in the gaseous condition a moment before its combination with oxygen takes place*.

42. In order to prevent explosion of the mixture of cyanogen and oxygen, it is preferable that the jets of the two gases meet from opposite sides before the slit of the spectral apparatus, forming there, if kindled, a brilliant, flat, vertical surface. The jet of cyanogen might be obtained directly from the retort, by the heating of which it may be regulated. Thus we get, all being properly arranged, a splendid and richly coloured spectrum. Especially we distinguish *eight groups of bright lines*, which, being all of the same general character, indicate at first sight the existence of vapour of carbon. We shall denote these groups, starting from the less refracted and proceeding to the more refracted ones, by *a*, *b*, *c*, *d*, *e*, *f*, *g*, *h*. The group *a* is formed by five, *b* by six, *c* by four, *d* by five, *e* by seven, *f* by three, *g* by seven, and *h* by three bright lines. But these lines, of a measurable breadth and a quite different appearance, are not to be confounded with the bright lines which, in the case of nitrogen and sulphur, for instance, constitute spectra of the second order. In each group the first line is the brightest; the following, which are nearer to one another, decrease in intensity, and under less favourable circumstances the last ones are not seen. Hence the groups, according to an expression of Mr. Attfield, have the appearance of a portico. The red group (*a*) is not always seen distinctly (less distinctly in the present case than in the case of other gaseous com-

---

* Mr. Attfield has the merit of having first stated that spectra hitherto attributed to compound gaseous substances, are to be referred to the vapour of carbon itself (Philosophical Transactions for 1862, p. 221).

pounds of carbon); the group *f* is very faint, the group *g* beautifully violet, *h* rather ultra-violet.

43. The whole spectrum, except its red extremity, is divided into large shaded fields. The shadow increases from the less to the more refracted part of each field; from its brighter less refracted part arise the bright lines of one group, the first of these lines towards the darkest extremity of the preceding field. As well as in the former cases of nitrogen and sulphur, the shadow is produced by dark transversal lines on a coloured ground. But here the distance of the shading-lines from each other varies even in the same field. Towards the bright, *i. e.* the less refracted extremity of each field, the distance decreases, while at the same time the darkness and the breadth of the lines is diminished. The space between two consecutive lines appeared to be greatest in the field containing the group *c*, at a distance from *d* about twice as great as that from *c*. There we counted, on making use of two prisms and applying a magnifying-power of eighteen, the aperture of the slit being regulated in the ordinary way (13), nine shading-lines, including eight nearly equal small bands, the total breadth of which corresponded to five divisions of our arbitrary scale. Hence we computed the angular distance of two consecutive dark lines which we observed to be about five-fourths of the distance of the sodium-lines.

The dark shading-lines also appear within the bands bounded by the lines of the brighter characteristic groups. The band between the second and the third bright line of the yellow group *b*, the total breadth of which corresponds to four divisions of our arbitrary scale, was divided by dark lines into twelve smaller bands of about equal breadth. Accordingly the angular distance of two such consecutive lines is about two-thirds the distance of the two sodium-lines. The dark lines within the neighbouring band, bounded by the first and second bright line of the same group, were much nearer to one another, and their number too great to be counted with certainty.

44. Between the groups *f* and *g* there is indicated a particular distribution of light and shadow, which, being a faint copy of what takes place if olefiant gas be burned instead of cyanogen, will be better understood after we have described the spectrum of the new gas.

45. The least-refracted part of the spectrum, preceding the first line of the group *a*, essentially differs from the more refracted part already described. There are three fine red bands contiguous to the first bright line of the group, extending nearly to Hα, and beyond this hydrogen-line, after a dark space, two similar but not so well-defined bands. The breadth of these bands is nearly the same, and all are shaded in a similar way. Contrary to the distribution of shadow in the larger field, the shadow is strongest in the less refracted part of each band; in the most refracted part we observed two bright lines.

46. When the combustion of cyanogen took place in air, the bands we have just described were best developed, and new similar ones added. They extended from beyond Hα nearly to Hβ. The breadth of these bands slightly increases towards the violet end of the spectrum, their general description remaining the same. We especially counted seven such bands, the first of which is traversed by the double sodium-line, and the last

is bounded at the place formerly occupied by the second bright line of the characteristic group *c*.

When the flame of cyanogen is fed by air, we observe under favourable circumstances no traces of the groups *a* and *b*, the least-refracted bright line of the group *c* faintly appears, *d* is scarcely indicated, but the groups *e*, *f*, *g* are fully developed, especially the last one, of a fine violet colour.

46. In supplying the flame of cyanogen by air increasingly mixed with oxygen, we distinctly see two spectra overlying one another. One of these spectra (the spectrum of bands) giving way step by step to the other, the appearance is continually changed. The red bands only remained undisturbed, they became even more distinct by the increased intensity of the combustion. The adjacent group *a* is scarcely developed, evidently on account of an imperfect extinction of the overlying bands.

The superposition of the two spectra introduces new details into the general configuration of the resulting spectrum. Thus, for instance, at a certain intensity of combustion the interval between the first and second bright line of the group *b* is divided by four fine bright lines into five spaces, the breadth of which decreases towards the violet part of the spectrum. Thus also in the large field containing the group *c*, the influence of the spectrum of bands is rendered sensible by a particular distribution of shadow.

47. Secondly, we submitted to a closer examination olefiant gas, $H^4 C^4$, when burned either with oxygen or with air. We operated as we did in the former case of cyanogen; only the gas, prepared by heating a mixture of alcohol and sulphuric acid, was previously introduced into a gasometer.

The luminous cone which exhibits the spectrum of vapour of carbon is of a fine blue colour, especially if the flame is fed by oxygen.

48. In the spectrum thus obtained the characteristic groups *a*, *b*, *c*, and *d* appeared on a shaded ground. All these groups, especially the red one *a*, scarcely seen in the spectrum obtained by the combustion of cyanogen, are finely developed. The last line of *b* and *d* is slightly expanded; but there is no trace whatever either of the bands of the spectrum of cyanogen, if burned in common air, or even of the groups *e* and *g*. Instead of these groups there is quite a new configuration. Equally distant from the place which the groups occupied in the former spectrum, a small well-defined black band was seen, bounded on the more refracted side by a violet space, which, being of great brilliancy where it touches the band, was shaded gradually till the spectrum, not extending beyond the place of the group *g*, was extinguished. This violet space is traversed by well-defined dark lines, equally distant from each other, but more apart than the shading lines we described in former cases. The black band is bounded on its less refracted side by a bright line, having the breadth of the lines of the characteristic groups, which at a certain distance was preceded by a more diffused violet light, traversed, like the brilliant one on the opposite side, by dark but less distinct lines. Here also the faint group *f* appeared.

The distribution of light and shade producing the configuration just described is seen also, distinctly but faintly, in the spectrum we obtained by the combustion of cyanogen with oxygen, where at the same time the groups *e* and *g* are beautifully expressed (44).

49. Among the gases exhibiting the spectrum of vapour of carbon, when enclosed in our spectral tubes and decomposed by the heat of the discharge of RUHMKORFF's coil, we first select *oxide of carbon.* In operating with this gas as we did with nitrogen, we got, if the Leyden jar was intercalated, simultaneously the spectrum of vapour of carbon and the spectrum of oxygen; without the jar, the pure spectrum of vapour of carbon. In the last case the heat of the discharge is high enough to ignite vapour of carbon, but not sufficient to give the spectrum of oxygen. The single spectrum, as well as the combined one, is obtained accordingly *ad libitum*; whence we conclude that as the successive discharges pass through the spectral tube, the gas is alternately decomposed and recomposed again.

50. We shall in a few words describe the spectrum obtained without the jar, at a tension of the gas, when observed by means of the manometer before the spectral tube was sealed, of 32 millims.

Four characteristic groups only were seen, *a*, *b*, *c*, and *d*. When the current first passed, the band *a* appeared completely; after some time its two first lines only remained, rising as isolated bright lines from a dark ground; finally all the group disappeared. The groups *b*, *c*, and *d* remained nearly unchanged; there appeared only two bright lines of *c*, the place corresponding to the two following ones being very brilliant.

The whole spectrum was divided into large fields, similar to the fields we described in the case of the flame of cyanogen fed with oxygen. But in this case each field is bounded at its more refracted and shaded extremity by the first bright line of a characteristic group; the following lines, bordered by shading, rise from the lightest part of the adjacent field. In the new instance the fields are not bounded in the same way. After the group *a* has disappeared, there is a differently shaded dark space, extending to the place of the third bright line of that group. In the remaining part of the spectrum we may distinguish seven shaded fields. The first goes a little beyond the first bright line of the group *b*, where it is bounded by a transversal line, dividing the band formed by the first two lines of the group into a dark less refracted and a light more refracted part. Accordingly the first bright line rises from the dark end of the first field, the remaining lines from the light end of the second field. The second field does not reach the first bright line of the following group *c*, this line being nearly equally distant from the extremity of the field and the next line of the same group. The third field goes slightly beyond Hβ; the fourth to the first line of the group *d*; the fifth nearly to the place occupied by the fifth line of the group *e*; the sixth approaches the place of the group *f*; and the seventh extends to the fourth line of the group *g*. The fourth and sixth fields presented the appearance of pure channeled spaces, as described in the case of nitrogen.

51. If the heating-power of the discharge be too strong, spectral tubes enclosing oxide of carbon at a higher tension showed only three large shaded fields, without any traces of the characteristic groups. The first two of these fields are coincident with the second and third of the former fields; the third occupies the place of the fourth and fifth former fields united into one. Here the shading of the three large fields not being disturbed by any additional appearance, the transversal shading lines were observed most distinctly even in making use of four prisms and employing a magnifying power of 36. In observing especially the light and less refracted part of the first field close to its extremity, these lines, on account of their extreme subtleness, are scarcely to be perceived; when they begin to become well defined they are very near to each other; but towards the more refracted part of the field their distance increases simultaneously with their breadth, till, at some distance from the bright extremity, the dark expanded lines are resolved into small shaded bands*.

52. Spectral tubes containing *carbonic acid* instead of oxide of carbon gave essentially the same spectra. The increased quantity of oxygen of the decomposed gas may be observed by means of the interposed jar. In such tubes there was no carbon deposited, not even after a long passage of the discharge.

53. All compound gases enclosed in our spectral tubes are decomposed by the heat produced by the discharge of RUHMKORFF'S large induction coil; but instantly after the discharge passes, the recomposition takes place. The recomposition is prevented only by a sudden cooling of the elementary gases obtained by the decomposition. Thus, for instance, spectral tubes enclosing *cyanogen* are scarcely fitted for observation, the interior surface of their capillary part being instantaneously blackened by the deposited carbon. No carburetted hydrogen resists final decomposition by the passing current. We add only a few observations, made by means of spectral tubes.

54. The spectrum of the light hydrocarbon gas, $C^2 H^4$, obtained without the Leyden jar, at once showed the expanded bright lines of hydrogen and an imperfect spectrum of vapour of carbon, especially the brightest lines of the characteristic groups *b*, *c*, and *d*. By intercalating the jar, the hydrogen-spectrum, approaching to a continuous one, became quite predominant.

Olefiant gas, $C^4 H^4$, of a primitive tension of about 70 millims., gave, without the jar, a scarcely visible spectrum; by intercalating the jar, the three hydrogen-lines $H\alpha$, $H\beta$, $H\gamma$ appeared well defined, and the spectrum of vapour of carbon, with its groups *a*, *b*, *c*, *d*, and its shaded large fields, well developed.

Methyl, $C^2 H^3$, showed, without the jar, at once $H\alpha$, $H\beta$, $H\gamma$, and the characteristic groups *e* and *g*; with the interposed jar these two groups disappeared, and were replaced by the groups *a*, *b*, *c*, and *d*.

Acetylene, $C_4 H_2$, though according to BERTHELOT and MORREN formed from its elements when DAVY's charcoal light is produced within an atmosphere of hydrogen, when introduced into our tubes is nevertheless rapidly decomposed by the discharge, and most incompletely recomposed after the discharge has passed. The inside of the tubes is instantly blackened, and in the first moment only, along with the spectrum of hydrogen, we perceive the groups of carbon-lines seen in the case of olefiant gas.

---

* The same spectrum, but fainter, is obtained under quite different conditions. We have already noticed, in the introductory remarks, that in a spectral tube evacuated to the last degree by GEISSLER'S exhauster, vaporized carbon is indicated by its spectrum. The spectrum obtained is that described above (8).

55. Finally, RUHMKORFF's large induction coil was discharged between two electrodes of carbon, surrounded by an atmosphere of hydrogen. The four groups *a*, *b*, *c*, and *d* were obtained, constituting the spectrum of vaporized carbon.

56. In resuming, we are struck by the variety of appearances presented by ignited vapour of carbon when submitted to spectral analysis under different conditions. But, whatever may be this variety, it is impossible not to admit that all or nearly all of the various types of spectra we described are derived from the same source. We may distinguish four such types: 1st, the bands, especially seen when the flame of cyanogen is fed by air; 2ndly, the particular distribution of light and shadow near $H\beta$ when the flame of olefiant gas is fed by oxygen; 3rdly, the large fields shaded by transversal dark lines; 4thly, the characteristic groups of bright lines, *a*, *b*, *c*, *d*, *e*, *f*, *g*, *h*, which are to be ranged into two different sets, *a*, *b*, *c*, *d*, and *e*, *f*, *g*, *h*. It is a curious fact that all these different types, either fully developed or indicated only, are represented in the flame of cyanogen, if fed with oxygen, while in all the other cases we examined there are represented either a single type or two types, or even three,—namely, 1, the third type alone; 2, the first type, with the second set of groups; 3, the third type, with one set of groups (*a*, *b*, *c*, *d*); 4, the same type, with the other set (*e*, *g*); 5, the second and third types, with the first set of groups. There is no doubt that the different types correspond to different degrees of temperature,—the temperature being lowest when the bands are principally developed, lower in the case of the second set of groups than in the case of the first, lower in the case of the shaded large fields than in the case where the characteristic groups appear simultaneously.

In the present state of the question we are not able fully to explain the various types of spectra of carbon. It is only proved that all spectra which we referred to carbonic vapour do not contain any bright line belonging to another elementary gas. Either the well-known spectra of foreign admixed gases, of nitrogen, oxygen, hydrogen, for instance, do not appear at all; or if they do, they may be subtracted from the whole apparent spectrum.

It appears doubtful that the different types depend *solely* upon temperature. If so, the temperature varying in the different parts of the ignited vapour of carbon, different types may be seen simultaneously. We shall not now discuss the influence which the coexistence of foreign gases might have on the spectra of vapour of carbon, nor may we here decide whether or not, in the lower temperature of the flame, a gaseous compound of carbon, not being entirely decomposed, exhibits, with the spectrum of the vapour of carbon, simultaneously the spectrum of the undecomposed gas.

In the spectrum of cyanogen, for instance, we got no visible traces of the spectrum of nitrogen (originating from the decomposed gas), whether we supplied the flame by a jet of oxygen, or operated in open air; but in both cases there is no reason not to admit that the bands, which are not seen in the case of any other compound of carbon, were owing to the undecomposed cyanogen (see no. 61).

57. With regard to the spectrum of *hydrogen*, we first refer to former observations. The spectrum one of us obtained by sending the discharge of Ruhmkorff's small induction coil through one of his highly evacuated spectral tubes, constructed by M. Geissler, shows only three bright lines, which he denoted by Hα, Hβ, and Hγ. The beautiful red light of the ignited rarefied gas, divided into these three bright lines, even after having passed through the four prisms of Steinheil's spectral apparatus, remains highly concentrated. At a magnifying power of 72, the three bright lines or small bands thus obtained are well defined. Their apparent breadth is equal to the breadth of the slit; consequently, on further narrowing the slit, they approach gradually to mathematical lines. Hence we conclude that, under the above-mentioned conditions, the length of wave of the light constituting each of the three hydrogen-lines is constant, and remains so if by widening the slit the lines are expanded into bands. In referring the middle lines of such bands to the middle line of the direct image of the slit, we obtain its angle of refraction. It was proposed to employ these middle lines instead of Fraunhofer's dark lines of the solar spectrum in determining the indices of refraction[*]. This proceeding has since been proved to be very expedient[†].

58. Hydrogen permits the electric discharge to pass at a lower tension than other gases do. When Ruhmkorff's small induction coil was discharged through a spectral tube enclosing hydrogen, which was gradually rarefied to the highest tenuity to be reached by means of Geissler's exhauster, finally the beautiful red colour of the ignited gas became fainter, and passed gradually into an undetermined violet. When analyzed by the prism, Hα disappeared, while Hβ, though fainter, remained well defined. Accordingly light of a greater length of wave was the first extinguished[‡].

59. Hydrogen shows in the most striking way the expansion of its spectral lines, and their gradual transformation into a continuous spectrum. When the direct discharge of Ruhmkorff's large induction coil is sent even through the old spectrum tubes enclosing hydrogen, the formerly obtained spectrum is essentially altered. By increasing the power of the coil, the violet line Hγ first expands; while it continues to expand, the expansion of the bluish-green line Hβ becomes visible. Let the aperture of the slit be regulated so that the double sodium-line will separate into two single lines nearly touching one another. Then, the angular breadth of Hβ becoming two or three minutes, the breadth of Hγ is about double. The expansion takes place as well

---

[*] Poggendorff's 'Annalen,' vol. cvii. p. 497.

[†] Landolt: "Ueber die Brechungsexponenten flüssiger homologer Verbindungen," Poggendorff's 'Annalen,' vol. cxvii. p. 353.

[‡] Plücker: "Ueber recurrente Ströme und ihre Anwendung zur Darstellung von Gasspectren," Poggendorff's 'Annalen,' vol. cxvi. p. 51.

towards the less as towards the more refracted part of the spectrum. $H\alpha$ remains almost unchanged after $H\gamma$ has passed into an undetermined large violet band, and $H\beta$ extended its decreasing light on its two sides. On employing the Leyden jar, and giving to the gas in our new tubes a tension of about 60 millims., the spectrum is already transformed into a continuous one, with a red line at one of its extremities. At a tension of 360 millims. the continuous spectrum is highly increased in intensity, while the red line $H\alpha$, expanded into a band, scarcely rises from it. If the electric spark passes through hydrogen at the ordinary tension, the ignited gas on its way always gives the spectrum of the three expanded lines[*].

60. Even in the old spectral tubes enclosing highly rarefied hydrogen, the ground, from which the three characteristic lines rise, did not appear always of the same darkness; in some instances new bright lines appeared, especially in the neighbourhood of the sodium-line. In resuming the subject, we pointed out the existence of a *new hydrogen-spectrum*, corresponding to a lower temperature, but having no resemblance at all to the spectra of the first order of nitrogen, sulphur, &c. In this spectrum, of a peculiar character, if fully developed, we observe a great number of well-defined bright lines, almost too numerous to count and represent by an engraving, but brilliant enough to be examined at a magnifying power of 72, after the light has passed through four prisms.

---

[*] After FRAUNHOFER, and especially Dr. WHEATSTONE, directed the attention of philosophers to the electric spectrum, MASSON indicated the red hydrogen-line, but without referring in an explicit way to its origin. ÅNGSTRÖM first separated the spectrum of gas from the spectra of metal. In the diagram he gave of the hydrogen-spectrum, he represented, by means of curves, the intensity of light along the whole length of the spectrum, especially the maxima of intensity within the red, the green, and the violet. These maxima correspond to $H\alpha$, $H\beta$, $H\gamma$, here expanded into bands, the breadth of which, as well as their decreasing intensity towards both ends, are indicated by the extension and steepness of the curves. After one of us published his first researches on the spectra of ignited gases, M. VAN DER WILLIGEN, in operating with strong induced currents, determined in a similar way the maxima of intensity of the hydrogen-spectrum.

The spectra thus obtained are not calculated to prove the connexion existing between the bright lines of ignited gases or vapours and FRAUNHOFER's dark lines of the solar spectrum. Starting, in his first communication made to the Royal Swedish Academy, 1853, from the theoretical conception "that the dark lines of the solar spectrum are to be regarded as an inversion of the bright lines of the electric spectrum," M. ÅNGSTRÖM concluded the coincidence of $H\alpha$ with FRAUNHOFER's line C; but the diagram shows that this conclusion was not based on exact measurement. One of us, in his publication of 1859, not being guided by any theoretical view on this point, first announced the coincidence of $H\beta$ with FRAUNHOFER's F, and fixed the position of $H\gamma$ near G, of $H\alpha$ at a distance of two minutes from C. When at a later period he made use of STEINHEIL's large spectral apparatus, he pointed out at first sight the exact coincidence of $H\alpha$ with C, $H\gamma$ with a marked black line at some distance from G, towards F. In operating with spectral tubes, M. ÅNGSTRÖM confirmed these results. (The spectroscope employed in 1859 being a small and imperfect one, there was given to the slit an aperture of more than three minutes. The adjustment was made with regard to $H\beta$. Hence the error finally made in determining the position of $H\alpha$ may be fully explained, by the circumstance that the illuminated border of the slit was observed instead of the illuminated aperture itself.)—ÅNGSTRÖM: "Optische Untersuchungen," POGGENDORFF's 'Annalen,' vol. xciv.; "Ueber die FRAUNHOFER'schen Linien im Sonnenspectrum," Ibid. vol. cxvii. VAN DER WILLIGEN: "Over het electrische Spectrum, Verhandelingen der K. Hollandsche Academie (Natuurkunde vii. & viii.). PLÜCKER, POGGENDORFF's 'Annalen,' vol. cvii. p. 544.

61. On sending the direct discharge of RUHMKORFF'S coil through a tube of glass from one-fourth to one-eighth of an inch in diameter, provided with electrodes of platinum or of aluminium, enclosing hydrogen at a tension of 5 to 10 millims., a luminous thread of light of a bluish-white colour was seen passing along the axis of the tube, without touching the glass. When analyzed by the prism, it gave a faint spectrum of the above-mentioned numerous bright lines, especially within the red and the yellow. Among these lines neither Hα nor Hγ were seen; Hβ only appeared, but less bright than many of the other lines. By interposing the Leyden jar and gradually increasing its charge (12), all lines became brighter, Hβ surpassing all other lines in brilliancy; Hα appeared beautifully, Hγ fainter. Hence we conclude that the numerous bright lines belong neither to the vaporized metal of the electrodes, nor to the decomposed interior surface of the glass, but solely to the hydrogen, constituting a new spectrum of it. This spectrum may be seen simultaneously with the three characteristic lines Hα, Hβ, Hγ; but at an increased temperature, when these lines begin to expand, it entirely disappears.

62. We got only one spectrum of *oxygen* in operating exactly in the same way as we did in the case of nitrogen, with merely this difference, that under the same conditions a spectrum of equal brightness was obtained only by means of a stronger discharge. Accordingly if oxygen, enclosed in the spectral tube, be replaced by common air, the spectrum of the oxygen it contains does not appear until after interposing the Leyden jar.

We do not enter here into the detail of the oxygen-spectrum, but conclude with a general remark. Nearly all luminous lines of the spectra of the second order expand when the temperature of the ignited gas increases beyond a certain limit; but neither do all lines reach the same brightness before expanding, nor do the lines in the different parts of the spectrum expand at the same temperature. That is seen best in the spectrum of the second order of oxygen. The bright lines constituting the characteristic groups of its middle part oppose the greatest resistance to expansion. If they are best defined, the luminous lines towards the red extremity, most distinct at a lower temperature, are already expanded, while towards the violet extremity the luminous lines are scarcely developed; they will be brightly developed, become well defined, and extend very far, after the ignited oxygen reaches a temperature at which the groups of the middle part are expanded. Hence arises the difficulty of representing the oxygen-spectrum. A drawing exhibiting the well-defined lines successively developed in its different parts is rather an ideal image than a true representation of nature.

63. *Water* introduced into a small spectral tube was kept boiling till the last traces of air were expelled, and then, before all the water was evaporated, the tube was hermetically sealed. The direct discharge, if passing, scarcely rendered the tube luminous, but with the intercalated jar the peculiar red light of hydrogen appeared, exhibiting the characteristic lines Hα, Hβ, Hγ well defined. When these lines became gradually expanded, the lines of the oxygen-spectrum successively appeared with an increasing intensity,

finally rising from the hydrogen-spectrum transformed into a continuous one. Here the heat of the discharge is increased by the increased density of the vapour of water, and reciprocally the evaporation is accelerated by the rising temperature of the discharge. The vapour of water is decomposed by the discharge; the ignited hydrogen resulting from the decomposition exhibits a spectrum at a lower temperature than the resulting oxygen does. After the discharge ceases, oxygen and hydrogen are recomposed again to water.

64. *Phosphorus*, when treated like sulphur (35), exhibits a beautiful spectrum of the second order. Whatever may be the gradual change of the intensity of light produced by regulating as well the discharge as (by means of a lamp) the heat of the spectral tube, we get only one spectrum of bright lines successively developed. Among them there is one announcing at first sight the presence of vapour of phosphorus, a triple orange line, formed by two single lines of first intensity, and a third less bright one bisecting the interval between them. The other brightest lines are seen within the green.

We get no difference at all by introducing into the spectral tube either common or red phosphorus. After the current had passed for some time, common phosphorus was seen, within the tube, transformed into a subtle powder of the red kind.

65. *Chlorine*, *Bromine*, and *Iodine* were among the substances first submitted to spectral analysis by one of us. On resuming the subject we fully confirmed the formerly obtained results, that not any two of the numerous spectral lines, characterizing the three substances, were coincident.

By means of the electric current we got in all instances only spectra of the second order. We were especially desirous of ascertaining whether there existed a spectrum of iodine, corresponding to a lower temperature, the inverse or negative image of which agreed with the spectrum produced by absorption on sending sunlight (which, in order to prevent the influence of FRAUNHOFER's dark lines, may be replaced by the light of phosphorus in combustion) through a stratum of heated vapour of iodine. Thus, indeed, we obtain more than fifty shaded bands, the breadth of which decreases from the violet to the red, constituting a spectrum of the first order. The flame of hydrogen in open air was not fitted to ignite vapour of iodine introduced into it sufficiently. But by feeding the flame by oxygen we got a new spectrum. Large fields, shaded by dark transversal lines, differently bounded, but quite similar to the third type of the spectra of vapour of carbon, constituted a spectrum of the first order. But the spectrum we might have expected according to theory was not seen.

66. *Arsenic*, when treated like sulphur and phosphorus, gives a well-defined spectrum of the second order.

67. So does *mercury* when introduced into a spectral tube from which air is expelled, either by means of GEISSLER's exhauster, or by boiling the mercury within it. After a slight heating of the tube by means of an alcohol-lamp the discharge passes; and having once passed, it continues to do so, even without the lamp. Vapour of mercury

opposing a comparatively small resistance to the passing current, we found it useful to intercalate at the same time a Leyden jar and a stratum of air. Thus, indeed, by regulating as well the density of the vapour as the thickness of the stratum, we obtained the best-developed spectrum.

The least quantity of mercury, if vaporized, becomes visible by the passing current. Especially when mixed with other metals like arsenic, antimony, &c., we may detect even the least traces of it, which would entirely elude chemical analysis. Thus, for instance, we observed that arsenic, whatever may be its origin, is not free from mercury. After introducing a small quantity of it, which we heated by an alcohol-lamp when we placed it before the slit of the spectral apparatus, in a few moments four lines of great brightness, among which was a double yellow one, rose from a dark ground, but before the spectrum was fully developed it was abruptly replaced by another quite as brilliant. The first spectrum obtained belongs to vapour of mercury, first developed by evaporization, the second to arsenic, which increasingly vaporized at a higher temperature disputes the conduction of the discharge with the mercury, the vapour of which, according to its small existing quantity, reaches only a very low limit. The spectrum of arsenic remaining alone, gradually increased in brilliancy by the development and expansion of its bright lines. In cooling the spectral tube, by taking off the lamp, the spectrum of arsenic lost its extreme brilliancy; well-defined bright lines, the number of which gradually diminished, rose from a dark ground, and were replaced again by the spectral lines of mercury, till finally all light was extinguished.

68. The metals of alkalies, sodium, potassium, lithium, thallium show, even at the lower temperature of BUNSEN'S lamp, a spectrum of the second order, consisting of bright lines, the number of which is increased by the higher temperature of the current, while the principal ones are expanded.

69. Barium, strontium, calcium show, even in BUNSEN'S lamp, shaded bands, and a bright chief single line at the same time. This line, green in the case of barium, bluish violet in the case of strontium, violet in the case of calcium, fully exhibits the character of the bright lines in the spectra of the second order. The bands, if well developed, constitute a spectrum of the first order. We examined especially the spectrum of barium, by introducing its chloride into the hydrogen-flame. In making use of two prisms and employing a magnifying power of eighteen, we distinctly obtained the shading of the bands resolved into dark lines, finer and closer to one another than in former similar cases. Thus we proved that *the band-spectrum of baryta is in every respect a spectrum of the first order.*

70. Spectra of the first order were observed in the case of a few heavy metals only. Among these metals we mention in the first instance *lead*. We obtain its spectrum in BUNSEN'S lamp, but in order to get it beautifully developed we must make use of the oxyhydrogen flame. The spectra we obtained *were identically the same* whatever compound of lead was introduced into that flame. We especially examined its combinations with chlorine, bromine, iodine, and oxygen. In all cases we observed larger bands.

which by increased temperature were divided into smaller ones. Each band has a channeled appearance produced by fine dark lines, the darkness of which increases from the more to the less refracted extremity of the band, contrary to what takes place in the violet channeled spaces of nitrogen.

Chloride of lead, when examined within our spectral tubes, showed no traces of bands; they were replaced by bright lines. But on account of the great difficulty of vaporizing it, the spectrum of the second order, owing to lead, is best developed by the discharge of RUHMKORFF'S coil between two electrodes made from this metal and surrounded by an atmosphere of hydrogen. The spectrum of this gas being under these conditions nearly a continuous one (59), the bright lines of the lead-spectrum of the second order rise from a coloured ground. More than fifty lines were counted, although the fainter ones did not appear.

71. When either chloride or bromide or iodide of *copper* is introduced into the flame of BUNSEN'S lamp, we get spectra of bands, but these bands are not exactly the same, they differ from one another by additional bands*. In the oxyhydrogen flame the bands are better developed, but we did not succeed in resolving the shadows of the bands into dark lines. At the same time four lines of single refrangibility appeared. The number of these lines was increased and the number of bands reduced, when chloride of copper was examined within our spectral tubes. The well-known spectrum of the second order was fully developed, and every trace of bands extinguished, by discharging RUHMKORFF'S coil between two copper electrodes.

72. Finally, *manganese* exhibited a curious spectrum of the first order, most similar to that of carbon (third and fourth type (56)). The whole spectrum is equally divided into large fields, but these fields are shaded differently by fine transversal lines, the shadow increasing from the more to the less refracted extremity of each field. From the brighter less refracted part rise groups of bright lines, similar to the groups of carbon, but the lines of the groups are differently distributed.

When RUHMKORFF'S large coil was discharged between two electrodes made from manganese (we surrounded them with an atmosphere of hydrogen), a pure spectrum of the second order, free from any traces whatever of the former spectrum, was obtained.

EXPLANATION OF THE PLATES.

In determining the different spectra both of the first and the second order, the dispersing prisms occupied invariably the same position, corresponding to the minimum deviation of the green hydrogen-line Hβ, *i. e.* of FRAUNHOFER'S F. All spectra represented in the Plates are referred to the three hydrogen-lines Hα, Hβ, Hγ, and the double sodium-line Na. Generally two prisms of about 60° and 45° were employed, giving the distances of Hα and Na on one side and of Hγ on the other side from Hβ, by the following numbers of divisions of an arbitrary scale:

---

* This fact has been noticed by M. A. MITSCHERLICH with regard to the chloride and the iodide, and attributed by him to the undecomposed salt (POGGENDORFF'S 'Annalen,' 1862, vol. ii. p. 299).

139·6, 100·5–101, 88·5.

In the first Plate portions of all the coloured spectra are represented as they appear by making use of two additional prisms of 45°.

PLATE I.

contains spectra of the first order. The first spectrum, N, belonging to nitrogen, is taken under such conditions that both its extremities appear equally developed. To the whole spectrum is added a representation of two bands, C, of its more refracted part, obtained by means of the four prisms. Here a determined number of subtle dark transverse lines produce the channeled appearance. Likewise the configuration of two orange bands, A, and two green ones, B, is represented, exhibiting the character of the less refracted part of the spectrum (15–19, 27, 28).

S represents the spectrum of sulphur, as obtained by means of an exhausted bent spectral tube enclosing sulphur moderately heated by an alcohol lamp, and traversed by the charge without an interposed jar (35, 36).

Two green and two blue shaded bands, as seen by means of the four prisms, are represented by A and B.

C I shows the spectrum of vapour of carbon obtained by the combustion of cyanogen in oxygen. It exhibits within the large shaded fields groups of peculiar bright lines, the brilliancy of which it was impossible to represent. These groups are denoted by *a*, *b*, *c*, *d*, *e*, *f*, *g*, *h*. The red extremity becomes fainter when the heat of combustion increases, and even appears more distinct if the combustion takes place in air (41–46).

The configuration of one of the red bands, as seen when the four prisms are employed, is represented by A.

C II exhibits the spectrum of vapour of carbon obtained by means of spectral tubes enclosing oxide of carbon, the gas being decomposed by the electric discharge (49, 50). On taking away all characteristic groups, the remaining part of the spectrum, consisting only of three large shaded fields, is that obtained if the density of the gas be greater and the discharge too strong (51), as well as in the case of imperceptible traces of decomposed carbonic combinations (8).

C III shows the less refracted part of the brightest of the large shaded fields (51).

C IV exhibits a peculiar distribution of light and shade within the violet, scarcely indicated in C I, but well developed when olefiant gas instead of cyanogen is burnt in oxygen (48).

PLATES II. & III.

represent spectra of the second order, on a scale one-third larger than the scale of Plate I.

In Plate II. N shows the second spectrum of nitrogen (20–23), O the spectrum of oxygen (63), S the second spectrum of sulphur (37, 38), Se of selenium (39).

In Plate III. I shows the spectrum of iodine, Br of bromine, Cl of chlorine. Some remarks may be added here with regard to the conditions under which the spectra are obtained.

Iodine was introduced into a bent spectral tube, and the tube exhausted as far as possible. While more recently tubes have been constructed which do not allow the discharge of Ruhmkorff's large coil to pass, not even at a very short distance of the electrodes, the same effect will scarcely be obtained if iodine is enclosed in the tube. Accordingly the very first moment the phenomena described in art. 8 take place; but soon after, vapour of iodine is developed, and by the heating power of the discharge we get, without the Leyden jar, a spectrum of mere iodine, consisting of very well-defined lines on a dark ground. After the interposition of the jar these lines became more brilliant, but remained well defined, and their number increased. Then the position and the intensity of the lines of the middle part were determined, while the red extremity was not at all developed, and the violet one most imperfectly. If the density of the vapour is increased by heating the tube by means of an alcohol lamp, the lines determined are expanded, while the ground becomes illuminated. The brilliancy so increases that the eye can scarcely bear it, till at last the discharge ceases to pass. While the middle part approaches to continuity, a certain number of delicate brilliant red lines, seen in the diagram, appear, and do not lose their distinctness as long as the discharge passes. Towards the violet extremity new lines likewise appear, but though that extremity becomes most brilliant, we were not able to get the lines well defined. Accordingly the position of the expanded lines is approximately indicated by dotted lines.

A drop of bromine was introduced into a small exhausted spectral tube. The tension of its vapour being too great to allow the discharge to pass, the vaporized fluid was expelled till the remaining vapour obtained a tension of about 6 centimetres. But by and by the vapour of bromine, combined with the platinum of the electrodes, was deposited on the interior surface of the tube, and after some time, evidently from want of sufficient conducting matter, the beautiful spectrum fainted almost suddenly. The spectrum was taken with the interposed jar. In this case $H\alpha$ and $H\beta$ are simultaneously seen, but expanded, indicating traces of remaining water. The lines of oxygen are not seen. Without the jar hydrogen is not indicated. Then four bright lines, belonging to bromine, appear in the neighbourhood of $H\alpha$. While, with the interposed jar, they are fully expanded like this hydrogen-line, a less refracted subtle line appears, always remaining most distinct. The blue and violet extremity of the spectrum is better defined than in the case of iodine.

The spectrum of chlorine is taken under similar conditions with the spectrum of bromine. The spectral tube most carefully exhausted was several times filled with chlorine and exhausted again. The final tension of the remaining gas was about 6 centimetres, as it was in the former case.

P exhibits the spectrum of phosphorus (64).

We conclude with a general remark regarding more or less all the spectra of the second order represented in Plates I. & II. The intensity attributed to the different bright lines constituting these spectra corresponds to the condition in which they are best developed. There seems to be a general rule that all luminous lines become brighter and are finally expanded, when the heating-power of the discharge continually increases. But for different lines the intensity does not rise in the same ratio: thus lines less brilliant at first than others may afterwards surpass them in brilliancy. The intensity attained by the different luminous lines before they are expanded greatly differs; lines may disappear by expansion, while others of the same spectrum do not yet appear. The least-refracted lines generally resist expansion the most.

II. *On the Osteology of the genus* Glyptodon. *By* THOMAS H. HUXLEY, *F.R.S.*

Received December 30, 1863,—Read January 28, 1864.

Part I.—The history of the discovery and determination of the remains of the *Hoplophoridæ.*
Part II.—A description of the skeleton of *Glyptodon clavipes,* OWEN (*Hoplophorus Selloi,* LUND ?).
§ 1. Description of the Skull.
§ 2. Description of the Vertebral Column.

PART I.—*The history of the discovery and determination of the remains of the* Hoplophoridæ, *or animals allied to, or identical with,* Glyptodon clavipes.

THE earliest notice of the discovery of the remains of *Glyptodon*-like animals is contained in the following extract from a letter, addressed to M. AUGUSTE ST. HILAIRE by Don DAMASIO LARAÑAGA, Curé of Monte Video, which appears in a note at p. 191 of the fifth volume of the first edition of CUVIER'S 'Ossemens Fossiles,' published in 1823:—

"I do not write to you about my *Dasypus* (*Megatherium,* Cuv.), because I propose to make it the subject of a memoir which, I trust, may not be unworthy of the attention of those European savants who take an interest in fossils. I will merely say that I have obtained a femur, which was found in the Rio del Sauce, a branch of the Saulis Grande. It weighs about seven pounds, and may be six or eight inches wide. In all points it resembles the femur of an Armadillo. I will send you one of its scales. The tail, as you have seen, is very short and very large; it also possesses scutes, but they are not arranged in rings, or in whorls. These fossils are met with, almost at the surface, in alluvial, or diluvial, formations of a very recent date. It would seem that similar remains exist in analogous strata near Lake Merrim, on the frontier of the Portuguese colonies."

CUVIER expresses no opinion as to the accuracy, or otherwise, of Don DAMASIO LARAÑAGA'S identification of his *Dasypus* with the *Megatherium,* an identification which, it will be seen, was erroneous.

The volume of the Transactions of the Royal Academy of Sciences of Berlin for the year 1827 contains a memoir by Professor WEISS* upon the collections of fossils and minerals gathered in South America by SELLOW, accompanied by five plates, four of which display excellent representations of various portions of the dorsal and caudal dermal armour, and of part of a femur, of one or more species of *Glyptodon.* Some of these fossils (the fragments of the dorsal dermal armour) were obtained at three feet from the surface, in the marly clay of which the banks of the Arapey Chico (a branch of the Arapey Grande, an affluent of the Uruguay) are formed. The skeleton of the *Megatherium* now at Madrid was found in a similar clay which underlies Buenos Ayres. The femur and the fragment of caudal armour were procured from the banks of the Quegnay, a more northern affluent of the Uruguay than the Arapey.

---

* Ueber das südliche Ende des Gebirgzuges von Brasilien in der Provinz San Pedro do Sul und der Banda Oriental oder dem Staate von Monte Video: nach den Sammlungen des Herrn FR. SELLOW, von Herrn WEISS (Gelesen in der Akademie der Wissenschaften am 9. August 1827, und 5. Juni 1828).

WEISS remarks upon these fossils (*l. c.* p. 276) "that it can hardly be doubted that they belonged to no other animal than the *Megatherium*. CUV. CUVIER himself published, in a note to p. 191 of his 'Recherches sur les Ossemens Fossiles,' t. v. 1^e partie, the first information which he received, in 1823, that his *Megatherium* was a loricated animal. M. LARAÑAGA, parish priest of Monte Video* (from whom this information was derived, and in whose house M. SELLOW, in 1822, saw two fragments of the armour, one belonging to the back and the other to the tail, which were found between Monte Video and Maldonado, in a gully opening into the Arroyo de Solis), believed the animal to be an Armadillo, *Dasypus*; CUVIER had already pointed out the similarity of the extremities to this genus and to *Myrmecophaga*. However, the armour plates found on the Arapey show no trace of a zonary arrangement, and the fragments possessed by M. LARAÑAGA also leaving a doubt on this point, it may remain an open question whether the Megatherium possessed a veritably jointed armour, or whether it was not more probably provided with a solid shield."

The figures show, and Professor WEISS remarks upon, the raised conical form of the marginal pieces of the carapace.

In the course of his description of the parts of the skeleton of a Megatherium sent to this country by Sir WOODBINE PARISH, Mr. CLIFT† remarks, "In these latter instances the osseous remains were accompanied by an immense shell or case, portions of which were brought to this country; but most of the bones associated with the shell crumbled to pieces after exposure to the air, and the broken portions preserved have not been sufficiently made out to be, at present, satisfactorily described. Representations, however, of parts of the shell in question are given in the plate annexed."

The plate (46) to which reference is here made exhibits views of the inner and outer surfaces of parts of the carapace of a *Glyptodon*. In a note (p. 437) Mr. CLIFT mentions that casts of the principal bones in question have been sent, among other places, to the Jardin des Plantes at Paris.

The next work upon this subject in the order of time, is the very valuable essay communicated by Professor E. D'ALTON to the Berlin Academy in 1833‡. SELLOW had been compelled by the local authorities to send to Rio Janeiro all the bones and the finest pieces of the carapace, which he discovered in association with the fragments of dermal armour figured by Weiss*; but, by good fortune, these additional materials at length found their way into the Berlin Museum, and afforded D'Alton the materials for his memoir, in the first section of which the pieces of the carapace of the fossil animal are described; while the second section is devoted to an account of the structure of the dermal armour of living Armadillos, and the third to a description of the fossil bones found in juxtaposition with that dermal armour.

---

* ["A friend of natural history and, in every way, an estimable man, who has now unfortunately become blind," writes M. SELLOW regarding him to M. von OLFERS on the 10th October 1829. We can therefore no longer look for the appearance of his promised essay on these fossil remains.]

† "Some account of the Remains of the Megatherium sent to England from Buenos Ayres by WOODBINE PARISH, jun., Esq., F.G.S., F.R.S." By WILLIAM CLIFT, Esq., F.G.S., F.R.S. Read June 13, 1832. Transactions of the Geological Society, vol. iii. 2nd series.

‡ "Ueber die von dem verstorbenen Herrn SELLOW aus der Banda Oriental mitgebrachten fossilen Panzer-Fragmente und die dazu gehörigen Knochen-Ueberreste," with four plates. The volume of the 'Abhandlungen der Königlichen Akademie der Wissenschaften,' in which this essay appears, was published in 1835.

The results of the comparison of the fossil armour with that of existing Armadillos are thus stated:—

"If we compare these fossil dermal plates with those of living species of *Dasypus*, it becomes obvious that all the peculiarities of the former may be paralleled by the latter; but with this difference, that while, as appears from Sellow's report, all the fossil plates belonged to one and the same animal, their peculiarities are not all found associated together in any one living species. The majority of the fossil plates which were distant from the margin, *e. g.* those represented by Weiss in figs. 1, 4, & 5, and many described above, exhibit the greatest similarity to the dermal plates of *Dasypus niger*; and thence it may be concluded that the epidermis of the *Dasypus* of the ancient world (if for brevity's sake I may so name the animal), like that of the *Dasypus niger*, was divided differently from the bony plates, and that strong hairs were arranged in the interstices of the epidermic scales.

"The pieces which belonged to the edge, or the pointed marginal scutes (Zacken), most nearly resemble those of *D. Poyou* (fig. 12 of our first Plate), and *D. grandis* shows a somewhat similar formation. In addition, the thoracic shield and the moveable zones of *D. villosus* (fig. 18) are also provided with pointed marginal scutes; and, according to Azara, the *Tatou pichey* exhibits similar structures. But in all the animals provided with such pointed scutes, they are directed from above, and forwards, downwards, and backwards; and therefore some of the fragments may be referred to the left, and some to the right side.... From the preceding comparisons it follows that the fossil scutes are similar to those of the thoracic and pelvic shields of different living Armadillos, although they differ from them in many respects. But if objections should still be raised to regarding the animal which bore the fossil armour as an Armadillo (Gürtelthier), two replies may be made. In the first place, neither the entire skeleton nor the perfect shell of the animal have been obtained. Of the skeleton, the vertebral column, the ribs, and sternum are wanting—or exactly those parts which the moveable zones (Gürtel) would have covered. Secondly, the moveable zones themselves, although among the characteristic features of the Armadillos, are of less importance than was formerly believed, as AZARA has already pointed out."

---

* Professor Owen writes (On the *Glyptodon clavipes*, Geol. Trans. vol. iii. pp. 82, 83), "The portions of the tessellated bony armour figured by Professor Weiss, pl. 1 and 2, and described at p. 277 of his memoir, were obtained by Sellow on the Arapey-Chico in the province of Monte Video; but no bones either of the Megatherium, or any other animal, are mentioned as having been associated with them. A third series of fossils, in which fortunately some bones of the extremities were discovered associated with the tessellated bony case, was presented to Sellow by the President of the province of San Pedro, with the information that they had been originally discovered in the proximity of Rio Janeiro."

This, however, appears to be a misapprehension of the state of the case. The armour figured by Weiss in pl. 1 and 2 of his memoir, and the "third series of fossils" were associated together; and so far from the President of the province of San Pedro having presented anything to Sellow, it was Sellow who was obliged to present the fossils to the President, or at any rate, to dispose of them according to his orders. "Denn die Aufforderung des damaligen Präsidenten der Provinz San Pedro, des Visconde des S. Leopoldo, nöthigte ihn [Sellow] den hauptsächlichsten Theil dieser fossilen Ueberreste nach Rio Janeiro abzuliefern."

It is therefore sufficiently obvious that the fossils were not found *at* Rio Janeiro, but were sent *to* that place from Arapey-Chico.

The state of the bones indicated that they appertained to a young animal, the epiphyses being distinct. Those described belonging to the fore limb are, a part of the scapula (?), the distal end of the left humerus, the radius and ulna, nearly perfect, and eighteen bones of the fore foot. Of the latter, five belonged to the carpus, of which the three proximal are interpreted by D'ALTON as the *semilunare* (Mondbein), *cuneiforme* (das dreieckige Bein), and *pisiforme* (Erbsenbein). I shall endeavour to show, in the course of my description of the specimen which forms the subject of this memoir, that the determinations of the *semilunare* and *cuneiforme* are perfectly correct, but that the so-called *pisiforme* is not rightly named. The distal bones are, according to D'ALTON'S interpretation, which I can fully confirm, the *magnum* and the *unciforme*.

Two entire metacarpal bones, and fragments of another, are considered by the author of the memoir to correspond with the third, fourth, and fifth of an ordinary five-toed fore foot; but they are really the second, third, and fourth, Professor D'ALTON having taken the surface of the cuneiform, which articulates with the fifth metacarpal, for the surface of articulation with the pisiform. The phalanges of the digits belonging to these metacarpal bones, and three of their sesamoid bones, are carefully described and figured.

The resemblances of the bones of the forearm with those of the existing Armadillos are pointed out, especial weight being laid upon the extension of the cuneiform round the unciform, and its articulation with what D'ALTON supposes to be the fifth metacarpal; and certain analogies of the fore foot with that of the mole are indicated.

A fragment of the distal end of a leg-bone, the seven tarsal bones, the four outer metatarsal bones; their digits, except the ungual phalanges; and some other bones of the hind foot, in a more or less fragmentary state, are described and figured, and attention is drawn to the remarkably short and strong character of the foot.

In conclusion D'ALTON remarks, "Though, as I have endeavoured to show above, there is a certain agreement between the *manus* of the fossil animal and that of the Armadillos, yet the foot shows us no greater similarity than may be observed between it and many other five-toed animals. Hence the osteology of the primeval animal does not afford a sufficient confirmation of the view which we derived from the consideration of the carapace, viz. that the bones, together with the fragments of dermal armour, might have belonged to an animal nearly allied to the Armadillos, or perhaps even to a very large, probably extinct, species of *Dasypus*. The fossil bones are too few to afford a safe foundation for so decided an opinion respecting the zoological affinities of the animal. A tolerably perfect skeleton is necessary in order to enable us, from the bones alone, to draw a safe conclusion as to the structure of the remainder of an animal."

Singularly enough, D'ALTON does not mention the *Megatherium* throughout this paper, which however affords, by implication, an ample demonstration that the bony armour described has nothing to do with that animal*.

In 1836, LAURILLARD, in editing the eighth volume of the second edition of CUVIER'S 'Ossemens Fossiles,' appends the following note to the letter of Don D. LARAÑAGA, quoted above:—

"It is very possible that the *Megatherium* was, in fact, covered by a scaly cuirass; but the great fragments which have been found must not be hastily attributed to it; for the plaster casts sent from London† prove that an Armadillo of gigantic size coexists with the *Megatherium* on the plains of Buenos Ayres. These characteristic fragments consist of a calcaneum, an astragalus, and a scaphoid, which depart from those of existing Armadillos only in size, and by purely specific differences."

In 1836, then, it was clearly made out that the cuirassed extinct animal of South America is *not* the *Megatherium* and *is* allied to the Armadillos. However, Dr. BUCKLAND, whose Bridgewater Treatise appeared in this year, and who therefore could hardly have been acquainted with the views of D'ALTON and of LAURILLARD, still associated the dermal armour with the Megatherium—supporting his views by an elaborate and ingenious teleological argument, which, like most reasonings of the kind, appeared highly satisfactory. But, in 1837, all further doubt upon the subject was removed by the discoveries of Dr. LUND, who, in that year, despatched to Copenhagen the second of the remarkable series of memoirs in which he reconstructed the ancient Fauna of Brazil‡. In this paper Dr. LUND established the genus *Hoplophorus* upon the dermal armour and certain bones of an edentate quadruped closely allied to, if not identical with, the "Dasypus" of LARAÑAGA.

Hoplophorus euphractus, the sole species of the new genus described in the memoir, was estimated by its discoverer to be of the size of an ox, and to have been provided with a carapace most nearly resembling that of *Tolypeutes*, but of an astonishing thickness. The extremities are said to have the general structure of those of the Armadillos, the feet being short and thick, with remarkably broad and short nails; so that they must have resembled those of an Elephant, or a Hippopotamus. The skull was sloth-like, and its jugal arch exhibited the structure characteristic of those animals. The teeth were similar to the molars of *Capybara*, but simple instead of being made up of many plates.

---

* Thus MÜLLER says in his memoir on the hind foot, cited below, "In der letzten Abhandlung ist von Herrn D'ALTON bewiesen, dass der Panzer nicht dem *Megatherium* angehört."

† *Vide supra*, p. 32. Mr. PENTLAND appears to have been led to the same opinion by the examination of these casts in 1835. See Transactions of the Geological Society, vol. vi. ser. 2nd, p. 85, and Mr. PENTLAND'S letter to M. ARAGO in the 'Comptes Rendus' for March 11, 1839.

‡ "Blik paa Brasiliens Dyreverden för sidste Jordomvæltning. Anden Afhandling: Patte dyrene. Lagoa Santa, 16de Novbr. 1837," published in 'Det Kongelige Danske Videnskabernes Selskabs Naturvidenskabelige og Mathematiske Afhandlingar,' Ottende Deel, 1841, p. 70. A notice of LUND'S labours, containing the names of his genera, is to be found in the 'Oversigt over det Kongelige Danske Videnskabernes Selskabs Fordhandlingar i Aaret 1838,' published by ÖRSTED, the Secretary of the Academy.

Professor BRONN, publishing the second edition of his 'Lethæa Geognostica' in the spring of 1838, and unacquainted with LUND'S labours, proposed the name of *Chlamydotherium* for the animal to which the carapace described by WEISS and D'ALTON belonged, in case the foot should really appertain to it; and *Orycterotherium*, in case the foot should belong to a different animal.

In March of the same year, it appears that M. VILARDEBO, Director of the Museum of Monte Video, and M. ISABELLE published conjointly, in Nos. 2551, 2553, and 2555 of a journal, the 'Universal,' an account of an animal which they had discovered on the Pedernal, in the Department of Canelones*.

After removing a thin layer of clay, these observers met with a shield formed of pieces of bone separated from one another by a slight interval; these pieces, 25 to 50 millimetres in diameter, and varying in thickness from 12 to 40 millimetres, were hexagonal: the largest occupied the dorsal region of the carapace, and the smallest its lateral regions. Each polygon presented a central disk (14 to 27 millimetres in diameter), from whence radiated six or eight lines, between which as many quadrangular areæ were left. These pieces of bone were symphysially united so as to form a very regular mosaic: the carapace appeared to be fringed with conical pieces forming a semicircle of 24 centimetres. The carapace was about 4 metres wide, and was as convex as a cask. The bones discovered in it were lumbar vertebræ and pelvic bones. In another place was discovered a femur about 0·57 metre long, with many plates of the carapace, and a tail formed of a single mass of bone (covered nevertheless by pieces soldered together), in the middle of which were widely separated caudal vertebræ. The tail was more than 0·50 metre long, and more than 0·36 metre in diameter at the base.

The authors discuss the question—to what class do these fossils belong?—with much sagacity, and conclude by expressing the opinion that they appertain to a species of *Dasypus*, which they term *D. antiquus*, and which they briefly characterize as follows: "*Cingulis dorsalibus nullis: verticillis caudalibus nullis.*"

The volume of the Transactions of the Danish Academy, already cited, contains another communication from Dr. LUND, dated Lagoa Santa, September 12, 1838, in which he speaks of the fossils described by D'ALTON, and identifies the animal to which they belonged, generically, with *Hoplophorus*, though he regards it as a distinct species, and names it *Hoplophorus Selloi*. Accompanying this paper are sundry figures of parts of the carapace and of bones of the hind foot of *Hoplophorus*.

Dr. LUND returns to the subject in a long letter addressed to M. V. AUDOUIN, dated the 5th of November 1838 (extracts from which are published in the 'Comptes Rendus' for the 15th of April 1839), which contains an enumeration, with brief descriptive notices, of the seventy-five species of fossil Mammalia which this untiring explorer had extracted in the preceding five years from the caverns of Brazil. Among the rest the writer describes:—

---

* See the Bulletin de la Société Géologique de France, t. xi. p. 159 (1840).

"6°. *Hoplophorus*, a genus very remarkable for the heavy proportions of its species, for their gigantic size, as well as for the singular manner in which it combines different types of organization; however, their characters approximate them most nearly to the Sloth family. These strange animals were armed with a cuirass which covered all the upper part of the body, and which was composed of little hexagonal scutes, except in the middle of the body, where the scutes took a quadrate form, and were disposed in innumerable transverse bands. The bones of the trunk, as well as the great bones of the extremities, are also very similar to those of the Tatous, and particularly to those of the Cachicames; but the bones which compose the feet are so shortened and have their articular faces so flattened, that nothing similar is to be seen in any animal skeleton, and that it is inconceivable how such feet should have been used in digging. The form of the teeth also indicates that these singular animals could feed only on vegetable substances, and it is to be supposed that they grazed after the fashion of the great Pachyderms. However this may be, the *Hoplophorus*, of which M. LUND describes two species, present the peculiarity, hitherto regarded as special to the Sloth, of having a descending branch to the zygomatic arch. These two species were as large as an ox. Fragments of the skeletons have already been described by MM. WEISS and D'ALTON of Berlin."—*Loc. cit.* pp. 572, 573.

A summary of LUND's researches, despatched by him from Lagoa Santa on November 5, 1838, and published in the Annales des Sciences Naturelles for 1839, under the title of "Coup d'œil sur les espèces éteintes de mammifères de Bresil: extrait de quelques mémoires présentés à l'Académie Royale des Sciences de Copenhague," gives a substantially similar account of *Hoplophorus*. The species *Hoplophorus Selloi* is identified with the cuirassed animal described and figured by WEISS and D'ALTON.

The sixth volume of the second series of the Transactions of the Geological Society contains an elaborate memoir by Professor OWEN* on the bones associated with the dermal armour, figured by Mr. CLIFT in the memoir already cited; and on certain teeth, upon which the genus *Glyptodon* was founded by the same writer, in Sir WOODBINE PARISH's work on Buenos Ayres†.

Professor OWEN considers these remains to be specifically identical with those collected by SELLOW, and described by WEISS and D'ALTON; so that if LUND was right in ascribing the same fossils to his genus *Hoplophorus*, *Glyptodon* becomes a synonym of the latter.

In the memoir under consideration the general form and the minute structure of the teeth, the distal end of the humerus, the radius, two phalanges of the fore foot, "the anchylosed distal extremities of the tibia and fibula, an astragalus, calcaneum, scaphoides, cuboides, external cuneiform bone, the three phalanges of the second toe, and the middle and distal phalanges of the third and fourth toes, with a few sesamoid bones," all belonging to the left side, are described; while the tooth and the bones of the leg and foot are figured.

---

* "Descriptions of a tooth and part of the skeleton of the *Glyptodon clavipes*, a large quadruped of the edentate order, to which belongs the tessellated bony armour described and figured by Mr. CLIFT in the former volume of the Transactions of the Geological Society, with a consideration of the question whether the Megatherium possessed an analogous dermal armour." By RICHARD OWEN, Esq., F.G.S., F.R.S. (Read March 23rd, 1839: an abstract of this paper appeared in No. 62 of the 'Proceedings.')

† 'Buenos Ayres and the provinces of the Rio de la Plata,' 1838, p. 178 *e*.

Professor OWEN considers that the dental characters "seem to indicate a transition from the *Edentata* to the pachydermatous *Toxodon*," and sums up his general conclusions as to the affinities of *Glyptodon* thus:—

"It may be concluded, therefore, that the extinct edentate animal to which belongs the fossil tessellated armour described by WEISS, BUCKLAND, and CLIFT, cannot be called an Armadillo, without making use of an exaggerated expression, and still less a species of *Megatherium*; but that it offers the type of a distinct genus, which was much more nearly allied to the Dasypodoid than to the Megatherioid families of Edentata, and most probably connected that order of quadrupeds with the heavy coated Rhinoceros of the Pachydermatous group" (*l. c.* p. 96).

In the same year (1839) Professor D'ALTON proposed for the animal, the remains of which he had originally described, the name of *Pachypus*; so that by this time no fewer than six names had been applied to mammals all of which are certainly closely allied to the *Hoplophorus* of LUND, whether they are, or are not, generically identical with it, and which may therefore be appropriately termed *Hoplophoridæ*.

In 1845 Professor OWEN returned to the *Glyptodon* question, in the 'Descriptive and illustrated Catalogue of the Fossil Organic Remains of Mammalia, and Aves contained in the Museum of the Royal College of Surgeons of England.'

It is here stated (p. 107) that "those specimens of the present genus which were presented to the College by Sir WOODBINE PARISH are from a low marshy place, about five feet below the surface, in the bank of a rivulet, near the Rio Matanza, in the Partido of Canuelas, about twenty miles to the south of the city of Buenos Ayres." The parts thus found associated are not stated, with the exception of the bones of the left hind leg and foot (p. 111), to have belonged to the same individual. They consist of a molar tooth, part of the left ramus of the lower jaw, a fragment of the humerus, the left radius, a metacarpal bone and two phalanges, the shaft and distal epiphyses of the femur (?), the anchylosed distal ends of the tibia and fibula, and numerous bones of the left hind foot. These had already been described and figured in the Geological Society's Transactions.

As new specimens, there are described and figured an almost entire carapace of *Glyptodon clavipes*, from the Pampas of Buenos Ayres, and many dermal bones, all of which are marked "Purchased," and appear not to have been accompanied by bones of the endoskeleton. Nos. 551, 552, 554, 555, 556, 557 are fragments of carapace, all presented by Sir WOODBINE PARISH, and obtained from the locality mentioned above. They are ascribed by Professor OWEN to no less than three distinct species, however, viz. *Glyptodon clavipes*, *G. reticulatus*, and *G. ornatus*; a fourth species, *G. tuberculatus*, is based upon purchased specimens, from the Pampas of Buenos Ayres, the precise locality of which is not stated.

The fact that the dermal ossicles of three species of *Glyptodon* were found in the same locality as the bones described, and the absence of any evidence demonstrating the association of the ossicles ascribed to *G. clavipes*, rather than those attributed to the other species, with the bones, throws, it will be observed, some doubt upon the certainty of that ascription, and opens the question whether the bones really belonged to one form of carapace or to another.

Of the Plates which illustrate the 'Catalogue,' the first contains a side view, partly restored, of the *Glyptodon clavipes*; the second, views of the carapace and tail; the third, of the skull; the fourth and fifth, of parts of the carapace; and the description of the Plates comprises accounts of the structure of the skull and of the tail, parts which had not been received until after the printing of the body of the catalogue.

In what locality the skull and the tail were obtained, and upon what evidence they are ascribed to the particular species, *G. clavipes*, is not stated. The lower jaw and the defensive bony covering of the skull in plate 1 "are restored on the authority of an original sketch of an entire specimen of this species of *Glyptodon* transmitted to Sir WOODBINE PARISH from Buenos Ayres." The bones of the fore foot are given in outline after D'ALTON.

On the 8th of June, 1846, the late JOHANNES MÜLLER read a short paper to the Berlin Academy upon the bones of the leg and hind foot described by D'ALTON, which had been worked out and mounted by the help of Professor OWEN'S memoir. This paper, accompanied by an excellent plate, was published in 1849*.

The number of the 'Comptes Rendus' for August 28, 1855, contains a "Description d'un nouveau genre d'Edenté fossile renfermant plusieurs espèces voisines des Glyptodons, et classification méthodique de treize espèces appartenant à ces deux genres," by M. L. NODOT, Director of the Museum of Natural History at Dijon; and this essay, enlarged and illustrated with plates, appeared two years later in the 'Mémoires de l'Académie Impériale de Dijon,' Deuxième Série, tom. v. 1857†.

M. NODOT, in his introductory remarks, states that Vice-Admiral DUPETIT brought back from Monte Video, in 1846, a great number of fossil bones which had been collected by Dr. NUMEZ on the banks of the river Lujan, and were given to the Vice-Admiral by the orders of the Dictator ROSAS. Admiral DUPETIT presented most of these remains to the Museum of the Jardin des Plantes in Paris; but dying before he had disposed of all, his widow bestowed two boxes full of detached dermal ossicles on the Dijon Collection. Out of these, by dint of four months' constant toil, M. NODOT reconstructed the carapace.

---

* "Bemerkungen über die Fussknochen des fossilen Gürtelthiers (*Glyptodon clavipes*, Ow.)," Abhandlungen d. Konigl. Akad. d. Wissenschaften, 1849.

† Under the title "Description d'un nouveau genre d'Edenté fossile renfermant plusieurs espèces voisines du Glyptodon, suivie d'une nouvelle méthode de classification applicable à toute l'histoire naturelle et spécialement à ces animaux. Avec un atlas de douze planches lithographiées."

Subsequent investigations in the store-rooms of the Jardin des Plantes revealed almost the whole of the tail, and many important parts of the skeleton, of what M. NODOT believed to be the same individual animal, mixed up, however, with fragments of *Mylodon*, *Megatherium*, and *Scelidotherium*. Besides these, M. NODOT found the tolerably complete extremity of the tail of another individual of the same genus in the Geological Gallery, and the right half of a lower jaw with the teeth, which he judged to belong to this individual.

The bones which M. NODOT, guided as it would seem chiefly by their colour, identifies as belonging to the same individual with the carapace, are, "the lateral and posterior part of the cranium, the occiput, the meatus auditorius, the zygomatic arch and its long apophysis, three alveoli, and the sagittal crest; the atlas, the axis, the vertebra of the fifth ring of the tail; the two femora entire; the tibiæ and fibulæ anchylosed; the calcanea; the astragali; the other tarsal bones; the left metatarsus; the three external toes of the left hind foot; the left radius; the ungual phalanx of one of the digits of the fore foot; and the ungual phalanx of an internal toe of the hind foot." The carapace and the tail are fully described by M. NODOT, who considers their peculiarities sufficient to justify him in establishing for these remains the new genus *Schistopleuron*.

How far he was justified in so doing is a point which must be discussed at the end of this memoir; but there can be no question that "*Schistopleuron*" is one of the *Hoplophoridæ*, closely allied to *Glyptodon clavipes*; and hence M. NODOT's descriptions of the mandible, sternum, and femur constitute substantial additions to our knowledge of the organization of that family.

The mandible is unlike the sketch furnished to Professor OWEN and adopted by him, but very like that which will be described below. The first piece of the sternum and the first two ribs were so anchylosed together as to leave no trace of their primitive separation.

On the 14th of November, 1862, I presented to this Society a "Description of a new Specimen of *Glyptodon*, recently acquired by the Royal College of Surgeons of England," which was published in the fifty-third Number of the 'Proceedings of the Royal Society.' The remains of the specimen, described briefly in this preliminary notice and, in full, in the present memoir, were presented to the Royal College of Surgeons by Señor Don MAXIMO TERRERO, having been discovered in 1860 on the estate of his brother, Señor Don JUAN N. TERRERO, which is situated on the banks of the river Salado, in the district of Monte, in the Province of Buenos Ayres, and about eighty miles due south of the city of that name.

No portions of any other animal, nor any duplicate bones, have been discovered among the osseous relics the description of which has been entrusted to me by the authorities of the College of Surgeons—a circumstance which justifies the belief that they all belonged to one and the same animal, and gives them a peculiar value, the more especially as there can be little doubt of the specific identity of the new specimen with the animal to which the skull ascribed by Professor OWEN to *Glyptodon clavipes* belongs.

I have thus been enabled to add to what was already known of *Glyptodon clavipes*, descriptions of the most essential peculiarities of the fore part of the skull, the entire palate, the mandible, the greater part of the spinal column, the pelvis, and the complete fore and hind feet, and to announce the existence, in this animal, of a conformation of the spinal column hitherto unknown in the Mammalian, and, indeed, in the Vertebrate series—the last cervical and two anterior dorsal vertebræ being anchylosed together into a single osseous mass articulated by ginglymi with the rest of the vertebral column. As another very remarkable peculiarity of this genus, I have pointed out the extraordinary characters of the pelvis, and the fact that the cuneiform bone in the carpus articulates with two metacarpal bones, the fourth and fifth, while the unciform does not articulate with the fifth at all.

Since the appearance of my paper in the 'Proceedings of the Royal Society,' and indeed not until the months of May and June 1863, M. SERRES, apparently unacquainted with what had been done in these matters, has redescribed the joint between the second and third dorsal vertebræ, though he appears to be still unaware of the existence of the 'trivertebral bone.' In addition, M. SERRES makes known the interesting circumstance, that the posterior edge of the *manubrium* of the sternum, anchylosed (as M. NODOT had pointed out, though M. SERRES does not refer to him) with the first pair of ribs, presents two concave articular facets, by which it was united with the rest of the sternum, which must have presented two convex surfaces adapted to the foregoing in order to allow of a movement of flexion. M. SERRES is of opinion that this mechanism is intended to allow of the retraction of the head: "Il est donc vraisemblable qu'au moment du danger, peut-être même que dans le repos ou le sommeil, le *Glyptodon* fléchissait le col pour ramener la tête sous la coupole de la carapace"*.

In his second communication to the Academy, M. SERRES still speaks of the "anchylosis of the first two dorsal vertebræ" only†.

Professor BURMEISTER, Director of the Museum at Buenos Ayres, has been good enough to communicate to me a letter, addressed by him to the Editor of the 'Nacion Arjentina' on the 5th July, 1863, commenting upon a lecture upon the Glyptodon which I delivered before the President and Council of the Royal College of Surgeons, which was published in the Medical Times and Gazette for the 28th of February and

---

* "Note sur deux articulations ginglymoïdes nouvelles existant chez le Glyptodon, la première entre la deuxième et la troisième vertèbre dorsale, la seconde entre la première et la deuxième pièce du sternum. Par M. SERRES" (Comptes Rendus, May 11, 1863).

† "Deuxième Note sur le développement de l'articulation vertébro-sternale du Glyptodon, et les mouvemens de flexion et d'extension de la tête chez cet animal fossile. Par M. SERRES" (Comptes Rendus, June 1, 1863).

7th of March, 1863, and which contains the substance of the statements previously published in the 'Proceedings' of this Society.

Professor **BURMEISTER** affirms that the skeleton of the *Glyptodon* in the Museum of Buenos Ayres is much more perfect than that in the Royal College of Surgeons; that it has the seven cervical vertebræ complete; and that the five middle cervical vertebræ are anchylosed together, while the seventh is very delicate and fragile. Under these circumstances, it would appear that Professor **BURMEISTER** considers the trivertebral bone (my description of which he confirms) to be composed of the three anterior dorsal vertebræ.

Professor **BURMEISTER** is further of opinion that the peculiar mechanism of the joint formed by the trivertebral bone with the rest of the spinal column has not that respiratory function which I have ascribed to it; but, with M. **SERRES**, he thinks that its object is to allow of the application of the cephalic shield to the anterior aperture of the shield of the body. Professor **BURMEISTER** goes on to remark—

"As little do I agree with Mr. **HUXLEY** as to the immobility of the ribs, which are wholly wanting in the London skeleton. The skeleton of the Museum of Buenos Ayres has nine ribs, three of which being complete, prove that they possess a certain mobility, moving downwards and backwards on their articulations with the spinal column, as in other Mammalia, but without doubt in a manner somewhat different from the ordinary way."

I am at a loss to divine on what grounds Professor **BURMEISTER** ascribes to me the opinion that the ribs are immoveable, and why he affirms that they are wholly wanting in the London skeleton. What I have stated is, that the first rib is immoveable; and so far from the ribs being wholly wanting, I have particularly mentioned their presence*, and have alluded to the characters of the first†.

Professor **BURMEISTER** adds that I am in error in supposing that the dorso-lumbar vertebræ were immoveably united. I believe, however, from Professor **BURMEISTER**'s own words, that my description is substantially accurate. These words are:—

"There exists a moveable place between the dorsal and the lumbar vertebræ, though the mobility is not so complete as that of the three first anchylosed vertebræ upon the following ones. In this part, the skeleton of Buenos Ayres presents a complete column, formed by eleven vertebræ incorporated into a solid piece, of a very peculiar form, with three crests in the upper part, the two lateral of which bear the ribs in articular excavations. The total number of dorsal vertebræ and of ribs is therefore fourteen. Then follow on these the lumbar vertebræ, all anchylosed together and immoveably united with the sacrum."

I do not venture to doubt the accuracy of Professor **BURMEISTER**'s description of the specimen under his own eyes; but nevertheless, as will be seen by-and-by, it is also true that the account I have given of the *Glyptodon* in the College Museum is quite accurate. And indeed, as Professor **BURMEISTER** admits that all the dorsal and all the lumbar vertebræ respectively were anchylosed together, with only an imperfect mobility at the junction of the two solid masses, I do not see how, in any important respect, his view of the matter differs from mine.

---

* Proceedings of the Royal Society, *l. c.* p. 317.

† Ibid. p. 319.

The last criticism which Professor BURMEISTER offers, refers to what he terms my error in ascribing five toes to the fore foot, when, as he affirms, it possesses only four. Professor BURMEISTER states that I have figured five toes to the foot of the *Glyptodon* in the lecture already referred to; but he is mistaken; only four toes are there represented, numbered, according to the digits of the typical foot which they represent, 2, 3, 4, 5. In the 'Proceedings' (p. 325) I have expressly stated—

"The trapezium possesses only a very small double articular facet on its palmar face. If this gave support to a metacarpal, it must have been very small; and as at present neither it nor any of the hallucal phalanges have been discovered, it is possible the pollex may have been altogether rudimentary. In any case the pollex must have been so much smaller and more slender in proportion than that of *Dasypus*, that the animal must have had a practically tetradactyle fore foot."

The errors, therefore, to which Professor BURMEISTER adverts, appear to me to arise to a great extent from his not having rightly comprehended my statements; and in part, it may be, from our having to deal with different objects.

PART II.—*Description of the Skeleton of* Glyptodon clavipes, OWEN (*Hoplophorus Selloi*, LUND?).

The materials which have been available for the following description of the osteology of *Glyptodon* are, in the first place, the skeleton referred to in the previous section as having been presented by Señor TERRERO to the Royal College of Surgeons; secondly, the detached parts which have been already described by Professor OWEN, and are now contained in the Museum of the Royal College of Surgeons; thirdly, some fragmentary specimens in the British Museum; and fourthly, photographs of a skeleton of *Glyptodon* in the Museum of Turin. The two latter sources of information, however, are of altogether secondary importance, and will be adduced merely in confirmation of the results obtained from the study of the two former series of materials,—in treating of which, I shall speak of the fragments of *Glyptodon clavipes* described by Professor OWEN as the "type specimen," and of the skeleton presented by Señor TERRERO as the "new specimen."

§ 1. *Description of the Skull of* Glyptodon clavipes.

In the new specimen* the anterior part of the skull, from a line drawn transversely, immediately behind the zygomatic processes, to the anterior end of the snout, is in a remarkably good state of preservation—the boundaries of the anterior nares, the anterolateral parts of the maxillary bones, the nasal, and the fore part of the frontal, bones being quite uninjured. Behind the imaginary transverse line in question this cranium

* Plate IV. figs. 1 & 3, Plate V., and Plate VI. figs. 1, 2, 4, & 5.

is very imperfect—the entire roof and sides, and the greater part of the base of the skull being absent, while a small portion only of the sphenoidal region is preserved.

Of the facial bones, those entering into the palate are preserved almost in their entirety, and one ramus of the lower jaw is nearly complete. This skull therefore supplies almost all those parts which were wanting in the cranium of the type specimen, in which the whole of the roof of the skull, from the nasal bones to the supraoccipital inclusive, most of the exoccipital, alisphenoidal, and orbitosphenoidal regions of the lateral walls, and of the basioccipital, basisphenoidal, and presphenoidal parts of the base, together with the temporal bones, are in good condition, while the premaxillary, maxillary, and palatine bones, with the mandible, are absent.

In order to give a tolerably complete view of the structure of the skull, I shall, in the first place, describe that of the new specimen; I shall next proceed to a comparison of the parts common to this fossil and the skull of the type specimen, in order to demonstrate the specific identity of the two; and then I shall endeavour to supply what is wanting in the new specimen by information derived from the study of the type.

The skull of the new specimen of Glyptodon clavipes.—The anterior nares have a trapezoidal form, the upper of the two parallel sides of the trapezoid being nearly three times as long as the lower, so that the two lateral boundaries converge from the roof towards the base of the nares (Plate VI. fig. 1).

The upper boundary of the anterior nares is formed by the anterior edges of the thick nasal bones, which are bevelled obliquely from below upwards, and so rounded off laterally that the contour of the two forms a large arc of a circle, the chord of which measures 3·4 inches (Plate IV. fig. 1). The upper surface of each nasal bone is rough and perforated by many vascular foramina, which open forward; and the two nasal bones are separated by a suture, which can be traced backwards in the middle line for 2·2 inches, and then comes to an abrupt termination. I presume that the extent of this suture indicates the distance to which the nasal bones reach backwards; but there are no traces of the nasofrontal, or nasomaxillary sutures. The middle of the under surface of each nasal bone presents a strong, rounded, longitudinal ridge, on each side of which there is an equally distinct concavity, and the apposed slightly thickened inner edges of the two nasal bones form a third, less marked, median ridge. The expanded upper edge of the perpendicular plate of the ethmoid embraces this middle ridge, while the nasal turbinal bones are continuous with the ridges on each side of it (Plate VI. fig. 1).

A well-marked notch, or sinuosity, separates the upper from the lateral contour of the anterior nares; and, about an inch below this, the inner surface of the outer wall of the nostril exhibits a rounded elevation or thickening. Still more inferiorly, the wall of the nasal cavity is somewhat excavated, so as to present a thin anterior edge, which passes into the trough-like lower boundary, constituted by the palatine portions of the præmaxillæ. These are separated throughout their whole length in the middle line (a distance of rather more than an inch) by a fissure less than one-tenth of an inch in diameter posteriorly, but twice as wide in front, the præmaxillæ becoming more distant by the divarication of their anterior and internal angles. The thick and rough anterior edges of the præmaxillæ diverge obliquely from one another, both forwards and outwards and upwards and outwards, at a very obtuse angle, the interval between their anterior and external terminations amounting to 1·5 inch (Plate IV. fig. 3). Viewed laterally, the anterior ends of the nasal bones are seen to project about half an inch beyond the upper part of the lateral boundary of the nares, which slopes upwards and backwards with a slight forward concavity from the palatine portion of the præmaxilla (Plate V. fig. 1).

The nasal cavity is divided, longitudinally, by a very strong osseous septum, which extends to the posterior end of the premaxillary fissure below, and to within 0·4 inch of the anterior contour of the nasal bones above (Plate VI. fig. 1). This septum terminates, in front and below, in a thin jagged edge; but above, it expands into a broad plate 1·2 inch wide, presenting a deep and broad notch above, into which, as I have previously stated, the conjoined median edges of the nasal bones are received. The septum is about 2·6 inches high in front; and of this height 2·2 inches, or about five-sixths, is formed by the perpendicular plate of the ethmoid, while the rest belongs to the vomer (*Vo.*). The ethmoidal plate is thin in front, thicker in the middle, and thin again posteriorly. The lower half is somewhat excavated on each side, from above downwards; it ends in an inferior edge, or rather surface, 0·7 inch in diameter, anchylosed with the upper edge of the vomer, which has, in front, a corresponding thickness. The floor of the anterior part of the nasal cavity (*i. e.* as far as the level of the fourth alveolus) is concave from side to side, and convex from before backwards, its convexity corresponding with, but being much more strongly marked than, the concavity of the arched roof of the palate.

At about 2 inches from the anterior boundary, a sharp longitudinal ridge commences upon the floor of each division of the nasal cavity, and extends backwards, for a distance of about 1½ inch, to the summit of the arch formed by that floor (Plate VI. fig. 1, *a*). Each ridge has a sloping convex external face, and a perpendicular concave inner face, 0·2 inch high. Between the latter and the side of the vomer, which is excavated for a corresponding distance from above downwards, lies a canal, a quarter of an inch wide, and open above and at its ends. The floor of each nasal chamber rises gradually into its lateral wall; and upon this, about three-fourths of an inch from the floor, appears a ridge which, at about an inch from the antero-lateral margin of the nostril (or just above the anterior end of the ridge on its floor), passes backwards into the commencement of the inferior spongy bone (Plate VI. fig. 1, *b*). The root of attachment of this bone to the maxilla is, as usual, a narrow and thin, though long, bony plate, which on its free, or inner, side is continued into two scroll-like lamellæ, an upper and a lower. The upper scroll comes much further forward than the lower, and is a stout plate of bone, slightly concave inwards and convex outwards. In front, it ends in a thin free edge. Superiorly, its margin is folded over outwards, and becomes anchylosed with the lateral wall of the nasal chamber.

The inferior lamella commences about an inch behind the superior one. It is thick, convex inwards and concave outwards, and its inferior edge becomes much thickened as it curves outwards. It is attached to the maxilla by an anterior and superior thin, and a posterior and inferior, much thicker, plate of bone. Three passages, consequently, lie between the lateral walls of the nasal chamber and the 'scrolls' of the inferior turbinal,—an upper, long, narrow, and flattened from side to side; a middle, reniform in section; and an inferior, rounded in contour. The ridges upon the under surfaces of the nasal bones are continued, as I have stated above, into two thick plates of lamellated bone (Plate VI. fig. 1, *c*), which increase in depth from before backwards and pass into what are, probably, the superior ethmoidal turbinals. Their inner surfaces are flattened and parallel with the sides of the perpendicular plate of the ethmoid. Their outer surfaces, irregularly concave, are separated by but a narrow interval from the concave faces of the superior scrolls of the inferior turbinal bone.

The posterior view of this fragmentary skull (Plate VI. fig. 2) affords a further insight into the arrangement of the bones which contribute to the formation of the olfactory chambers. The aspect presented is that of a transverse section taken just in front of the anterior end of the cranial cavity. The comparatively thin posterior part of the *lamina perpendicularis* of the ethmoid (*Eth*) is seen abutting, above, against the frontal bones (*Fr*), and, below, becoming connected with the vomer (*Vo*), the posterior nearly straight free edge of which bone ends on the floor of the nostrils, at the level of the posterior margin of the third molar tooth, and thence slopes obliquely upwards and backwards.

The ethmovomerine plate, however, is not free from all lateral connexion with the turbinal bones, as is commonly the case; but a thin plate of bone, convex forwards and concave backwards, passes, on each side, from the vomer and the lamina perpendicularis to the lateral masses of the ethmoid. The inner surfaces of these are marked by broad flattened grooves, directed forwards and downwards, and separated by sharp ridges, which, in the recent state, were probably produced into delicate plates of bone.

The lower portion of the lateral mass of the ethmoid, which represents the middle turbinal, is continuous with the inferior turbinal. The upper portion, representing the superior turbinal, is similarly continuous with the nasal turbinal. The superior turbinal of each side forms the floor of a considerable cavity (Plate VI. fig. 2), which is walled in, externally and above, by the frontal bone, and represents a frontal sinus. A rounded dome (*a*) of bone projects backwards from the anterior wall of this cavity, which appears to communicate with the nasal fossæ only by a few foramina, situated around the margins of the dome.

The palate (Plate IV. fig. 3) is singularly narrow, seeing that its length, measured in a straight line, is about 9½ inches, while its width, between the outer edges of the alveoli, nowhere exceeds 3 inches. The longitudinal contour of the palate is concave anteriorly, convex posteriorly (Plate V. fig. 1). The crown of the arch of the anterior concave portion is opposite the hinder margin of the third alveolus; from thence the roof of the palate slopes, downwards and forwards, to the free premaxillary edge. From the same point it slopes, downwards and backwards, to the level of the hinder margin of the fifth alveolus, while behind the sixth it ascends, somewhat abruptly, to its posterior termination.

Throughout the posterior two-thirds of its length, the palate is slightly and evenly concave from side to side; but, from the third alveolus forwards, its middle part rises to form a median convexity, which ends by a rough, abruptly truncated ridge (Plate IV. fig. 3, *a*), behind the premaxillary fissure. It forms, in fact, the posterior boundary of a transverse fissure ending in a notch, or short canal, at each extremity, which represents the anterior palatine foramen, and which, taken together with the intermaxillary fissure, simulates very closely the form of a T. A deep groove (*b*) separates the raised part of the palate from the alveolar margin, and ends, behind, in a canal which burrows into the substance of the bone opposite the anterior edge of the third tooth on both sides. On the left side, however, the hinder part of the groove is bridged over by a bar of bone. Large foramina are situated, along a line continuing the groove, opposite the third and fourth alveoli; but no such apertures appear in the posterior part of the palate until quite its hinder extremity is reached, when, on each side, two crescentic fossæ (Plate IV. fig. 3, *c*), wider in front than behind, lie on the inner side of the last alveolus, and appear to separate the palatine from the maxillary bones. They end cæcally above.

The bony palate exhibits no distinct sutures, except a trace of a maxillary suture behind the anterior palatine foramen, and of a palatine suture, which widens behind into a cleft, separating the arcuated, divergent inner and posterior boundaries of the palatine bones. The free surfaces of the bony masses which bound the palate, posteriorly, are so smooth and unbroken, that I suspect the pterygoid bones must be represented in them.

As the palate presents very nearly the same width throughout, while the roof-bones of the skull are always much wider than it, it follows that any vertical section of the skull, perpendicular to its long axis, in the palatine region, would exhibit a trapezoidal form, like that of the anterior nares—the predominance of the upper side over the lower being still more marked. But in the antorbital region the roots of the zygomatic processes are so large, and stand out so much from the sides of the head, that the skull, viewed in front, looks almost like a cube, with its lower face produced forwards and downwards into a truncated wedge (Plate VI. fig. 1). The only trace of a suture visible upon any part of the sides of the facial wedge is an almost obliterated one (Plate V. fig. 1, *a*), which runs from a slight notch, opposite the level of the anterior palatine foramen and in front of the first alveolus, upwards and slightly backwards, and marks off the ascending process of the præmaxilla from the maxilla. This ascending process, very narrow in the middle, widens above and joins the nasal bone, so that the circumference of the anterior nares is completed by the præmaxillæ and nasal bones only.

Opposite the second and third alveoli, the maxillary bone, as I have stated above, widens out and expands into the root of a stout zygomatic arch, whence a process, nearly 6 inches long by 2 inches wide, passes directly downwards. The process is much flattened from before backwards (Plate VI. fig. 1), and is arched from above downwards (Plate V.

fig. 1), so as to be convex in front and concave behind. Its inner edge is thick and rounded, except towards its termination, where it presents some slight irregularities or digitations. The outer edge is comparatively thin and rugose; it is bevelled off inferiorly, and more obliquely on the right side than on the left. The inner part of the front face of the process looks almost directly forwards, and is very smooth; but the outer part of that face looks outwards more than forwards, and is rugose (Plate VI. fig. 1). The hinder, concave face of the process (Plate VI. fig. 2) is divided by an oblique ridge (*b*), which passes from its superior and external to its inferior and internal angle into two areæ—an inner, smooth, and an outer, rough and tuberculated. The superior and external part of the process, where it was doubtless continued into the zygoma, is evidently fractured. The root of the zygoma is perforated near its origin by a large, oval, infraorbital canal, the lower edge of which is rather more than an inch distant from the lower margin of the root of the zygoma. The canal is short, and is directed forwards and outwards.

The lachrymal foramen is a round aperture, placed upon the anterior edge of the orbit, 1·6 inch above the infraorbital canal (Plate V. fig. 1, *b*).

The internal walls of eight alveoli, on each side, are preserved. The external walls of the anterior four upon the left side, and of the anterior three upon the right side, are almost entire; but, posteriorly, the external walls of all the other alveoli, upon each side, are broken away (Plate V. fig. 1).

Measured in a straight line, the eight alveoli occupy a space of 8 inches, and each alveolus is, on an average, 0·9 inch long. The teeth which occupy the alveoli are sensibly equal in long diameter; but the anterior tooth is much narrower than the others, measuring only 0·35 inch in this direction, while the other teeth have a transverse diameter of 0·6 inch, or nearly double that of the first.

None of the teeth are entire upon the right side. Of the left series, the crowns of the first, third, fourth, and sixth are in very good condition, while the second is much damaged; of the fifth, only the middle lobe exists, and of the seventh only the two anterior lobes (Plate IV. fig. 3).

The alveoli are exceedingly long, and the outer walls of the third and fourth, on each side, are so much broken away, that the whole length of their alveoli can be observed and measured. The fourth is 4·5 inches long, and bends outwards and forwards as it passes upwards, to terminate nearly on a level with the lachrymal foramen. The tooth which filled the alveolus must have had a corresponding length and curvature; for the two longitudinal ridges of bone, which partially subdivide the alveolus into three chambers near its free end, are continued quite up to its closed extremity, and are lined by a shell of dental substance, which gradually thickens below and becomes continuous with the body of the tooth (Plate V. fig. 1. 4, 4′).

The third alveolus presents the same general curvatures as the fourth, but is inclined somewhat further outwards at its upper end, which lies close to, and about an inch above, the hinder end of the infraorbital foramen.

The wall of the upper end of the first alveolus has been broken through on the right side. It lies on a level with the upper edge of the infraorbital foramen, and immediately behind the premaxillary suture.

From what remains of the hinder alveoli and teeth, I suspect they become more and more nearly straight posteriorly.

The external vertical contour of each tooth must be very similar to that of the maxillary surface between the upper end and the edge of the alveolus.

The lateral faces of all the teeth are divided by two longitudinal grooves, placed opposite to one another on the two sides of each tooth, into three lobes.

In the first tooth these grooves are very shallow, so that the thickness of the tooth, between the grooves, is far greater than the depth of a groove. In all the other teeth, the thickness of the teeth between the grooves, or of the isthmus by which the lobes of each tooth are connected, is much less than the depth of a groove.

The view of the palate (Plate IV. fig. 3) shows that lines following the planes of the anterior surfaces of each of the four anterior teeth are directed inwards and forwards; while in the sixth and seventh teeth, if not in all four posterior ones, such lines are directed inwards and backwards. The anterior surfaces of all the teeth, but the first, are concave, the posterior surfaces convex. The grinding-surfaces of all the teeth are directed a little outwards as well as downwards. Each surface is ridged in the middle and surrounded by a thin raised margin, and the general arrangement of the ridges is such that one is median, traversing the longitudinal axis of the grinding-surface, and three are disposed at right angles to these, in the longitudinal axes of the three lobes. The transverse ridges are continuous with the longitudinal, where they cut it (Plate V. figs. 3 & 4).

Sometimes a transverse ridge may be bifurcated at its extremity, or accessory branchlets may be given off from the transverse, or from the longitudinal, ridges.

A large pulp-cavity occupies the upper portion of each tooth; but as its walls begin sensibly to thicken at about the junction of the upper and middle thirds of the tooth, the pulp-cavity diminishes in a corresponding ratio, and, rather below the middle of the tooth, it becomes obliterated.

*The Mandible**.—The lower jaw of *Glyptodon* is very remarkable, partly on account of the trough-like projection of the symphysis, but more especially by reason of its great height in relation to its length. The height, as measured from any horizontal surface on which the jaw is allowed to rest, to the summit of the articular condyle, is 9·25 inches;

---

* Leaving aside for the present M. Nodot's "Schistopleuron," the only fragment of the lower jaw of *Glyptodon clavipes* yet described is that mentioned in the Catalogue of the Royal College of Surgeons under "No. 517. A fragment of the anterior part of the left ramus of the lower jaw, including portions of the sockets of the anterior teeth. The first is small and simple, and is situated close to the anterior termination of the dental canal; the second socket shows, by the two prominent vertical ridges on its anterior and posterior walls, that the tooth which it contained had the fluted form characteristic of the genus: the third socket, which is the most complete, differs from the preceding in a slight increase of size, and it shows that the tooth was implanted by an undivided base of considerable length, and of the same size and form as the exposed part or crown."

while the length, measured in a straight line, from the symphysis to the angle of the jaw, is not more than 10·75 inches. The horizontal ramus is very deep and thick, measuring about 3·25 inches vertically by 1·5 inch in thickness, while the ascending ramus is 3·5 inches wide by about 0·9 inch thick at thickest (Plate V. fig. 2).

The anterior end of the mandible is 2·9 inches wide and abruptly truncated, ending in a rugose edge, nowhere more than half an inch thick, which, at its extremities, bends round at a right angle into the upper margins of the rami (Plate VI. fig. 5). These, thick and rounded, ascend somewhat towards the first alveolus, which is 2·25 inches distant from the anterior end of the ramus. The symphysis, 5·7 inches long, appears to be formed by the sutural union, and not by the anchylosis of the rami; but the bone has been so broken that a large aperture occupies the middle of the symphysial space (Plate VI. figs. 4 & 5).

The exit of the inframaxillary canal is nearly half an inch wide, and is situated 1¾ inch below the upper margin of the jaw, and directly beneath the anterior boundary of the first alveolus. The anterior, or symphysial, contour of the mandible slopes, with a slight forward concavity, obliquely downwards and backwards to the level of the foramen; and is then continued, almost straight, or with a slight anterior convexity, to a point nearly in the same vertical line as the hinder edge of the third alveolus (Plate V. fig. 2).

The symphysial face is convex from side to side inferiorly, and gradually widens until, at its hinder end, its breadth amounts to 5·5 inches. Its outer boundary is formed by an obtuse longitudinal convexity, which runs along the middle of the outer face of the horizontal ramus, and dies away, posteriorly, at the commencement of the ascending ramus. From this ridge, or convexity, the summit of which corresponds with the greatest outside breadth of the jaw, the outer surface of the ramus slopes upwards and inwards to its alveolar margin (Plate VI. fig. 4). The inner face of each horizontal ramus is slightly concave from above downwards, passing, in front, into the excavated upper surface of the symphysis.

The general contour of the anterior half of the alveolar margin of the mandible is slightly convex upwards, in correspondence with the concavity of the opposed region of the maxilla (Plate V. fig. 2). The posterior half of the same margin is broken away; but it may be assumed that it was concave upwards, answering to the downward convexity of the hinder part of the maxillary alveolar edge.

The inner edges of the alveolar margins of the two rami are 2 inches apart. In the left ramus the series of alveoli is tolerably well preserved for 5½ inches, or to a point behind the anterior edge of the ascending ramus. From the character of the broken surface behind this point, however, it is obvious that the series of alveoli was continued along the inner surface of the ascending ramus, very nearly to the angle of the jaw, and considerably behind a line let fall perpendicularly from the articular condyle—an arrangement which, so far as I am aware, has no parallel among *Mammalia* (Plate VI. fig. 5).

As the whole length of the series of mandibular alveoli is about 8 inches, it is probable that the number of teeth was the same below as above, or eight on each side.

The external surface of the perpendicular ramus of the mandible is rugose, slightly convex from above downwards and from side to side, while its internal surface exhibits a corresponding concavity, which is exaggerated below by the inward projection of the posterior alveoli, and is divided by an elevation of its surface, which ascends obliquely from the alveolar margin towards the coronoid process, into an anterior and a posterior moiety. The apex of the coronoid process is broken away upon each side, but it seems not to have extended beyond the level of the articular condyle, from which it is separated by only a shallow notch.

The hinder margin of the perpendicular ramus, which is very thin inferiorly, thickens with the rest of the bone superiorly, and ends above in a transversely elongated condyle, which projects further upon the inner than on the outer side of the plane of the ramus (Plate V. fig. 2^a). Viewed laterally, this condyle has the form of a wedge, the base of which is 0·7 inch broad; its hinder face being slightly concave, while its anterior face, convex from above downwards, and slightly concave from side to side, looks forwards and upwards (Plate V. fig. 2). It is this face which bears the surface for articulation with the squamosal element of the skull, and is indeed coextensive therewith. The surface in question is 1·25 inch wide from side to side, and 0·6 inch broad or from above downwards, and is tolerably smooth, but not very different from the adjacent parts of the condyloid process.

The remains of five successive anterior teeth are observable in the alveoli of the left ramus of the mandible, and the socket of the sixth is clearly defined. Behind it, for a space of 1·8 inch, the inner wall of the ramus is broken away so completely that no trace of any alveolus is left. On the right side, the bone is nearly in the same state, but at a distance of 7·6 inches from the anterior edge of the most anterior alveolus, I observe a smooth vertically grooved surface of bone, which is situated nearly in the same plane as the outer walls of the other alveoli, and which I conceive to be part of the outer wall of the last alveolus.

The teeth of the mandible present the same trilobed form and other general characters of those of the maxilla, but very few are in a sufficiently entire state to furnish materials for description. The first and second, on the left side, and the third, upon the right side, however, have their grinding-surfaces entire, or nearly so (Plate VI. fig. 5).

The grinding-surface of the first tooth (left side) is 0·85 inch long and 0·4 inch wide at widest. It has a very different form from the first tooth of the maxilla, the two posterior ridges of the outer surface being much more developed.

The grinding-surface of the second tooth (left side) measures 0·9 inch by 0·45 inch; its outer ridges and grooves are also the better marked. The posterior surface of the tooth is flat or a little concave, and its plane is directed obliquely outwards and backwards.

The grinding-surface of the third tooth (right side) is 1·05 long, and the isthmuses which unite its prisms are much narrower than in the second tooth. Both the anterior and the posterior faces of the tooth are curved. The grinding-faces of all these teeth are inclined a little inwards as well as upwards, reversing the direction of the grinding-faces of the upper teeth.

The mandibular teeth seem to have been nearly straight, without either external or internal concavity. Their long axes are inclined rather backwards as well as downwards. The alveolus of the fourth tooth, on the right side, is laid open; and I judge from it that the fourth tooth must have had a length of about 3½ inches; and the others might have had the same dimensions, except the first, which is certainly shorter, probably not exceeding 2½ inches.

A considerable canal traverses the right ascending ramus from behind and below, upwards, forwards, and outwards. Its external aperture, oval, 0·3 inch wide, lies upon the outer face of the ramus, on a level with the alveolar margin, and rather nearer its anterior than its posterior edge (Plate V. fig. 2). The inner end of the canal, which is 1·7 inch long, terminates in the broken cancellous structure, on the outer side of what appears to be the remains of the last alveolus.

I cannot certainly discern any remains of a corresponding canal in the left ascending ramus.

All that remains to be described in this skull is a fragment of the basis cranii, consisting of part of the anchylosed basi- and pre-sphenoid bones. The presphenoid (Plate VI. fig. 2) is remarkable for the strong crest or spine into which the middle of its upper surface is produced, and which was not improbably continued into an ethmoidal *crista galli*. The posterior apertures of the passages for the optic nerves are ellipses, with their long axes directed upwards and outwards; they are about a quarter of an inch in diameter, and are continued into two canals, which are traceable, outwards and upwards, for about an inch in the substance of the orbitosphenoids. On each side, below and external to the optic foramina, are strong grooves which formed the inner portion of the confluent foramen rotundum and sphenorbital fissure. The front face of the presphenoid and the roots of the orbitosphenoids are excavated by deep sphenoidal sinuses.

Comparison of the Skull of the present specimen with that of the typical Glyptodon clavipes.—The principal parts which exist in both skulls, and may therefore serve as terms of comparison, are, 1, the nasofrontal region of the roof of the skull; 2, the descending zygomatic processes; 3, the alveoli; and 4, the basi- and pre-sphenoid.

1. The resemblances in size and general configuration between the nasofrontal regions of the two skulls are so obvious that I need hardly dwell upon them at any length. The present specimen differs from the type in the more rounded contour of the nasal bones, in the persistence of the nasal suture, in the less rugosity and squareness of the supraorbital prominences, and in the far less marked definition of the temporal ridges; but none of these characters appear to me to have more than an individual importance, and I am inclined to suspect that they depend largely on the less advanced age of the present specimen.

2. The zygomatic processes have the same length (measured from the infraorbital foramen) in each case. They are slightly narrower in the type specimen; in other respects

the zygomatic processes of the two specimens do not differ more than those of opposite sides in the same specimen.

3. In the typical specimen the upper ends of the three anterior alveoli, on each side, are preserved; they occupy just the same space as the three anterior alveoli of the present specimen.

4. The presphenoid in the type has the same crest, and the inner ends of the optic foramina are precisely the same distance apart.

When to these correspondences we add that the distance from the front edge of the nasals to the level of the posterior edges of the supraorbital prominences is the same in both skulls, and that the lower jaw of the new specimen would fit very fairly on to the typical skull, it will, I think, be admitted that there is sufficient evidence of the specific identity of the animals to which the two skulls belonged, and that the imperfections of the new specimen may be supplemented by the evidence afforded by the typical example.

Further data as to the Cranial Structure of Glyptodon *furnished by the typical skull.*—Professor OWEN ('Catalogue of Fossil Mammalia and Aves,' p. 384) thus describes the fragmentary skull of the typical specimen of *Glyptodon clavipes*:—

"The occipital condyle (*a*) presents a convexity in the vertical direction, which describes more than a semicircle, and is slightly convex transversely, but is narrower in that direction than it is in the *Mylodon*: it is directed in the *Glyptodon* backwards and obliquely outwards. The occipital foramen (*b*) is very large and transversely elliptical; its plane is inclined from below upwards and backwards 20° beyond the vertical line. The anterior condyloid foramen (*c*), though large, is relatively smaller than in the *Mylodon*, and is situated close to the anterior border of the condyle. The depression for the digastric muscle (*d*) is perforated and separated from the condyle by a wider tract of the paroccipital (*e*) than in the *Mylodon*; and the petromastoid (*f*) below the digastric depression presents a rough convexity, bounded posteriorly by a transverse ridge of the paroccipital instead of the hemispherical depression for the articulation of the stylohyoid bone which characterizes the skull of the *Mylodon*. The basioccipital (*g*) presents a median smooth concavity and two lateral rough depressions, which are continued on to the basisphenoid (*h*), and indicate the insertion of very powerful 'recti-capitis antici majores'; the obliterated suture between the basioccipital and basisphenoid forms a rough transverse ridge. The inequalities of this part of the basal region of the skull present a striking contrast to the broad smooth and even tract which the same part forms in the *Mylodon*. The sides of the concave under surface of the basisphenoid are bounded by longitudinal ridges, which have been broken off in the specimen. The petrous bone terminates by a prismatic pointed process in the foramen lacerum (*i*), which here gives passage both to the jugular vein and internal carotid. The foramen ovale (*k*) is circular, and of the same size as the anterior condyloid foramen. The foramen rotundum (*l*) is one inch and a half in advance of the foramen ovale, and opens with the commencement of a deep and long groove, which traverses the base of the pterygoid processes in the direction towards the antorbital foramen. The base of the zygomatic process supporting the articulation of the lower jaw (*m*) is brought much nearer the occiput than in the *Mylodon*, and is separated from the petromastoid by a deep excavation, perforated by wide apertures that seem to communicate with the tympanic cavity. The articular surface for the lower jaw is well defined, narrow in the axis of the skull, much extended transversely, gently convex in both directions. In the skull of a recent Armadillo (*Dasypus octocinctus*) the articulation for the lower jaw is almost flat, and on a level with the roof of the posterior perforated cavity. In the Priodon (*Dasypus gigas*, Cuv.) the articular surface is slightly concave, and extends longitudinally forwards from the posterior cavity. The zygomatic process of the malar bone bounds the outer and fore part of the surface, and extends forwards in the form of a laterally compressed plate of bone, and in the *Das. sexcinctus* forms a slight angular projection below the antorbital perforation. In the *Glyptodon*, the articulation for the lower jaw more resembles that in ordinary Pachyderms, and is thus conformable with the deviation from the Edentate structure manifested by the bones of the foot. But the most remarkable characteristic of the skull of the *Glyptodon*, by which it differs from the existing Armadillos and approaches the Megatherioids, is the long and strong process (*n*) which descends from the base or origin of the zygomatic process of the maxillary bone. This process is compressed, but in the opposite direction to that in the *Mylodon*, viz. from before backwards, instead of from side to side; it measures five inches in length from the antorbital perforation, one inch and three-fourths in breadth across the middle: the outer margin is entire, and as if folded back; the lower half of the inner margin is slightly notched, the extremity of the process curves backwards. Both anterior and posterior surfaces bear strong marks of the attachment of muscular fibres.

"The small remaining portion of the maxillary bone on the inner side of this process shows portions of three deep sockets (*o o*) of the same diameter throughout, indicating the implantation of molar teeth by a single excavated base, and showing two longitudinal ridges on both the outer and the inner side, which proves the teeth to have had the same fluted exterior which they present in the lower jaw, and of which the generic name of *Glyptodon* is expressive. The fractured anterior part of the basis cranii shows the large cavities for the olfactory bulbs, and the remains of a very extensive cribriform plate, the organ of smell being very largely developed.

"The posterior, or occipital surface of the skull slopes forward from the plane of the occipital foramen at an angle of 45°; in the small existing Armadillos it is vertical; in the *Glyptodon* it is divided by a strong median vertical ridge, and separated by a sinuous thicker transverse ridge from the upper surface of the skull. The posterior half of this region of the cranium is marked by the ridges bounding the origins of the temporal muscles, which almost meet along the middle or sagittal line. Part of the lambdoidal suture is seen at *p*; the other cranial sutures are obliterated. The temporal fossæ are pierced by numerous large vascular foramina. The anterior parts of the temporal ridges (*q*) diverge to the posterior angle of the supraorbital ridges. The frontal or interorbital part of the upper surface of the cranium is broad and nearly flat, smooth, and slightly concave at its posterior half, slightly convex, rough, and perforated by vascular foramina at its anterior half. The most prominent parts above the orbits are most rugose, and indicate a more intimate adhesion to the superincumbent osseous dermal helmet. The lachrymal foramen (*r*) is pierced immediately in front of the anterior border of the orbit.

"The difference in the development of the temporal muscles manifested by the *Glyptodon* and Mylodon in the position of the ridges in the fossil cranium indicates a corresponding difference in the power of mastication and in the density of the alimentary substances habitually selected by each species; the greater proportion of hard dentine in the teeth of the *Glyptodon*, and the greater number of the teeth, which appear to have been thirty-two, eight on each side of both jaws, coincide with the characters of the cranium, and support the inferences thence deducible."

It is necessary to make certain additions and qualifications to the above description. If we may be guided in the interpretation of the structure of the auditory region by the analogy of the existing *Euphractus*, the part which is there termed "paroccipital" (Plate IV. fig. 5, *h*) includes the true mastoid; the "perforated depression for the digastric muscle" (Plate IV. fig. 5, *f*) is the external auditory meatus; and that which is termed "petromastoid below the digastric depression" (Plate IV. fig. 5, *g*) is part of the tympanic element of the temporal bone. It would appear that, as in *Euphractus*, the tympanic bone sends a process outwards and backwards, the extremity of which comes into contact with the *pars mastoidea*, and so bounds the external auditory meatus externally and below; while it leaves between itself, the proper tympanic *bulla*, and the *pars mastoidea*, an aperture which communicates with the external auditory meatus. The latter is remarkably small for so large an animal. The "bulla," into which it opened, is broken away; but it is probable that a considerable part, if not the whole, of the rugose spaces supposed above to be for the insertion of "recti capitis antici," mark the place where the thick inner walls of the bullæ impinged upon the basioccipital. The *fenestra rotunda* is visible upon the under surface of the *pars petrosa* as an oval aperture 0·15 inch wide, the long axis of which is directed almost transversely to that of the skull. The *fenestra ovalis*, smaller, appears above the *fenestra rotunda*. The proper carotid canal probably traversed the anterior part of the internal wall of the bulla as in the Armadillos; the jugular vein most likely left the skull by a passage between the posterior and internal part of the bulla, the exoccipital, and the periotic.

The large apertures perforating the roof of the cavity which is situated behind the articular facet for the lower jaw, do not communicate with the tympanic chamber. They are probably venous channels, and they communicate internally with the cavity of the skull.

The articular facet for the lower jaw measures 1·8 inch along its greater, and 0·6 inch along its lesser diameter; its edges are well defined, and it has a somewhat kidney-shape, the hilus of the kidney being turned downwards (Plate IV. figs. 4 & 5, *e*). The general aspect of the facet is backwards and downwards, so that, when viewed laterally, its plane appears inclined more than 45° to a horizontal line. The long axis of the facet is nearly at right angles to the axis of the skull, but its outer half has a slight inclination forwards and outwards. It will be observed that the direction of this facet corresponds very well with that of the articular facet on the condyle of the lower jaw of the new specimen; and the nature of the articulation is such that the lower jaw must have had a purely hinge-like movement in a vertical plane, the doubly curved upper surface of each row of mandibular teeth being brought, with a simply crushing motion, against the correspondingly curved lower surface of the maxillary teeth in each masticatory act.

The "deep and long groove" into which, in the above description, the *foramen rotundum* is said to enter, requires particular notice. The *foramen rotundum* and the spheno-orbital fissure are represented by a rounded aperture 0·5 inch wide, situated immediately in front and to the inner side of the *foramen ovale*, and separated from it by only a narrow bar of bone. The small optic foramen, in like manner, lies immediately in front and to the inner side of this aperture, separated from it only by the lower root of the orbitosphenoid.

The alisphenoid is prolonged forwards as a broad plate, parallel with the orbitosphenoid, for about an inch; and thus the conjoined *foramen rotundum* and *fissura spheno-orbitalis* are continued outwards and forwards by a wide canal of the same length. Anteriorly, the alisphenoid ends in an arcuated free edge, and so forms the hinder part of the outer lip of a groove open inferiorly, the inner wall of which is constituted by the lateral mass of the ethmoid. The front part of the outer lip of the groove, separated from the other by a slight interval, is formed by a strong descending vertical plate of the frontal bone, ending below in a rugose edge, thicker behind than in front, which sweeps upwards and forwards towards the posterior part of the infraorbital prominence. It ceases at about three-quarters of an inch from that part.

The optic foramina are prolonged into canals directed forwards and outwards, each about an inch long, the anterior apertures of which open on the inner wall of the great passage just described, immediately behind the level of the anterior edges of the alisphenoids.

The optic nerves, which could hardly have been more than 0·1 inch in diameter, and were therefore very slender in relation to the size of the animal, must have been continued forwards between the frontal plate and the ethmoid for a distance of at least 3½ inches before they reached the eyeball.

Three other apertures are visible in the roof of the groove—one, about as large as the optic foramen, on its outer side, and three-quarters of an inch in front of the proper anterior end of the optic canal. The two others are smaller and situated close together, and rather on the inner side, half an inch in front of the former. These may be the ends of canals for the oculomotor nerves.

The remains of the expanded upper edge of a *lamina perpendicularis*, similar to that described in the new specimen, are visible, attached to the under surfaces of the nasal bones.

The inner surface of the right lateral portion of the ethmoid is marked by obliquely diverging ridges of bone, with which the plates of the inferior spongy bone were doubtless connected.

By combining the new specimen with this it is easy to ascertain approximatively the length of the cribriform plate. The former specimen, in fact, is broken through at a distance of six inches from the anterior end of the snout, but its posterior face does not exhibit any notable part of the anterior wall of the cranial cavity. The same distance (6 inches), therefore, measured off upon the roof of the type skull, should give the position of a line beyond which the cribriform plate certainly did not extend anteriorly. From the point thus defined to the anterior edge of the presphenoid is a distance of 1·75 inch, which must therefore represent the maximum length which the cribriform plate could have attained. The distance from the anterior edge of the presphenoid to the level of the posterior margins of the occipital condyles is 4·5 inches. The cribriform plate is rather shorter in proportion to the base of the skull in the *Glyptodon* than in the ordinary Armadillos, and its anterior part is situated far further back in relation to the antorbital processes.

The proper cranial cavity, or brain-case, is small when compared with the whole size of the skull, if the chambers which lodge the olfactory bulbs are left out of consideration. It is in fact only 4·5 inches long, 2·5 inches wide at widest, and about 1¾ inch high at highest. Its greatest width is situated beneath the occipital ridge, whence it narrows towards the olfactory outlet, which is about 1·25 inch wide. The immediate side walls and roof of the fore part of the cranial cavity are formed by a very thin inner table of bone, separated by a wide air-chamber from the denser and stouter outer table. This air-chamber does not appear to extend back beyond a transverse line connecting the two glenoidal facets.

Mr. FLOWER has obtained a cast of the cranial cavity, from which one is enabled to form an idea of the shape and size of the brain. The proportionally large cerebellum exhibits a prominent vermiform process, and is completely uncovered above by the cerebral hemispheres. The latter are quite smooth, and their upper contour is much arched, while their sides are flattened, and approach one another anteriorly. The absence of convolutions in the brain of so large an animal, together with the small absolute mass of the organ, leads one to suspect a great absence of intelligence in the *Glyptodon*.

Measurements of the Skulls.

A. *The new specimen.*

	inches.
Total length of the palate in a straight line	9·50
Width between the inner edges of the alveolar series	1·75
Width between the outer edges of the alveolar series, opposite third tooth	2·95
„ „ „ „ last tooth	2·8
„ „ „ „ first tooth	2·6
Hinder edge of the last alveolus in front of the posterior nares	0·5
Outer edge of the malar process to the centre of the palate	5·5
The extreme breadth of the skull therefore =	11·0
Vertical height of skull from frontal bones to palate at fourth tooth	6·0
From end of outer edge of orbit to the same point on the opposite side	7·2
Summit of the frontal region to the ends of the malar processes	9·5
Mandible:—	
Extreme length from the symphysis to the angle	10·7
Extreme height from the summit of a condyle to a flat surface on which the jaw rests	9·3
Depth of the horizontal ramus at the third tooth	3·2
Width at the symphysis	2·9
Width between the inner edges of the alveoli opposite the first tooth (remains the same throughout)	3·1
Width between the outer edges at the same point	3·1
Width between the outer edges at the third tooth	3·25

B. *The type specimen.*

Extreme length from nasal bones to the level of the occipital condyles	12·7
„ „ „ superior occipital ridge	10·5
Breadth at the front part of the orbits	6·8
„ at the interorbital constriction	4·3
„ across the occiput, about	5·8
Height of the occiput	2·6
Distance between the inner edges of the articular surfaces for the condyles of lower jaw	4·25

§ 2. *The Vertebral Column.*

The remains of this very interesting part of the organization of *Glyptodon* are, unfortunately, in a somewhat imperfect state, though enough exists to demonstrate its altogether unique character.

The Atlas.—Of this bone the mutilated right half is represented in Plate VII. fig. 1, giving the anterior, and fig. 2 the posterior aspect of the fragment.

The specimen exhibits rather more than the right half of the lower arch, and rather less than the corresponding portion of the upper arch of the bone. The right lateral mass, with its anterior and posterior articular facets, is almost entire, but the transverse process is broken off close to its origin. The inferior arch is a solid bar of bone with a straight upper and a convex lower contour; and somewhat thicker in the middle, both from above downwards and from before backwards, than at the sides. A section taken through the median plane of this part of the bone would have the shape of a spherical triangle; the lower or horizontal face convex, the anterior slightly concave, and the posterior and upper also concave.

The middle of the posterior and upper face of the inferior arch presents an oval articular facet (fig. 2, *a*) for the odontoid process of the axis, which, when entire, must have measured about 1·6 inch in width by 0·8 inch in antero-posterior length. It is slightly concave, both from before backwards and from side to side, and is bounded by a well-defined though narrow ridge. The outer end of this facet is half an inch distant from the inner and lower edge of the articular surface for the odontoid vertebra, upon the lateral mass of the atlas (fig. 2, *b*). This is a reniform surface with its inner and anterior side concave, while the outer and posterior aspect is convex. Its long axis is almost vertical, while the plane of its surface, which is a little concave both from above downwards and from side to side, is directed obliquely inwards and forwards. Lines drawn through the shorter axes of the two articular facets would intersect one another at a point very slightly in front of the anterior margin of the inferior arch. The foramen for the vertebral artery is situated on the outer side of the facet, opposite the junction of its middle and upper thirds, and nearly on the same level as a tubercle for the transverse ligament, situated on the inner side.

The foramen (fig. 2, *c*) leads into a canal which passes directly forwards, widening as it goes, and traverses the root of the transverse process. In front of this it presents a large oblique aperture, by which, however, it does not terminate. Instead of ending, it makes an abrupt turn upwards through the substance of the superior arch of the atlas, parallel with, and equidistant from, the anterior and posterior margins of that part, and ends by an oblique aperture in the outer part of the roof of the cavity of the atlas, and nearer the occipital than the odontoid edge. The upper face of the lateral mass of the atlas presents an elongated, irregular, transverse aperture, which communicates with the canal, and from the anterior and posterior margins of which broad and shallow grooves are continued.

The articular surface for the occipital condyle upon the anterior face of the lateral mass of the atlas (fig. 1) is much more concave from above downwards than that just described; and as it is neither concave nor convex from side to side, the surface may be regarded as a segment of a hollow cylinder, answering to rather less than half the circumference of such a figure. When the inferior arch of this atlas is made horizontal, this articular surface looks forwards and inwards. The inner and lower edges of the opposite occipital facets of the atlas must have been separated by a distance of about 1·9 inch.

The transverse process of the atlas is, as I have stated, broken off close to its origin; but the cancellated fractured surface, 2 inches long by more than half an inch wide superiorly, proves that the process was flattened from before backwards, and that it arose from the posterior half of the outer surface of the lateral mass of the bone. The surface of attachment of the process is almost perpendicular to that of the axis of the spinal canal, or, at most, has a very slight inclination from above downwards and forwards. The general plane of the process, on the other hand, as exhibited by an upper or an under view, is directed backwards and outwards. There are no means of judging how far the process may have extended outwards.

The Odontoid and immediately-following Cervical Vertebræ.—The fragment of this region of the vertebral column (figured in Plate IX. fig. 5 from without, fig. 6 from within, fig. 7 from behind, and fig. 8 from below) is composed of the right half of the neural arch of the axis, or odontoid, vertebra, anchylosed together with the arches of the third and fourth cervical vertebræ. It formed the right half of the roof and side walls of the neural canal in this region. The front face of the bone, thick and prismatic, is obliquely bevelled off to a rounded edge, which is concave anteriorly. The outer face is produced above into a tuberosity, the anterior part of which is perforated by a canal which traverses the whole thickness of the bone and opens on its inner face, near its upper end (fig. 5, *c*, fig. 6, *c'*). From the tuberosity a small ridge, partly broken away, leads forwards and inwards along the anterior face of the bone. A stouter ridge extends inwards near the posterior margin of the bone, from the same tuberosity. These two ridges were situated upon the proper upper surface of the arch, and probably joined the anchylosed spinous processes.

The lower part of the outer face presents a broken surface, with the outer terminations of three canals (figs. 5 & 8, *d*, *e*, *f*), the inner ends of which are visible on the inner or under surface of the bone (fig. 6, *d*, *e*, *f*) as they traverse its thickness obliquely from within outwards and downwards. The hindermost of these canals (*d*) is wide below, but narrows into a fissure above. The second, or middle, foramen (*e*) is wider, oval, and looks more downwards. The third (*f*) is much smaller than either of the other two. On the inner face of the bone (fig. 6) the aperture of the posterior canal (*d*) is longest. The middle canal opens upon nearly the same level; but the third, or anterior, canal takes a much shorter course through the bone, and thus its inner end is on a level below the others.

The aperture of the middle canal is situated at about the same distance from the anterior margin of the bone as the inner end of that canal (*c*, *c'*) which, I have stated, opens externally upon the tuberosity. A little aperture (*g*) in the same line with these two leads into the substance of the bone, and seems to have no external outlet. Lines drawn through the three apertures referred to, mark off an anterior segment of the bone from a middle segment, which is defined, by a line drawn from the inner end of the posterior canal below to another small aperture (*h*) above, from a hinder segment.

The posterior face of the bone exhibits, below, a large round aperture (fig. 7, *a*), leading into a passage which traverses the posterior canal just described, and debouches into the middle one.

Immediately beneath this foramen is a small concave articular surface, apparently a fragment of a much larger one.

Superiorly and internally the posterior face of the bone presents a deep fossa (fig. 7, *a*), bounded above and internally by a concave articular facet, the long axis of which is directed almost at right angles to the long axis of the bone.

The facet in question I take to correspond with the posterior oblique process or "post-zygapophysis" of the fourth cervical vertebra. The foramen on the posterior face is the aperture of the canal for the vertebral artery. The facet below it is part of an articular surface upon the inferior or "capitular" division of the transverse process, which is characteristic of the cervical vertebræ in Armadillos; and the middle and posterior canals are the intervertebral foramina for the third and fourth cervical nerves. The upper and inner foramina and canals represent the remains of the primitive interspaces between the several arches. The anchylosed spinous processes, and the bodies of the three coalesced vertebræ, are completely broken away, so that nothing can be said regarding their characters.

The fifth and sixth Cervical Vertebræ.—No remains of the fifth and sixth cervical vertebræ have been discovered among the bones sent by Señor TERRERO.

The "Trivertebral bone," or anchylosed seventh Cervical and first and second Dorsal Vertebræ (Plate VII. figs. 3, 4, 5, 6).—The three vertebræ which enter into the composition of this singular bone are very much depressed from above downwards, so that the neural canal is more than twice as wide as it is high; while the greatest depth of the whole bone, leaving the spinous process out of consideration, is hardly a fourth of its width. The inferior face of the bone is deeply concave from side to side; and as the floor of the neural canal is also concave, the part which corresponds with the centra of the anchylosed vertebræ has the form of a broad thin arched plate, thinnest in the middle. The superior arches of the vertebræ, which constitute the roof of the trivertebral bone, follow, in a general way, the contour of its floor; but they are much thicker; and, posteriorly, the roof of the trivertebral bone is produced, upwards and backwards, into a very thick short process, which probably represents the spinous processes of the two anterior dorsal vertebræ. The lateral parts of the trivertebral bone, which represent the anchylosed transverse processes of the vertebræ, are very thick and stout, especially in front. Viewed from above, or laterally, they are seen to be marked out by excavations into three portions, one for each primitive vertebral constituent of the bone. With the lateral excavations the heads of the two anterior ribs articulate.

So much for the general characters of this bone. A front view (Plate VII. fig. 5) exhibits the following features, worthy of more particular description. The lateral mass, which represents the transverse process of the first of the three vertebræ, presents an elongated oval articular facet (*a*), convex from above downwards and looking almost directly forwards, its long axis being horizontal and at right angles to the axis of the spinal canal. The facet is 1·8 inch long by 0·9 inch maximum height.

This articular facet is separated by a deep groove, into the bottom of which a large canal (*d*) opens, from two other articular surfaces (*b*, *c*), placed one immediately above the other, and also parted by a deep channel, which may be regarded as an internal branch of the groove.

The lower articular face (*c*), almost flat, looks inwards and forwards; and its long axis, which continues the direction of the floor of the neural canal, is inclined from above downwards and outwards.

The upper facet (*b*), also flat, and, elongated transversely, looks directly upwards. Its inner end is nearer the lower facet than its outer end; and a well-marked fossa or depression lies behind it. The upper articular surface certainly answers to the anterior oblique process or "prezygapophysis" of the seventh cervical vertebra. The nature of the lower and of the outer facet will only become obvious when the characters of the cervical vertebræ of recent Armadillos have been explained. The anterior face of the spinous process of the trivertebral bone exhibits two ridges, each convex towards the middle line, which divide the face into a middle and two lateral areæ.

The upper face of the bone (Plate VII. fig. 3) presents three pairs of foramina, terminating internally in canals which lead into the spinal canal, and externally opening into recurved grooves on the surface of the bone. The middle apertures are the largest, and the corresponding grooves more strongly defined and wider. The posterior apertures are smallest, and are situated quite close to the hinder margin. The surface of the bone between these apertures is rough and irregular. The margins of this face of the bone are produced into three processes which alternate with the foramina. The hindermost of these processes is the largest, and ends in a point which is somewhat recurved and bent down.

A side view of the trivertebral bone (Plate VII. fig. 6) shows that these processes are continued into irregular vertical ridges, between which two fossæ are enclosed. Of these, the anterior is much deeper and more capacious than the other. It is an irregular cavity subdivided by a vertical ridge into two, each of which presents a somewhat deeper fossa at its inner and lower end.

The second, shallower, fossa, which lies between the hinder face of the middle process and the front face of the posterior process, presents an elongated irregular articular facet on its anterior wall, and a more rounded articular surface on its posterior wall.

The second rib is received into this fossa, and articulates with both these facets.

The posterior face of the third process presents a small, slightly concave, oval articular face on its lower half, with which the third rib was doubtless connected.

The posterior aspect of the trivertebral bone (Plate VII. fig. 4) presents for notice, besides the features already mentioned, several others. The neural arch of the hindermost vertebra of the three overhangs; and its under face exhibits two oval slightly concave articular faces (*a*, *a*), the posterior oblique, or "postzygapophysial," surfaces of the second dorsal vertebra. These, however, are not carried upon distinct processes. The great spinous process seems completely to fill up the interval which properly exists between the postzygapophyses. The posterior face of this process is slightly excavated in the middle of its lower half. Its sides are also a little concave, so that the top swells out into a sort of knob with overhanging margins.

The posterior part of the floor of the trivertebral bone is broken away; but the hinder face of each lateral mass exhibits a transversely elongated articular surface (*b*, *b*), concave from above downwards, so as to resemble a segment of a hollow cylinder, the axis of which is directed from within outwards and very slightly backwards.

The inferior face of the trivertebral bone presents the arched surface, flatter behind than in front, of the continuously ossified central portions or bodies of the vertebræ, and, external to these, two pairs of apertures which perforate this face of the bone at its outer margin. The anterior of these apertures is very much larger than the posterior, and corresponds with the inner end of the middle transverse process, opening just behind the inner end of the first rib. Strictly speaking, the foramen seen upon the front face of the bone (Plate VII. fig. 5, *d*) forms one of this series of foramina (all of which are the terminations of short passages leading into the spinal canal); so that, as upon the upper, so on the under surface of the trivertebral bone, there are three pairs of foramina in communication with the spinal canal, and of these the middle pair are, in each series, the largest.

The homologies of the three vertebræ which compose the trivertebral bone are determined by the implantation of the head of the first rib into the great fossa between the lateral processes of the first and second. The vertebra which yields the anterior wall of the fossa is clearly the last cervical, and that which furnishes the posterior wall is the first dorsal. Hence the trivertebral bone is composed of the last, or seventh, cervical and the first and second dorsal vertebræ.

The remaining Dorso-lumbar Vertebræ.—Of these vertebræ thirteen are preserved. The anterior twelve have plainly been immoveably united together into a continuous arched tunnel or tubular bridge of bone, partly by anchylosis and partly by the manner in which their apposed surfaces interlock (Plate VIII. figs. 1–7).

The four anterior vertebræ (figs. 1, *d. l.* 3, 4, 5, 6) are so completely anchylosed together that almost all traces of their original distinctness are lost. Persistent sutures, of a character intermediate between a "harmonia" and a serrated suture, separate the fourth vertebra (*d. l.* 6) from the fifth, and the latter from the sixth; but the sixth and the seventh (*d. l.* 9) are completely fused into one bone. Between the eighth and ninth vertebræ a suture is interposed, and also between the ninth and the tenth, at least on the left side. The tenth and the eleventh (*d. l.* 13) are completely anchylosed above, while the suture seems to have persisted below*.

---

* It is convenient to speak of the first, second, &c. of the thirteen vertebræ which succeed the trivertebral bone; but it must be recollected that the first of these is the third of the dorso-lumbar series, the second the fourth dorso-lumbar, and so on, the number of any one of these vertebræ in the dorso-lumbar series being

Thus far, no trace of distinct articular processes is visible upon these vertebræ; but the hinder face of the eleventh vertebra (*d. l.* 13) presents certain irregular elevations and depressions, which interlock with corresponding ridges and cavities of the anterior face of the twelfth vertebra. The hinder face of the twelfth (*d. l.* 14) and the front face of the thirteenth vertebra (*d. l.* 15) are in like case. I shall return to the consideration of the character of these irregular articular elevations and depressions after describing the general form of the vertebræ.

In all but the first, second, third, eleventh, and thirteenth vertebræ, the parts representing the vertebral centra are broken away, but, when they remain, they are so similar to one another that their form was, doubtless, essentially the same throughout. Each centrum is a comparatively thin bony plate, bent so as to be convex downwards and concave upwards, and presenting a much flatter curve in the anterior than in the posterior part of the column. In front, the central plate is not more than 0·1 inch thick in the middle, but it becomes thicker posteriorly, so that the centrum of the eleventh vertebra is 0·45 inch thick; that of the thirteenth vertebra is 0·1 inch thinner. At the sides and above, the curved central part of the vertebra passes into the lateral processes and upper arches, which last are slightly concave downwards in the first vertebra, flat in the middle vertebræ, and somewhat arched again in the thirteenth. The contour of a transverse section of the spinal canal is a transversely elongated oval in the first vertebra (fig. 3), is more nearly round, but flattened at the top, in the middle vertebræ (*d. l.* 12), and is a vertically elongated oval in the thirteenth vertebra (*d. l.* 15).

The spinous and transverse processes of the vertebræ are represented by three crests or ridges of bone. One of these (Plate VIII. fig. 2, *a*, *b*), vertical, and situated in the middle line of the dorsal surfaces of the arches of the vertebræ, represents the spinous processes; while the lateral crests (*c*, *c*), directed obliquely upward and downwards, answer to transverse, accessory, and mammillary processes. As the latter ridges become directed more upwards towards the hinder part of the dorsal region, the total width of the column lessens, and the grooves between the middle and the outer ridges become deeper in the same direction. Thus, anteriorly, the column is fully six inches broad, while at the eleventh vertebra the distance from one external ridge to another is hardly half this amount.

The first vertebra (*d. l.* 3) is as broad and depressed as the trivertebral bone. Viewed in front (Plate VIII. fig. 3), the neural canal is seen not to take up more than one-fourth of the face of the bone, the rest of which is occupied by two broad expanded transverse processes, directed very slightly upwards as well as outwards. The under half of each of these processes presents an elongated articular facet (*a*, *a'*), convex from above downwards, slightly concave from side to side, which corresponds with, and is received into, the concave articular surfaces upon the hinder face of the trivertebral bone.

---

always greater by two than its number reckoned as one of the thirteen. In order to avoid confusion in describing each vertebra, I shall occasionally give after it its number in the dorsal lumbar series, *e. g.* (*d. l.* 3), (*d. l.* 6), by which it is indicated in the figures.

Seated upon the upper face of the neural arch are two other oval articular surfaces (*b*, *b'*), which answer to the postzygapophysial surfaces upon the under surfaces of the hinder part of the neural arch of the trivertebral bone.

The inner part of each of these articular faces is convex in all directions; the outer is concave from side to side, convex from before backwards; behind each lies a transverse

The outer ends of the transverse processes are obliquely truncated, and each presents two articular facets, an anterior and inferior, larger, and a posterior and superior smaller, which articulate with corresponding facets upon the capitulum and tuberculüm of the attached rib. A well-marked notch separates the hinder face of the transverse process of the first from that of the second vertebra; and the intervertebral foramen is situated on the same level as this notch, on the one hand, and the anterior inferior facet, on the other, or about halfway between the upper and lower faces of the bone.

The transverse process of the second vertebra (*d. l.* 4) presents two oval articular facets for the head of a rib, more nearly equal and more nearly on a level than those of the first vertebra. The transverse process of the third vertebra is broken on the left side; but on the right side, traces of an elongated costal facet are visible.

The ends of the lateral ridges representing the transverse processes of the fourth, fifth, sixth, and seventh vertebræ are broken away.

In the eighth, ninth, tenth, and eleventh vertebræ (Plate VIII. fig. 7, *d. l.* 10, 11, 12) they are preserved on the left side, broken away on the right; on the twelfth vertebra the corresponding ridges are broken on both sides.

I find no trace of articular surfaces for ribs on the lateral ridge continued along the eighth, ninth, tenth, and eleventh vertebræ, which, as I have stated, is entire on the left side; but the upper and inner surface of the ridge is rounded and marked by longitudinal striations (fig. 7). The outer surface is rough and irregular, opposite the anterior part of each vertebra, and raised into an irregular tubercle posteriorly.

The spinous processes of all the vertebræ are broken short off; that of the first is almost obsolete, being a mere ridge sloping back towards the second, into which it is continued. The anterior edge of the process is so much inclined backwards and upwards as to afford free play to the knobbed head of the spinous process of the trivertebral bone (fig. 2).

The spinous process of the second vertebra (*d. l.* 4) is 0·4 inch thick where it is broken through, and had probably a considerable height. A distinct interval separates it posteriorly from the thin anterior edge of the spinous process of the third vertebra, which is much thinner, and is anchylosed with its successors, as far as the eleventh inclusive, into a long continuous crest; slight traces of the original separation of the several spinous processes, however, are visible at the base of the crest, and they may have been distinct at their apices. The crest gradually increases in thickness to the sixth vertebra (*d. l.* 8) (where it attains 0·75 inch), and then gradually diminishes. The spinous process of the twelfth vertebra (*d. l.* 14) may have been distinct down to its base; and the posterior edge of the thin ridge, which is all that is left of the process, appears to incline upwards and forwards.

The foramina for the exit of the spinal nerves are not intervertebral in the ten anterior vertebræ, but perforate the bony substance of each vertebra nearer its posterior than its anterior boundary. Of these foramina there are two, on each side, for the five anterior vertebræ; one, larger, below the lateral apophysial ridge; and one, smaller, above, or upon, this ridge at the posterior boundary of each vertebra.

The larger foramen approaches the outer margin of the apophysial ridge, or seems to be situated higher up, in each successive vertebra from the first to the seventh. Beyond this point the level of the foramen descends somewhat. The eleventh vertebra (*d. l.* 13) appears to have possessed a simple intervertebral notch posteriorly, on the left side; but, on the right, a bar of bone is preserved, separating an anterior foramen from the rest of the notch, which receives a process of the twelfth vertebra. The arrangement appears to be the same in the twelfth vertebra (*d. l.* 14); that is to say, the apparent notch has been divided by a bar of bone into an anterior nervous foramen, and a posterior articular fossa.

I have briefly referred, above, to the articular surfaces of the eleventh and twelfth vertebræ, which are exceedingly irregular and distorted, apparently from partial anchylosis and filling up with osseous matter. A notion of their general character may best be obtained by the study of the posterior face of the twelfth vertebra (*d. l.* 14). On the upper part of the neural arch, on each side of the spine of this vertebra, irregular and partially obliterated posterior oblique processes, or postzygapophyses, are discernible. The zygapophysis is separated by a depression, or groove, directed from without obliquely downwards and inwards, from a wedge of bone which terminates the apophysial ridge. Inferiorly and externally, this wedge presents a slightly concave articular facet, separated by a deep fossa from a tuberosity with a rounded surface, which passes down into the body of the vertebra. On the same level as this fossa, there projects from the front surface of the vertebra a triangular process, which fits into a corresponding fossa of the eleventh vertebra. The front face of the thirteenth vertebra (*d. l.* 15), again, presents, on each side of the neural spine, pits, the floors of which answer to the anterior oblique processes, or prezygapophyses; outside of these are ridges, which fit into the fossæ between the postzygapophysis of the twelfth vertebra and the wedge-shaped process; external to the ridges are fossæ which receive those wedge-shaped processes; and external to and below these, again, are the remains of processes which were received into the deep fossæ mentioned above.

Except in the region of these articular processes, neither the anterior nor the posterior ends of the thirteenth vertebra (Plate VIII. figs. 6 & 7, *d l.* 15) are entire. Of the spinous process, only the base is left; it thins off anteriorly to a natural edge, which is inclined upwards and backwards, and seems to have been quite free. Posteriorly, it becomes rapidly thicker; but its mode of termination cannot be ascertained. The large nervous foramen perforates the wall of the vertebra, on a level with the articular processes, and bifurcates externally, so that one of its apertures ends above, and the other below, a stout bar of bone (Plate VIII. fig. 6, *a*), nearly an inch thick, which ends posteriorly in a raised curved ridge, forming the anterior boundary of a semicircular groove.

The spinal canal in the thirteenth vertebra is, as I have said, oval in shape, the long diameter of the oval (1·5 inch in length) being vertical, the short diameter (1·1 inch) transverse.

As, in the anterior part of the lumbo-sacral region, this canal has a very different shape, it is probable that two or three vertebræ are wanting in this portion of the spinal column.

The Sacrum and Coccygeal Vertebræ.—The "sacrum," composed of anchylosed lumbar, proper sacral, and coccygeal vertebræ, contains at fewest twelve, and perhaps thirteen vertebræ. The centra of the two hindermost lumbar vertebræ and of the two proper sacral vertebræ, which follow them (Plate IX. fig. 2), are thin and broad bony plates, flat above, and slightly concave from side to side below, exhibiting a most marked contrast to the semicylindrical form of the same part in the hindermost of the thirteen vertebræ described above. The plane of the plate formed by the centra of the anchylosed lumbar vertebræ is inclined, upwards and forwards, to pass into the general curve of the dorso-lumbar region. The plane of the centra of the two succeeding sacral vertebræ, on the other hand, is horizontal; and it is obvious, from the characters of the rest of the sacrum, that the centra of the following vertebræ, to the end of the sacral region, were arranged in an almost semicircular curve, the chord of which is about 18 inches long (Plate IX. fig. 3). The posterior face of the hindermost coccygeal vertebra (Plate IX. fig. 1, *a*) is broad, oval, and very slightly concave, like the face of an ordinary vertebral centrum; but the centrum of the penultimate coccygeal vertebra is much flatter and narrower; and this flattening and narrowing become still more marked in the centrum of the antepenultimate vertebra and of that which precedes it, or the fourth from the end. From this point to the two anterior sacral vertebræ the floor of the sacral canal is completely broken away, but there can be little doubt that the missing centra were represented by a broad and flat bony plate.

The neural arches are but imperfectly preserved, except in the lumbar region and the anterior part of the sacrum. They are thin, and are separated by large intervertebral foramina. In the lumbar vertebræ these foramina pass downwards and backwards into grooves which mark the sides of the central plate. Well-defined depressions upon the sides of the sacral crest lead upwards and backwards to the canals which pass between that crest and the ilia.

The four last coccygeal intervertebral foramina are still larger, and indicate the passage of large nerves to the muscles moving the tail.

The spinous processes of all the vertebræ which enter into the sacrum, up to the fourth from the end inclusively, are anchylosed together into a long and strong osseous crest (Plate IX. figs. 3 & 4), which expands above, so as to present a broad and very rugged superior face. This crest is 8 inches high in front, but slowly diminishes as it follows the curve of the centra posteriorly, to 5 inches. The spinous process of the penultimate coccygeal vertebra is very thick, but it is broken short off. It was probably not less than 4 inches high, and afforded a middle point of support for the dermal shield between the ischial protuberances (Plate IX. fig. 1).

The sides of the two anterior sacral vertebræ and the corresponding part of the sacral crest are anchylosed with the inner edges of the iliac bones, so that only a narrow oval space, left between these parts, near the upper edge of the crest, and the small canals above mentioned, allow of any communication between the region in front of, and that behind the ilia.

Behind this point the vertebræ are devoid of transverse processes as far as the fourth from the end. But the antepenultimate had a long and slender transverse process on each side; the penultimate possesses an equally long but much stouter process, and the last coccygeal vertebra has extremely thick processes of the same length. The enlarged distal ends of these processes unite with one another and with the inner surfaces of the ischia (Plate IX. figs. 1, 2, 4).

Caudal Vertebræ.—No caudal vertebræ existed among the remains of this specimen of *Glyptodon*.

Of the Vertebral Column as a whole.—It appears from the foregoing description that the atlas of the *Glyptodon* was moveable upon the odontoid vertebra; but that the latter was anchylosed with the third and fourth cervical vertebræ into one short bone, moveable upon the fifth cervical; of the fifth and sixth cervical vertebræ no remains exist. The seventh cervical is anchylosed with the first and second dorsal into a single "trivertebral bone," upon the front part of which the sixth cervical was certainly moveable; while the hinder part of it freely articulates with the third dorsal, so that the bone was capable of motion through a certain vertical arc.

Beyond this point, as far as the fourteenth dorso-lumbar vertebra, the vertebræ are so connected by complete, or partial, anchylosis, that it is impossible any motion should have taken place between them; and it is probable, though not so certain, that the fifteenth dorso-lumbar vertebra was similarly fixed.

Between this and the two hindermost lumbar vertebræ, which are completely anchylosed together and with the sacral vertebræ, there is a hiatus, but the condition of the two latter is not such as to lead to the supposition that the intermediate vertebræ were less firmly united than they.

The free cervical portion of the vertebral column must have been remarkably short, probably not exceeding 8 inches in length, and the cervical vertebræ were most likely arranged in a nearly straight line.

The trivertebral bone and the thirteen following dorso-lumbar vertebræ, when articulated together, form one great curve, concave downwards or towards the visceral cavity, the curve being much sharper in the anterior than in the posterior part of the column. Measured along its curvature, this part of the vertebral column is about 35 inches long.

At the anterior part of the sacral region the lumbar curve passes into the straight line of the two anterior sacral vertebræ, behind which commences the great sacro-coccygeal curve, concave towards the cavity of the pelvis. The lumbo-sacral is very nearly as long as the dorso-lumbar region, so that the vertebral column, from the last cervical to the last coccygeal, may be said to form two subequal arches with a common pier, formed by the proper sacral vertebræ.

DESCRIPTION OF THE PLATES.

PLATE IV.

Figs. 1 & 3. Upper and under views of the skull of the "new specimen" of *Glyptodon*

Figs. 2, 4, & 5. Upper, under, and side views of the hinder part of the skull of the "typical specimen" of *Glyptodon clavipes*.

All reduced to one-half the natural size.

PLATE V.

Fig. 1. Side view of the skull of the new specimen of *Glyptodon clavipes*.
Fig. 2. The left half of the mandible of the same, one-half the natural size.
Fig. 2^a. The ascending ramus of the mandible, viewed from behind.
Figs. 3 & 4. Grinding-surfaces of the teeth, of the natural size.

PLATE VI.

Fig. 1. Front view, and
Fig. 2. Back view of the skull of the new specimen of *Glyptodon clavipes*.
Fig. 3. View of the occipital face of the skull of the typical specimen.
Fig. 4. Front view, and
Fig. 5. Upper view of the mandible of the new specimen.

All reduced to one-half the natural size.

PLATE VII.

Figs. 1 & 2. Front and back views of the fragment of the atlas.

⁂ The artist has inadvertently inverted each figure, so that the lower side of the bone is turned upwards, and *vice versâ*.

Fig. 3. The trivertebral bone, seen from above.
Fig. 4. The trivertebral bone, from behind; *d*, the first rib, in place.
Fig. 5. The trivertebral bone, from in front.
Fig. 6. The trivertebral bone, viewed from the right side.

Fig. 7. The fragment of the first rib of the right side, viewed from without.
Figs. 8, 9, 10. Front, inner, and outer views of the fragment of the third left rib.

PLATE VIII.

Fig. 1. The third to the ninth dorso-lumbar vertebræ, viewed laterally.
Fig. 2. The same, viewed from above.
Fig. 3. The anterior face of the third dorso-lumbar vertebra.
Fig. 4. The posterior face of the sixth dorso-lumbar vertebra.
Fig. 5. The anterior face of the twelfth and thirteenth dorso-lumbar vertebræ. It is much mutilated, especially below and on the left side, none of the centrum of the twelfth vertebra remaining.
Fig. 6. The tenth to the fifteenth dorso-lumbar vertebræ, viewed laterally.
Fig. 7. The same, from above.

All reduced to one-half the natural size.

PLATE IX.

Fig. 1. Back view of the pelvis of *Glyptodon clavipes.*
Fig. 2. Front view of the same.
Fig. 3. Side view of the same.
Fig. 4. Upper view of the same.

All these figures are reduced to one-sixth the natural size.

Figs. 5–8. Outer, inner, back and under views of the fragment of the anchylosed odontoid, third, and fourth cervical vertebræ, one-half the natural size.

a the upper, and *b* the lower end of the bone in each figure, which is reduced to one-half the natural size.

III. *Investigations of the Specific Heat of Solid Bodies.*

By HERMANN KOPP. *Communicated by* T. GRAHAM, *Esq.*, *F.R.S.*

Received April 16,—Read May 12, 1864.

I. *Historical Introduction.*

1. ABOUT the year 1780 it was distinctly proved that the same weights of different bodies require unequal quantities of heat to raise them through the same temperature, or on cooling through the same number of thermometric degrees, give out unequal quantities of heat. It was recognized that for different bodies the unequal quantities of heat, by which the same weights of different bodies are heated through the same range, must be determined as special constants, and considered as characteristic of the individual bodies. This newly discovered property of bodies WILKE designated as their *specific heat*, while CRAWFORD described it as the comparative heat, or as the *capacity* of bodies *for heat.* I will not enter upon the earliest investigations of BLACK, IRVINE, CRAWFORD, and WILKE, with reference to which it may merely be mentioned that they depend essentially on the thermal action produced when bodies of different temperatures are mixed, and that IRVINE appears to have been the first to state definitely and correctly in what manner this thermal action (that is, the temperature resulting from the mixture) depends on the original temperature, the weights, and the specific heats of the bodies used for the mixture. LAVOISIER and LAPLACE soon introduced the use of the ice-calorimeter as a method for determining the specific heat of bodies; and J. T. MAYER showed subsequently that this determination can be based on the observation of the times in which different bodies placed under comparable conditions cool to the same extent by radiation. The knowledge of the specific heats of solid and liquid bodies gained during the last century, and in the first sixteen years of the present one, by these various methods, may be left unmentioned. The individual determinations then made were not so accurate that they could be compared with the present ones, nor was any general conclusion drawn in reference to the specific heats of the various bodies.

2. DULONG and PETIT'S investigations, the publication of which commenced in 1818, brought into the field more accurate determinations, and a general law. The investigations of the relations between the specific heats of the elements and their atomic weights date from this time, and were afterwards followed by similar investigations into the relations of the specific heats of compound bodies to their composition. In order to give a general view of the results of these investigations, it is desirable to present, for the elements mentioned in the sequel, a synopsis of the atomic weights assumed at different

times, and of certain numbers which stand in the closest connexion with these atomic weights.

	Berzelius's atomic weights.	Regnault's thermal atomic weights.	Usual equivalent weights.	Modern atomic weights.
Aluminium	Al = 13·7	Al = 13·7	Al = 13·7	Al = 27·4
Antimony	Sb = 61	Sb = 61	Sb = 122	Sb = 122
Arsenic	As = 37·5	As = 37·5	As = 75	As = 75
Barium	Ba = 68·5	Ba = 68·5	Ba = 68·5	Ba = 137
Bismuth	Bi = 105	Bi = 105	Bi = 210	Bi = 210
Boron	B = 10·9	B = 10·9	B = 10·9	B = 10·9
Bromine	Br = 40	Br = 40	Br = 80	Br = 80
Cadmium	Cd = 56	Cd = 56	Cd = 56	Cd = 112
Calcium	Ca = 20	Ca = 20	Ca = 20	Ca = 40
Carbon	C = 6	C = 12	C = 6	C = 12
Chlorine	Cl = 17·75	Cl = 17·75	Cl = 35·5	Cl = 35·5
Chromium	Cr = 26·1	Cr = 26·1	Cr = 26·1	Cr = 52·2
Cobalt	Co = 29·4	Co = 29·4	Co = 29·4	Co = 58·8
Copper	Cu = 31·7	Cu = 31·7	Cu = 31·7	Cu = 63·4
Fluorine	Fl = 9·5	Fl = 9·5	Fl = 19	Fl = 19
Gold	Au = 98·5	Au = 98·5	Au = 197	Au = 197
Hydrogen	H = 0·5		H = 1	H = 1
Iodine	I = 63·5	I = 63·5	I = 127	I = 127
Iridium	Ir = 99	Ir = 99	Ir = 99	Ir = 198
Iron	Fe = 28	Fe = 28	Fe = 28	Fe = 56
Lead	Pb = 103·5	Pb = 103·5	Pb = 103·5	Pb = 207
Lithium	Li = 7	Li = 3·5	Li = 7	Li = 7
Magnesium	Mg = 12	Mg = 12	Mg = 12	Mg = 24
Manganese	Mn = 27·5	Mn = 27·5	Mn = 27·5	Mn = 55
Mercury	Hg = 100	Hg = 100	Hg = 100	Hg = 200
Molybdenum	Mo = 48	Mo = 48	Mo = 48	Mo = 96
Nickel	Ni = 29·4	Ni = 29·4	Ni = 29·4	Ni = 58·8
Nitrogen	N = 7	N = 7	N = 14	N = 14
Osmium	Os = 99·6	Os = 99·6	Os = 99·6	Os = 199·2
Oxygen	O = 8		O = 8	O = 16
Palladium	Pd = 53·3	Pd = 53·3	Pd = 53·3	Pd = 106·6
Phosphorus	P = 15·5	P = 15·5	P = 31	P = 31
Platinum	Pt = 98·7	Pt = 98·7	Pt = 98·7	Pt = 197·4
Potassium	K = 39·1	K = 19·55	K = 39·1	K = 39·1
Rhodium	Rh = 52·2	Rh = 52·2	Rh = 52·2	Rh = 104·4
Rubidium	Rb = 85·4		Rb = 85·4	Rb = 85·4
Selenium	Se = 39·7	Se = 39·7	Se = 39·7	Se = 79·4
Silicium	Si = 21		Si = 14	Si = 28
Silver	Ag = 108	Ag = 54	Ag = 108	Ag = 108
Sodium	Na = 23	Na = 11·5	Na = 23	Na = 23
Strontium	Sr = 43·8	Sr = 43·8	Sr = 43·8	Sr = 87·6
Sulphur	S = 16	S = 16	S = 16	S = 32
Tellurium	Te = 64	Te = 64	Te = 64	Te = 128
Thallium	Tl = 204	Tl = 102	Tl = 204	Tl = 204
Tin	Sn = 59	Sn = 59	Sn = 59	Sn = 118
Titanium	Ti = 25	Ti = 25	Ti = 25	Ti = 50
Tungsten	W = 92	W = 92	W = 92	W = 184
Zinc	Zn = 32·6	Zn = 32·6	Zn = 32·6	Zn = 65·2
Zirconium	Zr = 33·6		Zr = 44·8	Zr = 89·6

For each of the previous columns the relation of the numbers to each other is alone important, and not the number which is taken as unit or starting-point. **Berzelius's** atomic weights and **Regnault's** thermal atomic weights are corrected with the nearest and most trustworthy experimental determinations, without alteration of the bases for the adoption of these numbers. The numerical relations presented in the above Table require, from the chemical point of view, no further explanation. The relations of these numbers to the specific heat form the subject of the investigations which are presented in the sequel.

3. The experiments by which DULONG and PETIT * showed, in the case of mercury various solid metals, and glass, that the specific heat increases with increasing temperature, were made by the method of mixtures. They determined at ordinary temperatures the specific heats of a greater number of elements by the method of cooling †. They found that when the numbers in the first column in § 2 corresponding to the elements Bi, Pb, Au, Pt, Sn, Zn, Cu, Ni, Fe, and S (the Berzelian atomic weights) are multiplied by the respective specific heats of these bodies, approximately the same number is obtained; and that approximately the same number is also obtained when $\frac{1}{2}$ Ag, $\frac{1}{2}$ Te, and $\frac{2}{3}$ Co are multiplied by their corresponding specific heats. They were of opinion that the atomic weights of the elements could and should be so selected that, when multiplied by the specific heats, they should give approximately the same number as product. This observation and this view, which DULONG and PETIT stated in 1819 in the following manner, "The atoms of all simple bodies have all exactly the same capacity for heat," have since that time been known as DULONG and PETIT'S *Law*.

I shall not here dwell upon POTTER'S investigations on the specific heat of metals and on the validity of DULONG and PETIT'S law ‡, but proceed directly to discuss NEUMANN'S investigations, which rank worthily by the side of those of DULONG and PETIT.

4. In his "Investigation on the specific heat of Minerals," NEUMANN (in 1831) first published § more accurate determinations of the specific heats of solid compounds. He investigated a large number of such compounds, especially those occurring in nature, partly by the method of mixture, and partly by the method of cooling; and he determined the sources of error in both these methods, and the corrections necessary to be introduced. In a postscript to this paper, he mentioned that he continued the investigations with an apparatus which, compared with that he had previously used, promised far greater accuracy in the individual results, without needing tedious and troublesome reductions. This apparatus, by means of which the specific heats of solid bodies, which may be heated in a closed space surrounded by steam, can be determined with great accuracy, he has not described ‖.

Of the general results of NEUMANN'S investigations, one must be particularly mentioned, that a dimorphous substance has the same specific heat in its two conditions. This he showed was the case with arragonite and calcite, and with iron pyrites and marcasite. But the most important is the discovery that in analogous compounds the products of the atomic weights into the specific heats are approximately equal. NEUMANN stated this last observation in the following manner:—"In bodies of analogous chemical composition the specific heats are inversely as the stochiometrical quantities, or, what is the same, stochiometrical quantities of bodies of analogous chemical composition have the same specific capacity for heat." NEUMANN adduced 8 carbonates, 4 sulphates, 4 sulphides (Me S), 5 oxides (Me O), and 3 oxides ($Me_2 O_3$), as showing this regularity, which is to be denoted as NEUMANN's law *.

---

* Annales de Chimie et de Physique, [2] vol. vii. p. 142.

† Ibid. vol. x. p. 395.

‡ Edinburgh Journal of Science, New Series, vol. v. p. 75, and vol. vi. p. 166. J. F. W. JOHNSTON'S remarks, vol. v. p. 278. I only know these papers from BERZELIUS'S 'Jahresbericht,' vol. xii. p. 17, and GEHLER'S 'Physicalisches Wörterbuch,' new edition, vol. x. part 1, p. 805 *et seq.*

§ POGGENDORFF'S 'Annalen,' vol. xxiii. p. 1.

‖ PAPE (POGGENDORFF'S 'Annalen,' vol. cxx. p. 337) has recently described this apparatus. I have had no

5. Soon after the publication of NEUMANN's researches in 1833, AVOGADRO published † a "Memoir on the Specific Heat of Solid and Liquid Bodies." He there gave a number of determinations of the specific heat of solid bodies made by the method of mixture. As far as can be ascertained by comparison with the most trustworthy of our newer determinations, these results are by no means so accurate as those of NEUMANN; but they are far more accurate than those which had been obtained up to about 1830, and many of them come very close to the best of our modern results. It would be unjust to AVOGADRO's determinations ‡ to judge them all by one case, in which he obtained a totally erroneous result (for ice, by a modified method); and by the circumstance that in a subsequent memoir § he gives specific heats for several elements as deduced from his experiments, which are decidedly incorrect ||. AVOGADRO recognizes the validity of DULONG and PETIT's law. With reference to the specific heats of compound bodies, he considers that he had established, with tolerable probability, that for solid and liquid bodies the same regularity prevails which he had previously deduced for gases from DULONG's experiments. That is, "that the specific heat of the atom of a compound body is equal to the square root of the integral or fractional number expressing the atoms or parts of atoms which go to form the atom of the compound body such as it exists in the solid or liquid state, taking as unity the specific heat of the atom of a simple body in the samestate." He observes that there is a difficulty incidental to the application of this law to solid and liquid bodies, which is not met with in the case of gaseous bodies, in which the composition by atoms or by volumes is held to be directly given by

---

opportunity of seeing NEUMANN's memoir cited by PAPE, "Commentatio de emendenda formula per quam calores corporum specifici ex experimentis methodo mixtionis institutis computantur." Regiomonti, 1834.

* The objections of REGNAULT (Ann. de Chim. et de Phys. [3] vol. i. p. 131) as to the inadequacy of the proofs adduced by NEUMANN in support of the law do not apply.

† Ann. de Chim. et de Phys. [2] vol. lv. p. 80, as an abstract from 'Memorie della Società Italiana delle Scienze residente in Modena,' t. xx. Fascicolo 2 di fisica'.

‡ They are also found in GMELIN's 'Handbuch der Chemie,' 4 Auflage, vol. i. in the Tables, pp. 215–218 *et seq.*

§ Ann. de Chim. et de Phys. [2] vol. lvii. p. 113.

|| I only know AVOGADRO's investigations from the abstracts published in the Ann. de Chim. et de Phys., and am not aware whether the bold corrections of AVOGADRO urged by REGNAULT (Ann. de Chim. et de Phys. [2] vol. lxxiii. p. 10) were used in all his experiments, or only in some.

observation. This difficulty consists in knowing what constitution is to be assigned to the body in question for the solid or liquid condition; this constitution, from the conclusions derived from his theoretical considerations, would often be different from that which the body has in the state of gas or vapour. His considerations led him to assume the atomic weights of many elements different from those which BERZELIUS had given: AVOGADRO described the atoms, to which the weights assumed by him refer, as *thermal atoms*.

6. R. HERMANN published in 1834 a memoir "On the Proportions in which Heat unites with the Chemical Elements and their Compounds, and on the Combining Weights considered as quotients of the capacity for Heat of Bodies into their Specific Gravities"[*]. He gives there a great number of determinations of the specific heat of solid bodies (of a few elements, but chiefly of compound bodies). He made a few experiments in which he used LAVOISIER and LAPLACE's calorimeter[†]; but by far the greater number of determinations are made by the method of cooling[‡]. Many of his results approach very closely to those which are at present considered accurate, but they are in so far untrustworthy that a considerable number among them are decidedly incorrect.

As for HERMANN's theoretical results, it must be borne in mind that, regarding matter as he does, not from the point of view of the atomic but of the dynamical theory, he puts the idea of combination weights in the place of the idea of atomic weights. The propositions which he endeavours to establish are the following. The quotients obtained by dividing the specific gravities of the elements[§] in the solid state by their specific gravities in the gaseous state, are either equal or stand to each other in simple ratios; they are 1, 2 . . . . . . 15 times as much as a certain base. The same is the case with the products of the specific gravities of the solid elements into their specific heats, that is, with their relative heat; and the number indicating the multiple for a given element is the same for both the above relations. It follows from this that the combining weights m of the elements are proportional to the quotients of their relative heats into their specific gravity in the solid condition; that the products of the specific heats and the combining weights for different elements are equal to a constant, and that from the known combining weight of an element its specific heat in the solid form may be calculated (it is equal to $\frac{0.375}{m}$, where m is the combining weight of the substance in question referred to oxygen $= 1$). For several elements (phosphorus, tellurium, cadmium, and silver for instance) atomic weights are taken which differ from those of BERZELIUS. In the case of the sulphides, the specific heats may be calculated from those of the constituents, assuming that the specific heats of the elements in these compounds are the same as in the free state. The same holds good for several chlorides and for basic metallic oxides, if the specific heats of chlorine and of oxygen, as given by the above formula, are taken as basis. But in acids a smaller specific heat must be taken for oxygen (one half in several acids and null in phosphoric acid); and there are even compounds (cassiterite, *e. g.*, or arsenious acid), in which the same element is contained partly with the normal and partly with the modified specific heat*. For oxygen salts it is to be assumed that both the acid and the base have the same specific heat as in the free state, and hence the specific heat of one constituent (of the acid, for instance) may be calculated, if that of the salt and that of the other constituent (the base) is known; and it is also found that the specific heat of chromic acid in the neutral and in acid chromate of lead is the same.

---

[*] Nouveaux Mémoires de la Société Impériale des Naturalistes de Moscou, vol. iii. p. 137.

[†] HERMANN tried to alter this apparatus so as to make it serve for measuring the change of volume which takes place when ice melts; but he did not further follow this application of the modified apparatus.

[‡] They are found not quite complete in GMELIN's 'Handbuch der Chemie,' 4 Auflage, in the Tables, pp. 215–218 *et seq.*

[§] HERMANN considers that the specific gravities of the elements in the state of gas or vapour are either obtained by observation, or may be theoretically deduced by assuming that they are in the ratio of the combining weights.

This memoir of HERMANN'S did not become much known. Unacquainted with it, other philosophers have subsequently developed independently similar opinions.

7. In 1835 RUDBERG described a method†, which, by ascertaining the heat developed when salts are dissolved in water, in experiments in which the proportion of the salt to the water was constant, but the temperature of the salt varied, should give a means of at once determining the specific heat of the salt, and of the heat which was either absorbed or became free. Yet the numbers which he obtained from his experiments for the specific heat of solid salts are undoubtedly erroneous.

DUMAS‡ (in 1838) discussed the possibility of determining the specific heat of organic bodies by the following process. A platinum vessel containing the substance in question, along with a thermometer, is to be heated to 30° or 40°, and then brought into a vessel provided with a second thermometer, and containing water, the temperature being about 5° or 6° lower than that of the surrounding room. When the temperature has risen to the same extent above that of the room, both thermometers are to be observed. I know no determinations made by this method.

8. In 1840 REGNAULT commenced the publication of a series of important investigations on specific heat which he had made. As they are generally known, I may be more brief in enumerating the contents of the individual publications. In the first which he published, REGNAULT developed§ the reasons which led him to prefer the method of mixture to other processes for determining the specific heats of solid bodies;

---

* HERMANN designates such compounds as hermaphrodites. He thinks that an acid and a base may have the same composition, and that they may form salts with each other. Cassiterite, for instance, he considers to be stannate of binoxide of tin.

† BERZELIUS'S 'Jahresbericht,' vol. xv. p. 63. POGGENDORFF'S 'Annalen', vol. xxxv. p. 474.

‡ DUMAS'S "Thèse sur la question de l'action du calorique sur les corps organiques" (Paris, 1838) Ann. der Pharm. und Chem. vol. xxviii. p. 151.

§ Ann. de Chim. et de Phys. [2] vol. lxxiii. p. 5.

he described his mode of executing this method, and published the results obtained for a great number of elements. In a second memoir [*] he gave the specific heats of several metallic alloys containing metals in simple atomic ratios, and of a great number of solid chemical compounds; and he published comprehensive experiments on the specific heat of carbon in its different conditions. The investigations announced in the first memoir [†] on the specific heat of organic compounds, as well as those promised in the second memoir [‡] on the specific heat of sulphur at different temperatures, have not to my knowledge been published. But in a third memoir [§] Regnault has investigated the difference in the specific heats of certain metals according as they are hardened or soft, and also with reference to sulphur according as it is in the native crystallized form, or has solidified a longer or shorter time after being melted; and he has more especially tried to impart greater certainty to the method of cooling. In his subsequent investigations, however, he has only used the method of mixture as being the more certain. These investigations [||] have given the specific heats of a large number of solid elements, and also of individual compounds.

By his investigations Regnault has removed some objections which seemed to affect Dulong and Petit's law, and has given a great number of new cases in which it applies. He considers [¶] this law to be universally valid, and discusses the reasons why for individual elements the specific heats found do not quite agree with the law, but only approximately. In his view the atomic weight of an element is to be so taken that it agrees with Dulong and Petit's law. He took the atomic weight of silver and of the alkaline metals half as great, and that of carbon twice as great as Berzelius had done. Yet with regard to selecting, by means of the specific heat, from among the numbers which the chemical investigations of an element has given as admissible, that which is the correct one, Regnault does not always express himself decidedly. In the case of carbon [**] and of silicium [††] he mentions the possibility of their disagreement with Dulong and Petit's law. He proved the validity of Neumann's law for a number of cases very considerably greater than that on which it had originally been based; and he expressed it in a much more general form [‡‡]. "In all compounds of analogous atomic composition, and similar chemical constitution, the specific heats are approximately inversely proportional to the atomic weights. Regnault designates the numbers agreeing with this law as thermal atomic weights. He has either determined them directly from the numbers found for the specific heats of the elements in the free state, applying DULONG and PETIT'S law, or indirectly by ascertaining the specific heat of solid compounds, assuming NEUMANN'S law; or finally (and only in a few cases), he has determined them by means of their probable analogies. These are the atomic weights given in the second column of the Table in § 2.

---

[*] Ann de Chim. et de Phys. [3] vol. i. p. 129.

[†] Ibid. [2] vol. lxxiii. p. 71.

[‡] Ibid. [3] vol. i. p. 205.

[§] Ibid. [3] vol. ix. p. 322.

[||] Ibid. [3] vol. xxvi. pp. 261 & 268; vol. xxxviii. p. 129; vol. xlvi. p. 257; vol. lxiii. p. 5. Comptes Rendus, vol. lv. p. 887.

[¶] Ann. de Chim. et de Phys. [2] vol. lxxiii. p. 66; further, [3] vol. xxvi. p. 261, and vol. xlvi. p. 257.

[**] Ibid. [3] vol. i. p. 205. But both before and after (Ibid. [2] vol. lxxiii. p. 71, and [3] vol. xxvi. p. 263) Regnault inclined to the view that carbon, with the equivalent=12, and the specific heat found for wood-charcoal, must be considered as obeying Dulong and Petit's law.

[††] Ibid. vol. lxiii. p. 30.

[‡‡] Ibid. vol. i. p. 199.

With regard to the relations of the specific heats of solid compounds to those of their constituents, REGNAULT has shown * that with metallic alloys, at a considerable distance from their melting-points, the specific heats may be calculated from those of their constituents in tolerable accordance with the experimental results, assuming that the specific heats of the metals are the same in the alloys as in the free state. The investigation, whether for true chemical compounds there is a simple relation between their specific heats and those of their constituent elements, REGNAULT has reserved † till the conclusion of his experiments on the specific heats of gaseous bodies ‡. To my knowledge he has published nothing for solid bodies. But in 1862, with reference to the relations which had been recognized between the specific heats and atomic weights of solid, simple or compound bodies, he spoke as follows §. "It is true that these laws, in the case of solid bodies, only apply approximately to simple bodies and those compounds of least complex constitution; for all others it is impossible to pronounce anything in this respect." From some remarks of REGNAULT in reference to carbon ‖ and silicium ¶ he considers it possible, or probable with certain elements, that they have a different specific heat in their compounds to that which they have in the free state.

9. In 1840 DE LA RIVE and MARCET published ** investigations on the specific heat of solid bodies. They made their determinations by the method of cooling. They found that, assuming BERZELIUS'S atomic weights, selenium, molybdenum, and wolfram fall under DULONG and PETIT'S law, which they consider as universally valid; but that carbon forms an exception, and they consider it as probable that its true atomic weight has not yet been ascertained. For several sulphides they found a greater specific heat than was calculated for them, assuming that their constituents have in them the same specific heat as in the free condition. They think that for solid as well as for liquid and gaseous compounds the law governing the specific heat is still unknown. A subsequent memoir by these physicists †† treated of the specific heat of carbon in its various conditions.

10. In 1840 ‡‡ H. SCHRÖDER made an investigation as to what volumes are to be assigned to the constituents of solid and liquid compounds when contained in those compounds. In his memoirs on the subject, he expressed the view that the specific heat of compounds depends on the specific heats of the constituents in that particular

---

* Ann. de Chim. et de Phys. [3] vol. i. p. 183.

† Ibid. p. 132.

‡ REGNAULT has made known the results of these experiments in 1853 by a preliminary account in the Comptes Rendus, vol. xxxvi. p. 676, and more completely in 1862 in his 'Relation des expériences pour déterminer les lois et les données physiques nécessaires au calcul des machines à feu,' vol. ii. p. 3.

§ Relation, &c. vol. ii. p. 289.

‖ Ann. de Chim. et de Phys. [3] vol. i. p. 205.

¶ Ibid. [3] vol. lxiii. p. 31.

** Ibid. [2] vol. lxxv. p. 113.

†† Ibid. [3] vol. ii. p. 121.

‡‡ POGGENDORFF'S 'Annalen,' vol. l. p. 553.

state of condensation in which they are contained in the compounds in question. In 1841 *, reasoning from the results of REGNAULT'S experiments, he endeavoured to show that the atomic heat (that is the product of the atomic weight into the specific heat) of a compound is equal to the sum of the atomic heats for the states of condensation in which the elements are contained in the compound, and to ascertain what atomic heats are to be assigned to certain elements in certain compounds. On the assumption that the atomic heat of metals in compounds is as great as in the free state, he endeavoured to determine the atomic heat of oxygen, sulphur, &c. in certain compounds of these elements with the metals; he came to the conclusion that an element (sulphur for instance) may in some compounds have an atomic heat different from that which it has in the free state; and the same element (sulphur or oxygen for instance) may have different atomic heats in different compounds; but the changes in the atomic heat of an element always ensue in simple ratios. I cannot here adduce the individual results which he obtained when he inferred the atomic heat of an element in a compound by subtracting from the atomic heat of the compound the atomic heat of the other elements in it, which he had calculated either from direct determinations of their specific heat, or from previous considerations. The essential part of SCHRÖDER'S conception is that in this manner the atomic heat of a body, as a constituent of a compound, may be indirectly determined; and the result is that the atomic heat, at any rate of some elements in compounds, is different from what it is in the free state, and may be different in different compounds, and that the changes are in simple ratios. SCHRÖDER considered also that there was probably a connexion between these changes and those of the volumes of the elements, without, however, stating how from the one change the other might be deduced.

11. L. GMELIN (in 1843) considered it as inadmissible, from the chemical point of view, to assign throughout such atomic weights to the elements as to make them agree with DULONG and PETIT'S law. Certain exceptions must be admitted. Comparing the specific heats of oxygen, hydrogen, and nitrogen for the gaseous state with the specific heats of other elements in the solid state, he came to the conclusion that if the numbers given in § 2 as the equivalents ordinarily assumed be taken as atomic weights, the atomic heat of hydrogen, of nitrogen, and by far the greater number of the elements is equal to about 3·2; several of them twice as great, that of oxygen one-half, that of carbon (as diamond) one-fourth as great. With reference to the dependence of the atomic heats of the compounds on those of the elements, GMELIN expressed the opinion ‡ that in general the elements on entering into compounds retain the atomic heats they have in the free state, but for individual elements, especially for oxygen and carbon, it must be assumed that their atomic heat changes in simple ratios with the compounds into which they enter.

---

* POGGENDORFF'S 'Annalen,' vol. lii. p. 269. † L. GMELIN'S 'Handbuch der Chemie,' 4th ed. vol. i. p. 217.

‡ Ibid. p. 222: compare an earlier remark of GMELIN which applies to this subject (1840) in the new edition of GEHLER'S 'Physikalisches Wörterbuch,' vol. ix. p. 1941.

12. Wœstyn was also of opinion* that the specific heats of the elements remain unchanged when they enter into chemical compounds. In 1848 he stated as a general proposition; "The quantity of heat necessary to raise the temperature of the atomic weight of a body through 1° is equal to the sum of the quantities of heat necessary to raise the temperature of the atoms, and fractions of atoms, through 1°". If A is the atomic weight and C the specific heat of a compound, $a_1, a_2, a_3 \ldots.$ the atomic weights†, and $c_1, c_2, c_3 \ldots.$ the specific heats of the elements contained in it, and $n_1, n_2, n_3 \ldots.$ the numbers which express how many atoms of each element are contained in an atom of the compound, then

$$AC = n_1a_1c_1 + n_2a_2c_2 + n_3a_3c_3 \ldots\ldots$$

As a proof of this law, he compared the calculated values of AC of several compounds (metallic iodides and sulphides) and alloys with the observed values, taking Regnault's determinations of the specific heats of the elements and of the compounds. It follows, further, from that proposition, that if the formula and the values for several compounds are compared with each other, there must be the same differences of the values AC for the same differences of formulæ. Wœstyn showed by a number of examples that this is so approximately. By means of this law, the product of the specific heat and the atomic weight for one constituent of a compound may be found, if this is known for the compound and the other constituents. Wœstyn deduced in this way the product for oxygen (by subtracting from the product for different metallic oxides that found for the metals, and from chlorate of potass that for chloride of potassium) to be 2·4 to 2·1 (O.=8), and for chlorine 3·0 to 3·5 (Cl.=17·75). Wœstyn finally expressed a doubt whether Neumann's law is universally applicable. He laid stress on the circumstance that when two elements give different products, the difference is also met with in the products for their analogous compounds; and, for instance, the greater products which mercury and bismuth have in comparison with other elements, are also met with in the compounds of these metals.

13. Garnier (in 1852) developed the view‡, that not only in the case of elements are the atomic weights A§ inversely proportional to the specific heats C, but that the same is the case with water‖ and solid compounds in whose atom n elementary atoms are contained, if the so-called mean atomic weight $\frac{A}{n}$ be compared with the specific heat C; for elements $A \times C = 3$, and for compound bodies $\frac{A}{n} \times C = 3$ (if O=8). He endeavoured to prove this from Regnault's determinations of specific heats. From the latter equation he calculated the specific heat for several compounds. In the case of the basic oxides, sulphides, chlorides, bromides, and iodides, his calculated results agree tolerably

---

* Ann. de Chim. et de Phys. [3] vol. xxiii. p. 295.

† Wœstyn based his considerations on Regnault's thermal atomic weights.

‡ Comptes Rendus, vol. xxxv. p. 278.

§ If Regnault's thermal atomic weights are taken.

‖ I shall in § 93 return specially to the question how often the specific heat of liquid water was compared with that of solid bodies.

with the observed ones; this is less the case with metallic acids and oxygen salts, for which calculation mostly gives results far too large. GARNIER* drew, further, from the above proposition the conclusion, that the atomic weight of hydrogen, chlorine, &c. must in fact be taken only half as great as the equivalent weight; for only by assuming this smaller atomic weight is the mean atomic weight such that its product with the specific heat is near 3.

In 1852 BANCALARI† repeated that the specific heat of an atom of a compound body (that is, its atomic heat) is equal to the sum of the specific heats of the individual constituent simple atoms, and showed, from a series of examples (oxides, chlorides, sulphates, and nitrates), that, according to that proposition, the atomic heats of many compounds may be calculated in tolerable approximation with those derived from REGNAULT'S experimental investigations, if, for the elements which he investigated, the atomic heats derived from his determinations be taken as a basis, that is, for oxygen ($O=8$) the atomic heat 1·89; for chlorine ($Cl=17{\cdot}75$) 3·21; for nitrogen ($N=7$) 3·11.

CANNIZARO (in 1858‡) has used the proposition, that, in the sense above taken, universally $\frac{AC}{n}=$a constant, for the purpose of ascertaining the value of n for the atomic weight of different compounds, and therewith ascertaining the atomic weight of elements which are contained in these compounds.

14. Besides those of REGNAULT, but few experimental determinations of the specific heats of solid bodies have been published. BEDE§ and BYSTRÖM|| have published investigations on the specific heat of several metals at different temperatures¶: both sets of experiments were made by the method of mixtures. From the year 1845, PERSON**, in his investigations on the specific heat of ice, then on the latent heats of fusion, and their relations to the specific heats in the solid and liquid condition, has determined the specific heat for several solid substances, especially also for some hydrated salts. He worked more especially by the method of mixture. He observed††, in the case of these

---

* Comptes Rendus, vol. xxxvii. p. 130.

† An abstract from Memorie della Accademia delle Scienze di Torino, [2] vol. xiii. p. 287, in the Archives des Sciences Physiques et Naturelles, vol. xxii. p. 81. I only know the contents of this memoir from this abstract.

‡ Il Nuovo Cimento, vol. vii. p. 321. PIAZZA also gives a statement of this speculation in his pamphlet, 'Formole atomistiche et typi chimici,' 1863. I only know this from a notice in the Bulletin de la Société Chimique de Paris, 1863.

§ An abstract from the Bulletin de l'Académie des Sciences de Belgique, vol. xxii. p. 473, and the Mémoires Couronnés par l'Académie de Belgique, vol. xxvii., appeared in the Bericht über die Fortschritte der Physik im Jahre 1855, dargestellt von der physicalischen Gesellschaft zu Berlin, p. 379.

|| Abstract from the Översigt of Stockholm Vetenskaps-Akademiens Förhandlingar, 1860, in the same Jahresbericht, 1800, p. 369.

¶ To the experiments of DULONG and PETIT on this subject, mentioned in § 3, POUILLET'S determinations of the specific heat of platinum at different temperatures must be added (Comptes Rendus, vol. ii. p. 782).

** Comptes Rendus, vol. xx. p. 1457; xxiii. pp. 162 & 366. Ann. de Chim. et de Phys. [3] vol. xxi. p. 295; xxiv. p. 129; xxvii. p. 250; xxx. p. 78.

†† PERSON expressed this in 1845 (Comptes Rendus, vol. xx. p. 1457), with regard to his determinations of

salts, that their specific heats may be calculated in close approximation with those found experimentally on the assumption that the constituents, anhydrous salt and water considered as ice, have the same specific heats in them as in the free state. By the same method, ALLUARD* (in 1859) determined the specific heat of napthalene. SCHAFARIK†, lastly, has executed by the method of mixtures a series of experiments on the determination of the specific heats of vanadic, molybdic, and arsenious acids.

Quite recently (1863), PAPE‡ has published investigations on the specific heat of anhydrous and hydrated sulphates. He worked by the method of mixture, which he modified in the case of salts rich in water, by placing them in turpentine, and observing the increase of temperature produced in the salt and in the liquid by immersing heated copper. As a more general result, PAPE finds that for hydrated sulphates of analogous formulæ, the products of the specific heats and the equivalents are approximately equal; and further, that with sulphates containing different quantities of water, the product of the specific heat and the equivalent increases with the quantity of water, in such a manner, that to an increase of each one equivalent there is a corresponding increase in the product.

15. In the preceding paragraphs I have collated, as far as I know them, the investigations on the specific heat of solid bodies, on the relations of this property to the atomic weight, and on the connexion with the chemical composition of a substance. The views which have been expressed relative to the validity of DULONG and PETIT'S § and of NEUMANN'S laws, and also as to the question whether the elements enter into chemical compounds with the same specific heats which they have in the free state or with modified ones, have been various and often discordant. In this respect it may be difficult to express an opinion which has not been already either stated or hinted at, or which at any rate cannot be naturally deduced from a view previously expressed.

The results to which my investigations on the specific heats of solid bodies have led me are the following:—Each solid substance, at a sufficient distance from its melting-point, has a specific heat, which may vary somewhat with physical conditions (temperature, greater or less density, amorphous or crystalline conditions, &c); yet the variations are never so great as must be the case if a variation in the specific heat of a body is to

---

the specific heat of crystallized borax and of ordinary phosphate of soda. He has subsequently published the results of his experiments for the latter salt (Ann. de Chim. et de Phys. [3] vol. xxvii. p. 253), but I cannot find the number which he found for crystallized borax.

* Ann. de Chim. et de Phys. [3] vol. lvii. p. 438.

† Berichte der Wiener Akademie der Wissenschaften, vol. xlvii. p. 248.

‡ POGGENDORFF'S 'Annalen,' vol. cxx. pp. 337 & 579.

§ The universal validity of this law was also defended by BREDOW, "On the relation of the Specific Heat to the Chemical Combining Weight." Berlin, 1838. I only know this paper from the mention of it in the new edition of GEHLER'S 'Physicalisches Wörterbuch,' vol. x. p. 818. It is also admitted by MANN, in his attempt to deduce this law from the undulatory theory of heat. (1857: SCHLÖMILCH and WITZSCHEL'S 'Zeitschrift für Mathematik und Physik,' II. Jahrgang, p. 280); and by STEFAN, in his investigation on the bearing of this law on the mechanical theory of heat (1859: Berichte der Wiener Akademie, vol. xxxvi. p. 85).

be held as a reason for explaining why the determinations of the specific heats of solid elements do not even approximately obey DULONG and PETIT's law, nor those of solid compounds of analogous chemical constitution NEUMANN's law. Neither law is universally valid, although I have found that NEUMANN's law applies in the case of many compounds of analogous atomic composition, to which, on account of their totally different chemical deportment, different formulas are assigned; and even in cases in which these laws have hitherto been considered as essentially true, the divergences from them are material. Each element has the same specific heat in its solid free state and in its solid compounds. From the specific heats to be assigned to the elements, either directly from experimental determination, or indirectly by calculation on the basis of the law just stated, the specific heats of their compounds may be calculated. I show the applicability of this by a great number of examples.

In reference to this calculation of the specific heats of solid bodies I may here make a remark. The agreement between the results of calculation and experiment is often only approximate; it is then natural to urge that the two ought really to agree more closely. To that the question may be allowed: What means are there of even approximately predicting and calculating beforehand the specific heat of any inorganic or organic solid compound when nothing but its empirical formula is given? to which among the numbers 0·1, 0·2, 0·3 . . . . . . may it come nearest? The cases in which differences exist between calculation and observation, enumerated in § 103 to 110, may be set against *this* uncertainty.

My proof of the propositions given above is based on determinations made by earlier inquirers, and on a not inconsiderable number of my own. I first describe the method by which I worked, and then give the results which I have obtained by its means.

PART II.—DESCRIPTION OF A METHOD OF DETERMINING THE SPECIFIC HEAT OF SOLID BODIES.

16. I have worked by the method of mixture. It is not necessary for me to discuss the advantages which this method has over that of the ice-calorimeter, at any rate in requiring smaller quantities; nor, as compared with the method of cooling, need I discuss the uncertainties and differences in the results for the same substance, which are incidental to the use of this method, and which REGNAULT has detailed*.

The method of mixtures has been raised by NEUMANN and by REGNAULT to a high degree of perfection. Although by NEUMANN's method it is possible to determine more accurately the temperature to which the body investigated is heated, REGNAULT's method allows larger quantities to be used. REGNAULT's process gives the specific heats of such substances as can be investigated by it as accurately as can at all be expected in the determination of this property. In the case of copper and steel, it is not merely possible to determine their specific heats by its means, but also to say whether and how far there is a difference in the first metal according as it has been heated or hammered; and in the second, according as it is soft or hard. It may be compared with a goniometer, which not only measures the angles of a crystal, but also the differences in the angle produced by heat; or it may be compared to a method for determining the specific gravity of a body, by which not only this property, but also its changes with the temperature may be determined. But along with such methods, simpler ones, though perhaps less accurate, have also their value. Which method is the most convenient or which ought to be used in a given case, depends on the question to be decided by the experiment, or on the extent to which the property in question is constant in the substance examined.

---

* Ann. de Chim. et de Phys. [2] vol. lxxiii. p. 14; [3] vol. ix. p. 327.

In regard to the relations of the specific heat of solid bodies to their atomic weight and to their composition, REGNAULT'S determinations have shown that both DULONG and PETIT'S and NEUMANN'S law are only approximate, and that even the accuracy in determining the specific heat which REGNAULT attempted, and obtained, could not show that these laws were quite accurate.

Although the description of REGNAULT'S mode of experimenting is so widely known, yet it cannot be said to have become the common property of physicists, or to have found an entrance into the laboratories of chemists, to whom the determination of the specific heat is interesting from its relation to the atomic weight. Very few experiments have been made by this method other than the determinations of REGNAULT. The method depends on the use of an apparatus which is tolerably complicated and takes up much room. Each experiment requires a long time, and for its performance several persons are required. REGNAULT has usually worked with very considerable quantities of the solid substance, and in by far the majority of cases at temperatures (usually up to 100°) which many chemical preparations, whose specific heats it is important to know, do not bear. In the sequel I will describe a process, for the performance of which the apparatus can be readily constructed, and for which one operator is sufficient; by which, moreover, the determination of specific heat can be made with small quantities of the solid substance and at a moderate temperature. But the method as I have used it has by no means the accuracy of that of REGNAULT. In § 18 I shall discuss the advantages for which some of the accuracy which characterizes REGNAULT'S method is sacrificed; but I may here remark that the results obtained by the method which I have used are capable of increased accuracy, provided the experiments are executed on a larger scale and within greater ranges of temperature.

17. The principle which forms the basis of my method is as follows:—To determine the total increase of temperature produced when a glass containing the substance to be investigated, covered by a liquid which does not dissolve it, the whole previously warmed, is immersed in cold water; to subtract from the total increase of temperature that due to the glass and the liquid in it, and to deduce from the difference, which is due to the solid substance, its specific heat.

If, in regard to gain or loss of heat, the glass, in so far as it comes in contact with water, is equivalent to x parts of water, if f is the weight of the liquid in it, y its specific heat, m the weight of the solid substance, M the weight of the water in a calorimeter, including the value in water of the immersed part of a thermometer and of the calorimeter, T the temperature to which the glass and its contents have been heated before immersion in water, and T′ the temperature to which the glass sinks when immersed in the water, while the temperature of the latter rises from t to t', then the specific heat (sp. H.) of the solid substance is

$$\text{sp. H} = \frac{M(t'-t)-(x+fy) \,.\, (T-T')}{m(T-T')}.$$

In the sequel I shall discuss more specially the manner in which the individual magnitudes in this equation were determined: I will first give a description of the apparatus and method which I used*.

The glass vessel in which the substance is confined (Plate XX. *a* in fig. 1) is a tube of glass, the bottom of an ordinary test-tube. In it fits, but not air-tight, a cork *c*, which is pressed between two small brass plates that are screwed to a wire *b*. The solid substance to be investigated, in the form of thin cylinders, or in small pieces the size of a pea, along with a liquid of known specific heat, which does not dissolve it, are placed in the tube in such a manner that the liquid covers the solid substance, and that there is a space between the liquid and the cork when it is inserted. The glass, when the cork is fitted, may be suspended to the balance by the wire *b*. Three weighings (1) of the empty glass, (2) after introducing the solid substance, and (3) after introducing the liquid, give the weight of the solid substance (m) and of the liquid (f).

The heating apparatus (fig. 1) serves to raise the temperature of the glass with its contents. The glass is dipped in a mercury-bath A near its upper edge, and retained by a holder E. The mercury-bath, which consists of a cylindrical glass vessel, is suspended by means of a triangle round the neck of the vessel in an oil-bath B, which stands on a tripod C, and can be heated by a spirit-lamp D. A thermometer *d*†, fixed to the holder F, is also immersed in the mercury-bath.

The flame of the spirit-lamp may be regulated so that the thermometer *d* indicates the same temperature for a long time ‡. If it may be assumed that the contents of the glass *a* have also risen to this temperature, then the wire *b* being firmly held in the right-hand by its hook, and the clamp of the holder E in the left, the glass *a* is rapidly removed from the heating vessel to the calorimeter H (fig. 2). This is almost the only part of the entire experiment which really requires much practice; the transference of the glass *a* from the one vessel to the other must be effected in an instant, and none of the liquid in the glass must touch the cork.

---

* All figures on the Plate are one-third of the natural size.

† Fig 7 shows in section how the glass with its contents and the thermometer dip in the mercury-bath and this in the oil-bath.

‡ In order to obtain temperatures constant at about 50°, a spirit-lamp with a thin wick is used, and this is pressed in the sheath so that the alcohol-vapour above it burns with a very small flame. The position of the wick and the intensity of the flame may be conveniently regulated if the upper part of the wick is surrounded by a spiral of thin copper wire whose ends project from the sheath.

The calorimeter H stands upon a support G (fig. 2)*, on which there is an oval metal plate *o*. In this there are three depressions, in which fit the three feet of the calorimeter (they are made of very thin hard brass wire). The calorimeter is oval-shaped, and is made of the very thinnest brass plate. In it a brass stirrer fits, made of two parallel plates of brass of the same thinness, which are joined below by thin wires, and provided with a thin wire ending in a little button *i*, which serves as handle. The plates of the stirrer are perforated in such a manner that the glass *a* and a thermometer can be passed through them. Fig. 4 shows more distinctly the construction of the stirrer, also the section of the calorimeter.

For the experiments, the calorimeter is always filled, as nearly as possible, with the same quantity of water†. The stirrer is immersed, and a thermometer *f* dipping in the water gives its temperature, which is kept uniform by an upward and downward uniform motion of the stirrer. When the tube *a* is brought into the water of the calorimeter, it is fastened‡ in the clamp of the holder K, which is arranged like the pincettes used for blowpipe experiments, so that it stands on the bottom of the calorimeter, and then the stirrer is set to work. This motion of the stirrer, and therewith of the water, must be moderate and uniform in all experiments; this is of some importance for the uniformity and comparability of the experiments. The temperature indicated by the thermometer *f* rises and soon attains its maximum, which continues for some time, and can be observed with certainty. With this the experiment is concluded. The tube *a* can be taken from the calorimeter, dried, and used for a new experiment.

The increase of temperature produced in the calorimeter by the tube *a* and its contents, would be incorrectly given if the warmth of the body of the operator, who moves the stirrer and observes the thermometer, acted on the calorimeter. This is prevented by a glass screen *g g g g*, fig. 2, which is fitted in the brackets *h h*, and above which the handle of the stirrer projects.

18. This process for determining the specific heat of solid bodies, the details of which are more minutely discussed in the sequel, has advantages over those hitherto prin-

---

* In making the experiment, the actual distance between the calorimeter and the heating apparatus must be greater than is indicated in the figure, but not so great that the glass *a* cannot, by a rapid motion of the arm, be transferred from the mercury-bath to the calorimeter.

† This is most conveniently effected by laying across it a bridge with a stem directed downwards (fig. 3), and adding water until it touches the point of the stem; and the calorimeter, which now contains almost the requisite quantity of water, is placed on the balance, and the filling completed by means of the dropping-flask (fig. 8). The construction of the latter is readily intelligible: it is held by the cork between two fingers, and by approaching the hand to the bottom of the flask water commences to drop. When the flask is not in use the tube, which fits air-tight in the cork, is raised, so that it does not dip in the water, and thus the water is prevented from escaping.

‡ Fig. 5 shows in a section the glass *a*, with its contents, and the thermometer *f* immersed in the water of the calorimeter.

cipally used, which I will here mention. The use of the mercury-bath makes it possible readily to produce, and maintain for any adequate length of time, any temperature desirable in such experiments. The mercury-bath* shares with the air-bath the advantage that, to the substance heated in it (in this case the tube and contents), nothing adheres when it is removed which might influence the thermal effect in the calorimeter. It has over the air-bath the advantage, that any body placed in it takes the temperature of the surrounding medium much more quickly through its entire mass. The communication of heat to the solid substance is materially promoted by the circulation of the liquid between its particles; the time necessary for the entire contents of the glass to become equally heated is a very short one†. Moreover this very circulation of the liquid between the particles of the solid ensures a quicker and more uniform transmission of the heat of the contents of the glass to the water of the calorimeter; the maximum temperature of this water is soon attained‡, although the transmission of the excess of temperature must take place through the sides of the glass.

---

* In 1848 I already used such a one for heating liquids enclosed in glass tubes, in determining their specific heats (POGGENDORFF's 'Annalen,' vol. lxxv. p. 98).

† In experiments on the scale on which I made them, when the mercury-bath had once been raised to the requisite temperature, it only required ten minutes' immersion of the glass in the bath to impart to it the temperature of the bath. A more prolonged heating was found to be useless in all cases in which I tried it. In the experiments to be subsequently described, the heating was continued about ten minutes; in most cases less would have been sufficient. In REGNAULT's experiments (Ann. de Chim. et de Phys. [2] vol. lxxiii. p. 22), in which the substance (in much larger quantities it is true) was heated in a space nearly surrounded by steam, a thermometer placed in the substance indicated after about two hours an almost constant position (always one or two degrees lower than the temperature of the steam); and then it was found convenient to continue this heating for at least an hour, in order to see that the temperature did not change, and to be certain that the substance had the temperature indicated by the thermometer throughout its entire mass. In NEUMANN's experiments, the space in which the substance to be heated is contained is smaller and more completely surrounded by vapour. The time necessary for heating the substance uniformly must be smaller, and the temperature must be nearer that of the surrounding vapour. According to PAPE (POGGENDORFF's 'Annalen,' vol. cxx. p. 352), a thermometer placed in the above space, if surrounded by steam for forty-five to sixty minutes, gives exactly the temperature of this steam.

‡ In several experiments I determined the time which elapsed between immersing the glass with contents in the water of the calorimeter and its attaining a maximum. Under the circumstances, which I subsequently give more specially, and which, as far as possible, were maintained in all experiments, this time was always less than two minutes, if the liquid could circulate between compact pieces of the solid substance. What I have said above justifies, I think, my not having made, in experiments with such substances, a correction for the loss of heat which the calorimeter experiences between the moment of immersing the glass and the establishment of a maximum temperature. In substances which form a fine powder or a porous mass, or in general in cases in which the circulation stagnates, the maximum temperature is more slowly attained, the above loss of heat is more considerable, and the numbers for the specific heats are then somewhat too small. I shall recur to this again in enumerating the experiments in § 41 with chromium, and in § 52 with chloride of chromium. In a few cases I have endeavoured to diminish this error, and to promote the circulation of the liquid by pressing the porous substance into small disks. I must leave it as an open question whether more accurate results would not be obtained for such substances if they were formed by means of a suitable cement into compact masses, and then the thermal action of the cement thus added taken into account.

The apparatus which I have just described is very simple. It is readily constructed; the chief point is to have two thermometers which have been compared with each other, one of them (*f*) graduated in tenths of a degree, while on the other (*d*) the tenth of a degree can be observed with certainty. The apparatus does not require much space; yet, while the experiment is being made, rapid changes in the temperature of the surrounding air must be avoided. One observer only is required (all the experiments described in the sequel have been made without assistance). The experiments which I shall communicate prove that, by means of this apparatus, the specific heat of solid substances, even when only small quantities are taken (in most cases I worked with only a few grammes), may be determined with an accuracy not much less than that attained with larger quantities in more complicated processes.

19. Yet, it is true, the accuracy of the results obtained by this process appears to be inferior to that attainable by the use of NEUMANN'S or of REGNAULT'S methods. I have investigated many substances, determinations of which have also been made by these physicists. I do not find that the numbers I have obtained deviate in one special direction from those which these physicists have found, which moreover sometimes differ considerably among themselves*; but that the certainty of the results I have obtained is less, is shown by the fact that the results of different experiments with the same substance agree less closely with one another than do those of REGNAULT and of NEUMANN.

That my determinations are less accurate is probably least due to the circumstance that I did not use certain corrections, for instance, that I did not allow for the loss of heat in the calorimeter between the time when the heated body was immersed and the maximum temperature was attained†. I have endeavoured to diminish the uncertainty of the results from this source by having the temperature of the water in the calorimeter, before immersing the heated body, somewhat lower than that of the surrounding air. I have endeavoured to ensure comparability in my results for different substances by always operating as much as possible under the same circumstances; that is, I endeavoured always to produce in the water of the calorimeter the same excess of temperature over that of the surrounding air. Without depreciating the interest and value of such corrections, I think that their application may be omitted if their practical importance is inconsiderable, and the increased difficulties which they necessitate proportionally large. It must be considered, in reference to such corrections, how far the accuracy, which the results obtained by their means claim, is not more apparent than real‡. And further, that these corrections, where the conditions for their application really exist, are not considerable; while, where they exert a considerable influence on the result, they may be uncertain, because the suppositions made in their development are less applicable. But more especially can such corrections be disregarded when, as in the case with my determinations, other circumstances diminish more materially the accuracy of the results to be obtained.

---

* PAPE, in POGGENDORFF'S 'Annalen,' vol. cxx. p. 579, discusses the probable causes of these differences.

† Another correction, which appears to me to be more important for the experiments in question, is, that the contents of the glass at the time at which the temperature of the water is at its maximum may be at a somewhat higher temperature. This I have approximately taken into account. Compare §§ 23 & 24.

‡ It is unnecessary to adduce examples where such corrections, proceeding from as comprehensive a basis as

Such circumstances in my experiments are, that I worked on a small scale in every respect. I could only heat the solid investigated together with a liquid to 50°, and in many cases not even to this. In NEUMANN'S and in REGNAULT'S experiments, on the contrary, the solid was usually heated to near 100°, and the difference in temperature, T—T′ (compare § 17), obtained in the latter experiments was usually thrice as great as in mine. In REGNAULT'S experiments (in NEUMANN'S the details are not given) the quantity of substance taken was, on the average, twenty times as much, and the weight of water in the calorimeter about eighteen times as much as in mine*: hence in the latter experiments the unavoidable accidental errors of observation must be greater than in the former.

But there is a still more important circumstance which makes the accuracy to be hoped for from my experiments less than that to be expected from REGNAULT'S and NEUMANN'S experiments. In the latter methods the increase in the temperature of the water of the calorimeter is entirely, or is almost entirely produced by the solid under examination. In my experiments, on the contrary, this increase is produced by the glass, the solid, and the liquid in the glass. The thermal action due to the solid is only a part of the entire thermal action observed, and if from the latter that due to the liquid and to the glass is subtracted, all uncertainties in the assumptions as to the thermal action of the

---

possible, lose their significance from necessary simplifications, and their practical importance becomes finally very slight. The amount of correction is then to be pronounced as having no influence on the final result. It is more important to take into account the following. The trustworthiness of the specific heat to be assigned to any particular compound depends upon the certainty of the determination of the physical property, and upon the certainty of the knowledge of the composition of the body in question; that is, in how far this compound corresponds to a given formula. The greatest trouble which can be taken in that determination, the consideration of all sources of error which are possible in the physical experiment, the most complete exposition of the corrections which by developing conclusions from more or less certain assumptions may be formulated in one expression, and the most conscientious application of these corrections,—all this may be paralyzed by the circumstance that the composition of the body in question is not, as it were, the ideal, not corresponding accurately to the formula. The partial substitution, if even to a very small extent, of one constituent by an isomorphous one, the attraction of water by a hygroscopic substance before and during the experiment, the presence of some mother-liquor in a crystallized salt, the loss of some water in drying a hydrated substance, so that this has not exactly the composition corresponding to the formula,—all these sources of error, which can scarcely be taken into account, may easily exercise an influence on the final result, whose magnitude far exceeds that of certain corrections applied to the physical part of the determination. It lies in the nature of the case that in such investigations, in some cases bodies of well-known, in other cases bodies of less well-known composition are taken. I tried to be certain what substances could be considered as of definite composition and what of doubtful composition, especially where the relations between the specific heat and the atomic weight or chemical composition were under discussion.

* About sixty solids have been investigated both by REGNAULT and myself; for about thirty the weights which he used in his determinations are twenty times as much as in mine or more.

liquid and that of the glass are concentrated on the remainder, on the thermal action of the solid substance from which its specific heat is to be deduced. The results obtained by my method are less accurate when the residue is only a small fraction of the total result from which it is deduced. In individual cases, where this was unavoidable, I shall have to remark upon it.

It may be said in favour of my method that, for a number of solid substances, no other method yet attempted is applicable either at all or with more prospect of a successful result. But this is less important than the proof furnished by my examination of very many substances, whose specific heat has been already determined by NEUMANN and by REGNAULT, that the specific heat of bodies may be determined by my method with an accuracy quite sufficient for many comparisons. But there are cases in which it is even advantageous not to heat the solid alone, but in conjunction with a liquid, and to bring them together into the water of the calorimeter. The chemical nature of the solid may necessitate this; as, for example, when it readily alters on being heated in the air (compare § 34 in reference to amorphous boron); its physical structure may also render it desirable, as for instance if the substance has a large surface as compared with its mass, or is so porous that the thermal action due to humectation, and first observed by POUILLET *, takes place. REGNAULT has shown that this may be considerable †; he states that for this reason the specific heat of some substances is found about $\frac{1}{30}$ too great. He appears to have estimated this thermal action by ascertaining the increase of temperature produced in the water of the calorimeter when the porous substance, whose temperature is that of the water and of the surrounding air, is dipped in it. But this action is probably far more considerable if, while heated, it is immersed in the water, because it then contains less air confined on its surface and in its pores ‡, and surface action can then act more intensely upon the liquid. The influence of this source of error cannot be measured exactly. It is unequal in different substances. In platinum it is small (REGNAULT found by his method that the specific heat of spongy platinum did not materially differ from that of massive pieces), while it may be con-

---

* Ann. de Chim. et de Phys. [2] vol. xx. p. 141.

† Ann. de Chim. et de Phys. [3] vol. i. p. 133. REGNAULT preferred to immerse the heated porous substances, when they could be obtained in coherent pieces, directly in the water of the calorimeter. If they were enclosed in thin tubes and immersed, the equalization of temperature proceeded too slowly. REGNAULT abstained from enclosing at the same time a sufficient quantity of water in the tube to promote the circulation, because in that case the thermal action of the solid was only a fraction of that of the water added, on which the entire source of error falls. REGNAULT found also (ibid. p. 142) that in immersing anhydrous baryta, strontia, and lime in most carefully dehydrated oil of turpentine, there is such a thermal action that no useful result is to be obtained by his method for these oxides.

‡ To the examples already known, which show what influence temperature exerts on the quantity of air absorbed in a porous body, REGNAULT has added a very instructive one (Ann. de Chim. et de Phys. [3] vol. lxiii. p. 32). If amorphous boron, formed into disks by pressure in a steel mortar, was strongly cooled and then immersed in the water of the calorimeter (at the mean temperature), so considerable a disengagement of absorbed air was produced, that REGNAULT was compelled to give up the determination of the specific heat by this method.

siderable for porous charcoal (in fact POUILLET'S experiments make this probable). This source of error is excluded in my method.

20. In order to appreciate the trustworthiness of the results arrived at by my mode of experiment, it is important to state with what amount of accuracy the data of observation and the ancillary magnitudes were determined. I will give this statement in what now follows.

For observing the temperature of the water in the calorimeter I used thermometers made by GEISSLER of Bonn, which the kindness of Professor BUFF, Director of the Physical Cabinet in Giessen, placed at my disposal. In these thermometers the tube consists of a fine glass thread drawn out at the lamp. The bulb is cylindrical, very thin in the glass, and contains but little mercury. On one (*b*) 1° C. corresponds to a length of almost 5 millims. on the scale, and on the other (*r*) to almost 4·5 millims. Tenths of a degree can be read off directly on the scale, and it is easy to learn to estimate hundredths safely. I have repeatedly compared these two thermometers, between 7° and 24°, with two normal thermometers of my own construction, which agree very well with each other, and on one of which a division corresponds to 0°·4878, and the other to 0°·4341. The differences of the indications between the GEISSLER'S thermometers and these could be considered as constant within those limits; for the differences thus observed all the readings made with the GEISSLER'S thermometers had to be corrected to make them comparable with the indications of the normal thermometer.

The temperature of the mercury-bath was ascertained by means of one of these normal thermometers, and the indications of this thermometer immersed in the bath (*d* in fig. 1.) corrected for the lower temperature of the mercury thread out of the bath; this latter temperature was given with adequate approximation by the second thermometer, *e*.

21. The weight of the thin sheet-brass calorimeter, together with stirrer, was 11·145 grms.* Taking the specific heat of brass, according to REGNAULT, at 0·09391, the calorimetric value in water of this mass of metal is 1·046 grm. Considering that the calorimeter in the experiments was not quite filled with water, but about $\frac{1}{6}$th remained empty, even after introducing the tube, I put the value in water at 0·872.

In determining the calorimetric water value of the immersed parts of the thermometers *r* and *b*, the following experiments were made. The weight of water in the calorimeter, together with the reduced weight of the metal, was 30·87 grms. When the thermometer *r* heated to 33°·86 was immersed, the temperature rose from 10°·73 to 10°·85; the immersion of the thermometer *b* at a temperature of 37°·53 caused a rise from 10°·61 to 10°·76. In both cases the temperature of the water was indicated by means of the other thermometer, the reduced value of which might be neglected under these circumstances. These experiments gave 0·16 as the reduced value of the thermometer τ, and 0·17 as the reduced value of the thermometer b. The thermometers have very nearly the same dimensions. Hence I put the reduced value of the calorimeter (that is, of the part of the metal concerned), of the stirrer, and of the immersed part of the thermometer at 1·04 grm. Even if this determination is a few tenths out, it is scarcely appreciable as compared with the quantity of water in the calorimeter. In all following experiments this was between 25·85 and 25·95 grms.

---

* At the beginning of these investigations. During their progress the calorimeter was cleaned and dried with bibulous paper a countless number of times, so that its weight diminished by about 0·04 grm. in the course of the experiments. In determining the weight of water used in each experiment, the weight which the calorimeter actually had at the time was taken as basis.

All the subsequent determinations depend on fixing differences of weights and of temperatures. The accuracy of the results depends on the precision with which both kinds of magnitudes are ascertained; and it is useless to determine the weights to $\frac{1}{1000}$ or nearer, if the differences in temperature cannot be determined more accurately than to $\frac{1}{200}$ or $\frac{1}{300}$. I have weighed to centigrammes instead of to milligrammes, by which the time necessary for the weighings was much shortened, and their accuracy not materially lessened.

22. The reduced value x remained to be determined of the glasses (cylindrical tubes of thin glass, see § 17), or, rather, of that part which was immersed in the water of the calorimeter, the quantity of which was always the same. In the following, T is the temperature to which the glass in the mercury-bath was heated (compare fig. 1), M the quantity of water in the calorimeter + the reduced value in water of the other parts of the latter, which required to be taken into account, t the temperature of the water in the calorimeter when the glass was immersed (fig. 2), and τ the temperature to which the water became heated, and which must be considered as that to which the glass cooled*. We have then

$$x=\frac{M(\tau-t)}{T-\tau}.$$

In my experiments I used three glasses, which may be called 1, 2, and 3. To ascertain the reduced value of glass 1, I made the following determinations:—

Temperature of Air 15°·8.

T.	τ.	t.	M.	x.
°	°	°	grms.	
78·54	17·23	15·72	26·98	0·664
74·38	17·16	15·78	26·97	0·651
75·51	17·14	15·72	26·92	0·655
76·06	17·15	15·73	26·945	0·649
77·32	17·22	15·74	26·96	0.664
			Mean . .	0·657

---

* If the cork which closes the glass, and by means of the wire passing through it enables it to be handled, is moist, incorrect and discordant values are obtained for it, owing to the evaporation of water in the empty glass so long as this is in the mercury-bath, and to the condensation of aqueous vapour in the glass when it is immersed in the calorimeter.

I subsequently made a second series of experiments to determine the reduced value for glass 1, which gave the following results:—

Temperature of the Air 19°·9–19°·8.

T.	τ.	t.	M. grms.	x.
78°·50	21°·32	19°·93	26·99	0·656
81·86	21·47	20·03	26·98	0·643
80·42	21·43	20·02	26·98	0·645
79·77	21·42	20·03	26·935	0·642
80·14	21·51	20·12	26·955	0·639
			Mean . .	0·645

The mean of these two means, 0·657 and 0·645, gives as the reduced value in water of glass 1, 0·651 grm.

To obtain the water value for glass 2, I made the following determinations:—

Temperature of the Air 12°·0–12°·5.

T.	τ.	t.	M. grms.	x.
75°·87	13°·53	12°·43	26·94	0·475
77·05	13·46	12·31	26·96	0·488
76·71	13·68	12·54	26·975	0·488
75·97	13·76	12·65	26·95	0·481
78·60	13·83	12·62	26·95	0·503
			Mean . .	0·487

The reduced value for glass 2 is hence = 0·487 grm. This glass broke before I made a second series of experiments to ascertain its reduced value.

I made two series of experiments to determine the reduced value of glass 3. The first gave the following results:—

Temperature of the Air 19°·3–19°·5.

T.	τ.	t.	M. grms.	x.
81°·00	20°·33	19°·31	26·98	0·454
80·03	20·83	19·84	26·965	0·451
80·22	20·93	19·94	26·98	0·451
84·06	21·04	20·02	26·945	0·436
81·90	20·93	19·93	26·975	0·442
			Mean . .	0·447

The second series of experiments gave the following results:—

Temperature of the Air 19°·9–19°·8.

T.	τ.	t.	M.	x.
°	°	°	grms.	
80·41	21·08	20·06	26·965	0·464
79·64	21·10	20·09	26·965	0·465
79·98	21·12	20·12	26·96	0·458
80·22	21·12	20·12	26·985	0·457
79·53	21·10	20·12	26·965	0·452
80·52	21·13	20·14	26·96	0·450
			Mean . .	0·458

The reduced value of glass 3 = 0·453 grm., the average of the mean numbers of both series of experiments.

23. In those experiments in which a glass containing a liquid and perhaps a solid substance is immersed, while warm, in the water of the calorimeter, it may be asked if, when the water has become heated to a certain maximum temperature, the contents of the glass have actually cooled to the same temperature. In earlier experiments made by the method of mixture, it was at once assumed that the temperature assumed by the water of the calorimeter after immersing the solid was actually that also to which the immersed body sank. Neumann has taken into account that the immersed body, when the water shows its maximum temperature, may have a somewhat higher temperature *. Avogadro has also taken this into account †, and Regnault has also allowed for this circumstance in the case in which the mass, immersed in the water of the calorimeter, is a bad conductor of heat ‡. A correction for this fact is certainly inconsiderable and unnecessary if the immersed body conducts heat well, and the range of temperature through which it cools in the liquid is great. This interval of temperature was in my experiments considerably smaller than in those of Neumann and of Regnault; and as in my experiments the excess of heat of the contents of the glass had to pass through its sides to the water of the calorimeter, it might be doubted whether, when the temperature of the water was at its maximum, this temperature could be considered as that of the contents of the glass.

I have endeavoured to answer these questions experimentally. A glass, such as was used for holding the solid investigated and a liquid, was filled with water, and a perforated cork fitted, by means of which the glass could be handled, and which permitted the introduction of a thermometer into the water within the glass. The glass filled with water was warmed, and then placed in the calorimeter filled with water; a thermometer A, passing through the cork, showed the temperature of the water in the glass;

---

* In the memoirs mentioned in § 4, Pape has also discussed and applied the correction to be made for the above circumstance (Poggendorff's 'Annalen,' vol. cxx. p. 341).

† Ann. de Chim. et de Phys. [2] vol. lv. p. 90.

‡ Ibid [2] vol. lxxiii. p. 26.

a second, B, showed that of the calorimeter water. If the glass filled with the warmer water is immersed in the cold water, the following circumstances are observed[*]. A sinks very rapidly, while B rises more slowly; if B shows the maximum temperature for the water of the calorimeter (this temperature being called t'), A gives a higher temperature (T) for the contents of the glass. B then slowly sinks and A follows it, while the difference between t' and T' always becomes smaller. In the two following series of experiments I have endeavoured to determine by how much, under certain conditions, the temperature T' of the water in the glass exceeds the maximum temperature t' of the water in the calorimeter when this maximum temperature as such is observed. I obtained the following results: the temperature of the air in the experiments was 13°·2–13°·5.

Experiments with Glass 1.			Experiments with Glass 2.		
T'.	t'.	Difference.	T'.	t'.	Difference.
15°·51	15°·13	0°·38	15°·71	15°·50	0°·21
14·96	14·72	0·24	15·96	15·65	0·31
16·11	15·94	0·17	15·16	14·91	0·25
15·56	15·36	0·20	14·76	14·47	0·29
14·24	14·05	0·19	14·66	14·33	0·33
15·96	15·64	0·32	15·56	15·24	0·32

A closer agreement in the numbers expressing the difference between T' and t' is difficult to attain, since a certain time is necessary to observe the occurrence of the maximum temperature, and during the time in which the thermometer B remains constant, the thermometer A still sinks; according to the moment at which the maximum temperature is considered to be established, this difference may be obtained different, and the smaller the later the observation is made. Moreover the magnitude of this difference between T' and t' depends on the difference between t' and the temperature of the air. I have always endeavoured to work under the same circumstances, and especially to arrange the experiments so that the maximum temperature of the water in the calorimeter did not exceed by more than 2° the temperature of the air[†]. For these experiments and the apparatus which I used, I assumed, on the basis of the preceding experiments, that if the water of the calorimeter had assumed its maximum temperature t', the contents of the glass were 0°·3 higher; that is, I put throughout T', the temperature to which the contents of the glass immersed in the calorimeter had fallen, $=t'+0^\circ\cdot3$.

24. It is a matter of course that, in introducing this correction for obtaining the tem-

---

[*] In these experiments, in order to ensure uniformity in the temperature of the water, the stirrer was kept in continual motion, and the same process followed as in ascertaining the specific heat.

[†] A preliminary experiment shows how cool the water in the calorimeter ought to be. Water which is somewhat cooler than the surrounding air, may be kept in stock for such experiments by placing it in a cylindrical flask covered externally with filtering paper, and standing in a dish of water, so that the paper is always moist. To warm the water in the calorimeter, it was merely necessary, with apparatus of the dimensions I used, to lay the hand on it for a short time.

perature of the contents of the glass at the time the maximum temperature has been attained in the calorimeter, it is unnecessary to give the indications of T′ in hundredths of a degree; and since the temperature T, to which the glass with its contents was heated in the mercury-bath, only serves to deduce the difference T−T′, it is unimportant in giving this temperature to do so in hundredths of a degree. The accuracy of the determinations of specific heat, in so far as it depends on determinations of temperature, is limited by the accuracy with which the difference of T−T′ and $t'-t$ are determined (where t is the original temperature of the water in the calorimeter, and the other letters have the meanings previously assigned to them). To have one of these differences very accurately, while the other is much less accurately determined, avails nothing for the accuracy of the final results. It is at once seen that in my experiments, and especially in those of NEUMANN and REGNAULT, the hundredths of a degree have a greater significance for the small difference $t'-t$, than the tenths of a degree for the great difference T−T′.

The correction for educing the value of T′, which I have just discussed, is of course more important the smaller the difference T−T′; for most of my experiments in which this difference is about 30°, the significance of this correction is inconsiderable, if the contents of the glass be a good conductor. I give a few numbers. The experiments given in § 25 on the specific heat of mercury, which, by using this correction, give it at 0·0335 in the mean, give it =0·0331 if this correction is neglected, that is, T′ made=t'. The fourth series of experiments, given in § 27, for determining the specific heat of coal-tar naphtha A, give it at 0·425 when this correction is made, and at 0·420 when it is omitted. The first series of experiments in § 33, for determining the specific heat of sulphur, give it at 0·159 when this correction is used, and at 0·152 when it is neglected. Whether in all such cases T′ is put $=t'$, or$=t'^{\circ}+0^{\circ}\cdot3$, is of inconsiderable importance. The correction in question is inadequate if the substance in the glass is a bad conductor; for example, when the solid in the glass is a pulverulent or porous mass, in which the moistening liquid stagnates (compare § 18). That, under such circumstances, the numbers obtained for the specific heat are found somewhat too small must be remembered in § 41 in the case of chromium, and in § 52 in the case of chloride of chromium. Too small numbers are also obtained, if in the experiments the maximum temperature of the cooling water exceeds that of the air by much more than 2°. Such experiments are not comparable with the others, for example, with those made for the purpose of ascertaining the ancillary magnitudes occurring in the calculation of the results; for them this correction is inadequate, and the loss of heat which the contents of the calorimeter experiences between the time which elapsed between immersing the glass and the establishment of the maximum temperature is too great. By individual examples in § 25 in the case of water, in § 39 in the case of copper, and § 41 in the case of iron, I shall call to mind how this source of error may give somewhat too small numbers for the specific heat; but I have always tried to avoid this error, since I saw its importance in my first preliminary experiments.

25. I first attempted to test my method by some experiments in which water or mercury was placed in the calorimeter. For the specific heats of these liquids the following numbers were obtained, calculated by the formula

$$\text{sp. H}=\frac{M(t'-t)-x(T-T')}{f(T-T')},$$

in which the signification of *f* is manifest from what follows, that of the other letters from what has been given before.

In the experiments in which a readily vaporizable liquid was contained in the glass, such as water, or coal-tar naphtha, a sensible formation of vapour took place, although the temperature did not exceed 50°. If the glass containing the liquid was heated in the mercury-bath (compare fig. 7), vapour was formed in the empty space below the cork which served as stopper; if the glass was then brought into the water of the calorimeter, this vapour condensed and settled partially on the stopper. The stopper did not act materially on the water of the calorimeter (see fig. 5). The quantity of liquid in the glass which acted directly on the water of the calorimeter, decreased somewhat in each experiment; but this decrease is very inconsiderable. In the following experiments *f* denotes first the weight of the liquid in the glass at the commencement of the experiment, and at last its weight at the end of the experiments, that is, after subtracting the liquid which had vaporized and condensed on the stopper. After the end of the experiment the stopper was dried to remove the liquid, and by another weighing of the glass, together with its contents and stopper, the weight of the liquid still contained in the glass was obtained. The decrease of weight of the liquid in the glass was always found to be inconsiderable, and might without any harm have been neglected; for the last experiment of a series I have always taken the diminished weight of the liquid into account, but for those between the first and the last I have neglected the diminution of the weight of the liquid in the glass. What I have here said explains a remark of frequent subsequent use, "after drying the stopper." In reference to the influence of the formation of vapour on the accuracy of the results obtained for the specific heat of the individual substances, compare § 38.

Two series of experiments in which water was contained in the glass, gave the following results for the specific heat of this liquid:—

Experiments with Glass 1. Temperature of the Air 19°·0.

T	T'.	*t'*.	*t*.	M.	*f*.	*x*.	sp. H.
°	°	°	°	grms.	grms.	grm.	
45·2	20·9	20·62	16·83	26·945	3·43	0·651	1·035
46·6	21·2	20·92	17·03	26·935	„	„	1·013
47·4	21·3	20·96	17·03	26·965	3·42*	„	0·997

* After drying the stopper.

Experiments with Glass 3. Temperature of the Air 19°·0.

T.	T'.	t'.	t.	M.	f.	x.	sp. H.
				grms.	grms.	grm.	
46°·8	21°·1	20°·76	17°·03	26·95	3·445	0·453	1·004
46·8	21·1	20·83	17·12	26·985	„	„	0·999
47·0	21·2	20·93	17·22	26·935	3·435*	„	0·996

The value found for the specific heat of the contents of the glass comes very near the number 1, assumed for the specific heat of water†.

Determinations in which mercury was contained in the glass gave the following results for the specific heat of the contents of the glass.

Experiments with Glass 1. Temperature of the Air 13°·8–14°·4.

T.	T'.	t'.	t.	M.	f.	x.	sp. H.
				grms.	grms.	grm.	
51°·1	16°·8	16°·50	13°·41	26·945	53·015	0·651	0·0335
48·5	16·8	16·48	13·64	26·95	„	„	0·0333
45·2	16·5	16·20	13·63	26·965	„	„	0·0333

Experiments with Glass 2. Temperature of the Air 13°·8–14°·4.

T.	T'.	t'.	t.	M.	f.	x.	sp. H.
				grms.	grms.	grm.	
50°·0	17°·1	16°·79	13°·74	26·935	60·015	0·487	0·0335
45·6	16·7	16·41	13·72	26·935	„	„	0·0337

The mean of these five determinations gives 0·0335 for the specific heat of mercury, in accordance with the results found by other observers for this metal (0·0330 between 0° and 100°, Dulong and Petit; 0·0333, Regnault).

26. For the liquid which is to be placed in the glass along with the substance whose specific heat is to be investigated, I could in many cases use water; but many substances, the

---

* After drying the stopper.

† In § 24 it was mentioned that the numbers obtained for the specific heat of the contents of the glass are somewhat too small, if the maximum temperature of the water in the calorimeter, t', exceeds the temperature of the air by much more than 2°. As an example I give the following determinations, in which the glass used contained water.

Experiments with Glass 1. Temperature of the Air 13°·5–13°·8.

T.	T'.	t'.	t.	M.	f.	x.	sp. H.
				grms.	grms.	grm.	
46°·5	18°·1	17°·81	13°·64	26·94	3·40	0·651	0·976
43·9	16·7	16·38	12·33	26·955	„	„	0·989

Experiments with Glass 2. Temperature of the Air 13°·5–13°·8.

T.	T'.	t'.	t.	M.	f.	x.	sp. H.
				grms.	grms.	grm.	
49°·1	18°·3	18°·03	13°·37	26·94	3·66	0·487	0·981
47·6	18·3	18·04	13·66	26·99	„	„	0·969
47·0	17·5	17·22	12·73	26·97	3·65*	„	0·991

* After drying the stopper.

determination of which is important, dissolve in water, and hence I had to use a different liquid. Coal-tar naphtha has the advantage that it is a mobile liquid, does not dissolve most salts, and does not resinify in contact with the air; but besides the disagreeable odour, with continuous working, respiring air charged with its vapour appears to act injuriously on the organs of the voice. As compared with water, coal-tar naphtha has the disadvantage, that its specific heat must be specially determined, and any possible uncertainty in this is transferred to the determination of the specific heat of the solid substance; but the thermal action of a given volume of naphtha is only about $\frac{1}{3}$ that of the same volume of water[*]; and in experiments in which the thermal action of a solid substance is determined, along with that of the necessary quantity of liquid which is contained with that substance in a glass, the thermal action due to the solid is a larger fraction of the total if coal-tar naphtha is used than if water is the liquid, which is a favourable circumstance in the accurate determination of specific heat. As it was more especially important for me to obtain comparability in the results for specific heat, I have, for a great many substances which are insoluble in water, and for whose investigation water might have been used, also employed coal-tar naphtha. Water was used for a few substances which are soluble in coal-tar naphtha (sulphur, phosphorus, sesquichloride of carbon, for instance). Several substances I determined both with water and with naphtha; the results thus obtained agree satisfactorily. To the question as to whether any possible change in the specific heat of naphtha with the temperature can be urged against the use of this liquid, I shall return in § 29.

27. The coal-tar naphtha A which I principally used in the subsequent experiments was prepared from the commercial mixture of hydrocarbons $\mathrm{Ꞓ}_n\mathrm{H}_{2n-6}$, by purifying it by means of sulphuric acid, treating the portion which distilled between 105° and 120° with chloride of calcium for six days, then again rectifying it, and collecting separately that which passed between 105° and 120°. This liquid had the specific gravity 0·869 at 15°; in determining its specific heat I made four series of experiments, two at first when I was engaged on experiments in which I used this naphtha, and two towards the end.

I.—Experiments with Glass 1. Temperature of the Air 12°·1–12°·9.

T.	T'.	t'.	t.	M.	f.	x.	sp. H.
°	°	°	°	grms.	grms.	grm.	
46·1	13·8	13·51	11·24	26·99	2·875	0·651	0·433
48·6	14·0	13·71	11·24	26·945	2·875 [†]	„	0·443
45·5	14·1	13·83	11·59	26·97	2·975 [‡]	„	0·439
45·3	14·3	14·01	11·80	26·94	2·970 [†]	„	0·428
					Mean	. . .	0·436

---

[*] The specific heat of the coal-tar naphtha A, with which I made most of my experiments, is 0·431, and its specific gravity at 15°=0·869.

[†] After drying the stopper.

[‡] After adding some naphtha.

II.—Experiments with Glass 2. Temperature of the Air 12°·1–12°·7.

T.	T'.	t'	t.	M. grms.	f. grms.	x. grm.	sp. H.
49°·0	13°·8	13°·53	11°·02	26·955	3·28	0·487	0·438
45·9	14·1	13·83	11·50	26·93	3·48 *	,,	0·427
43·3	14·2	13·86	11·73	26·95	,,	,,	0·427
46·6	14·5	14·23	11·85	26·95	3·475 †	,,	0·435
					Mean	. . .	0·432

III.—Experiments with Glass 1. Temperature of the Air 16°·7.

T.	T'.	t'.	t.	M. grms.	f. grms.	x. grm.	sp. H.
51°·4	18°·6	18°·32	16°·02	26·98	2·895	0·651	0·429
51·5	18·4	18·06	15·73	26·97	,,	,,	0·431
51·5	18·4	18·14	15·81	26·985	,,	,,	0·431
51·0	18·5	18·22	15·93	26·96	2·88†	,,	0·434
					Mean	. . .	0·431

IV.—Experiments with Glass 3. Temperature of the Air 16°·7.

T.	T'.	t'.	t.	M. grms.	f. grms.	x. grm.	sp. H.
51°·7	18°·7	18°·43	16°·22	26·935	3·195	0·453	0·423
50·7	18·6	18·32	16·14	26·935	,,	,,	0·431
50·7	18·6	18·27	16·13	26·95	,,	,,	0·421
50·2	18·6	18·26	16·14	26·93	3·18 †	,,	0·426
					Mean	. . .	0·425

The average of the means of these four series of experiments, 0·436, 0·432, 0·431, 0·425, gives 0·431 as the specific heat of the coal-tar naphtha A between 14° and 52°; this value is taken in calculating the experiments in the following section.

28. If it were only a question as to the determination of the specific heat of this naphtha, the method described in the preceding might be advantageously replaced by another. For by this method the specific heat of the liquid must be found somewhat too great, owing to the fact that in the empty space in the glass under the stopper a distinct quantity of vapour is formed, which condenses when the glass is dipped in the water of the calorimeter (compare § 25). Direct experiments ‡, in which this formation of vapour was almost entirely avoided, have shown that the method used for the previous determinations, that is, the use of glasses for heating the liquid in which a

---

* After adding some naphtha.

† After drying the stopper.

‡ I determined the specific heat of coal-tar naphtha A, using a glass in which only very little vapour could form above the heated liquid. This glass (which I used in experiments for the determination of the specific heat of liquid compounds) had a narrow neck, and was filled so that there was very little space in which vapour could form; the calorimetric value of this glass, in so far as it was immersed in the water of the

relatively considerable space above the liquid remains empty, gives the specific heat of readily vaporizable liquids somewhat too high, but that at the same time this influence of the formation and condensation of vapour is very small in the conditions under which I worked*.—The number 0·431 obtained in the previous determinations expresses the thermal action due to the cooling of 1 grm. naphtha A through 1° in my experiments, which thermal action depends to by much the greatest extent on the specific heat of this liquid, and only to a very small extent on the condensation of the previously formed vapour. In calculating the experiments communicated in the third section, that number is taken as the expression for the thermal action of naphtha, which is put as proportional to the weight of the latter. This is, strictly speaking, not accurate, in so far as the thermal action arising from condensation of vapour only depends on the magnitude of the empty space and the temperature, and not on the quantity of naphtha in the glass. But the small possible inaccuracy due to this cause in my experiments is not to be compared with other uncertainties. The manner in which I have taken into account the naphtha contained in the glass corresponds most accurately to the actual conditions of the experiment, when this thermal action is most considerable (only naphtha in the glass); and if my mode of calculation less satisfies these conditions (less naphtha in the glass), the entire amount is less considerable, and the influence of that which might be missed in that calculation, a vanishing quantity.

29. My experiments have been made at very different temperatures. The temperature of the air was often something under 10°, sometimes above 20°. These numbers represent the limits to which the liquid in the glass, together with the solid substance cooled in the calorimeter. In most experiments I heated the glass with its contents to about 50°, in some cases not so high. Now, for the various intervals of temperature within which the liquid in the glass cooled, can its specific heat be assumed to be always the same? For water this may be done, and for coal-tar naphtha I did not

---

calorimeter (comp. fig. 6), was = 0·688 grm. A series of experiments in which this glass was used to determine the specific heat of the naphtha A gave the following results:—

Temperature of the Air 15°·5–15°·6.

T.	T'.	t'.	t.	M.	f.	x.	sp. H.
°	°	°	°	grms.	grms.	grm.	
52·5	17·8	17·53	14·93	26·945	3·205	0·688	0·415
49·6	17·4	17·13	14·73	26·955	„	„	0·412
50·9	17·6	17·29	14·83	26·96	„	„	0·407
50·5	17·6	17·26	14·83	26·975	„	„	0·407
51·6	17·7	17·38	14·84	26·985	„	„	0·416
50·9	17·8	17·47	15·03	26·94	„	„	0·405
						Mean . . .	0·410

* This is seen from the experiments on water communicated in § 25, and from the subsequent determinations in the next section, in which water was contained in the glass along with the solid substance.

doubt it while engaged in my experiments. I first, when they were finished, became acquainted with REGNAULT's[*] investigations on the specific heat of liquids at various temperatures; according to these experiments the specific heat of some liquids considerably increases with the temperature. I have not directly investigated coal-tar naphtha in this respect, but it is probable that the specific heat of this mixture of hydrocarbons $\mathrm{C}_n \mathrm{H}_{2n-6}$, alters but little with the temperature, and it is certain that this change is without influence on the accuracy of my determinations of the specific heats of solid substances. REGNAULT's experiments[†], made by the method of cooling, show no change for benzole, $\mathrm{C}_6 \mathrm{H}_6$, between 20° and 5°, while there is a distinct change in the case of alcohol. For pure benzole[‡] I found the specific heat by the method of mixture to be 0·450 between 46° and 19°; REGNAULT[§] found it between 71° and 21° to be 0·436. These numbers, obtained with different preparations, are not indeed comparable for a decision of the question just discussed, but they render improbable a considerable increase in the specific heat of benzole with the temperature. What I more especially lay weight upon is this: the specific heats of solids which I have determined at various temperatures, by their agreement with the numbers previously found by others, do not indicate any influence of a change of specific heat of naphtha with the temperature.

30. My stock of the naphtha, discussed in § 27, was used before I had investigated all the solid substances, for which a determination of the specific heat appeared necessary. Another quantity of the same coal-tar naphtha was subjected to the same treatment as indicated there, and the portion passing over between 105° and 120° used for the remainder of the experiments. To ascertain the specific heat of this naphtha B, I made the four following series of experiments:—

I.—Experiments with Glass 1. Temperature of the Air 18°·1–18°·3.

T.	T'.	t'.	t.	M.	f.	x.	sp. H.
°	°	°	°	grms.	grms.	grm.	
51·5	19·6	9·33	17·22	26·96	2·70	0·651	0·419
52·7	19·9	19·64	17·49	26·95	,,	,,	0·413
50·5	19·8	19·54	17·51	26·99	,,	,,	0·420
49·9	20·0	19·73	17·75	26·995	2·695[‖]	,,	0·422
					Mean	. . .	0·418

---

[*] Relation des expériences . . . . pour déterminer les lois et les données physiques nécessaires au calcul des machines à feu, vol. ii. p. 262 (1862).

[†] Ann. de Chim. et de Phys. [3] vol. ix. pp. 336 & 349.

[‡] POGGENDORFF's 'Annalen,' vol. lxxv. p. 107.

[§] Relation, etc. . . . ., vol. ii. p. 283.

[‖] After drying the stopper.

II.—Experiments with Glass 3. Temperature of the Air 18°·1–18°·3.

T.	T'.	t'.	t.	M. grms.	f. grms.	x. grm.	sp. H.
51°·4	19°·7	19°·36	17°·32	26·94	3·085	0·453	0·415
51·5	19 9	19·63	17·56	26·965	„	„	0·426
49·1	19·9	19·61	17·73	26·955	„	„	0·416
50·5	20·1	19·82	17·86	26·98	3·08 *	„	0·418
					Mean	. . .	0·419

III.—Experiments with Glass 1. Temperature of the Air 17°·8–18°·3.

T.	T'.	t'.	t.	M. grms.	f. grms.	x. grm.	sp. H.
52°·2	19°·8	19°·49	17°·27	26·97	2·80	0·651	0·427
50·6	20·0	19·73	17·64	26·96	„	„	0·425
51·2	20·2	19·92	17 82	26·98	„	„	0·420
51·3	20·2	19·86	17·76	26·99	„	„	0·418
50·4	20·2	19·86	17·85	26·95	2·785 *	„	0·410
					Mean	. . .	0·420

IV.—Experiments with Glass 3. Temperature of the Air 18°·4.

T.	T'.	t'.	t.	M. grms.	f. grms.	x. grm.	sp. H.
50°·2	19°·7	19°·43	17°·33	26·96	3·31	0·453	0·424
50·1	20·1	19·77	17·66	26·99	„	„	0·416
52·5	20·2	19·87	17·65	26·96	„	„	0·423
50·1	20·1	19·83	17·82	26·95	„	„	0·409
51·4	20·2	19·93	17·82	26·97	3·29 *	„	0·417
					Mean	. . .	0·418

The average of the means of these four series of experiments, 0·418, 0·419, 0·420, 0·418, gives 0·419 for the specific heat of coal-tar naphtha B between 20° and 50°.

In the preceding method of experiment, whether water or naphtha of the kind described is contained in the vessel, a temperature much higher than 50° cannot be employed; for otherwise the quantity of liquid evaporating and condensing on the stopper becomes far too considerable. Perhaps with hydrocarbons of higher boiling-points higher temperatures might be ventured upon: I have no experiments on this subject.

PART III.—DETERMINATION OF THE SPECIFIC HEAT OF INDIVIDUAL SOLID SUBSTANCES.

31. By the method whose principle and mode of execution have been discussed in the preceding, I have determined the specific heat of a large number of solid substances. I

---

* After drying the stopper.

should have liked to include a still larger number of bodies in my investigations; but a limit was put by the straining of the eyes from constant reading of finely divided scales, and by the injurious action which the long-continued working with coal-tar naphtha produces.

My crystallographic collection furnished me with much material for investigating the specific heat of both naturally occurring and artificially prepared substances, but for much more I have to thank others. By far the greater part of the chemical preparations investigated I obtained from the Laboratory of the University of Giessen, through the kindness of the Director, Professor Will, and of the assistants, Professor Engelbach, to whom my thanks are especially due, Drs. Körner and Dehn. Professor Wöhler, of Göttingen, placed a number of chemical preparations at my disposal. Professor Bunsen, of Heidelberg, has helped me to the investigation of some rubidium-compounds. Platinum and iridium I have been furnished with by M. Heræus, the proprietor of the well-known platinum-manufactory in Hanau. I have had a very large number of minerals from the mineral collection of the University of Giessen, through the kindness of the Director, Professor Knop; and to obtain the necessary quantity of dioptase, Professors Blum of Heidelberg, and Dunker of Marburg, have contributed.

32. The signification of the letters in the statement of the following experiments and their calculation is clear from § 17; in reference to the value of the numbers for M, compare § 21, for x § 22, for T′ § 23, for y § 27 and § 30.

It would require too much space always to give the comparison of my results with those of other observers. I can only do this in individual cases where there are considerable differences and their discussion is of importance. For other substances, where there are recent observations by trustworthy observers, the Tables in § 82 to § 89 give data for comparison.

33. *Sulphur*: pieces of transparent (rhombic) crystals from Girgenti. I made three series of experiments with this substance.

I.—Experiments with Water. Glass 1. Temperature of the Air 13°·2.

T.	T′.	t′.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
45·8	15·5	15·24	11·74	26·95	4·16	1·765	1·000	0·651	0·168
46·0	16·2	15·93	12·52	26·935	„	„	„	„	0·160
45·2	16·0	15·73	12·42	26·945	„	„	„	„	0·153
45·8	16·4	16·05	12·74	26·96	„	1·75[*]	„	„	0·153
							Mean . . .		0·159

---

[*] After drying the stopper: compare § 25.

II.—Experiments with Water. Glass 2. Temperature of the Air 13°·2.

T.	T′.	t′.	t.	M.	m.	f.	y.	x.	sp. H.
				grms.	grms.	grms.		grm.	
45°·8	16°·4	16°·07	12°·36	26·96	4·815	2·09	1·000	0·487	0·171
47·3	16·6	16·33	12·46	26·95	„	„	„	„	0·170
44·1	16·5	16·15	12·74	26·925	„	„	„	„	0·156
45·1	16·6	16·28	12·77	26·96	„	2·07*	„	„	0·159
							Mean	. . .	0·164

Both these series of determinations are from the time when I first worked at this subject. Towards the end, when I had acquired tolerable readiness, I made a third series, which agreed very closely with the results previously obtained.

III.—Experiments with Water. Glass 3. Temperature of the Air 17°·2.

T.	T′.	t′.	t.	M.	m.	f.	y.	x.	sp. H.
				grms.	grms.	grms.		grm.	
43°·7	19°·1	18°·83	15°·79	26·99	4·92	2·065	1·000	0·453	0·166
43·5	19·1	18·84	15·84	26·97	„	„	„	„	0·162
43·3	19·2	18·92	15·92	26·94	„	„	„	„	0·170
43·1	19·2	18·87	15·93	26·98	„	2·05*	„	„	0·166
							Mean	. . .	0·166

Taking the mean of the means obtained in the three series of experiments, 0·159, 0·164, 0·166, we obtain 0·163 as the specific heat of rhombic sulphur between 17° and 45°. By the method of cooling, Dulong and Petit found the specific heat of sulphur at the mean temperature to be 0·188; Neumann found 0·209 by the method of mixture; for sulphur which had been purified by distillation, fused and cast in rolls, Regnault found[†] the specific heat between 14° and 98° to be 0·2026. In these experiments a development of heat depending on a change from amorphous sulphur into rhombic-crystallized appears to have cooperated, and to have caused the circumstance observed by Regnault, that after immersing the heated sulphur in the water of the calorimeter, the maximum temperature was only set up after an unusually long time. Sulphur which has solidified after being melted, usually contains an admixture of amorphous sulphur, the greater the more the melting-point has been exceeded, which at the ordinary temperature passes slowly, at 100° more rapidly, into crystallized, accompanied by disengagement of heat. The transformation of the sulphur set up by the heating, and continued in the water of the calorimeter, brought about this slow appearance of the maximum temperature, and made the specific heat appear too great; for Regnault's subsequent determinations[‡], also made between 97° and 99° and the mean temperature, gave it considerably less: 0·1844 for freshly melted sulphur (in which

---

[*] After drying the stopper.

[†] Ann. de Chim. et de Phys. [2] vol. lxxiii. p. 50.

[‡] Ibid. [3] vol. ix. pp. 326 & 344.

superfusion had been avoided?); 0·1803 for sulphur which had been melted two months; 0·1764 for what had been melted two years (and which had then given 0·2026); 0·1796 for sulphur of natural occurrence. The difference between the latter result and my own doubtless depends, partially at least, on the fact that REGNAULT's determination was made between 14° and 99° (the latter of which temperatures is very near the melting-point of rhombic sulphur); mine was made between 17° and 45° [*].

Tellurium: crystalline pieces [†].

Experiments with Naphtha A. Glass 3. Temperature of the Air 18°·6–19°·1.

T.	T'.	t'.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
51·8	20·4	20·07	17·96	26·93	10·80	1·93	0·431	0·453	0·0486
51·3	20·3	20·02	17·93	26·98	,,	,,	,,	,,	0·0495
51·5	20·7	20·36	18·33	26·93	,,	,,	,,	,,	0·0454
51·0	20·7	20·43	18·43	26·955	,,	1·91 [‡]	,,	,,	0·0466
							Mean	. . .	0·0475

34. *Boron.*—I have made some experiments with this substance, which have some interest for the question whether this body has essentially different specific heats in its different modifications; but the results are not very trustworthy, owing to the spongy nature of the amorphous boron and the doubtful purity of the crystallized variety.

The *amorphous Boron* [§] which I investigated was pressed in small bars, and had stood several days *in vacuo* over sulphuric acid.

Experiments with Naphtha A. Glass 1. Temperature of the Air 17°·0–17°·2.

T.	T'.	t'.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grm.	grms.		grm.	
49·0	18·7	18·73	16·36	26·955	1·52	2·515	0·431	0·651	0·246
48·1	18·6	18·55	16·23	26·965	,,	,,	,,	,,	0·254
48·0	18·6	18·64	16·33	26·95	,,	,,	,,	,,	0·252
47·9	18·7	18·72	16·42	26·95	,,	2·49 [‡]	,,	,,	0·262
							Mean	. . .	0·254

Even if the results of the individual experiments agree tolerably with each other they are not very trustworthy; for the quantity of boron (only 1½ grm.) is very small, and the amount of heat due to the boron is a very small part of the total (comp. § 19). Yet I do not consider the result of the above series of experiments (that between 18° and 48° the specific heat of amorphous boron is about 0·254) as being very far from the truth. There are no considerable accidental errors of observation in these experiments, to judge from their agreement with one another. Of the constants for calculating the experiments, x and y must be taken into account in regard to any possible uncertainty. It has been assumed that x=0·615 and y=0·431; if we took x=0·63 and y=0·41, the specific heat as the mean of four experiments would be =0·30; if x were 0·67 and y 0·45, the specific heat would be 0·21. But from what has been communicated in § 22 and § 27 in reference to the determination of x and y, it cannot be assumed that any possible uncertainty in reference to these values can reach either of the above limits. It can be assumed with the greater certainty that the specific heat of amorphous boron is between 0·2 and 0·3 and nearly 0·25, because x and y could not simultaneously both be found too great or too small (if x had been too small y would have been too great, and *vice versâ*).

---

[*] There is nothing known certainly as to whether the different modifications of sulphur have essentially different specific heats. MARCHAND and SCHEERER's experiments on brown and yellow sulphur made by the method of cooling, compare in Journal für Prakt. Chemie, vol. xxiv. p. 153.

[†] "Obtained from Vienna, and obviously distilled."—WÖHLER.

[‡] After drying the stopper.

[§] "Prepared from boracic acid by sodium, and treated with hydrochloric acid."—WÖHLER.

Crystallized Boron[*].

Experiments with Naphtha A. Glass 3. Temperature of the Air 18°·9–18°·7.

T.	T′.	t'.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
50·9	20·8	20·52	18·53	26·94	2·82	1·53	0·431	0·453	0·237
51·3	20·8	20·52	18·52	26·975	„	„	„	„	0·233
51·5	20·8	20·53	18·53	26·985	„	„	„	„	0·229
51·4	20·8	20·46	18·43	26·99	„	1·52[†]	„	„	0·222
							Mean . . .		0·230

Hence the specific heat of the crystallized (adamantine) boron investigated is 0·230 between 21° and 51°; it is pretty near that found for amorphous boron, 0·254. REGNAULT found[‡] (between 98° and 100° and the mean temperature) 0·225 for a specimen of crystallized boron prepared by ROUSSEAU; 0·257 for one prepared by DEBRAY; 0·262 for one obtained from DEVILLE; and 0·235 for a specimen of graphitic boron prepared by DEBRAY. The specific heat of amorphous boron could not be determined by REGNAULT's method, because, when heated to 100° in air, it partially oxidizes into boracic acid with disengagement of heat (three experiments, in which the quantity of boracic acid formed was determined, and its specific heat, but not the thermal action due to the formation of hydrated boracic acid in immersion in water allowed for, gave respectively 0·405, 0·348, and 0·360, which numbers REGNAULT does not consider as even approximately representing the specific heat of amorphous boron), and when greatly cooled disengages a quantity of air when immersed in warmer water, which renders the results uncertain.

---

[*] "Made in Paris, probably by ROUSSEAU, and doubtless by melting borax with aluminium. To conclude from its external appearance, it probably contained some aluminium and carbon: compare the analysis in Ann. der Chem. und Pharm. vol. ci. p. 347."—WÖHLER.

[†] After drying the stopper.

[‡] Ann. de Chim. et de Phys. [3] vol. lxiii. p. 31.

35. *Phosphorus.*—I have only made a few determinations with ordinary yellow phosphorus, which was cast in sticks.

Experiments with Water. Glass 1. Temperature of the Air 10°·9.

T.	T′.	t′.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grms.		grm.	
38·8	13·5	13·20	10·05	26·95	3·075	2·065	1·000	0·651	0·208
33·8	12·9	12·62	10·03	26·97	„	„	„	„	0·204
35·5	13·2	12·91	10·17	26·93	„	2·06*	„	„	0·195
							Mean	. . .	0·202

The specific heat of yellow phosphorus, as deduced from these determinations, is somewhat greater than that found by Regnault, doubtless because in my experiments the upper limit of temperature, T′, was nearer the melting-point of phosphorus, 44°. Compare § 82.

Antimony.—Purified by Liebig's method; crystalline pieces.

I.—Experiments with Naphtha A. Glass 2. Temperature of the Air 14°·7.

T.	T′.	t′.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	°	grms.	grm.		grm.	
46·4	16·0	15·65	13·42	26·945	12·245	1·925	0·431	0·487	0·0539
44·9	15·9	15·64	13·54	26·98	„	„	„	„	0·0520
44·2	15·8	15·53	13·52	26·96	„	1·91*	„	„	0·0496
							Mean	. . .	0·0518

II.—Experiments with Water. Glass 1. Temperature of the Air 15°·8–16°·1.

T.	T′.	t′.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grms.		grm.	
45·0	17·9	17·60	14·22	26·945	11·835	2·095	1·000	0·651	0·0519
45·1	17·9	17·64	14·25	26·96	„	„	„	„	0·0519
45·0	17·9	17·64	14·25	26·965	„	„	„	„	0·0530
45·4	18·1	17·76	14·34	26·955	„	2·085*	„	„	0·0542
							Mean	. . .	0·0528

From these determinations, the average of the means of both series of determinations, 0·0518 and 0·0528, the number 0·0523 is the specific heat of antimony between 17° and 45°.

Bismuth.—Purified by melting with nitre, and cast in small bars. In the case of this metal also, I have made a series of determinations with coal-tar naphtha in the glass, and one with water.

---

* After drying the stopper.

I.—Experiments with Naphtha A. Glass 3. Temperature of the Air 18°·9–18°·8.

T.	T'.	*t'*.	*t*.	M.	*m*.	*f*.	*y*.	*x*.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
50·8	20·6	20·33	18·33	26·99	20·71	1·70	0·431	0·453	0·0291
50·3	20·7	20·42	18·43	26·955	„	„	„	„	0·0302
50·1	20·6	20·33	18·37	26·955	„	„	„	„	0·0292
50·9	20·7	20·40	18·42	26·955	„	1·685*	„	„	0·0284
							Mean . . .		0·0292

II.—Experiments with Water. Glass 1. Temperature of the Air 16°·7–16°·8.

T.	T'.	*t'*.	*t*.	M.	*m*.	*f*.	*y*.	*x*.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
45·2	18·7	18·44	15·25	26·97	19·43	1·995	1·000	0·651	0·0309
45·5	18·9	18·57	15·36	26·965	„	„	„	„	0·0313
45·0	18·9	18·64	15·47	26·975	„	„	„	„	0·0324
46·0	18·1	18·82	15·56	26·99	„	1·985*	„	„	0·0327
							Mean . . .		0·0318

From these determinations we get for the specific heat of bismuth between 30° and 48° the number 0·0305.

36. *Carbon.*—It is known how different are the numbers obtained for the specific heat of carbon in its different forms. I have determined the specific heat for comparatively only a few of the modifications of carbon—for gas-carbon, for natural and artificial graphite. Before the experiment each of these substances was strongly heated for some time in a covered porcelain crucible, and then allowed to cool, and immediately transferred into the glass for its reception, and, after weighing, naphtha poured over it.

Gas-carbon from a Paris gas-works; very dense, of an iron-grey colour, and left very little ash when calcined†. It was used in pieces the size of a pea, and two series of experiments were made.

---

* After drying the stopper.

† This carbon, as well as the above-mentioned varieties of graphite, was analyzed in the Laboratory at Giessen by Mr. Huber. The gas-carbon gave, when placed in a platinum boat and burned in a stream of oxygen,—

	I.	II.	III.	IV.	V.
Carbon........	97·19	98·25	97·73	98·08	98·55
Hydrogen	0·53	0·15	0·68	0·37	1·00
Ash	0·61	0·62	0·73	0·23	0·69
	98·33	99·02	99·14	98·68	100·24

I. Experiments with Naphtha A. Glass 1. Temperature of the Air 18°·9–19°·2.

T.	T′.	t′.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
52·9	20·8	20·53	18·13	26·955	3·135	1·825	0·431	0·651	0·184
52·6	20·9	20·63	18·26	26·98	„	„	„	„	0·185
51·7	20·7	20·42	18·06	26·97	„	„	„	„	0·196
52·4	20·9	20·58	18·23	26·98	„	1·805*	„	„	0·186
							Mean . . .		0·188

II.—Experiments with Naphtha A. Glass 3. Temperature of the Air 20°·5–20°·8.

T.	T′.	t′	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
52·6	22·6	22·33	20·23	26·985	3·345	1·935	0·431	0·453	0·180
52·2	22·5	22·23	20·14	26·985	„	„	„	„	0·183
52·3	22·5	22·20	20·12	26·965	„	„	„	„	0·179
52·5	22·6	22·31	20·22	26·955	„	1·91*	„	„	0·182
							Mean . . .		0·181

These determinations give as the average of the means of both sets of experiments the number 0·185 as the specific heat of gas-carbon between 22° and 52°.

Natural graphite from Ceylon. Left very small quantities of ash when calcined†.

I.—Experiments with Naphtha A. Glass 3. Temperature of the Air 18°·9–19°·2.

T.	T′.	t′.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grms.		grm.	
51·4	20·8	20·48	18·13	26·975	4·025	2·085	0·431	0·453	0·179
51·4	20·8	20·51	18·13	26·99	„	„	„	„	0·186
51·8	20·8	20·54	18·15	26·975	„	„	„	„	0·181
52·0	20·8	20·54	18·13	26·99	„	2·06*	„	„	0·183
							Mean . . .		0·183

---

* After drying the stopper.

† In Mr. Huber's analyses this substance was placed in a platinum boat, then burned in a porcelain tube in oxygen.

	I.	II	III.
Carbon........	..	99·11	98·52
Hydrogen	..	0·17	0·06
Ash	0·26	0·27	0·51
		99·55	99·09

The residual porous ash left after the combustion was tolerably white, with admixed red particles.

II.—Experiments with Naphtha A. Glass 1. Temperature of the Air 19°·0–18°·7.

T.	T'.	t'.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
53·9	21·1	20·77	18·22	26·97	3·515	1·935	0·431	0·651	0·174
52·2	21·0	20·73	18·31	26·96	„	„	„	„	0·176
52·1	21·2	20·86	18·52	26·94	„	„	„	„	0·158
53·0	21·0	20·73	18·32	26·97	„	„	„	„	0·155
52·8	21·0	20·73	18·33	26·965	„	1·91*	„	„	0·160
							Mean	. . .	0·165

III.—Experiments with Naphtha A. Glass 3. Temperature of the Air 19°·9–20°·0.

T.	T'.	t'.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grms.		grm.	
51·6	21·9	21·55	19·33	26·97	3·90	2·05	0·431	0·453	0·174
51·3	22·0	21·71	19·52	26·955	„	„	„	„	0·174
51·5	22·0	21·70	19·52	26·97	„	„	„	„	0·168
51·5	21·9	21·63	19·42	26·96	„	2·04*	„	„	0·175
							Mean	. . .	0·173

The average of the means of these three series of determinations, 0·183, 0·165, and 0·173, gives 0·174 as the specific heat of Ceylon graphite between 21° and 52°.

Iron graphite from Oberhammer, near Sayn, separated upon black ordnance iron. Thin, very lustrous laminæ, freed from iron by treatment with aqua regia as much as possible, yet not completely†.

---

* After drying the stopper.

† This iron graphite, according to Mr. Huber's analyses, in which it was also burned in oxygen in a platinum boat placed in a porcelain tube, gave the following results:—

	I.	II.	III.
Carbon........	97·01	96·12	96·37
Hydrogen	..	0·12	0·18
Ash	4·88	4·87	3·99
	101·89	101·11	100·54

It is probable that both in this graphite and in that of natural occurrence, the hydrogen is not essential, but arises from hygroscopic moisture. The residual ash contained porous particles consisting of sesquioxide of iron and silica, and also small pellets, covered externally with a layer of magnetic oxide of iron: these dissolved in hydrochloric acid at first quietly, and afterwards under disengagement of hydrogen; and in the solution small blisters of graphite could be perceived. It is owing to the oxidation of the iron that the sum of the constituents in all cases exceeds 100.

I. Experiments with Naphtha A. Glass 3. Temperature of the Air 19°·0–18°·7.

T.	T'.	t'.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grms.		grm.	
52·5	20·8	20·53	18·21	26·955	2·51	2·445	0·431	0·453	0·186
52·9	21·1	20·84	18·54	26·98	"	2·565*	"	"	0·156
51·4	20·9	20·64	18·43	26·94	"	"	"	"	0·157
52·0	20·9	20·60	18·33	26·99	"	2·545†	"	"	0·168
							Mean	. . .	0·167

II.—Experiments with Naphtha A. Glass 1. Temperature of the Air 19°·9–20°·0.

T.	T'.	t'.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grms.		grm.	
52·1	21·9	21·57	19·32	26·94	2·48	2·205	0·431	0·651	0·164
51·7	22·0	21·66	19·45	26·97	"	"	"	"	0·163
51·5	22·0	21·73	19·54	26·98	"	"	"	"	0·162
51·5	22·0	21·66	19·46	26·945	"	2·19†	"	"	0·167
							Mean	. . .	0·164

The average of the means of both these series of experiments, 0·167 and 0·164, gives 0·166 as the specific heat of iron graphite between 22° and 52°.

The results previously known in reference to the specific heat of carbon, differ greatly for its different conditions, as also do the results obtained by different inquirers and by different methods for the same condition. But even leaving out of consideration the numbers obtained by De la Rive and Marcet by the method of cooling, there are still considerable differences between Regnault's results, obtained by the method of mixture, and my own. Regnault found for animal charcoal 0·261, for wood-charcoal 0·241, for gas-carbon 0·209, for natural graphite 0·202, for iron graphite 0·197, for diamond 0·1469; his experiments gave greater numbers for the same substance than my own. I think that exactly for a substance like carbon in its less dense modifications, my method promises more accurate results than that of Regnault. Even in mine, the substance, after being strongly heated before the experiment, might absorb gases or aqueous vapour, which would make the specific heat too great. But in Regnault's method this source of error might also operate, and more especially also the source of error due to the disengagement of heat when porous substances are moistened by water. These sources of error, which affect the determination of the specific heat of the various modifications of carbon and make it too high, have the more influence the looser and the more porous the substance investigated. I believe that the only certain determination of the specific heat of carbon is that of diamond, and all other determinations are too high, owing to various circumstances, and in Regnault's experiments with wood and animal charcoal, &c., owing to the heat disengaged when these substances are moistened by water.

---

* After some more naphtha had been added.

† After drying the stopper.

37. *Silicium.*—I have investigated this substance in four different modifications.

Amorphous Silicium *.—For the experiments picked coherent pieces were used, which had stood for several days *in vacuo* over sulphuric acid.

Experiments with Naphtha A. Glass 3. Temperature of the Air 19°·2.

T.	T'.	t'.	t.	M.	m.	f.	y.	x.	sp. H.
				grms.	grm.	grms.		grm.	
51°·5	20°·7	20°·38	18°·13	26·95	1·095	2·88	0·431	0·453	0·251
50·0	20·8	20·54	18·46	26·975	„	„	„	„	0·208
50·4	21·0	20·66	18·55	26·98	„	„	„	„	0·221
50·5	20·9	20·59	18·52	26·935	„	2·87†	„	„	0·177
						Mean . . .			0·214

The very discordant results of these experiments are very little trustworthy; the quantity of silicium, 1 grm., was too small, and its thermal action inconsiderable as compared with that of the other substances immersed with it in the water of the calorimeter.

Graphitoidal Silicium ‡.

Experiments with Naphtha A. Glass 3. Temperature of the Air 16°·7–17°·2.

T.	T'.	t'.	t.	M.	m.	f.	y.	x.	sp. H.
				grms.	grms.	grm.		grm.	
51°·0	18°·8	18°·51	16°·34	26·965	3·155	1·83	0·431	0·453	0·182
52·3	19·1	18·82	16·59	26·975	„	„	„	„	0·181
51·1	18·9	18·62	16·44	26·98	„	„	„	„	0·185
50·4	18·8	18·52	16·43	26·95	„	1·81†	„	„	0·174
						Mean . . .			0·181

Crystallized Silicium.—Grey needles §.

Experiments with Naphtha A. Glass 1. Temperature of the Air 19°·1.

T.	T'.	t'.	t.	M.	m.	f.	y.	x.	sp. H.
				grms.	grms.	grm.		grm.	
53°·8	21°·1	20°·83	18°·53	26·94	2·395	1·955	0·431	0·651	0·168
52·6	21·0	20·74	18·52	26·975	„	„	„	„	0·168
52·3	21·0	20·72	18·52	26·98	„	„	„	„	0·168
51·9	21·0	20·66	18·53	26·975	„	1·935†	„	„	0·156
						Mean . . .			0·165

---

* "Prepared from silicofluoride of potassium by means of sodium."—WÖHLER.

† After drying the stopper.

‡ "Obtained by melting silicofluoride of potassium, or sodium, with aluminium; the aluminium was then extracted with hot hydrochloric acid, and the oxide of silicium with fluoric acid."—WÖHLER.

§ "This silicium was prepared from the silicofluoride of potassium, or sodium, by sodium and zinc, and the lead (from the zinc) removed by nitric acid. Whether it was afterwards treated with hydrofluoric acid I cannot say, but probably so. It was quite unchanged when heated in the vapour of hydrochlorate of chloride of silicium (passed by means of hydrogen). Probably it contained, however, like all silicium reduced by zinc, a trace of iron, which appears when it is heated in chlorine. An experiment with another portion of such silicium gave, however, so little iron that its quantity could not be determined."—WÖHLER.

Fused Silicium**.

Experiments with Naphtha A. Glass 1. Temperature of the Air 18°·9–18°·7.

T.	T'.	*t'.*	*t.*	M.	*m.*	*f.*	*y.*	*x.*	sp. H.
				grms.	grms.	grm.		grm.	
49°·0	20°·5	20°·24	18°·40	26·97	4·17	1·555	0·431	0·651	0·142
50·5	20·7	20·43	18·52	26·96	,,	,,	,,	,,	0·139
49·7	20·6	20·27	18·42	26·965	,,	,,	,,	,,	0·136
50·8	20·7	20·43	18·52	26.94	,,	1·145†	,,	,,	0·136
							Mean . . .		0·138

38. *Tin*: reduced from the oxide, cast in small bars.

I.—Experiments with Naphtha A. Glass 1. Temperature of the Air 17°·8–18°·8.

T.	T'.	*t'.*	*t.*	M.	*m.*	*f.*	*y.*	*x.*	sp. H.
				grms.	grms.	grm.		grm.	
51°·4	19°·8	19°·46	17°·14	26·965	14·835	1·385	0·431	0·651	0·0493
51·4	19·9	19·62	17·23	26·98	,,	,,	,,	,,	0·0539
51·3	20·0	19·72	17·34	26·95	,,	,,	,,	,,	0·0540
51·5	20·3	20·03	17·65	26·995	,,	1·365†	,,	,,	0·0553
							Mean . . .		0·0531

II.—Experiments with Water. Glass 1. Temperature of the Air 15°·5–15°·9.

T.	T'.	*t'.*	*t.*	M.	*m.*	*f.*	*y.*	*x.*	sp. H.
				grms.	grms.	grm.		grm.	
45°·1	17°·5	17°·24	14°·13	26·975	14·62	1·595	1·000	0·651	0·0543
46·4	17·5	17·24	13·94	26·985	,,	,,	,,	,,	0·0571
45·6	17·6	17·34	14·14	26·99	,,	,,	,,	,,	0·0574
45·7	17·6	17·34	14·14	26·95	,,	1·58†	,,	,,	0·0573
							Mean . . .		0·0565

The average of the means of these two series of observations gives 0·0548 as the specific heat of tin between 19° and 48° at 0·0548.

Platinum: several pieces of fused platinum and of thick platinum wire.

Experiments with Naphtha A. Glass 1. Temperature of the Air 17°·8–18°·2.

T.	T'.	*t'.*	*t.*	M.	*m.*	*f.*	*y.*	*x.*	sp. H.
				grms.	grms.	grm.		grm.	
53°·5	20°·4	20°·14	17°·23	26·96	23·625	2·225	0·431	0·651	0·0322
52·8	20·0	19·65	16·73	26·975	,,	,,	,,	,,	0·0335
51·5	20·0	19·73	16·95	26·96	,,	,,	,,	,,	0·0326
50·9	20·0	19·74	17·05	26·96	,,	2·205†	,,	,,	0·0316

I have also made a few experiments with a piece of fused *iridium* which M. HERÆUS gave me.

---

* WÖHLER had obtained it from DEVILLE; it formed a cylindrical piece.

† After drying the stopper.

Experiments with Naphtha A. Glass 3. Temperature of the Air 17°·8–18°·2.

T.	T′.	t′.	t.	M. grms.	m. grms.	f. grms.	y.	x. grm.	sp. H.
51°·8	19°·5	19°.24	16°·93	26·995	16·66	2·04	0·431	0·453	0·0359
51·0	19·6	19·26	16·95	26·97	,,	,,	,,	,,	0·0391?
50·0	19·5	19·24	17·06	26·965	,,	,,	,,	,,	0·0357
50·5	19·6	19·34	17·13	26·93	,,	2·03 *	,,	,,	0·0359

Excluding the second experiment, which is obviously uncertain, these determinations give 0·0358 as the specific heat of iridium. This iridium was not free from metals of smaller atomic weight and greater specific heat. For various specimens of impure iridium, Regnault (Ann. de Chim. et de Phys. [2] vol. lxxiii. p. 53; [3] vol. xlvi. p. 263; vol. lxiii. p. 16) found 0·0368, 0·0363, 0·0419, and for almost pure iridium 0·0326.

39. *Silver*: pure, cast in bars.

Experiments with Naphtha A. Glass 3. Temperature of the Air 18°·9–19°·1.

T.	T′.	t′.	t.	M. grms.	m. grms.	f. grm.	y.	x. grm.	sp. H.
52°·1	21°·1	20°·82	18°·15	26·975	21·51	1·585	0·431	0·453	0·0552
51·5	21·1	20·77	18·14	26·99	,,	,,	,,	,,	0·0557
51·4	20·9	20·62	17·94	26·98	,,	,,	,,	,,	0·0574
50·9	21·0	20·65	18·06	26·95	,,	,,	,,	,,	0·0557
51·0	21·1	20·83	18·25	26·965	,,	1·565 *	,,	,,	0·0558
							Mean	. . .	0·0560

Copper.—Commercial copper wires †.

I.—Experiment with Naphtha A. Glass 1. Temperature of the Air 13°·2.

T.	T′.	t′.	t.	M. grms.	m. grms.	f. grm.	y.	x. grm.	sp. H.
44°·3	15°·9	15°·64	12°·64	26·985	16·505	1·675	0·431	0·651	0·0895
46·2	15·1	14·82	11·43	26·97	,,	,,	,,	,,	0·0949
45·7	15·2	14·91	11·63	26·97	,,	,,	,,	,,	0·0926
47·7	15·2	14·93	11·43	26·98	,,	1·67 *	,,	,,	0·0930
							Mean	. . .	0·0925

---

* After drying the stopper.

† With reference to what has been said in § 24, I here communicate a series of experiments (one of my earliest) where t' was much more above the temperature of the air than usual, and hence too small numbers were obtained for the specific heat of the substance in question.

Experiments with Naphtha A. Glass 2. Temperature 13°·8.

T.	T′.	t′.	t.	M. grms.	m. grms.	f. grm.	y.	x. grm.	sp. H.
45°·6	16°·5	16°·23	13°·02	26·98	18·33	1·96	0·431	0·487	0·0897
48·5	16·9	16·64	13·21	26·97	,,	,,	,,	,,	0·0870
43·7	16·5	16·15	13·21	26·98	,,	1·95*	,,	,,	0·0867

After drying the stopper.

II.—Experiments with Naphtha B. Glass 3. Temperature of the Air 19°·4–19°·0.

T.	T′.	*t*′.	*t*.	M.	*m*.	*f*.	*y*.	*x*.	sp. H.
				grms.	grms.	grm.		grm.	
55°·0	21°·9	21°·62	18°·06	26·96	19·725	1·56	0·419	0·453	0·0909
54·1	21·4	21·11	17·60	26·965	,,	,,	,,	,,	0·0906
53·6	21·2	20·86	17·36	26·99	,,	,,	,,	,,	0·0917
54·2	21·3	20·96	17·44	26·98	,,	,,	,,	,,	0·0902
51·7	21·2	20·85	17·55	26·965	,,	1·545 *	,,	,,	0·0921
							Mean	. . .	0·0911

III.—Experiments with Water. Glass 1. Temperature of the Air 18°·4–18°·7.

T.	T′.	*t*′.	*t*.	M.	*m*.	*f*.	*y*.	*x*.	sp. H.
				grms.	grms.	grm.		grm.	
49°·7	20°·8	20°·50	16°·17	26·95	18·26	1·625	1·000	0·651	0·0965
50·0	20·6	20·32	15·93	26·96	,,	,,	,,	,,	0·0958
49·5	20·8	20·50	16·22	26·93	,,	,,	,,	,,	0·0953
47·9	20·9	20·62	16·64	26·945	,,	1·615 *	,,	,,	0·0934
							Mean	. . .	0·0953

According to these determinations, the mean of the average results 0·0925, 0·0911, 0·0953, the number 0·093 represents the specific heat of copper between 20° and 50°.

40. *Lead*: reduced from sulphate of lead and cast in small bars.

I.—Experiments with Naphtha A. Glass 1. Temperature of the Air 18°·9–18°·8.

T.	T′.	*t*′.	*t*.	M.	*m*.	*f*.	*y*.	*x*.	sp. H.
				grms.	grms.	grm.		grm.	
50°·5	20°·6	20°·33	18°·23	26·995	19·93	1·465	0·431	0·651	0·0308
50·5	20·7	20·43	18·35	26·975	,,	,,	,,	,,	0·0302
50·9	20·7	20·44	18·35	26·965	,,	,,	,,	,,	0·0293
50·5	20·6	20·32	18·24	26·94	,,	1·445 *	,,	,,	0·0302
							Mean	. . .	0·0301

II.—Experiments with Water. Glass 1. Temperature of the Air 15°·5–15°·9.

T.	T′.	*t*′.	*t*.	M.	*m*.	*f*.	*y*.	*x*.	sp. H.
				grms.	grms.	grm.		grm.	
46°·0	17°·5	17°·21	14°·02	26·96	24·845	1·56	1·000	0·651	0·0325
45·3	17·6	17·32	14·23	26·985		,,	,,	,,	0·0322
45·9	17·7	17·42	14·25	26·945	,,	,,	,,	,,	0·0329
46·1	17·9	17·61	14·43	26·985	,,	1·55 *	,,	,,	0·0339
							Mean	. . .	0·0329

The mean of the averages of both series of experiments, 0·0301 and 0·0329, gives for the specific heat of lead between 19° and 48° the number 0·0315

---

* After drying the stopper.

Zinc: purified, cast in small bars.

I.—Experiments with Naphtha A. Glass 3. Temperature of the Air 17°·8–18°·9.

T.	T'.	*t'*.	*t*.	M.	*m*.	*f*.	*y*.	*x*.	sp. H.
				grms.	grms.	grm.		grm.	
51°·5	20°·5	20°·22	17°·23	26·995	15·555	1·745	0·431	0·453	0·0899
51·1	20·3	19·95	16·96	26·985	,,	,,	,,	,,	0·0909
51·7	20·6	20·25	17·24	26·99	,,	,,	,,	,,	0·0905
50·9	20·5	20·23	17·25	26·945	,,	1·72 *	,,	,,	0·0930
							Mean	. . .	0·0911

II.—Experiments with Water. Glass 1. Temperature of the Air 16°·0–16°·5.

T.	T'.	*t'*.	*t*.	M.	*m*.	*f*.	*y*.	*x*.	sp. H.
				grms.	grms.	grm.		grm.	
43°·0	17°·7	17°·43	13°·82	26·98	14·25	1·855	1·000	0·651	0·0943
43·1	18·1	17·84	14·26	26·965	,,	,,	,,	,,	0·0951
42·7	18·1	17·82	14·32	26·96	,,	,,	,,	,,	0·0933
42·7	18·4	18·05	14·54	26·99	,,	,,	,,	,,	0·0977
42·9	18·5	18·23	14·74	26·97	,,	1·845 *	,,	,,	0·0956
							Mean	. . .	0·0952

These determinations give 0·0932 as the mean of the means of the two series of determinations for the specific heat of zinc between 19° and 47°.

Cadmium: cast in small bars.

Experiments with Naphtha A. Glass 1. Temperature of the Air 18°·9–19°·1.

T.	T'.	*t'*.	*t*.	M.	*m*.	*f*.	*y*.	*x*.	sp. H.
				grms.	grms.	grm.		grm.	
53°·7	21°·0	20°·72	18°·24	26·955	13·335	1·555	0·431	0·651	0·0542
51·6	20·9	20·56	18·23	26·97	,,	,,	,,	,,	0·0544
51·9	20·8	20·47	18·12	26·98	,,	,,	,,	,,	0·0538
52·3	20·8	20·52	18·14	26·975	,,	1·535 *	,,	,,	0·0544
							Mean	. . .	0·0542

Magnesium: metallic globules and masses comminuted†.

Experiments with Naphtha A. Glass 1. Temperature of the Air 18°·6–19°1.

T.	T'.	*t'*.	*t*.	M.	*m*.	*f*.	*y*.	*x*.	sp. H.
				grms.	grms.	grm.		grm.	
53°·3	20°·6	20°·32	17°·74	26·995	3·485	1·42	0·431	0·651	0·249
51·8	20·6	20·26	17·83	26·97	,,	,,	,,	,,	0·240
51·0	20·6	20·33	17·94	26·99	,,	,,	,,	,,	0·247
51·6	21·0	20·72	18·33	26·96	,,	1·40 *	,,	,,	0·244
							Mean	. . .	0·245

---

* After drying the stopper.

† "The magnesium was prepared by the methods of DEVILLE and CARON, and WÖHLER. The reguline masses were not remelted, but treated with dilute hydrochloric acid, then washed with water and dried at a gentle temperature."—ENGELBACH.

Iron: pieces of iron wire.

I.—Experiments with Naphtha A. Glass 2. Temperature of the Air 13°·2.

T.	T'.	t'.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
46·6	16·2	15·92	12·52	26·97	17·565	1·46	0·431	0·487	0·108
45·4	15·1	14·83	11·33	26·95	„	„	„	„	0·114
46·0	15·1	14·77	11·22	26·935	„	„	„	„	0·113
46·2	15·2	14·91	11·34	26·98	„	1·455 *	„	„	0·113
							Mean	. . .	0·112

II.—Experiments with Water. Glass 1. Temperature of the Air 16°·8–17°·2.

T.	T'.	t'.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
43·2	18·8	18·46	15·02	26·985	15·57	1·425	1·000	0·651	0·111
42·9	19·1	18·84	15·47	26·975	„	„	„	„	0·112
43·6	19·3	19·04	15·62	26·99	„	„	„	„	0·111
42·5	19·3	19·01	15·72	26·985	„	1·42 *	„	„	0·113
							Mean	. . .	0·112

The means of both series of experiments give for the specific heat of iron between 17° and 44° the number 0·112.

With reference to what has been said in § 24, the following series of experiments made at the beginning of my investigation are given, in which *t'* exceeded the ordinary temperature much more than usual, and hence the numbers for the specific heat of iron were found somewhat too small.

Experiments with Naphtha A. Glass 1. Temperature of the Air 13°·8.

T.	T'.	t'.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
48·1	16·4	16·12	12·73	26·93	15·57	1·185	0·431	0·651	0·111
44·5	16·3	15·97	13·03	26·905	„	„	„	„	0·106
45·7	16·6	16·26	13·23	26·97	„	„	„	„	0·106
47·0	16·7	16·43	13·23	26·96	„	1·17 *	„	„	0·103

Another source of error which may make the numbers for the specific heat of the substance investigated too small, has been discussed in § 18 and 24,—the circumstance, namely, that the substance may fill the glass so densely as to impede the circulation of the liquid, or make it impossible. This circumstance made the numbers for the specific heat of *chromium*, which were obtained from the following series of observations, too small. The chromium was reduced from chloride of chromium according to Wöhler's method by means of zinc (Ann. der Chem. und Pharm. vol. cxi. p. 230); the heavy, finely crystalline powder deposits in the glass as a dense mass impeding the circulation. The following results were obtained:—

---

* After drying the stopper.

Experiments with Naphtha A. Glass 3. Temperature of the Air 19°·8–19°·1.

T.	T′.	t′.	t.	M.	m.	f.	y.	x.	sp. H.
				grm.	grms.	grms.		grm.	
51°·2	21°·6	21°·34	18°·96	26·965	6·725	2·405	0·431	0·453	0·101
51·2	21·6	21·33	18·95	26·97	,,	,,	,,	,,	0·101
50·8	21·5	21·24	18·92	26·945	,,	,,	,,	,,	0·096
51·8	21·5	21·22	18·81	26·99	,,	2·36 *	,,	,,	0·101

As the atomic weight of chromium is somewhat smaller than that of iron, it is to be supposed that the specific heat of chromium is somewhat greater than that of iron.

Aluminium: a piece of a small bar †.

Experiments with Naphtha A. Glass 3. Temperature of the Air 18°·6–18°·4.

T.	T′.	t′.	t.	M.	m.	f.	y.	x.	sp. H.
				grms.	grms.	grm.		grm.	
52°·3	20°·9	20°·64	18°·03	26·98	5·916	1·45	0·431	0·453	0·197
51·9	20·7	20·44	17·83	26·995	,,	,,	,,	,,	0·200
52·2	20·9	20·62	17·95	26·97	,,	,,	,,	,,	0·207
51·0	20·8	20·47	17·93	26·975	,,	1·435 *	,,	,,	0·202
							Mean	. . .	0·202

42. *Hemisulphide of Copper*, $\mathrm{Ꞓu_2S}$ ‡. *Copper-glance* was investigated; a dense specimen with conchoidal fracture from Liberty Mine in Maryland and a crystallized specimen of unknown locality, which also I tested as to its purity.

Experiments with Naphtha A. Glass 1. Temperature of the Air 16°·7.

T.	T′.	t′.	t.	M.	m.	f.	y.	x.	sp. H.
				grms.	grms.	grm.		grm.	
52°·6	19°·0	18°·72	15°·74	26·995	8·775	1·595	0·431	0·651	0·120
52·0	18·9	18·58	15·65	26·995	,,	,,	,,	,,	0·120
52·6	19·0	18·72	15·74	26·99	,,	,,	,,	,,	0·120
51·6	18·8	18·53	15·63	26·96	,,	1·58 *	,,	,,	0·120
							Mean	. . .	0·120

---

* After drying the stopper.

† "By remelting Paris aluminium, by which it became poorer in iron; contains probably still some iron and silicium."—Wöhler.

‡ All formulæ of compounds whose specific heat is discussed in the following are written under the assumption of the new atomic weights (see § 2).

Sulphide of Mercury, HgS. Pieces of a sublimed cake of cinnabar *.

Experiments with Naphtha A. Glass 1. Temperature of the Air 20°·3–21°·1.

T.	T′.	t′.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
50·9	22·2	21·94	19·79	26·95	13·44	1·565	0·431	0·651	0·0516
51·8	22·3	22·02	19·80	26·95	,,	,,	,,	,,	0·0523
51·2	22·4	22·05	19·92	26·98	,,	,,	,,	,,	0·0499
51·8	22·4	22·14	19·93	26·98	,,	1·55 †	,,	,,	0·0528
							Mean	. . .	0·0517

Sulphide of Zinc. ZnS. Fragments of crystals of black *Zinc-blende* from Bohemia.

Experiments with Naphtha A. Glass 1. Temperature of the Air 14°·1.

T.	T′.	t′.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
50·8	16·3	16·02	13·18	26·975	7·00	1·64	0·431	0·651	0·123
46·7	16·1	15·83	13·33	26·935	,,	,,	,,	,,	0·120
44·1	15·9	15·63	13·32	26·94	,,	,,	,,	,,	0·121
44·8	16·2	15·93	13·63	26·94	,,	,,	,,	,,	0·116
43·1	15·9	15·63	13·42	26·97	,,	1·625 †	,,	,,	0·120
							Mean	. . .	0·120

Sulphide of Lead, PbS. Cleavage fragments of *Galena* from the Harz.

Experiments with Naphtha A. Glass 1. Temperature of the Air 14°·5–14°·9.

T.	T′.	t′.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
51·3	16·4	16·05	13·34	26·93	13·835	1·78	0·431	0·651	0·0486
48·6	16·4	16·05	13·54	26·975	,,	,,	,,	,,	0·0495
45·7	16·1	15·83	13·53	26·95	,,	,,	,,	,,	0·0489
48·4	16·2	15·94	13·44	26·925	,,	1·765 †	,,	,,	0·0490
							Mean	. . .	0·0490

---

* This cinnabar was found, on being tested, to be free from admixed uncombined sulphur. In experiments with another specimen of beautiful crystalline appearance, I obtained considerably greater numbers for the specific heat.

Experiments with Naphtha A. Glass 1. Temperature of the Air 16°·3–16°·6.

T.	T′.	t′.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
53·0	18·5	18·23	15·72	26·975	9·805	1·72	0·431	0·651	0·0582
51·5	18·4	18·14	15·76	26·96	,,	,,	,,	,,	0·0557
52·0	18·4	18·13	15·73	26·99	,,	,,	,,	,,	0·0546
51·6	18·5	18·16	15·81	26·97	,,	1·70†	,,	,,	0·0542

But the Naphtha which had been in contact with this cinnabar, left on evaporation a considerable quantity of sulphur, the admixture of which made the specific heat too large.

† After drying the stopper.

43. *Sulphide of Copper and Iron*, $\mathrm{Cu\,Fe\,S_2}$, or $\mathrm{Cu_{\frac{1}{2}}\,Fe_{\frac{1}{2}}\,S}$. Crystals and fragments of crystalline masses of *Copper pyrites* from Dillenburg.

Experiments with Water. Glass 1. Temperature of the Air 17°·2–17°·5.

T.	T'.	t'.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
47·5	19·1	18·82	15·22	26·975	7·365	1·825	1·000	0·651	0·128
48·0	19·4	19·12	15·44	26·985	,,	,,	,,	,,	0·135
47·6	19·5	19·23	15·65	26·975	,,	,,	,,	,,	0·131
48·1	19·6	19·25	15·64	26·985	,,	,,	,,	,,	0·128
47·6	19·5	19·23	15·64	26·94	,,	1·81*	,,	,,	0·133
							Mean . . .		0·131

Bisulphide of Iron, $\mathrm{Fe\,S_2}$. Small crystals and crystalline fragments of *Iron pyrites* from Dillenburg.

I.—Experiments with Naphtha A. Glass 2. Temperature of the Air 13°·3.

T.	T'.	t'.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
47·1	16·0	15·66	12·74	26·92	10·11	1·81	0·431	0·487	0·125
46·2	15·9	15·61	12·77	26·93	,,	,,	,,	,,	0·124
47·1	16·0	15·74	12·87	26·97	,,	,,	,,	,,	0·121
47·9	16·2	15·87	12·95	26·93	,,	1·795*	,,	,,	0·121
							Mean . . .		0·123

II.—Experiments with Water. Glass 3. Temperature of the Air 17°·4–17°·5.

T.	T'.	t'.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grms.		grm.	
47·1	19·7	19·43	15·33	26·97	10·145	2·295	1·000	0·453	0·127
47·5	19·7	19·42	15·23	26·955	,,	,,	,,	,,	0·130
47·6	19·8	19·47	15·33	26·965	,,	,,	,,	,,	0·125
47·4	19·8	19·52	15·36	26·945	,,	2·28*	,,	,,	0·131
							Mean . . .		0·128

The average of the means of both these series of experiments, 0·123 and 0·128, makes the specific heat of iron pyrites between 18° and 47°=0·126.

44. *Suboxide of Copper*, $\mathrm{Cu_2O}$. A crystalline fine-grained *Red copper-glance* of conchoidal fracture was used for investigation.

---

* After drying the stopper.

Experiments with Naphtha A. Glass 3. Temperature of the Air 16°·7.

T.	T′.	t′.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
51·6	18·7	18·36	15·80	26·97	8·67	1·635	0·431	0·453	0·109
51·0	18·6	18·26	15·73	26·995	„	„	„	„	0·110
50·8	18·6	18·26	15·72	26·96	„	„	„	„	0·112
52·3	18·6	18·33	15·66	26·95	„	1·625 *	„	„	0·113
							Mean . . .		0·111

Oxide of Copper, CuO. Granular freshly ignited oxide of copper.

Experiments with Naphtha A. Glass 1. Temperature of the Air 17°·1–17°·9.

T.	T′.	t′.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
51·1	19·2	18·86	16·23	26·965	6·295	1·85	0·431	0·651	0·123
52·0	19·3	18·95	16·23	26·985	„	„	„	„	0·126
51·1	19·4	19·11	16·43	26·94	„	„	„	„	0·132
50·8	19·4	19·07	16·43	26·97	„	1·83 *	„	„	0·131
							Mean . . .		0·128

Oxide of Lead, PbO. Larger pieces of litharge freed by the sieve from the finer particles.

Experiments with Naphtha A. Glass 3. Temperature of the Air 17°·4–17°·6.

T.	T′.	t′.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grms.		grm.	
51·5	19·1	18·83	16·51	26·965	10·17	2·11	0·431	0·453	0·0559
50·4	19·1	18·84	16·63	26·95	„	„	„	„	0·0532
49·2	19·0	18·73	16·56	26·98	„	„	„	„	0·0567
48·5	19·0	18·73	16·63	26·985	„	2·10 *	„	„	0·0554
							Mean . . .		0·0553

Oxide of Mercury, HgO. Crystalline pieces of *Mercurius præcipitatus per se*, freed by the sieve from finer particles.

Experiments with Naphtha A. Glass 1. Temperature of the Air 17°·4–17°·6.

T.	T′.	t′.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
53·1	19·3	19·03	16·64	26·985	8·45	1·925	0·431	0·651	0·0506
52·0	19·1	18·83	16·46	26·975	„	„		„	0·0547
51·5	19·1	18·83	16·53	26·935	„		„	„	0·0510
50·4	19·1	18·82	16·56	26·965	„	1·915 *	„	„	0·0557
							Mean . . .		0·0530

Hydrate of Magnesia, $MgO+H_2O$. Transparent cleavage laminæ of *Brucite* from Texas in Pennsylvania. Dried at 40°–50°.

---

* After drying the stopper.

Experiments with Naphtha A. Glass 3. Temperature of the Air 17°·2.

T.	T′.	t′.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grms.		grm.	
51·9	19·4	19·13	16·02	26·985	3·59	2·29	0·431	0·453	0·318
52·2	19·5	19·23	16·12	26·99	„	„	„	„	0·314
48·2	19·3	19·04	16·32	26·95	„	„	„	„	0·305
49·2	19·6	19·32	16·53	26·985	„	2·27 *	„	„	0·310
							Mean	. . .	0·312

45. *Spinelle*, $MgAl_2O_4$†. Transparent crystalline grains from Ceylon of octahedral form.

I.—Experiments with Naphtha A. Glass 1. Temperature of the Air 11°·5.

T.	T′.	t′.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
45·6	13·8	13·52	10·88	26·925	5·025	1·325	0·431	0·651	0·202
44·1	13·5	13·23	10·68	26·965	„	„	„	„	0·204
46·0	13·8	13·46	10·84	26·96	„	„	„	„	0·193
44·8	13·9	13·55	11·04	26·975	„	1·32 *	„	„	0·193
							Mean	. . .	0·198

II.—Experiments with Naphtha A. Glass 2. Temperature of the Air 11°·5.

T.	T′.	t′.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
45·7	14·1	13·83	11·47	26·935	5·025	1·265	0·431	0·487	0·195
46·1	13·8	13·54	11·14	26·95	„	„	„	„	0·193
46·2	13·2	12·85	10·33	26·975	„	„	„	„	0·205
48·0	13·8	13·45	10·93	26·95	„	1·26 *	„	„	0·190
							Mean	. . .	0·196

I subsequently received another quantity of spinelle grains, also from Ceylon, and have made the following series of experiments with this material.

III.—Experiments with Naphtha A. Glass 1. Temperature of the Air 15°·5.

T.	T′.	t′.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
46·6	17·7	17·36	14·53	26·94	7·53	1·34	0·431	0·651	0·187
47·5	17·8	17·46	14·53	26·96	„	„	„	„	0·190
47·6	17·8	17·54	14·63	26·965	„	„	„	„	0·187
48·4	17·8	17·54	14·54	26·95	„	1·32 *	„	„	0·189
							Mean	. . .	0·188

---

* After drying the stopper.

† Abich's analysis of red spinelle from Ceylon (Rammelsberg's 'Handbuch der Mineralchemie,' p. 161), gave the following results compared with those calculated by the above formula:—

	Al_2O_3.	Cr_2O_3.	MgO.	FeO.	SiO_2.	Total.
Analysis........	69·01	1·10	26·21	0·71	2·02	99·05
Calculation......	71·99	„	28·01	„	„	100·00

These determinations give as the average of the means of the three series of experiments (0·198, 0·196, and 0·188) 0·194 for the specific heat of spinelle between 15° and 46°.

Chrome Iron Ore, $Mg_{\frac{1}{2}} Fe_{\frac{1}{2}} Cr_{\frac{3}{2}} Al_{\frac{1}{2}} O_4$*. Fragments of granular pieces, partly distinctly crystalline, of chrome iron ore from Baltimore.

Experiments with Naphtha A. Glass 1. Temperature of the Air 14°·2–13°·8.

T.	T′.	*t*′.	*t*.	M.	*m*.	*f*.	*y*.	*x*.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
47·6	16·4	16·12	13·14	26·97	7·625	1·63	0·431	0·651	0·163
46·9	16·5	16·24	13·38	26·985	”	”	”	”	0·155
46·8	16·4	16·13	13·24	26·925	”	”	”	”	0·158
46·4	16·4	16·13	13·28	26·955	”	1·61 †	”	”	0·159
							Mean . . .		0·159

Magnetic Iron Ore, $Fe_3 O_4$. Small crystals and crystalline fragments from Pfitsch in Tyrol.

I.—Experiments with Naphtha A. Glass 1. Temperature of the Air 11°·0.

T.	T′.	*t*′.	*t*.	M.	*m*.	*f*.	*y*.	*x*.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
45·1	13·9	13·64	10·54	26·96	9·07	1·43	0·431	0·651	0·156
47·4	13·8	13·53	10·23	26·97	”	”	”	”	0·152
49·1	14·1	13·84	10·42	26·98	”	”	”	”	0·151
47·6	14·1	13·83	10·54	26·92	”	1·415 †	”	”	0·152
							Mean . . .		0·153

II.—Experiments with Water. Glass 3. Temperature of the Air 19°·5–19°·4.

T.	T′.	*t*′.	*t*.	M.	*m*.	*f*.	*y*.	*x*.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
43·5	21·6	21·32	18·02	26·985	10·625	1·925	1·000	0·453	0·159
42·7	21·6	21·32	18·13	26·99	”	”	”	”	0·160
43·0	21·6	21·33	18·12	26·97	”	1·91 †	”	”	0·158
							Mean . . .		0·159

These determinations give as the mean of the averages of the two sets of experiments, 0·156 for the specific heat of magnetic iron ore between 18° and 45°.

---

* The admissibility of this formula for the ore investigated follows from the following comparison of the results calculated from it, with those which Abich had obtained (Rammelsberg's 'Handbuch der Mineralchemie,' p. 172) by the analysis, *a* of compact, *b* of crystallized chrome iron ore from Baltimore.

		Cr_2O_3.	Al_2O_3.	Fe O.	Mg O.	Total.
Analysis	*a*	55·37	13·97	19·13	10·04	98·51
	b	60·04	11·85	20·13	7·45	99·47
Calculation		58·32	13·11	18·37	10·20	100·00

† After drying the stopper.

46. ***Sesquioxide of Iron,*** **$Fe_2 \Theta_3$. Crystals and crystalline pieces of *specular iron* from St. Gotthard.**

I.—Experiments with Naphtha A. Glass 1. Temperature of the Air 12°·4–12°·3.

T.	T′.	t′.	t.	M.	m.	f.	y.	x.	sp. H.
				grms.	grms.	grm.		grm.	
47°·0	14°·8	14°·47	11°·38	26·97	7·51	1·74	0·431	0·651	0·158
46·4	14·7	14·43	11·43	26·975	„	„	„	„	0·153
45·8	14·7	14·44	11·52	26·925	„	„	„	„	0·150
45·8	15·0	14·73	11·83	26·98	„	1·72 *	„	„	0·153
							Mean	. . .	0·154

II.—Experiments with Water. Glass 1. Temperature of the Air 19°·5.

T.	T′.	t′.	t.	M.	m.	f.	y.	x.	sp. H.
				grms.	grms.	grm.		grm.	
44°·1	21°·5	21°·17	17°·81	26·97	8·845	1·935	1·000	0·651	0·161
43·6	21·6	21·26	18·01	26·985	„	„	„	„	0·158
42·5	21·5	21·23	18·12	26·985	„	„	„	„	0·159
42·8	21·6	21·33	18·22	26·98	„	1·92 *	„	„	0·157
							Mean	. . .	0·159

The specific heat of specular iron between 18° and 45°, according to these determinations, is 0·157, the mean of the averages of both series of experiments 0·154 and 0·159.

Iserine, $Fe_{\frac{5}{4}} Ti_{\frac{3}{4}} \Theta_3$ †. Indistinct crystalline grains from the Iserwiese in the Riesengebirge.

Experiments with Naphtha A. Glass 2. Temperature of the Air 14°·2–13°·8.

T.	T′.	t′.	t.	M.	m.	f.	y.	x.	sp. H.
				grms.	grms.	grm.		grm.	
46°·6	17°·1	16°·77	13°·43	26·975	11·145	1·415	0·431	0·487	0·176
47·0	16·7	16·43	12·97	26·98	„	„	„	„	0·178
46·5	16·6	16·33	12·93	26·93	„	„	„	„	0·176
47·0	16·9	16·56	13·15	26·98	„	1·39 *	„	„	0·177
							Mean	. . .	0·177

---

* After drying the stopper.

† This formula corresponds to the composition assumed by Rammelsberg (Handbuch der Mineralchemie, pp. 413, 1015) for iserine from the Iserwiese, namely, 3 $(FeO\ TiO_2)+Fe_2O_3$.

Oxide of Chromium, $\text{Ɛr}_2\,\text{Ɵ}_3$. Crystalline crusts prepared from oxychloride of chromium.

Experiments with Naphtha A. Glass 3. Temperature of the Air 19°·1.

T.	T′.	t′.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
52·1	21·5	21·23	18·53	26·955	5·405	2·255	0·431	0·453	0·176
51·5	21·2	20·93	18·22	26·955	,,	,,	,,	,,	0·181
53·1	21·4	21·06	18·25	26·945	,,	,,	,,	,,	0·178
52·1	21·2	20·94	18·23	26·99	,,	2·245 *	,,	,,	0·175
							Mean . . .		0·177

Hydrated Sesquioxide of Manganese $\text{Mn}_2\,\text{Ɵ}_3+\text{H}_2\,\text{Ɵ}$ †. Fragments of good crystals of *Manganite* from Ihlefeld in the Harz, dried at 40° to 50°.

Experiments with Naphtha A. Glass 3. Temperature of the Air 14°·6–14°·4.

T.	T′.	t′.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
47·0	17·1	16·82	13·83	26·985	8·31	1·855	0·431	0·453	0·174
45·6	17·0	16·69	13·83	26·94	,,	,,	,,	,,	0·173
45·7	17·0	16·73	13·85	26·92	,,	1·845*	,,	,,	0·174
							Mean . . .		0·174

I made subsequently another series of experiments with a specimen from the same locality dried at the ordinary temperature.

Experiments with Naphtha A. Glass 3. Temperature of the Air 17°·7–17°·4.

T.	T′.	t′.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
52·0	20·5	20·15	17·06	26·95	8·04	1·77	0·431	0·453	0·178
52·3	20·3	20·02	16·86	26·975	,,	,,	,,	,,	0·180
51·9	20·1	19·77	16·65	26·965	,,	,,	,,	,,	0·178
51·6	20·1	19·84	16·80	26·995	,,	1·75*	,,	,,	0·174
							Mean . . .		0·178

The specific heat of manganite between 19° and 49° is 0·176, the mean of the averages of both series of determinations.

---

* After drying the stopper.

† "Manganite dried at about 80°–90°, and then kept for half a day over sulphuric acid, gave in a water-determination, in which the water was collected in a chloride of calcium tube, 9·96 per cent. of water."—KNOP. The above formula requires 10·23 per cent. of water.

47. *Binoxide of Manganese*, $\mathrm{Mn}\,\Theta_2$. Pyrolusite from Ilmenau, dried at 100°–110°*.

Experiments with Naphtha A. Glass 1. Temperature of the Air 14°·4–14°·5.

T.	T'.	t'.	t.	M.	m.	f.	y.	x.	sp. H.
				grms.	grms.	grms.		grm.	
51°·6	17°·0	16°·70	13°·41	26·955	6·32	2·06	0·431	0·651	0·162
48·5	16·9	16·63	13·63	26·945	,,	,,	,,	,,	0·161
45·9	16·9	16·61	13·86	26·93	,,	,,	,,	,,	0·161
44·0	16·9	16·64	14·13	26·97	,,	2·04 †	,,	,,	0·153
							Mean . . .		0·159

Titanic Acid, $\text{Ŧi}\,\Theta_2$. I have investigated the one quadratic modification, rutile, and the rhombic modification Brookite or Arkansite; I had no material for the investigation of anatase, the other quadratic modification.

Rutile. Fragments of crystals from Saxony and from France.

Experiments with Naphtha A. Glass 1. Temperature of the Air 13°·5–13°·7.

T.	T'.	t'.	t.	M.	m.	f.	y.	x.	sp. H.
				grms.	grms.	grm.		grm.	
47°·9	16°·0	15°·73	12°·63	26·95	8·055	1·60	0·431	0·651	0·159
47·6	16·1	15·78	12·73	26·97	,,	,,	,,	,,	0·158
45·2	15·9	15·56	12·73	26·965	,,	,,	,,	,,	0·156
45·6	16·1	15·84	13·01	26·965	,,	1·58 †	,,	,,	0·156
							Mean . . .		0·157

Brookite or Arkansite. Beautiful small crystals from hotsprings in Arkansas, purified by treatment with hydrochloric acid from adherent oxide of iron.

Experiments with Naphtha A. Glass 1. Temperature of the Air 16°·1–16°·3.

T.	T'.	t'.	t.	M.	m.	f.	y.	x.	sp. H.
				grms.	grms.	grm.		grm.	
47°·1	18°·2	17°·94	15°·22	26·97	8·00	1·415	0·431	0·651	0·160
49·3	18·5	18·23	15·22	26·96	,,	,,	,,	,,	0·161
49·2	18·7	18·40	15·52	26·935	,,	,,	,,	,,	0·160
49·0	18·6	18·31	15·43	26·96	,,	1·395 †	,,	,,	0·163
							Mean . . .		0·161

---

[*] This pyrolusite was not pure binoxide, but probably contained some manganite also. In experiments made by Mr. Oeser in the Giessen laboratory, this pyrolusite, dried at 100° to 110°, gave, when heated in a current of dry air, the water being collected in a chloride of calcium apparatus, 1·21 per cent. of water; treated with oxalic acid, as much carbonic acid was disengaged as corresponded to 95·36 per cent. of binoxide. As the specific heat of manganite (0·176) does not very much differ from that found for pyrolusite (0·159), I neglected to introduce a correction for the small quantity of manganite.

[†] After drying the stopper.

Binoxide of Tin, $\mathrm{Sn}\,\Theta_2$. Fragments of crystals of tinstone from Saxony.

Experiments with Naphtha A. Glass 2. Temperature of the Air 14°·5.

T.	T′.	t′.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
50·4	17·0	16·66	13·52	26·99	14·495	1·71	0·431	0·487	0·0906
46·6	16·4	16·14	13·33	26·925	„	„	„	„	0·0884
45·1	16·4	16·05	13·35	26·96	„	„	„	„	0·0905
45·7	16·3	16·04	13·32	26·98	„	1·695 *	„	„	0·0882
							Mean . . .		0·0894

48. *Silicic Acid*, $\mathrm{Si}\,\Theta_2$. Pieces of transparent quartz (rock-crystal) from the Grimsel.

I.—Experiments with Naphtha A. Glass 1. Temperature of the Air 17°·7–17°·4.

T.	T′.	t′.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
53·8	20·1	19·83	17·03	26·99	4·885	1·58	0·431	0·651	0·186
52·5	19·8	19·53	16·77	26·96	„	„	„	„	0·193
51·8	19·7	19·43	16·77	26·98	„	„	„	„	0·185
51·7	19·7	19·42	16·76	26·945	„	„	„	„	0·186
52·7	19·7	19·35	16·64	26·96	„	1·56 *	„	„	0·182
							Mean . . .		0·186

II.—Experiments with Naphtha A. Glass 3. Temperature of the Air 19°·1–19°·4.

T.	T′.	t′.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
51·5	21·0	20·74	18·36	26·985	5·135	1·635	0·431	0·453	0·185
51·0	21·1	20·79	18·45	26·96	„	„	„	„	0·185
52·6	21·2	20·92	18·45	26·955	„	„	„	„	0·187
52·6	21·2	20·89	18·42	26·97	„	1·62 *	„	„	0·189
							Mean . . .		0·187

III.—Experiments with Naphtha B. Glass 3. Temperature of the Air 17°·8–17°·9.

T.	T′.	t′.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
50·0	20·0	19·69	17·27	26·98	5·645	1·70	0·419	0·453	0·175
50·5	19·9	19·64	17·14	26·97	„	„	„	„	0·184
50·0	20·1	19·82	17·40	26·99	„	„	„	„	0·181
50·0	20·0	19·66	17·22	26·975	„	1·685 *	„	„	0·178
							Mean . . .		0·180

* After drying the stopper.

IV.—Experiments with Water. Glass 1. Temperature of the Air 17°·8–18°·3.

T.	T'.	t'.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
47·6	19·7	19·37	15·72	26·945	5·02	1·93	1·000	0·651	0·188
47·9	19·9	19·57	15·92	26·95	„	„	„	„	0·186
47·6	20·0	19·65	16·03	26·985	„	„	„	„	0·191
47·3	20·0	19·67	16·08	26·98	„	1·915 *	„	„	0·196
							Mean	. . .	0·190

The average of these four means, 0·186, 0·187, 0·180, 0·190, gives 0·186 as the specific heat of quartz between 20° and 50°.

It was interesting to determine also the specific heat of amorphous silicic acid. I accordingly made experiments with opal and with hyalite, taking into account the water contained in these minerals. If the quantity of silica in the mineral taken is *m*, that of the water in it *w*, and *z* the specific heat of the water contained in the mineral, then, taking the other symbols in the sense hitherto assigned to them, the specific heat of the silica in the mineral can be calculated by the formula

$$\text{sp. H} = \frac{M(t'-t)-(x+fy+wz)(T-T')}{m(T-T')}.$$

But though the quantity of water contained in the (air-dried) minerals investigated is so small (scarcely exceeding 4 per cent.), the specific heat of silicic acid is found to be very different, according as (α) the specific heat *z* is put equal to 1, that of liquid water, (β) or equal to 0·48, that of solid water or ice (which is at least correct for far the greater part of the water of these minerals, *vide* § 97). I give as follows, under α and β, the numbers resulting from both calculations.

Noble Opal from Honduras: yellowish, colourless in small pieces. The air-dried mineral contained 4·3 per cent. of water; in the following experiments 4·12 grms. of opal were used, containing, therefore, 3·943 grms. of anhydrous substances (*m*) and 0·177 grm. of water (*w*).

Experiments with Naphtha B. Glass 3. Temperature of the Air 18°·5–18°·7.

T.	T'.	t'.	t.	M.	m.	w.	f.	y.	x.	sp. H.	
°	°	°	°	grms.	grms.	grm.	grm.		grm.	α.	β.
50·4	20·6	20·34	18·10	26·98	3·943	0·177	1·69	0·419	0·453	0·175	0·198
52·6	20·6	20·32	17·84	26·985	„	„	„	„	„	0·191	0·214
51·9	20·6	20·32	17·92	26·98	„	„	„	„	„	0·185	0·209
51·3	20·6	20·32	17·96	26·955	„	„	1·67 *	„	„	0·188	0·211
								Mean	. . .	0·185	0·208

Hyalite from Steinheim near Hanau. Small limpid spheroidal masses. The air-dried mineral contained 3·65 per cent. of water. In the following experiments 3·795 grms. of hyalite were used, which therefore contained 3·656 grms. of anhydrous substance (m) and 0·139 grm. of water (w).

---

* After drying the stopper.

Experiments with Naphtha B. Glass 1. Temperature of the Air 17°·8–17°·9.

T.	T'.	t'.	t.	M.	m.	w.	f.	y.	x.	sp. H.	
°	°	°	°	grms.	grms.	grm.	grm.		grm.	α.	β.
50·4	19·8	19·50	17·26	26·98	3·656	0·139	1·345	0·419	0·651	0·170	0·190
50·8	19·8	19·51	17·23	26·98	,,	,,	,,	,,	,,	0·172	0·192
50·4	19·8	19·53	17·27	26·97	,,	,,	,,	,,	,,	0·175	0·194
51·4	19·8	19·53	17·21	26·98	,,	,,	1·33*	,,	,,	0·173	0·193
								Mean . . .		0·173	0·192

In another series of experiments 4·475 grms. of hyalite were used, containing 4·312 grms. anhydrous substance (m) and 0·163 grm. water (w).

Experiments with Water. Glass 1. Temperature of the Air 17°·1–17°·2.

T.	T'.	t'.	t.	M.	m.	w.	f.	y.	x.	sp. H.	
°	°	°	°	grms.	grms.	grm.	grm.		grm.	α.	β.
43·5	18·9	18·55	15·41	26·97	4·312	0·163	1·88	1·000	0·651	0·174	0·193
42·7	19·1	18·83	15·79	26·99	,,	,,	,,	,,	,,	0·182	0·201
42·7	19·2	18·87	15·84	26·955	,,	,,	,,	,,	,,	0·181	0·201
42·9	19·2	18·94	15·92	26·955	,,	,,	1·865*	,,	,,	0·175	0·195
								Mean . . .		0·178	0·197

The specific heat of amorphous silica must lie between the numbers standing under α and β, and coming nearer those under β. It does not seem to differ materially from that found for crystallized silica.

49. *Molybdic Acid*, Mo Θ_3. Greyish-white powder, which, when heated in a porcelain crucible, became permanently bright grey: the results are not trustworthy.

Experiments with Naphtha A. Glass 3. Temperature of the Air 19°·5–20°·1.

T.	T'.	t'.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grms.		grm.	
51·4	20·9	20·64	18·44	26·99	2·27	2·65	0·431	0·453	0·155
51·3	21·3	21·04	18·88	26·97	,,	,,	,,	,,	0·153
51·5	21·4	21·12	18·94	26·995	,,	,,	,,	,,	0·159
51·2	21·4	21·06	18·93	26·96	,,	2·635*	,,	,,	0·149
							Mean . . .		0·154

---

* After drying the stopper.

Tungstic Acid, $\mathrm{W}\Theta_3$. Yellow powder.

Experiments with Naphtha A. Glass 1. Temperature of the Air 19°·5–20°·1.

T.	T′.	t′.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
52·1	21·3	21·02	18·60	26·98	6·89	1·965	0·431	0·651	0·0902
52·8	21·5	21·16	18·73	26·99	„	„	„	„	0·0868
50·5	21·4	21·14	18·84	26·965	„	„	„	„	0·0919
51·9	21·6	21·29	18·93	26·985	„	1·95[*]	„	„	0·0886
							Mean . . .		0·0894

Of the above pulverulent metallic acids only small quantities were used, and their thermal action was only a small proportion of the whole thermal action observed. The results can only be considered as approximations to the true specific heat.

50. *Chloride of Sodium*, Na Cl. Pure chloride of sodium fused.

Experiments with Naphtha A. Glass 1. Temperature of the Air 10°·9–11°·5.

T.	T′.	t′.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
45·8	12·3	11·97	9·34	26·91	3·65	1·57	0·431	0·651	0·215
45·5	12·7	12·44	9·88	26·94	„	„	„	„	0·212
45·7	13·0	12·74	10·20	26·99	„	1·56[*]	„	„	0·212
							Mean . . .		0·213

Almost clear pieces of rock-salt, sharply dried.

Experiments with Naphtha A. Glass 2. Temperature of the Air 10°·9–11°·5.

T.	T′.	t′.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grms.		grm.	
44·8	12·6	12·32	9·63	26·95	3·955	2·025	0·431	0·487	0·225
45·8	13·0	12·73	10·04	26·935	„	„	„	„	0·214
44·6	13·3	13·01	10·43	26·95	„	2·015[*]	„	„	0·219
							Mean . . .		0·219

Chloride of Potassium, K Cl. Pure salt fused[†].

I.—Experiments with Naphtha A. Glass 2. Temperature of the Air 12°·1–12°·2.

T.	T′.	t′.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
46·3	14·0	13·73	11·24	26·98	3·665	2·265	0·431	0·487	0·168
45·7	14·2	13·86	11·44	26·99	„	„	„	„	0·167

---

[*] After drying the stopper.

[†] These experiments with fused chloride are more trustworthy than those with crystallized salt, which, however, are very near; for the latter, in loose crystals, only in small quantity, filled the glass used in the determinations. The experiments with sharply dried crystallized chloride of potassium gave the following results:—

II.—Experiments with Naphtha A. Glass 2. Temperature of the Air 10°·9.

T.	T'.	t'.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
46·0	12·7	12·41	9·98	26·95	3·685	1·915	0·431	0·487	0·178
45·6	12·8	12·53	10·15	26·96	,,	,,	,,	,,	0·175
46·4	13·0	12·74	10·34	26·955	,,	,,	,,	,,	0·169
45·0	12·9	12·64	10·34	26·975	,,	1·90 *	,,	,,	0·170

The mean of the preceding six determinations gives 0·171 as the specific heat of chloride of potassium between 13° and 46°.

Chloride of Rubidium, Rb Cl. Pure salt fused.

Experiments with Naphtha A. Glass 2. Temperature of the Air 14°·3–14°·5.

T.	T'.	t'.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
47·9	16·1	15·84	13·64	26·96	5·22	1·835	0·431	0·487	0·112
46·0	16·2	15·92	13·83	26·975	,,	,,	,,	,,	0·118
44·3	16·2	15·93	14·00	26·94	,,	,,	,,	,,	0·110
43·8	16·4	16·13	14·26	26·98	,,	1·82*	,,	,,	0·109
							Mean	. . .	0·112

51. *Chloride of Ammonium*, NH_4Cl. I have made five series of experiments with different forms of this salt.

Chloride of Ammonium, crystallized from pure aqueous solution in very small octahedra.

I.—Experiments with Naphtha A. Glass 1. Temperature of the Air 12°·1–11°·8

T.	T'.	t'.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grm.	grms.		grm.	
51·3	13·7	13·43	10·39	26·96	1·445	2·255	0·431	0·651	0·387
44·9	13·7	13·44	10·93	26·99	,,	,,	,,	,,	0·380
44·6	14·0	13·70	11·26	26·905	,,	2·245*	,,	,,	0·365
							Mean	. . .	0·377

I.—Experiments with Naphtha A. Glass 1. Temperature of the Air 12°·1–12°·2.

T.	T'	t'.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grms.		grm.	
44·1	13·7	13·39	11·11	26·945	1·795	2·485	0·431	0·651	0·166
47·0	14·1	13·84	11·42	26·96	,,	,,	,,	,,	0·145

II.—Experiments with Naphtha A. Glass 1. Temperature of the Air 12°·9.

T.	T'	t'.	t.	M.	m.	f.	y.	x.	sp. H.
45·6	14·5	14·22	11·90	26·945	2·365	2·125	0·431	0·651	0·187
45·7	14·4	14·14	11·90	26·98	,,	,,	,,	,,	0·154
46·5	14·7	14·43	12·14	26·955	,,	2·115*	,,	,,	0·160

---

* After drying the stopper.

II.—Experiments with Naphtha A. Glass 2. Temperature of the Air 12°·9.

T.	T'.	t'.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grm.	grms.		grm.	
47·0	14·5	14·24	11·45	26·93	1·88	2·495	0·431	0·487	0·399
45·0	14·8	14·46	11·93	26·98	„	„	„	„	0·371
45·1	14·8	14·46	11·93	26·99	„	2·485*	„	„	0·370
								Mean . . .	0·380

Only a small quantity of this finely crystallized chloride of ammonium goes into the glasses which I used for the experiments. Hence I also investigated chloride of ammonium in more compact pieces.

Long fibrous pieces from a sublimation cake:

III.—Experiments with Naphtha A. Glass 2. Temperature of the Air 12°·1–11°·8.

T.	T'.	t'.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grms.		grm.	
45·5	13·9	13·63	10·73	26·97	2·76	2·20	0·431	0·487	0·377
45·1	14·2	13·92	11·07	26·97	„	„	„	„	0·381
44·2	14·2	13·93	11·20	26·98	„	2·19*	„	„	0·371
								Mean . . .	0·376

From the so-called "gas liquor," NOELLNER has prepared a very pure chloride of ammonium, apparently in quadratic trapezoedra. With such crystals, 8 to 10 millims. long, I made the following determinations:—

IV.—Experiments with Naphtha A. Glass 1. Temperature of the Air 14°·1–13°·8.

T.	T'.	t'.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grm.	grms.		grm.	
48·5	15·9	15·63	12·84	26·99	1·978	2·085	0·431	0·651	0·384
44·7	16·0	15·73	13·32	26·93	„	„	„	„	0·360
44·8	16·0	15·70	13·32	26·97	„	2·075*	„	„	0·346
								Mean . . .	0·363

Finally, I examined chloride of ammonium which had crystallized, from a solution containing urea, in beautiful transparent cubes of 2 to 3 millims. in the side.

V.—Experiments with Naphtha A. Glass 2. Temperature of the Air 14°·1–13°·8.

T.	T'.	t'.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grms.		grm.	
45·2	16·0	15·73	13·05	26·92	2·595	2·34	0·431	0·487	0·376
44·4	16·1	15·83	13·25	26·975	„	„	„	„	0·371
45·7	16·4	16·08	13·45	26·96	„	2·33*	„	„	0·358
								Mean . . .	0·368

The mean of the means of the five series of determinations, 0·377, 0·380, 0·376, 0·363, 0·368, gives 0·373 for the specific heat of chloride of ammonium between 15° and 45°.

---

* After drying the stopper.

52. *Chloride of Mercury*, $\mathrm{Hg\,Cl_2}$. Well-dried crystals.

Experiments with Naphtha A. Glass 1. Temperature of the Air 9°·2.

T.	T′.	*t′*.	*t*.	M.	*m*.	*f*.	*y*.	*x*.	sp. H.
°	°	°	°	grms.	grms.	grms.		grm.	
45·2	11·5	11·17	8·86	26·985	6·07	1·885	0·431	0·651	0·0636
44·3	11·2	10·90	8·50	26·99	,,	2·105*	,,	,,	0·0657
46·1	11·5	11·21	8·72	26·915	,,	2·10†	,,	,,	0·0628
							Mean . . .		0·0640

Chloride of Magnesium, $\mathrm{Mg\,Cl_2}$. Pieces of a beautiful preparation which had solidified with crystalline structure after being melted.

Experiments with Naphtha A. Glass 1. Temperature of the Air 13°·2.

T.	T′.	*t′*.	*t*.	M.	*m*.	*f*.	*y*.	*x*.	sp. H.
°	°	°	°	grms.	grms.	grms.		grm.	
47·5	14·8	14·53	12·13	26·98	2·235	2·01	0·431	0·651	0·207
46·4	15·0	14·72	12·43	26·98	,,	,,	,,	,,	0·201
45·6	15·1	14·84	12·63	26·96	,,	2·115*	,,	,,	0·175
46·9	15·3	15·03	12·73	26·945	,,	2·105†	,,	,,	0·180
							Mean . . .		0·191

Chloride of Barium, $\mathrm{Ba\,Cl_2}$. Pieces of a specimen which was of a dead white colour after solidifying.

Experiments with Naphtha A. Glass 1. Temperature of the Air 14°·4.

T.	T′.	*t′*.	*t*.	M.	*m*.	*f*.	*y*.	*x*.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
46·2	16·2	15·87	13·64	26·975	6·795	1·72	0·431	0·651	0·0902
48·0	16·3	16·02	13·64	26·96	,,	,,	,,	,,	0·0930
47·1	16·3	16·03	13·73	26·945	,,	,,	,,	,,	0·0912
46·4	16·2	15·94	13·73	26·97	,,	1·705†	,,	,,	0·0865
							Mean . . .		0·0902

Crystallized Chloride of Barium, $\mathrm{Ba\,Cl_2+2H_2O}$. Crystals dried *in vacuo*.

Experiments with Naphtha A. Glass 3. Temperature of the Air 16°·1–16°·8.

T.	T′.	*t′*.	*t*.	M.	*m*.	*f*.	*y*.	*x*.	sp. H.
°	°	°	°	grms.	grms.	grms.		grm.	
45·5	17·6	17·34	15·04	26·975	5·055	2·14	0·431	0·453	0·168
47·1	17·8	17·50	15·03	26·955	,,	,,	,,	,,	0·177
47·0	18·0	17·74	15·33	26·975	,,	,,	,,	,,	0·171
46·2	18·2	17·94	15·63	26·965	,,	2·125†	,,	,,	0·169
							Mean . . .		0·171

---

* After adding some more naphtha. (The naphtha was apparently sucked up by the crystals of chloride of mercury, hence more naphtha was added. The liquid formed a smeary border at the side of the glass, but there was no deliquescence of the crystals in the naphtha.)

† After drying the stopper.

Chloride of Chromium, $\mathrm{Ꞓr_2\,Cl_6}$. Violet insoluble chloride of chromium twice boiled out with water, washed and dried at 130°. As a porous mass this substance is but ill suited for an accurate determination of the specific heat. I pressed it, by means of a glass rod, in a glass tube into small disks, between which the naphtha could circulate. The object of this is to prevent a stagnation of the liquid absorbed by the solid mass, in consequence of which the water of the calorimeter assumes its maximum more slowly, and hence the specific heat is found too low (compare §§ 18 & 24); but this object is not quite attained in this way*.

Experiments with Naphtha A. Glass 1. Temperature of the Air 11°·4–11°·5.

T.	T'.	t'.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grms.		grm.	
47·5	13·2	12·86	10·32	26·93	3·165	2·095	0·431	0·651	0·139
47·5	13·0	12·73	10·13	26·97	„	„	„	„	0·151
43·8	12·9	12·63	10·33	26·945	„	„	„	„	0·143
46·0	13·0	12·65	10·21	26·94	„	2·085†	„	„	0·140
							Mean . . .		0·143

I should have liked to determine the specific heat of a solid metallic chloride of the formula $\mathrm{R\,Cl_3}$, and tried with chloride of antimony, but it coloured naphtha yellow when poured upon it, and became itself milky white, forming a heavy layer below the naphtha, and fused completely a little above 40°.

53. *Chloride of Zinc and Chloride of Potassium*, $\mathrm{Ƶn\,K_2\,Cl_4}$. Crystals dried at 100° to 110°‡.

Experiments with Naphtha A. Glass 1. Temperature of the Air 14°·3–14°·5.

T.	T'.	t'.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grms.		grm.	
48·7	16·2	15·93	13·53	26·915	3·01	2·02	0·431	0·651	0·155
47·1	16·3	16·04	13·77	26·965	„	„	„	„	0·155
46·5	16·4	16·12	13·92	26·955	„	„	„	„	0·150
44·1	16·4	16·14	14·13	26·94	„	2·00†	„	„	0·147
							Mean . . .		0·152

---

* The above source of error was of more importance, and the experiments gave far lower numbers for the specific heat of chloride of chromium when this body was not formed in disks, but just placed in the vessel and moderately lightly pressed. The following results were obtained in this manner:—

Experiments with Naphtha A. Glass 2. Temperature of the Air 11°·5.

T.	T'.	t'.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grms.		grm.	
46·4	13·4	13·12	10·52	26·915	2·425	3·035	0·431	0·487	0·134
45·6	13·8	13·53	11·04	26·985	„	„	„	„	0·131
45·7	13·8	13·52	11·02	26·99	„	„	„	„	0·132
45·6	13·8	13·48	11·02	26·95	„	3·015†	„	„	0·123

† After drying the stopper.

‡ "These crystals were deposited from a solution which contained for one equivalent of chloride of potassium

Hydrated Chloride of Copper and Potassium, $\text{Ꞓu}\,K_2Cl_4+2H_2\text{Ɵ}$. Air-dried crystals.

Experiments with Naphtha A. Glass 3. Temperature of the Air 17°·0–17°·2.

T.	T'.	t'.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
51·4	19·1	18·80	16·33	26·95	4·085	1·86	0·431	0·453	0·197
50·4	19·0	18·66	16·26	26·94	,,	,,	,,	,,	0·197
50·0	19·1	18·77	16·43	26·955	,,	,,	,,	,,	0·193
49·2	19·0	18·68	16·35	26·95	,,	1·84*	,,	,,	0·204
							Mean	. . .	0·197

Chloride of Tin and Potassium, $\text{Sn}\,K_2Cl_6$. Crystals dried at 105°.

Experiments with Naphtha A. Glass 3. Temperature of the Air 16°·4–17°·3.

T.	T'.	t'.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
50·1	18·3	17·97	15·70	26·96	5·305	1·77	0·431	0·453	0·134
51·1	18·7	18·42	16·12	26·93	,,	,,	,,	,,	0·131
49·5	18·7	18·36	16·19	26·955	,,	,,	,,	,,	0·129
49·1	18·8	18·52	16·34	26·965	,,	1·76*	,,	,,	0·137
							Mean	. . .	0·133

Chloride of Platinum and Potassium, $\text{Ᵽt}\,K_2Cl_6$. Well-formed small crystals.

Experiments with Naphtha A. Glass 2. Temperature of the Air 11°·5–11°·2.

T.	T'.	t'.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
44·3	13·2	12·91	10·55	26·93	7·25	1·55	0·431	0·487	0·122
46·1	13·4	13·06	10·67	26·975	,,	,,	,,	,,	0·113
47·9	13·5	13·18	10·68	26·975	,,	,,	,,	,,	0·111
48·1	13·5	13·23	10·76	26·98	,,	1·535*	,,	,,	0·107
							Mean	. . .	0·113

---

at least two equivalents of chloride of zinc. In the analyses (the potassium was not determined) there were—

Found........ 24·0 per cent. Zinc, 49·3 and 49·6 Cl.
Calculated .... 22·85 per cent. *Zn*, 49·75 per cent. Cl, and 27·40 K.

"The crystals were only pressed between paper, and hence were impregnated with some mother-liquor, which explains the excess of zinc found."—ENGELBACH.

* After drying the stopper.

54. *Fluoride of Calcium*, $\mathrm{Ca\,Fl_2}$. Cleavage pieces of fluor-spar from Münsterthal in Baden.

Experiments with Naphtha A. Glass 1. Temperature of the Air 18°·4–19·1.

T.	T′.	t′.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
50·5	20·7	20·42	17·67	26·985	5·675	1·56	0·431	0·651	0·206
49·9	20·4	20·07	17·33	26·94	”	”	”	”	0·208
50·1	20·5	20·22	17·43	26·97	”	”	”	”	0·215
49·9	20·6	20·26	17·53	26·965	”	”	”	”	0·209
50·5	20·8	20·49	17·75	26·98	”	1·54*	”	”	0·207
							Mean	. . .	0·209

Cryolite, $\mathrm{Al\,Na_3\,Fl_6}$. Comminuted cryolite from Greenland, smartly dried.

Experiments with Naphtha A. Glass 3. Temperature of the Air 19°·2–19°·5.

T.	T′.	t′.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
50·6	21·5	21·21	18·44	26·975	5·55	1·775	0·431	0·453	0·243
50·0	21·5	21·15	18·43	26·965	”	”	”	”	0·244
49·6	21·5	21·17	18·53	26·965	”	”	”	”	0·237
50·6	21·6	21·27	18·56	26·985	”	”	”	”	0·235
51·0	21·6	21·34	18·62	26·99	”	1·75*	”	”	0·232
							Mean	. . .	0·238

55. *Cyanide of Mercury*, $\mathrm{Hg\,C_2\,N_2}$. Well-dried crystals.

Experiments with Naphtha A. Glass 2. Temperature of the Air 9°·2.

T.	T′.	t′.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
45·2	11·2	10·86	8·34	26·935	6·555	1·955	0·431	0·487	0·100
47·0	11·5	11·23	8·62	26·965	”	”	”	”	0·098
49·5	11·7	11·43	8·64	26·955	”	”	”	”	0·099
43·7	11·5	11·22	8·84	26·95	”	1·94*	”	”	0·101
							Mean	. . .	0·100

Cyanide of Zinc and Potassium, $\mathrm{Zn\,K_2\,C_4\,N_4}$. Distinct crystals. I made four series of experiments with this substance.

Crystals dried *in vacuo*.

I.—Experiments with Naphtha A. Glass 2. Temperature of the Air 11°·8–11°·5.

T.	T′.	t′.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grms.		grm.	
44·9	13·8	13·53	11·13	26·96	2·515	2·195	0·431	0·487	0·257
48·0	13·9	13·64	11·13	26·93	”	”	”	”	0·218
46·9	13·9	13·57	11·12	26·94	”	”	”	”	0·225
45·0	13·9	13·63	11·34	26·975	”	2·175*	”	”	0·223
							Mean	. . .	0·231

* After drying the stopper.

II.—Experiments with Naphtha A. Glass 2. Temperature of the Air 12°·4–12°·3.

T.	T'.	t'.	t.	M.	m.	f.	y.	x.	sp. H.
				grms.	grms.	grms.		grm.	
45°·5	14°·5	14°·15	11°·83	26·97	2·465	2·225	0·431	0·487	0·232
46·7	14·5	14·22	11·74	26·97	,,	,,	,,	,,	0·256
45·2	14·3	13·96	11·72	26·945	,,	2·17*	,,	,,	0·215
45·2	14·5	14·23	11·95	26·92	,,	,,	,,	,,	0·234
							Mean . . .		0·234

Crystals dried at 100°.

III.—Experiments with Naphtha A. Glass 1. Temperature of the Air 11°·8–11°·5.

T.	T'.	t'.	t.	M.	m.	f.	y.	x.	sp. H.
				grms.	grms.	grm.		grm.	
46°·6	13°·5	13°·20	10°·74	26·955	2·415	1·665	0·431	0·651	0·263
48·5	13·8	13·53	10·96	26·99	,,	,,	,,	,,	0·261
44·3	13·6	13·26	11·05	26·99	,,	,,	,,	,,	0·238
45·2	13·6	13·32	11·04	26·93	,,	1·655†	,,	,,	0·240
							Mean . . .		0·251

IV.—Experiments with Naphtha A. Glass 1. Temperature of the Air 11°·2–11·3.

T.	T'.	t'.	t.	M.	m.	f.	y.	x.	sp. H.
				grms.	grms.	grm.		grm.	
49°·4	13°·3	13°·04	10°·43	26·94	2·255	1·78	0·431	0·651	0·235
46·7	13·4	13·11	10·62	26·98	,,	,,	,,	,,	0·266
49·2	13·6	13·33	10·72	26·955	,,	,,	,,	,,	0·247
48·0	13·5	13·22	10·73	26·97	,,	1·765†	,,	,,	0·237
							Mean . . .		0·246

The specific heat of cyanide of zinc and potassium between 14° and 46° is 0·241 as the mean of the averages of the four series of determinations, 0·231, 0·234, 0·251, 0·246.

Crystallized Ferrocyanide of Potassium, $\text{Fe}\,K_4\,C_6\,N_6+3\,H_2\,\Theta$. Fragments of air-dried crystals.

Experiments with Naphtha A. Glass 1. Temperature of the Air 19°·2.

T.	T'.	t'.	t.	M.	m.	f.	y.	x.	sp. H.
				grms.	grms.	grm.		grm.	
50°·6	21°·3	21°·03	18°·46	26·98	3·425	1·69	0·431	0·651	0·288
51·3	21·1	20·82	18·22	26·98	,,	,,	,,	,,	0·275
51·0	21·0	20·74	18·14	26·97	,,	,,	,,	,,	0·280
51·0	21·1	20·84	18·26	26·965	,,	1·675†	,,	,,	0 278
							Mean . . .		0·280

* After removing some naphtha on the stopper.

† After drying the stopper.

Ferridcyanide of Potassium, $\mathrm{Fe\,K_3\,\Theta_6\,N_6}$. Well-formed crystals, smartly dried.

Experiments with Naphtha A. Glass 2. Temperature 13°·2.

T.	T'.	t'.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grms.		grm.	
48·5	15·3	15·01	12·23	26·95	3·63	2·025	0·431	0·487	0·247
45·1	15·0	14·66	12·20	26·92	„	„	„	„	0·232
47·1	15·5	15·23	12·68	26·975	„	„	„	„	0·225
44·4	15·3	15·00	12·64	26·98	„	2·015*	„	„	0·229

Mean . . . 0·233

56. *Nitrate of Soda*, $\mathrm{Na\,N\Theta_3}$. Crystallized salt, briskly dried.

Experiments with Naphtha A. Glass 2. Temperature of the Air 11°·8.

T.	T'.	t'.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grms.		grm.	
47·2	14·3	13·95	11·02	26·91	3·645	2·25	0·431	0·487	0·258
46·2	14·9	14·55	11·82	26·945	„	„	„	„	0·245
46·5	14·3	14·02	11·13	26·93	„	„	„	„	0·263
44·3	14·1	13·84	11·15	26·945	„	2·235*	„	„	0·261

Mean . . . 0·257

Fused Salt.

Experiments with Naphtha A. Glass 1. Temperature of the Air 11°·8.

T.	T'.	t'.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
47·8	13·9	13·62	10·57	26·98	3·92	1·66	0·431	0·651	0·271
43·9	14·3	14·03	11·43	26·065	„	„	„	„	0·256
43·6	14·6	14·33	11·83	26·925	„	„	„	„	0·243
46·4	14·5	14·22	11·43	26·965	„	1·65*	„	„	0·254

Mean . . . 0·256

Nitrate of Potass, $\mathrm{K\,N\,\Theta_3}$. Smartly dried crystallized salt.

Experiments with Naphtha A. Glass 1. Temperature of the Air 12°·1–12°·4.

T.	T'.	t'.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
44·2	14·2	13·88	11·43	26·93	3·105	1·845	0·431	0·651	0·242
46·5	14·4	14·14	11·56	26·99	„	„	„	„	0·233
45·6	14·3	14·03	11·53	26·97	„	„	„	„	0·228
44·7	14·0	13·74	11·31	26·98	„	1·83*	„	„	0·224

Mean . . . 0·232

---

* After drying the stopper.

Fused Salt.

Experiments with Naphtha A. Glass 2. Temperature of the Air 12°·1–12°·4.

T.	T'.	t'.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grms.		grm.	
46·6	14·5	14·20	11·53	26·94	3·745	2·035	0·431	0·487	0·234
45·9	14·4	14·14	11·56	26·935	,,	,,	,,	,,	0·225
46·1	14·3	14·03	11·44	26·96	,,	,,	,,	,,	0·222
44·7	14·1	13·83	11·32	26·96	,,	2·02*	,,	,,	0·228
							Mean . . .		0·227

57. *Nitrate of Ammonia*, $N_2H_4\Theta_3$. Vitreous transparent pointed crystals, like those of nitre; dried *in vacuo* over sulphuric acid.

I.—Experiments with Naphtha A. Glass 2. Temperature of the Air 10°·9.

T.	T'.	t'.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grms.		grm.	
32·3	12·7	12·43	10·53	26·92	2·555	2·41	0·431	0·487	0·424
31·1	12·8	12·52	10·66	26·945	,,	,,	,,	,,	0·475
29·2	12·6	12·33	10·63	26·92	,,	,,	,,	,,	0·482
33·5	13·1	12·81	10·74	26·93	,,	2·405*	,,	,,	0·473
							Mean . . .		0·463

II.—Experiments with Naphtha A. Glass 2. Temperature of the Air 14°·4–15°·0.

T.	T'.	t'.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grms.		grm.	
32·4	15·9	15·57	14·02	26·96	2·025	2·29	0·431	0·487	0·455
30·8	15·7	15·44	14·03	26·975	,,	,,	,,	,,	0·449
31·5	16·0	15·66	14·23	26·95	,,	,,	,,	,,	0·435
32·9	16·2	15·93	14·37	26·97	,,	,,	,,	,,	0·449
							Mean . . .		0·447

The specific heat of nitrate of ammonia between 14° and 31° is as the mean of the averages of both series of experiments, 0·463 and 0·447, =0·455. The crystals were quite unchanged at this temperature. In these experiments the difference of temperature T−T' was but small, and it would not be surprising to find even greater deviations among the individual results than are exhibited by the above numbers in the last column. Nitrate of ammonia cannot be heated much above 30°, because it then undergoes a molecular change, which apparently is accompanied by disengagement of heat. This was observed in a series of experiments in which the heat was raised to 45° or 48°; the crystals which, dried *in vacuo*, were originally of a vitreous lustre and transparent, became, like the crystals dried at 100°, milky-white, porous.

* After drying the stopper.

and absorbent of naphtha. In these experiments the following numbers were obtained.

Experiments with Naphtha A. Glass 2. Temperature of the Air 12°·1–12°·4.

T.	T'.	t'.	t.	M.	m.	f.	y	x.	sp. H.
°	°	°	°	grms.	grms.	grms.		grm.	
44·9	14·8	14·53	11·23	26·935	2·69	2·295	0·431	0·487	0·549
45·9	14·9	14·62	11·23	26·94	„	„	„	„	0·546
47·6	14·6	14·32	10·70	26·925	„	2·445*	„	„	0·531
46·4	15·0	14·73	11·24	26·98	„	2·425†	„	„	0·545

The numbers for the specific heat of nitrate of ammonia are throughout greater than those found between 14° and 31°; and probably because through the heating to 45° or 48° the change was set up *during* the experiments. Experiments with nitrate of ammonia in which, by drying at 100°, this change had been effected before making the experiments, gave numbers which more closely approach the first set, though somewhat greater, and on the whole not very concordant. I obtained in a series of experiments the following results with dull milky crystals dried at 100°.

Experiments with Naphtha A. Glass 1. Temperature of the Air 9°·7.

T.	T'.	t'.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grms.		grm.	
45·0	12·3	11·95	8·96	26·975	2·03	1·77	0·431	0·651	0·519
45·6	12·3	12·03	9·01	26·935	„	„	„	„	0·507
44·9	12·6	12·26	9·32	26·965	„	1·90*	„	„	0·485
45·1	12·5	12·24	9·31	26·98	„	„	„	„	0·470
45·4	12·6	12·33	9·32	26·965	„	2·08‡	„	„	0·457

Crystals dried at 100°–110°, which apparently had been softened, gave the following numbers.

Experiments with Naphtha A. Glass 1. Temperature of the Air 12°·1–12°·4.

T.	T'.	t'.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grms.		grm.	
44·6	14·2	13·93	11·03	26·97	2·095	1·91	0·431	0·651	0·524
43·6	14·4	14·13	11·42	26·935	„	„	„	„	0·489
47·8	14·8	14·54	11·44	26·975	„	2·04*	„	„	0·479
46·5	14·6	14·32	11·23	26·96	„	2·02†	„	„	0·520

I do not know the nature of the change which nitrate of ammonia undergoes just above 40°.

* After adding some naphtha. † After drying the stopper. ‡ After more naphtha.

58. **_Nitrate of Strontia_, $\mathrm{Sr}\,\mathrm{N}_2\,\Theta_6$. Crystallized, dried at 100°.**

Experiments with Naphtha A. Glass 3. Temperature of the Air 14°·9–16°·0.

T.	T'.	t'.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grms.		grm.	
46·0	16·6	16·33	13·95	26·955	4·575	2·10	0·431	0·453	0·180
46·8	17·1	16·83	14·43	26·95	,,	,,	,,	,,	0·179
46·7	17·1	16·84	14·44	26·935	,,	,,	,,	,,	0·180
47·9	17·2	16·93	14.43	26·975	,,	2·085*	,,	,,	0·185
							Mean	. . .	0·181

Nitrate of Baryta, $\mathrm{Ba}\,\mathrm{N}_2\,\Theta_6$. Crystals dried at 100°.

Experiments with Naphtha A. Glass 2. Temperature of the Air 13°·3–13°·4.

T.	T'.	t'.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grms.		grm.	
48·7	15·3	15·23	12·52	26·98	4·995	2·255	0·431	0·487	0·149
48·5	15·4	15·13	12·43	26·985	,,	,,	,,	,,	0·149
47·1	15·5	15·23	12·72	26·955	,,	,,	,,	,,	0·137
46·1	15·6	15·32	12·85	26·95	,,	2·24*	,,	,,	0·146
							Mean	. . .	0·146

Nitrate of Lead, $\mathrm{Pb}\,\mathrm{N}_2\,\Theta_6$. Crystals dried at 100°.

Experiments with Naphtha A. Glass 1. Temperature of the Air 13°·3–13°·4.

T.	T'.	t'.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
46·8	15·7	15·35	12·73	26·925	7·955	1·675	0·431	0·651	0·113
48·2	15·8	15·53	12·82	26·98	,,	,,	,,	,,	0·111
48·1	16·1	15·83	13·22	26·965	,,	,,	,,	,,	0·104
45·0	15·9	15·57	13·15	26·99	,,	1·655*	,,	,,	0·111
							Mean	. . .	0·110

59. *Chlorate of Potass*, $\mathrm{K}\,\mathrm{Cl}\,\Theta_3$. Pure well-dried crystals.

Experiments with Naphtha A. Glass 1. Temperature of the Air 16°·4–17°·3.

T.	T'.	t'.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grms.		grm.	
50·6	18·4	18·12	15·63	26·97	2·485	2·18	0·431	0·651	0·199
50·0	18·6	18·25	15·83	26·945	,,	,,	,,	,,	0·196
48·3	18·8	18·45	16·22	26·95	,,	,,	,,	,,	0·180
48·4	18·8	18·53	16·24	26·96	,,	2·165*	,,	,,	0·202
							Mean	. . .	0·194

---

* After drying the stopper.

Crystallized Chlorate of Baryta, $Ba\,Cl_2\,\Theta_6+H_2\,\Theta$. Crystalline crusts, dried *in vacuo*.

Experiments with Naphtha A. Glass 1. Temperature of the Air 14°·3–14°·4.

T.	T'.	t'.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grms.		grm.	
46·7	16·1	15·83	13·53	26·97	3·02	2·135	0·431	0·651	0·151
46·2	16·2	15·92	13·62	26·915	,,	,,	,,	,,	0·163
46·5	16·1	15·76	13·45	26·95	,,	,,	,,	,,	0·158
46·5	16·1	15·83	13·53	26·99	,,	2·13*	,,	,,	0·157
							Mean	. . .	0·157

Perchlorate of Potass, $K\,Cl\,\Theta_4$. Well-formed crystals.

Experiments with Naphtha A. Glass 2. Temperature of the Air 11°·5.

T.	T'.	t'.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grms.		grm.	
46·6	13·7	13·43	11·02	26·93	3·205	2·115	0·431	0·487	0·179
45·7	13·6	13·33	10·94	26·98	,,	,,	,,	,,	0·190
44·9	13·7	13·43	11·10	26·955	,,	,,	,,	,,	0·192
44·0	13·6	13·33	11·04	26·945	,,	2·095*	,,	,,	0·199

Permanganate of Potass, $K\,Mn\,O_4$. Crystals.

Experiments with Naphtha A. Glass 1. Temperature of the Air 11°·5.

T.	T'.	t'.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
44·3	13·7	13·43	11·02	26·955	3·655	1·83	0·431	0·651	0·187
45·6	13·7	13·43	10·94	26·955	,,	,,	,,	,,	0·181
46·0	13·8	13·51	11·03	26·99	,,	,,	,,	,,	0·175
46·2	13·7	13·44	10·95	26·935	,,	1·815*	,,	,,	0·173
							Mean	. . .	0·179

60. *Metaphosphate of Soda*, $Na\,P\,\Theta_3$. Prepared as a transparent vitreous mass by igniting phosphate of soda and ammonia.

Experiments with Naphtha A. Glass 2. Temperature of the Air 14°·4–14°·5.

T.	T'.	t'.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
49·1	16·7	16·37	13·54	26·92	4·70	1·845	0·431	0·487	0·227
48·3	16·8	16·45	13·75	26·975	,,	,,	,,	,,	0·219
43·1	16·5	16·23	13·96	26·92	,,	,,	,,	,,	0·216
43·3	16·7	16·44	14·23	26·935	,,	1·83*	,,	,,	0·205
							Mean	. . .	0·217

---

* After drying the stopper.

Phosphate of Silver, $Ag_3 P \Theta_4$: yellow powder dried at 110°. This substance, in the quantity I used, is but ill fitted for procuring accurate results. I have made two series of experiments with it, but the results obtained thereby are only to be considered as rough approximations.

I.—Experiments with Naphtha A. Glass 1. Temperature of the Air 20°·5–20°·8.

T.	T'.	t'.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grms.		grm.	
51·4	22·5	22·19	20·16	26·99	3·775	2·105	0·431	0·651	0·0895
52·0	22·4	22·14	20·12	26·955	,,	,,	,,	,,	0·0745 ?
51·5	22·5	22·16	20·13	26·965	,,	,,	,,	,,	0·0872
51·5	22·5	22·15	20·14	26·985	,,	2·095*	,,	,,	0·0839
							Mean† . . .		0·0869

II.—Experiments with Naphtha A. Glass 3. Temperature of the Air 16°·3–16°·6.

T.	T'.	t'.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grms.		grm.	
51·1	18·4	18·12	15·72	26·955	4·545	2·555	0·431	0·453	0·0933
51·5	18·4	18·13	15·73	26·995	,,	,,	,,	,,	0·0887
51·8	18·5	18·22	15·76	26·94	,,	,,	,,	,,	0·0959
51·6	18·6	18·33	15·93	26·98	,,	2·54*	,,	,,	0·0911
							Mean . . .		0·0923

The mean of both these means gives 0·0896 as the specific heat of phosphate of silver. This number, as already remarked, is but little trustworthy. But it may be concluded from these experiments that the specific heat of phosphate of silver cannot differ much from 0·09.

Phosphate of Potass, $K H_2 P \Theta_4$. Clear crystals dried at 110°.

Experiments with Naphtha A. Glass 1. Temperature of the Air 14°·9–16°·0.

T.	T'.	t'.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
46·8	16·9	16·56	14·21	26·96	3·95	1·575	0·431	0·651	0·200
48·0	17·2	16·89	14·43	26·965	,,	,,	,,	,,	0·209
47·5	17·4	17·09	14·71	26·96	,,	,,	,,	,,	0·203
48·0	17·2	16·92	14·43	26·995	,,	1·56*	,,	,,	0·218
							Mean . . .		0·208

---

* After drying the stopper.

† Excluding the second experiment.

Arseniate of Potass, $K H_2 As \Theta_4$. Clear crystals dried at 105°.

Experiments with Naphtha A. Glass 2. Temperature of the Air 14°·3–14°·4.

T.	T'.	*t'*.	*t*.	M.	*m*.	*f*.	*y*.	*x*.	sp. H.
°	°	°	°	grms.	grms.	grms.		grm.	
47·1	16·2	15·93	13·43	26·96	4·455	2·05	0·431	0·487	0·182
47·5	16·2	15·92	13·43	26·975	„	„	„	„	0·174
45·1	16·1	15·84	13·54	26·955	„	„	„	„	0·172
45·5	16·3	16·01	13·70	26·955	„	2·045*	„	„	0·172
							Mean . . .		0·175

61. *Carbonate of Soda*, $Na_2 \text{Ȼ} \Theta_3$. Fused salt.

Experiments with Naphtha A. Glass 2. Temperature of the Air 15°·5.

T.	T'.	*t'*.	*t*.	M.	*m*.	*f*.	*y*.	*x*.	sp. H.
°	°	°	°	grms.	grms.	grms.		grm.	
48·0	17·7	17·35	14·54	26·935	4·575	2·08	0·431	0·487	0·244
47·9	17·7	17·43	14·63	26·95	„	„	„	„	0·244
48·1	17·7	17·40	14·53	26·985	„	„	„	„	0·254
48·1	17·7	17·43	14·63	26·965	„	2·055*	„	„	0·243
							Mean . . .		0·246

Carbonate of Potass, $K_2 CO_3$. Fused salt.

Experiments with Naphtha A. Glass 1. Temperature of the Air 15°·5.

T.	T'.	*t'*.	*t*.	M.	*m*.	*f*.	*x*.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.
47·4	17·4	17·14	14·75	26·975	3·045	1·96	0·651	0·215
47·5	17·4	17·12	14·73	26·975	„	„	„	0·212
47·3	17·4	17·14	14·82	26·95	„	„	„	0·196
45·6	17·5	17·21	15·02	26·96	„	1·95*	„	0·200
						Mean . . .		0·206

Carbonate of Rubidium, $Rb_2 \text{Ȼ} \Theta_3$. Fused salt.

Experiments with Naphtha A. Glass 2. Temperature of the Air 15°·5.

T.	T'.	*t'*.	*t*.	M.	*m*.	*f*.	*y*.	*x*.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
49·3	17·7	17·38	14·80	26·965	6·855	1·95	0·431	0·487	0·127
47·1	17·4	17·13	14·70	26·955	„	„	„	„	0·128
46·8	17·6	17·33	14·94	26·97	„	„	„	„	0·128
45·8	17·6	17·33	15·16	26·93	„	1·93*	„	„	0·110
							Mean . . .		0·123

---

* After drying the stopper.

62. *Carbonate of Lead*, $Pb\,CO_3$. *Cerussite* from Washington mine, Davidson county, North Carolina: beautiful clear crystals.

Experiments with Naphtha A. Glass 1. Temperature of the Air 13°·8.

T.	T'.	t'.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
49·2	16·3	16·03	13·16	26·95	11·42	1·90	0·431	0·651	0·0772
49·8	16·0	15·68	12·72	26·94	,,	,,	,,	,,	0·0779
47·4	15·9	15·60	12·80	26·94	,,	,,	,,	,,	0·0810
46·5	15·9	15·64	12·94	26·97	,,	,,	,,	,,	0·0797
43·2	15·8	15·55	13·14	26·96	,,	1·885*	,,	,,	0·0795
							Mean	. . .	0·0791

Carbonate of Lime, $Ca\,CO_3$. I have investigated both the rhombic and the rhombohedral modification.

Arragonite. Fragments of clear crystals from Bilin, in Bohemia

Experiments with Naphtha A. Glass 1. Temperature of the Air 13°·8.

T.	T'.	t'.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
51·1	16·8	16·53	13·25	26·965	6·445	1·94	0·431	0·487	0·195
46·6	16·0	15·70	12·73	26·98	,,	,,	,,	,,	0·201
45·8	16·1	15·83	12·94	26·975	,,	,,	,,	,,	0·216
44·0	16·0	15·74	13·03	26·965	,,	,,	,,	,,	0·200
44·3	15·9	15·63	12·86	26·955	,,	1·92*	,,	,,	0·204
							Mean	. . .	0·203

Calcareous Spar. Cleavage pieces of transparent specimens from Auerbach, on the Bergstrasse.

Experiments with Naphtha A. Glass 1. Temperature of the Air 14°·4–14°·7.

T.	T'.	t'.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
49·5	15·5	15·24	12·13	26·98	5·425	1·48	0·431	0·651	0·217
49·6	16·3	15·96	13·00	26·96	,,	,,	,,	,,	0·204
48·2	16·1	15·83	12·94	26·915	,,	,,	,,	,,	0·209
45·2	16·2	15·94	13·42	26·93	,,	1·465*	,,	,,	0·195
							Mean	. . .	0·206

---

* After drying the stopper.

63. *Magnesian Spar*, $Ca_{\frac{1}{2}}\,Mg_{\frac{1}{2}}\,CO_3$*. Specimens of magnesian spar from the Zillerthal.

Experiments with Naphtha A. Glass 3. Temperature of the Air 15°·1–15°·9.

T.	T'.	t'.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
48·9	17·7	17·43	14·52	26·96	6·195	1·76	0·431	0·453	0·210
48·3	17·9	17·60	14·77	26·96	,,	,,	,,	,,	0·210
47·0	17·9	17·64	15·02	26·995	,,	1·745†	,,	,,	0·198
							Mean . . .		0·206

Spathic Iron, $Fe_{\frac{8}{11}}\,Mn_{\frac{2}{11}}\,Mg_{\frac{1}{11}}\,CO_3$‡. Cleavage pieces of reddish crystals from Bieber, Hesse Cassel.

Experiments with Naphtha A. Glass 1. Temperature of the Air 14°·6–14°·4.

T.	T'.	t'.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
47·7	17·0	16·74	13·92	26·98	6·56	1·78	0·431	0·651	0·162
45·6	16·9	16·63	13·94	26·93	,,	,,	,,	,,	0·169
46·1	16·9	16·55	13·83	26·965	,,	1·765†	,,	,,	0·168
							Mean . . .		0·166

64. *Zircon*, $Zr\,Si\,O_4$, or $Zr_{\frac{1}{2}}\,Si_{\frac{1}{2}}\,O_2$. Hyacinth crystals from Ceylon.

Experiments with Naphtha A. Glass 1. Temperature of the Air 18°·4–19°·8.

T.	T'.	t'.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
51·2	20·6	20·33	17·46	26·945	9·69	1·32	0·431	0·651	0·135
50·2	20·8	20·54	17·83	26·955	,,	,,	,,	,,	0·131
51·0	21·0	20·74	18·01	26·97	,,	,,	,,	,,	0·127
52·0	21·2	20·87	18·03	26·96	,,	,,	,,	,,	0·131
51·1	21·3	21·03	18·24	26·93	,,	1·30†	,,	,,	0·135
							Mean . . .		0·132

---

* The results of my analysis of this spar (Ann. der Chem. und Pharm. lxxxi. 50) are, compared with the numbers required by the above formula, as follows:—

	$CaO\,CO_2$.	$MgO\,CO_2$.	$FeO\,CO_2$[a].	Total.
Found	54·3	42·2	3·7	100·2
Calculated	54·3	45·7	,,	100·0

† After drying the stopper.

‡ The numbers found in my analysis of this spathic iron (Ann. der Chem. und Pharm. lxxxi. 51) are given below, compared with those calculated on the above formula.

	$FeO\,CO_2$.	$MnO\,CO_2$.	$CaO\,CO_2$.	$MgO\,CO_2$.	X[b].	Total.
Found	73·7	19·0	0·9	6·6	0·7	100·9
Calculated	74·7	18·6	,,	6·7	,,	100·0

---

[a] With some $MnO\,CO_2$.

[b] Insoluble in aqua regia.

Chrysolite, $\mathrm{Mg}_{\frac{20}{11}}\mathrm{Fe}_{\frac{2}{11}}\mathrm{SiO}_4$*. From Dockweiler in the Eifel. Transparent to translucent bright green crystalline fragments.

Experiments with Naphtha A. Glass 1. Temperature of the Air 19°·2–19°·5.

T.	T′.	t′.	t.	M. grms.	m. grms.	f. grm.	y.	x. grm.	sp. H.
51°·3	21°·4	21°·14	18°·53	26·985	5·84	1·475	0·431	0·657	0·183
50·4	21·4	21·13	18·55	26·965	,,	,,	,,	,,	0·191
50·9	21·5	21·17	18·54	26·985	,,	,,	,,	,,	0·193
50·9	21·5	21·16	18·55	26·96	,,	,,	,,	,,	0·189
49·9	21·4	21·13	18·63	26·975	,,	1·45†	,,	,,	0·187
						Mean	.	. .	0·189

Olivine, $\mathrm{Mg}_{\frac{20}{11}}\mathrm{Fe}_{\frac{2}{11}}\mathrm{SiO}_4$‡. From a spheroidal mass surrounded by lava from the Eifel.

Experiments with Naphtha A. Glass 1. Temperature of the Air 19°·0–19°·6.

T.	T′.	t′.	t.	M. grms.	m. grms.	f. grm.	y.	x. grm.	sp. H.
51°·5	21°·6	21°·26	18°·53	26·975	6·37	1·425	0·431	0·651	0·188
51·4	21·3	20·97	18·22	26·975	,,	,,	,,	,,	0·188
51·5	21·6	21·25	18·52	26·975	,,	,,	,,	,,	0·188
52·1	21·8	21·52	18·72	26·97	,,	1·41†	,,	,,	0·194
						Mean	.	. .	0·187

65. *Wollastonite*, CaSiO_3. Pure pieces of Wollastonite from Finnland.

Experiments with Naphtha A. Glass 1. Temperature of the Air 17°·2.

T.	T′.	t′.	t.	M. grms.	m. grms.	f. grm.	y.	x. grm.	sp. H.
51°·0	19°·4	19°·12	16°·33	26·955	5·31	1·81	0·431	0·651	0·179
50·5	19·1	18·76	16·01	26·945	,,	,,	,,	,,	0·175
50·0	19·2	18·92	16·19	26·98	,,	,,	,,	,,	0·181
50·7	19·4	19·13	16·40	26·97	,,	1·785†	,,	,,	0·176
						Mean	.	. .	0·178

---

* An analysis by Professor KNOP gave the following results, which are collated with the numbers required by the above formula:—

	$\mathrm{SiO_2}$.	MgO.	FeO.	$\mathrm{Al_2O_3}$.	Total.
Found	40·95	50·82	8·83	trace	100·60
Calculated	41·15	49·87	8·98	,,	100·00

† After drying the stopper.

‡ This olivine has the same composition as the above chrysolite. Professor KNOP found for this olivine the following numbers, which are compared with those required by the above formula:—

	$\mathrm{SiO_2}$.	MgO.	FeO.	$\mathrm{Al_2O_3}$.	Total.
Found	41·85	49·10	8·75	trace	99·70
Calculated	41·15	49·87	8·98	,,	100·00

Diopside, $\text{Ca}_{\frac{1}{2}}\,\text{Mg}_{\frac{1}{2}}\,\text{Si}\,\text{O}_3$. Fragments of a greenish and white crystal of the characteristic aspect of the diopside from Schwarzenstein in the Tyrol.

Experiments with Naphtha A. Glass 1. Temperature of the Air 16°·3–16°·5.

T.	T'.	*t'.*	*t.*	M.	*m.*	*f.*	*y.*	*x.*	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
48·1	18·7	18·42	15·65	26·99	6·17	1·55	0·431	0·651	0·186
49·4	18·4	18·13	15·22	26·98	,,	,,	,,	,,	0·185
51·8	18·6	18·25	15·13	26·98	,,	,,	,,	,,	0·185
50·8	18·8	18·54	15·53	26·925	,,	1·53*	,,	,,	0·186
							Mean	. . .	0·186

Dioptase, $\text{Cu}\,\text{SiO}_3+\text{H}_2\text{O}$. Fine crystals from the Kirgisensteppe.

Experiments with Naphtha A. Glass 3. Temperature of the Air 16°·7–16°·4.

T.	T'.	*t'.*	*t.*	M.	*m.*	*f.*	*y.*	*x.*	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
49·8	18·9	18·63	16·04	26·94	5·545	1·80	0·431	0·453	0·186
50·3	19·1	18·76	16·17	26·95	,,	,,	,,	,,	0·182
50·3	18·9	18·64	16·05	26·99	,,	,,	,,	,,	0·180
48·5	18·9	18·58	16·13	26·945	,,	1·79*	,,	,,	0·181
							Mean	. . .	0·182

Orthoclase, $\text{Al}_2\,\text{K}_2\,\text{Si}_6\,\text{O}_{16}$. Cleavage pieces of a flesh-coloured reddish orthoclase from Aschaffenburg.

Experiments with Naphtha A. Glass 3. Temperature of the Air 18°·4–19°·1.

T.	T'.	*t'.*	*t.*	M.	*m.*	*f.*	*y.*	*x.*	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
50·6	20·2	19·86	17·42	26·945	5·185	1·78	0·431	0·453	0·182
49·6	20·3	20·00	17·63	26·95	,,	,,	,,	,,	0·185
51·1	20·5	20·15	17·71	26·94	,,	,,	,,	,,	0·179
51·2	20·5	20·21	17·73	26·965	,,	1·77*	,,	,,	0·186
							Mean	. . .	0·183

Albite, $\text{Al}_2\,\text{Na}_2\,\text{Si}_6\,\text{O}_{16}$. Fragments of white crystals from Pfunders, in Tyrol.

Experiments with Naphtha A. Glass 3. Temperature of the Air 18°·7–19°·8.

T.	T'.	*t'.*	*t.*	M.	*m.*	*f.*	*y.*	*x.*	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
52·4	20·3	20·04	17·44	26·955	4·835	1·84	0·431	0·453	0·194
50·7	20·8	20·53	18·14	26·975	,,	,,	,,	,,	0·188
50·1	20·9	20·63	18·30	26·935	,,	,,	,,	,,	0·187
52·0	21·1	20·82	18·33	26·955	,,	,,	,,	,,	0·192
50·4	21·3	21·04	18·73	26·97	,,	1·82*	,,	,,	0·187
							Mean	. . .	0·190

---

* After drying the stopper.

66. *Borate of Soda*, $Na_2B_4\Theta_7$. Beautiful transparent vitreous pieces of fused borax.

Experiments with Naphtha A. Glass 2. Temperature of the Air 14°·4.

T.	T′.	*t′*.	*t*.	M.	*m*.	*f*.	*y*.	*x*.	sp. H.
°	°	°	°	grms.	grms.	grms.		grm.	
46·6	16·6	16·33	13·67	26·95	4.475	2·005	0·431	0·487	0·232
46·8	16·6	16·33	13·65	26·98	„	„	„	„	0·233
46·5	16·6	16·33	13·73	26·965	„	„	„	„	0·222
46·6	16·8	16·54	13·93	26·945	„	1·99*	„	„	0·227
							Mean	. . .	0·227

Hydrated Borate of Soda, $Na_2B_4\Theta_7+10H_2\Theta$. Crystallized borax dried in the air.

Experiments with Naphtha A. Glass 3. Temperature of the Air 16°·3–16°·5.

T.	T′.	*t′*.	*t*.	M.	*m*.	*f*.	*y*.	*x*.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
50·9	18·7	18·43	15·43	26·98	3·38	1·745	0·431	0·453	0·387
50·3	18·4	18·13	15·15	26·95	„	„	„	„	0·388
49·1	18·5	18·16	15·33	26·96	„	„	„	„	0·381
49·5	18·8	18·45	15·61	26·945	„	1·73*	„	„	0·383
							Mean	. . .	0·385

67. *Tungstate of Lime*, $CaW\Theta_4$. Crystals of *Scheelite* from Zinnwald in Bohemia.

Experiments with Naphtha A. Glass 1. Temperature of the Air 16°·7–16°·4.

T.	T′.	*t′*.	*t*.	M.	*m*.	*f*.	*y*.	*x*.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
50·3	19·3	19·00	16·27	26·96	11·575	1·34	0·431	0·651	0·0990
49·5	19·1	18·84	16·22	26·96	„	„	„	„	0·0946
50·5	19·0	18·71	15·94	26·97	„	„	„	„	0·0988
48·6	19·0	18·66	16·12	26·99	„	1·325*	„	„	0·0945
							Mean	. . .	0·0967

Wolfram, $Fe_{\frac{2}{5}}Mn_{\frac{3}{5}}W\Theta_4$†. Fragments of crystals from Altenberg in the Erzgebirge.

Experiments with Naphtha B. Glass 1. Temperature of the Air 19°·1–19°·0.

T.	T′.	*t′*.	*t*.	M.	*m*.	*f*.	*y*.	*x*.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
52·1	21·1	20·83	18·14	26·985	11·455	1·525	0·419	0·651	0·0918
52·9	21·2	20·92	18·14	26·975	„	„	„	„	0·0939
54·0	21·2	20·92	18·04	26·97	„	„	„	„	0·0941
54·8	21·4	21·13	18·23	26·945	„	1·51*	„	„	0·0921
							Mean	. . .	0·0930

* After drying the stopper.

† According to KERNDT's analysis of the wolfram of Altenberg (Rammelsberg's 'Handbuch der Mineral. Chemie,' p. 308).

Molybdate of Lead, $\text{Pb M }\Theta_4$. Comminuted crystals of Wülfenite (Gelbbleierz) from Bleiberg in Carinthia.

Experiments with Naphtha A. Glass 3 Temperature of the Air 17°·6–17°·4.

T.	T′.	t′.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grms.		grm.	
50·2	19·3	18·95	16·45	26·98	8·69	2·32	0·431	0·453	0·0840
50·0	19·2	18·92	16·43	26·97	„	„	„	„	0·0837
48·6	19·1	18·84	16·47	26·935	„	„	„	„	0·0818
49·3	19·3	19·01	16·62	26·98	„	2·295*	„	„	0·0814
						Mean	.	. .	0·0827

68. *Chromate of Lead*, $\text{Pb Cr }\Theta_4$. For the investigation pieces of artificially prepared chromate of lead were used, which after fusion had solidified to an aurora-red mass of a fibrous crystalline structure, and with crystal needles on the surface.

Experiments with Naphtha A. Glass 3. Temperature of the Air 17°·1–17°·9.

T.	T′.	t′.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
50·0	19·0	18·74	16·22	26·975	10·60	1·93	0·431	0·453	0·0857
50·1	19·2	18·92	16·34	26·985	„	„	„	„	0·0931
49·6	19·2	18·93	16·42	26·975	„	„	„	„	0·0889
49·9	19·3	19·02	16·44	26·99	„	1·915*	„	„	0·0940
						Mean	.	. .	0·0900

Chromate of Potass, $K_2\,\text{Cr }\Theta_4$. Crystals of the neutral salt dried at 105°.

Experiments with Naphtha A. Glass 1. Temperature of the Air 16°·1–16°·8.

T.	T′.	t′.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
49·1	18·0	17·69	15·13	26·985	4·995	1·535	0·431	0·651	0·182
45·7	17·8	17·49	15·14	26·975	„	„	„	„	0·192
47·3	17·9	17·62	15·13	26·995	„	„	„	„	0·195
48·2	18·2	17·93	15·43	26·955	„	1·525*	„	„	0·188
						Mean	.	. .	0·189

Acid Chromate of Potass, $K_2\,\text{Cr}_2\,\Theta_7$ Crystals of the so-called bichromate.

Experiments with Naphtha A. Glass 1. Temperature of the Air 19°·1–19°·5.

T.	T′.	t′.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
53·3	21·1	20·83	18·33	26·97	4·275	1·58	0·431	0·651	0·178
51·5	21·1	20·82	18·42	26·95	„	„	„	„	0·186
51·6	21·1	20·76	18·33	26·96	„	„	„	„	0·191
52·6	21·2	20·93	18·45	26·975	„	1·555*	„	„	0·189
						Mean	.	. .	0·186

* After drying the stopper.

69. *Sulphate of Soda*, $Na_2\,S\Theta_4$. Crystalline crusts briskly dried.

Experiments with Naphtha A. Glass 1. Temperature of the Air 11°·2–11°·4.

T.	T′.	*t′*.	*t*.	M.	*m*.	*f*.	*y*.	*x*.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
44·2	12·8	12·52	9·94	26·97	3·465	1·73	0·431	0·651	0·236
47·8	13·2	12·93	10·14	26·93	„	„	„	„	0·224
46·1	13·2	12·93	10·25	26·95	„	„	„	„	0·230
46·6	13·6	13·32	10·69	26·975	„	1·715*	„	„	0·219
							Mean . . .		0·227

Sulphate of Potass, $K_2\,S\Theta_4$. Crystal crusts sharply dried.

Experiments with Naphtha A. Glass 2. Temperature of the Air 11°·2–11°·4.

T.	T′.	*t′*.	*t*.	M.	*m*.	*f*.	*y*.	*x*.	sp. H.
°	°	°	°	grms.	grms.	grms.		grm.	
44·5	12·7	12·44	12·02	26·915	3·405	2·145	0·431	0·487	0·187
47·0	13·2	12·93	10·22	26·95	„	2·30†	„	„	0·200
45·9	13·3	13·02	10·41	26·95	„	„	„	„	0·200
43·1	13·3	13·03	10·67	26·95	„	2·275*	„	„	0·196
							Mean . . .		0·196

Acid Sulphate of Potass, $K\,H\,S\Theta_4$. Well-formed crystals dried at 100°‡. The salt became feebly red on the surface in contact with the coal-tar naphtha.

Experiments with Naphtha A. Glass 1. Temperature of the Air 17°·0–17°·2.

T.	T′.	*t′*.	*t*.	M.	*m*.	*f*.	*y*.	*x*.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
50·7	19·4	19·12	16·43	26·94	3·445	1·85	0·431	0·651	0·251
50·4	19·3	19·01	16·36	26·945	„	„	„	„	0·245
50·5	19·3	18·97	16·34	26·96	„	„	„	„	0·239
51·9	19·4	19·05	16·32	26·965	„	1·83*	„	„	0·239
							Mean . . .		0·244

70. *Sulphate of Ammonia*, $N_2\,H_8\,S\Theta_4$. I made two series of experiments with this salt. Crystals dried *in vacuo* over sulphuric acid.

I.—Experiments with Naphtha A. Glass 2. Temperature of the Air 10°·9–11°·3.

T.	T′.	*t′*.	*t*.	M.	*m*.	*f*.	*y*.	*x*.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
45·1	13·0	12·73	9·73	26·93	3·425	1·825	0·431	0·487	0·363
44·5	13·4	13·12	10·25	26·98	„	„	„	„	0·355
44·3	13·2	12·93	10·08	26·93	„	1·815*	„	„	0·350
							Mean . . .		0·356

---

* After drying the stopper. † After adding some naphtha.

‡ Dr. Engelbach found the quantity of potass in these crystals to be 33·70 and 34·13 per cent. Calculated from the above formula 34·61 per cent. are required.

Crystals dried at 120°.

II.—Experiments with Naphtha A. Glass 1. Temperature of the Air 10°·9–11°·31.

T.	T′.	t′.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
44·2	12·9	12·63	9·97	26·94	2·84	1·555	0·431	0·661	0·341
42·2	12·6	12·33	9·81	26·95	,,	,,	,,	,,	0·343
45·4	13·3	12·96	10·30	26·985	,,	,,	,,	,,	0·322
46·7	13·0	12·72	9·77	26·935	,,	1·535*	,,	,,	0·368
							Mean . . .		0·344

The mean of the means of both series of experiments, 0·356 and 0·344, gives for the specific heat of sulphate of ammonia between 13° and 45° the number 0·350†.

71. *Sulphate of Lead*, $PbSO_4$. Fragments of transparent crystals of lead-vitriol from Müsen, near Siegen.

Experiments with Naphtha A. Glass 1. Temperature of the Air 17°·6–17°·4.

T.	T′.	t′.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
48·3	19·6	19·33	16·90	26·975	12·575	1·47	0·431	0·651	0·0795
50·9	19·3	19·00	16·23	26·96	,,	,,	,,	,,	0·0858
49·9	19·3	19·01	16·33	26·985	,,	,,	,,	,,	0·0858
50·4	19·6	19·24	16·63	26·99	,,	1·45*	,,	,,	0·0798
							Mean . . .		0·0827

Sulphate of Baryta, $BaSO_4$. Cleavage pieces of crystal of heavy spar from the Auvergne.

I.—Experiments with Naphtha A. Glass 1. Temperature of the Air 15°·1–15°·9.

T.	T′.	t′.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
46·5	17·4	17·12	14·64	26·945	9·15	1·405	0·431	0·651	0·113
48·5	17·5	17·17	14·56	26·97	,,	,,	,,	,,	0·111
44·6	17·4	17·05	14·82	26·97	,,	1·395*	,,	,,	0·105
							Mean . . .		0·110

* After drying the stopper.

† I had made a third series of experiments with large dry transparent crystals of sulphate of ammonia, but in which t′ exceeded more than usual the temperature of the air, and hence numbers were found for the body investigated which are somewhat too small.

Experiments with Naphtha A. Glass 2. Temperature of the Air 9°·7.

T.	T′	t′.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grms.		grm.	
45·6	12·4	12·05	8·86	26·935	3·725	2·015	0·431	0·487	0·331
47·1	12·8	12·45	9·22	26·97	,,	,,	,,	,,	0·318
42·9	12·6	12·25	9·42	26·99	,,	,,	,,	,,	0·313
44·1	12·5	12·22	9·24	26·95	,,	,,	,,	,,	0·318
47·0	12·7	12·36	9·16	26·94	,,	1·985[a]	,,	,,	0·314

---

[a] After removing some naphtha from the stopper.

II.—Experiments with Naphtha A. Glass 1. Temperature of the Air 16°·7–17°·2.

T.	T'.	t'.	t.	M. grms.	m. grms.	f. grm.	y.	x. grm.	sp. H.
49°·9	19°·0	18°·65	16°·13	26·96	7·77	1·68	0·431	0·651	0·106
50·9	19·0	18·74	16·14	26·94	„	„	„	„	0·106
49·0	19·0	18·67	16·22	26·96	„	1·665*	„	„	0·107
							Mean	. . .	0·106

The mean of the means of these two sets of experiments gives 0·108 for the specific heat of heavy spar between 18° and 44°.

Sulphate of Strontia, $\mathrm{Sr\,S\Theta_4}$. Crystals of celestine from Dornburg, near Jena.

Experiments with Naphtha A. Glass 3. Temperature of the Air 15°·6–16°·1.

T.	T'.	t'.	t.	M. grms.	m. grms.	f. grm.	y.	x. grm.	sp. H.
50°·2	17°·8	17°·47	14°·74	26·965	7·63	1·90	0·431	0·453	0·137
50·5	17·7	17·43	14·64	26·955	„	„	„	„	0·134
51·4	17·8	17·51	14·64	26·995	„	„	„	„	0·135
52·7	17·9	17·55	14·61	26·955	„	1·875*	„	„	0·133
							Mean	. . .	0·135

72. *Sulphate of Lime*, $\mathrm{Ca\,S\Theta_4}$. Small crystalline pieces of anhydrite.

I.—Experiments with Naphtha A. Glass 1. Temperature of the Air 13°·2–13°·7.

T.	T'.	t'.	t.	M. grms.	m. grms.	f. grm.	y.	x. grm.	sp. H.
46°·1	15°·6	15°·33	12°·72	26·98	5·305	1·715	0·431	0·651	0·173
46·5	15·5	15·22	12·53	26·93	„	„	„	„	0·178
45·7	15·6	15·34	12·74	26·92	„	„	„	„	0·176
43·6	15·7	15·44	13·11	26·94	„	1·70*	„	„	0·163
							Mean	. . .	0·173

II.—Experiments with Water. Glass 3. Temperature of the Air 17°·9–18°·3.

T.	T'.	t'.	t.	M. grms.	m. grms.	f. grms.	y.	x. grm.	sp. H.
47°·5	19°·9	19°·62	15°·62	26·95	5·62	2·415	1·000	0·453	0·185
47·1	19·8	19·53	15·61	26·99	„	„	„	„	0·179
47·1	20·1	19·77	15·87	26·975	„	„	„	„	0·183
47·5	20·2	19·94	16·03	26·98	„	2·40*	„	„	0·180
							Mean	. . .	0·182

The average of the means of these determinations gives 0·178 as the specific heat of anhydrite between 18° and 46°.

---

* After drying the stopper.

Hydrated Sulphate of Lime, $\mathrm{\bar{C}a\,\bar{S}\bar{O}_4+2\,H_2\,O}$. Cleavage pieces of transparent *Gypsum* from Reinhardtsbrunn, in Thüringen.

Experiments with Naphtha A. Glass 2. Temperature of the Air 13°·2–13°·7.

T.	T'.	t'.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grms.		grm.	
47·2	15·6	15·29	12·32	26·94	4·335	2·115	0·431	0·487	0·261
47·4	15·8	15·53	12·57	26·99	„	„	„	„	0·261
45·7	15·8	15·53	12·73	26·96	„	„	„	„	0·260
44·2	16·0	15·73	12·13	26·94	„	2·095*	„	„	0·252
							Mean . . .		0·259

73. *Crystallized Sulphate of Copper*, $\mathrm{\bar{C}u\,\bar{S}\bar{O}_4+5\,H_2\,\bar{O}}$. Crystals of *Blue vitriol* dried in the air.

Experiments with Naphtha A. Glass 1. Temperature of the Air 14°·1–14°·2.

T.	T'.	t'.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
50·8	16·4	16·08	12·82	26·99	4·12	1·65	0·431	0·651	0·290
47·3	16·4	16·05	13·12	26·965	„	„	„	„	0·290
46·7	16·5	16·16	13·34	26·99	„	„	„	„	0·281
45·0	16·6	16·26	13·63	26·965	„	1·635*	„	„	0·277
							Mean . . .		0·285

Crystallized Sulphate of Manganese, $\mathrm{Mn\,\bar{S}\bar{O}_4+5\,H_2\,\bar{O}}$. Crystals of the salt isomorphous with blue vitriol.

Experiments with Naphtha A. Glass 2. Temperature of the Air 14°·1–14°·2.

T.	T'.	t'.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
48·5	16·7	16·42	13·23	26·945	4·12	1·97	0·431	0·487	0·332
45·7	16·4	16·14	13·24	26·945	„	„	„	„	0·323
46·5	16·7	16·43	13·53	26·98	„	„	„	„	0·313
44·0	16·8	16·53	13·85	26·945	„	1·955*	„	„	0·322
							Mean . . .		0·323

Crystallized Sulphate of Nickel, $\mathrm{\bar{N}i\,\bar{S}\bar{O}_4+6\,H_2\,\bar{O}}$. Crystals of quadratic nickel vitriol dried *in vacuo*.

Experiments with Naphtha A. Glass 1. Temperature of the Air 15°·6–16°·1.

T.	T'.	t'.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
52·5	18·0	17·74	14·61	26·97	3·60	1·655	0·431	0·651	0·307
50·3	17·7	17·42	14·37	26·995	„	„	„	„	0·322
51·5	17·7	17·36	14·24	26·985	„	„	„	„	0·313
52·8	18·1	17·82	14·62	26·94	„	1·63*	„	„	0·314
							Mean . . .		0·313

* After drying the stopper.

74. *Crystallized Sulphate of Magnesia*, $Mg\,SO_4+7\,H_2\,O$. Air-dried crystals of Epsom salt. I have made two series of experiments with this salt. In one the temperature did not exceed 40°, and in the other did not attain 50°. In both cases the crystals remained transparent and unchanged.

I.—Experiments with Naphtha A. Glass 3. Temperature of the Air 19°·8–19°·9.

T.	T'.	t'.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
38·5	21·6	21·29	19·77	26·96	3·175	1·845	0·431	0·453	0·371
39·3	21·6	21·32	19·73	26·945	,,	,,	,,	,,	0·369
38·7	21·6	21·34	19·83	26·98	,,	,,	,,	,,	0·357
37·7	21·6	21·27	19·85	26·935	,,	1·835*	,,	,,	0·356
							Mean	. . .	0·363

II.—Experiments with Naphtha A. Glass 1. Temperature of the Air 16°·1.

T.	T'.	t'.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
47·6	18·3	18·04	15·42	26·97	2·775	1·81	0·431	0·651	0·353
47·9	18·4	18·12	15·43	26·985	,,	,,	,,	,,	0·371
45·2	18·3	17·96	15·53	26·94	,,	,,	,,	,,	0·361
43·9	18·3	17·96	15·67	26·975	,,	1·795*	,,	,,	0·356
							Mean	. . .	0·360

These determinations give as the mean of the two series 0·362 for the specific heat of crystallized sulphate of magnesia below 50°†.

Crystallized Sulphate of Zinc, $Zn\,SO_4+7\,H_2\,O$. Transparent crystals of white vitriol, dried in the air. In the determinations a heat but little over 50° could be employed; towards 50° the crystals undergo decomposition in the coal-tar naphtha‡.

Experiments with Naphtha A. Glass 1. Temperature of the Air 13°·4.

T.	T'.	t'.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
28·7	14·6	14·33	12·93	26·945	3·55	1·655	0·431	0·651	0·369
30·7	14·9	14·62	13·13	26·95	,,	,,	,,	,,	0·332

This series of experiments had to be interrupted here. I subsequently made another set.

---

* After drying the stopper.

† Above 50° the salt with 7 at. water of crystallization undergoes decomposition. A series of experiments in which the temperature exceeded 50° gave the following results.

Experiments with Naphtha A. Glass 3. Temperature of the Air 20°·3–21°·1.

T.	T'.	t'.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
51·5	22·6	22·32	19·61	26·995	3·43	1·57	0·431	0·453	0·409
51·4	22·8	22·52	19·55	26·93	,,	,,	,,	,,	0·475
51·0	23·0	22·71	19·73	26·945	,,	,,	,,	,,	0·507
50·0	23·0	22·71	19·81	26·93	,,	1·56*	,,	,,	0·515

The results are as if more and more water in the free state had been eliminated. After the experiments the crystals were swollen, and externally milk white, still containing a clear nucleus inside.

‡ In the following series of experiments, in which a heat of towards 50° was employed, the crystals of white

Experiments with Naphtha A. Glass 1. Temperature of the Air 14°·4–15°·0.

T.	T′.	t′.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
30·9	15·7	15·43	14·03	26·93	3·49	1·645	0·431	0·651	0·321
32·3	16·0	15·65	14·13	26·96	,,	,,	,,	,,	0·331
30·8	15·8	15·52	14·03	26·95	,,	,,	,,	,,	0·377
32·8	16·1	15·83	14·23	26·97	,,	1·635*	,,	,,	0·352

In all these experiments the crystals employed remained clear. The mean of the six experiments gives 0·347 as the specific heat of crystallized sulphate of zinc.

Crystallized Sulphate of Iron, Fe SΘ_4+7 H$_2$ Θ. Dry crystals of green vitriol.

Experiments with Naphtha A. Glass 2. Temperature of the Air 16°·1.

T.	T′.	t′.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
47·9	18·6	18·32	15·56	26·93	3·47	1·91	0·431	0·487	0·354
47·5	18·6	18·25	15·55	26·925	,,	,,	,,	,,	0·347
46·0	18·5	18·21	15·64	26·955	,,	,,	,,	,,	0·348
44·6	18·4	18·13	15·73	26·96	,,	1·895*	,,	,,	0·336
							Mean	. . .	0·346

Crystallized Sulphate of Cobalt, Ɵo SΘ_4+7 H$_2$ Θ. Crystals of the salt isomorphous with green vitriol. In the following experiments the crystals remained transparent[†].

Experiments with Naphtha A. Glass 2. Temperature of the Air 13°·4–13°·2.

T.	T′.	t′.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
31·6	14·9	14·63	12·96	26·97	3·445	1·895	0·431	0·487	0·405
29·9	14·8	14·54	13·14	26·945	,,	,,	,,	,,	0·347
28·4	15·0	14·67	13·43	26·93	,,	,,	,,	,,	0·345
31·6	15·2	14·94	13·44	26·94	,,	1·885*	,,	,,	0·338
							Mean	. . .	0·343[‡]

---

vitriol undergo an essential change. At the end of the experiments they were opaque, and no longer detached, as before, but as if swollen up in the glass. These experiments gave the following numbers:—

Experiments with Naphtha A. Glass 1. Temperature of the Air 14°·8–14°·4.

T.	T′.	t′.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
47·4	17·0	16·74	13·62	26·94	3·465	1·695	0·431	0·651	0·399
47·6	17·0	16·72	13·62	26·945	,,	,,	,,	,,	0·389
45·1	16·9	16·63	13·77	26·975	,,	1·655§	,,	,,	0·396
43·8	17·1	16·83	14·22	26·99	,,	,,	,,	,,	0·368

[*] After drying the stopper.

[†] In a series of experiments, in which the temperature amounted to 50°, the crystals of sulphate of cobalt with seven atoms of water underwent a change; they were opaque, and stuck in the glass as if swollen up: and the numbers found for the specific heat were considerably greater.

[‡] Excluding the first experiment. The temperature of the glass, together with the solid substance and the liquid, exceeded in all experiments the final temperature of the water in the calorimeter only by about 15°.

[§] After removing some naphtha from the stopper.

75. *Crystallized Sulphate of Magnesia and Potass*, $\mathrm{Mg}\,K_2\,S_2\,\Theta_8+6\,H_2\,\Theta$. Well-shaped crystals.

Experiments with Naphtha A. Glass 3. Temperature of the Air 17°·0–17°·2.

T.	T'.	t'.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grms.		grm.	
51·0	19·4	19·13	16·43	26·99	4·135	1·735	0·431	0·453	0·267
51·0	19·3	19·02	16·33	26·965	,,	,,	,,	,,	0·263
50·0	19·3	19·02	16·43	26·96	,,	,,	,,	,,	0·260
50·2	19·4	19·06	16·44	26·95	,,	1·715*	,,	,,	0·266
							Mean	. . .	0·264

Crystallized Sulphate of Zinc and Potass, $\mathrm{Zn}\,K_2\,S_2\,\Theta_8+6\,H_2\,\Theta$. Well-shaped crystals; in both the following series they remained transparent and unchanged.

1.—Experiments with Naphtha A. Glass 1. Temperature of the Air 19°·8–19°·9.

T.	T'.	t'.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
40·2	21·7	21·37	19·73	26·925	3·965	1·535	0·431	0·651	0·271
40·6	21·7	21·42	19·75	26·935	,,	,,	,,	,,	0·269
40·2	21·7	21·38	19·73	26·955	,,	,,	,,	,,	0·275
39·8	21·7	21·40	19·83	26·925	,,	1·52*	,,	,,	0·260
							Mean	. . .	0·269

II.—Experiments with Naphtha A. Glass 2. Temperature of the Air 14°·8–14°·4.

T.	T'.	t'.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
48·9	16·9	16·64	13·63	26·94	4·365	1·98	0·431	0·487	0·273
47·2	16·8	16·50	13·63	26·92	,,	,,	,,	,,	0·275
48·0	16·9	16·61	13·69	26·98	,,	,,	,,	,,	0·273
45·7	16·9	16·63	13·96	26·97	,,	1·965*	,,	,,	0·267
							Mean	. . .	0·272

The mean of the means of both series of experiments gives 0·270 as the specific heat of crystallized sulphate of zinc and potass between 19° and 40°–50°.

Crystallized Sulphate of Nickel and Potass, $\mathrm{Ni}\,K_2\,S_2\,\Theta_8+6\,H_2\,\Theta$. Well-formed crystals.

Experiments with Naphtha A. Glass 2. Temperature of the Air 13°·3–13°·5.

T.	T'.	t'.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
49·1	16·1	15·84	12·77	26·94	4·775	1·945	0·431	0·487	0·247
45·1	15·6	15·34	12·61	26·96	,,	,,	,,	,,	0·245
45·5	15·8	15·46	12·73	26·945	,,	,,	,,	,,	0·241
44·0	15·6	15·32	12·69	26·975	,,	1·925*	,,	,,	0·247
							Mean	. . .	0·245

---

* After drying the stopper.

76. *Crystallized Sulphate of Alumina and Potass*, $\mathrm{Al}_2\,\mathrm{K}_2\,\mathrm{S}_4\,\Theta_{16}+24\,\mathrm{H}_2\,\Theta$. Transparent air-dried crystals of alum.

Experiments with Naphtha A. Glass 1. Temperature of the Air 17°·2–17°·4.

T.	T'.	*t'*.	*t*.	M.	*m*.	*f*.	*y*.	*x*.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
49·1	19·5	19·16	16·55	26·98	2·87	1·595	0·431	0·651	0·362
49·6	19·1	18·83	16·12	26·985	,,	,,	,,	,,	0·369
49·0	19·3	18·96	16·32	26·99	,,	,,	,,	,,	0·370
49·5	19·3	18·95	16·23	26·96	,,	1·58*	,,	,,	0·382
							Mean	. . .	0·371

Crystallized Sulphate of Chrome and Potass, $\mathrm{Cr}_2\,\mathrm{K}_2\,\mathrm{S}_4\,\Theta_{16}+24\,\mathrm{H}_2\,\Theta$. Air-dried crystals of chrome alum: they remained unchanged in the following experiments.

Experiments with Naphtha A. Glass 3. Temperature of the Air 17°·2–17°·4.

T.	T'	*t'*.	*t*.	M.	*m*.	*f*.	*y*.	*x*.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
50·9	19·3	19·03	16·14	26·95	3·70	1·875	0·431	0·453	0·325
50·6	19·4	19·06	16·23	26·965	,,	,,	,,	,,	0·320
50·9	19·5	19·23	16·34	26·995	,,	,,	,,	,,	0·331
51·4	19·6	19·34	16·46	26·97	,,	1·865*	,,	,,	0·320
							Mean	. . .	0·324

77. *Chloride of Carbon*, $\mathrm{C}_2\,\mathrm{Cl}_6$. The determination of the specific heat of this, the so-called sesquichloride of carbon, has given me much trouble.

I first investigated, in two series of experiments, a preparation which, after melting in a small glass tube, had solidified in porcelain-like white crusts†.

I.—Experiments with Water. Glass 1. Temperature of the Air 18°·5–18°·8.

T.	T'.	*t'*.	*t*.	M.	*m*.	*f*.	*y*.	*x*.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
53·5	20·5	20·22	16·16	26·94	3·765	1·61	1·000	0·651	0·280
52·2	20·4	20·10	16·18	26·945	,,	,,	,,	,,	0·282
52·0	20·7	20·43	16·83	26·97	,,	,,	,,	,,	0·269
52·6	20·8	20·45	16·61	26·965	,,	1·585*	,,	,,	0·271
							Mean	. . .	0·276

---

* After drying the stopper.

† Sesquichloride of carbon was prepared by continuously passing chlorine into crude chloride of ethylene in the sunlight, and washing the solidified product with water; it was then again treated with chlorine and washed with solution of soda and much water. The crystalline mass was afterwards repeatedly pressed between bibulous paper (by which a small quantity of an oily product was absorbed), dried in the air, then washed with cold alcohol, dried, and fused, and the parts which had crept up the sides separated when solid.—ENGELBACH.

II.—Experiments with Water. Glass 1. Temperature of the Air 17°·5–17°·4.

T.	T'.	t'.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
50·2	19·8	19·54	15·54	26·955	3·525	1·995	1·000	0·651	0·256
50·1	19·6	19·33	15·31	26·94	,,	,,	,,	,,	0·257
50·5	19·7	19·36	15·24	26·96	,,	,,	,,	,,	0·272
49·2	19·7	19·43	15·52	26·97	,,	,,	,,	,,	0·263
47·8	19·7	19·36	15·62	26·99	,,	1·965*	,,	,,	0·277
							Mean . . .		0·265

I should not have hesitated to take the number 0·27, the mean of the averages of both these series of determinations, as the normal specific heat of sesquichloride of carbon, and to consider it as sufficiently below the melting-point (according to FARADAY this is at 160°), if the connexion between the specific heat of solid bodies and their composition, discussed in § 96 *et seq.*, had not been known to me; but the specific heat of sesquichloride of carbon calculated therefrom is 0·177. This deviates from the number found in a manner which at first I could not understand. The idea that the specimen was impure was inadmissible†. To try whether the porcelain-like mass of sesquichloride which solidified on fusion had an essentially different specific heat from that not fused, I recrystallized the substance from ether, washed the crystals (which showed very distinctly the characteristic form of the body as described by BROOKE and LAURENT) with a little ether, and dried them at 100°. Dried at this temperature, without being melted, they were white, like porcelain, and gave now the following results.

III.—Experiments with Water. Glass 3. Temperature of the Air 18°·4–18°·7.

T.	T'.	t'.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grms.		grm.	
49·2	20·6	20·34	16·53	26·935	3·835	2·06	1·000	0·453	0·280
49·2	20·7	20·42	16·62	26·94	,,	,,	,,	,,	0·281
49·0	20·8	20·53	16·81	26·95	,,	2·05*	,,	,,	0·274
							Mean . . .		0·275

That is essentially the same specific heat as my earlier experiments gave. If now it was improbable that the specific heat of sesquichloride of carbon did not differ much from 0·27, I might, on the other hand, also consider it improbable that this compound would make an exception to the relation which I had found between specific heat and composition—a relation which holds good in hundreds of cases of solid bodies. Sesquichloride of carbon would be the only exception to the validity of this relation; but this single exception would be sufficient to disprove its universal applicability, and to leave it undecided when, and in how many cases, other such exceptions might occur.

---

* After drying the stopper.

† In the specimen I investigated, Mr. DEHN found 90·19 per cent. chlorine; the quantity calculated from the formula $\mathrm{C}_2\,\mathrm{Cl}_6$ is 89·88 per cent.

Although the great distance of the temperatures used in my experiments from the melting-point of sesquichloride of carbon made it improbable, it was yet possible that the specific heat of this body varies considerably at the temperatures which I used, and is only constant and normal at still lower temperatures. In the preceding experiments I had heated sesquichloride of carbon to 49°–52°; it was improbable that this body, at so great a distance from its melting-point (160°), should absorb latent heat in softening in appreciable quantity, yet the circumstance that this substance is brittle in the cold, but distinctly tougher at 50°, led me to determine the specific heat at lower temperatures than in the previous case. I made the two following series of experiments, *a* with sesquichloride crystallized from alcoholic, and *b* from ethereal solution: in both series the crystals dried at 100° were porcelain white in appearance.

a.—Experiments with Water. Glass 1. Temperature of the Air 17°·8.

T. °	T′. °	*t′*. °	*t*. °	M. grms.	*m*. grms.	*f*. grms.	*y*.	*x*. grm.	sp. H.
36·8	19·7	19·35	17·42	26·98	2·11	2·085	1·000	0·651	0·146
37·6	19·8	19·52	17·52	26·94	”	”	”	”	0·138
37·2	19·7	19·44	17·51	26·94	”	”	”	”	0·111
37·1	19·8	19·45	17·53	26·98	”	2·075*	”	”	0·127

b.—Experiments with Water. Glass 3. Temperature of the Air 17°·8.

T. °	T′. °	*t′*. °	*t*. °	M. grms.	*m*. grms.	*f*. grms.	*y*.	*x*. grm.	sp. H.
37·2	19·8	19·45	17·42	26·98	3·64	2·11	1·000	0·453	0·161
37·2	19·7	19·43	17·42	26·99	”	”	”	”	0·148
37·3	19·7	19·44	17·42	26·965	”	”	”	”	0·146
37·3	19·7	19·44	17·43	26·965	”	2·10	”	”	0·145

Both these series can only be considered as giving approximate results. In both the magnitude T−T′ is very small, not as much as 18°; in the series *a* the quantity of solid was moreover small, and its thermal action but a small fraction of the entire amount observed. The mean of the four experiments of the series *b* would give the specific heat between 20° and 37° at 0·15, and the first experiment of the series *a* agrees well with this. The specific heat here found between 20° and 37° comes very near that calculated from the composition, and is so much less than that found between 20° and 50°, that it is probable this substance may towards 50° absorb heat in softening, the amount of which may make the numbers for the specific heat too great.

To decide upon this point, I made two additional series of experiments in which, since the vessel containing sesquichloride of carbon and water could only be slightly heated

---

* After drying the stopper.

(not to 40°), and the difference of temperature T − T′ accordingly was small, I used all possible care. I thus obtained the following results.

a. Crystals obtained from ethereal solution dried at 100°: milky white.

Experiments with Water. Glass 1. Temperature of the Air 16°·1–15°·7.

T.	T′.	*t′*.	*t*.	M.	*m*.	*f*.	*y*.	*x*.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
37·1	18·1	17·84	15·64	26·94	3·58	1·845	1·000	0·651	0·174
37·1	18·2	17·92	15·73	26·99	,,	,,	,,	,,	0·176
37·2	18·0	17·72	15·63	26·985	,,	1·835*	,,	,,	0·165

Temperature of the Air 16°·1.

T.	T′.	*t′*.	*t*.	M.	*m*.	*f*.	*y*.	*x*.	sp. H.
43·7	18·2	17·93	14·93	26·995	3·58	1·835	1·000	0·651	0·193
43·5	18·2	17·93	14·95	26·97	,,	,,	,,	,,	0·193

Temperature of the Air 16°·2.

T.	T′.	*t′*.	*t*.	M.	*m*.	*f*.	*y*.	*x*.	sp. H.
51·9	18·4	18·12	13·86	26·995	3·58	1·82	1·000	0·651	0·269
48·6	18·1	17·77	13·84	26·975	,,	,,	,,	,,	0·281

b. Clear crystals obtained from ethereal solution, dried by passing a current of dry air over them at the ordinary temperature.

Experiments with Water. Glass 3. Temperature of the Air 16°·2–15°·7.

T.	T′.	*t′*.	*t*.	M.	*m*.	*f*.	*y*.	*x*.	sp. H.
°	°	°	°	grms.	grms.	grms.		grm.	
36·9	18·2	17·93	15·62	26·99	4·235	2·155	1·000	0·453	0·171
36·8	18·2	17·92	15·64	26·99	,,	,,	,,	,,	0·184
37·1	18·3	18·01	15·63	26·975	,,	2·145*	,,	,,	0·193

Temperature of the Air 16·1°·–16°·2.

T.	T′.	*t′*.	*t*.	M.	*m*.	*f*.	*y*.	*x*.	sp. H.
°	°	°	°	grms.	grms.	grms.		grm.	
43·4	18·1	17·84	14·63	26·99	4·235	2·145	1·000	0·453	0·195
43·4	18·2	17·90	14·70	26·96	,,	,,	,,	,,	0·195

Temperature of the Air 16°·2.

T.	T′.	*t′*.	*t*.	M.	*m*.	*f*.	*y*.	*x*.	sp. H.
52·0	18·9	18·63	14·05	26·955	4·235	2·125	1·000	0·453	0·272
47·3	18·1	17·83	13·73	26·945	,,	,,	,,	,,	0·285

In the last series of experiments, on heating to about 50° a change took place in the hitherto clear crystals; they became dull and resembled porcelain. By special experiments I found that transparent crystals of sesquichloride of carbon gradually heated in water underwent this change at 50°–52°.

These determinations leave no doubt that, as is the case with other substances[†], for

---

[*] After drying the stopper.

[†] I call to mind the experiments of PERSON, who found (Ann. de Chim. et de Phys. [3] vol. xxvii. p. 263) for the specific heat of bees' wax melting at 61°·8,

Between −21° and +3°	6° and 26°	26° and 42°	42° and 58°
0·4287	0·504	0·82	1·72

temperatures near their melting-points, so also with sesquichloride of carbon at a temperature of 50° (that is more than 100° from its melting-point), the specific heat (or rather the number which is obtained for this in determinations) rapidly and considerably increases. From the last two series of experiments the specific heat of sesquichloride of carbon is

	Between 18° and 37°.	Between 18° and 43°.	Between 18° and 50°.
Mean of experiments: *a* . . .	0·172	0·193	0·275
,, ,, *b* . . .	0·183	0·195	0·279
Average	0·178	0·194	0·277

The specific heat of sesquichloride of carbon increases much more between 43° and 50° than between 37° and 43°. It may be assumed that for temperatures below 37° the number found, 0·178, comes very near the true specific heat of this compound, that is, uninfluenced by heat of softening.

78. *Cane-sugar*, $\mathrm{C}_{12}\mathrm{H}_{22}\mathrm{O}_{11}$. Dried crystalline fragments of clear sugarcandy.

Experiments with Naphtha A. Glass 3. Temperature of the Air 20°·6.

T.	T'.	*t'*.	*t*.	M.	*m*.	*f*.	*y*.	*x*.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
49·9	22·2	21·93	19·75	26·96	3·165	1·625	0·431	0·453	0·306
51·4	22·6	22·26	20·03	26·94	,,	,,	,,	,,	0·295
51·4	22·6	22·30	20·05	26·965	,,	1·62*	,,	,,	0·302
							Mean	. . .	0·301

Fine loaf-sugar was recrystallized from water, the mother-liquor washed off with dilute alcohol, the pure white crystals dried at 100°. They gave the following results.

Experiments with Naphtha B. Glass 1. Temperature of the Air 18°·5–18°·7.

T	T'.	*t'*.	*t*.	M.	*m*.	*f*.	*y*.	*x*.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
51·5	20·9	20·62	18·16	26·945	2·915	1·54	0·419	0·651	0·299
51·6	20·7	20·43	17·95	26·95	,,	,,	,,	,,	0·297
50·3	20·6	20·33	17·94	26·985	,,	1·52*	,,	,,	0·303
							Mean	. . .	0·300

I also examined amorphous cane-sugar. Crystals dried at 100°, as used in the preceding experiment, were melted in an oil-bath at 160°–170°, and the fused mass allowed to cool in the closed tube. The resultant amorphous amber-like viscous mass, exactly resembling colophony, was comminuted (as rapidly as possible to avoid the absorption of moisture), and gave the following results.

---

* After drying the stopper.

Experiments with Naphtha B. Glass 1. Temperature of the Air 18°·0–18°·4.

T.	T′.	t′.	t.	M.	m.	f.	y.	x.	sp. H.
				grms.	grms.	grm.		grm.	
51°·4	20°·1	19°·82	17°·24	26·97	2·475	1·77	0·419	0·651	0·336
50·9	20·0	19·74	17·20	26·99	”	”	”	”	0·334
51·6	20·1	19·78	17·15	26·975	”	”	”	”	0·345
50·9	20·1	19·77	17·20	26·96	”	1·75*	”	”	0·357
						Mean	. . .		0·342

The pieces of amorphous sugar used for these experiments were clear even when the experiments were concluded. In the investigation of such a hygroscopic substance it is impossible to avoid with certainty any absorption of water; yet it seems to me improbable that the difference between the number 0·342 found for amorphous cane-sugar between 20° and 51°, and 0·301 for crystallized sugar between the same limits, depends on an absorption of water by the former; but it is probable that the greater specific heat found for amorphous sugar depends on the fact that at 50° even it contains some heat of softening. According to WÖHLER's observations, bodies in the amorphous condition have other, in general lower, fusing-points than those in the crystallized state†; crystallized cane-sugar melts at 160° C., amorphous between 90° and 100°; at the latter temperature the amorphous sugar may be drawn out in threads, but even at a lower temperature the softening begins.

Mannite, $C_6H_{14}O_6$. Crystallized mannite, dried at 100°, was melted in the oil-bath at 160°–170°, and the radiant crystalline mass was comminuted. It gave the following results‡.

Experiments with Naphtha B. Glass 3. Temperature of the Air 17°·1–17°·8.

T.	T′.	t′.	t.	M.	m.	f.	y.	x.	sp. H.
				grms.	grms.	grm.		grm.	
51°·1	19°·3	18°·92	16°·57	26·98	2·56	1·815	0·419	0·453	0·318
51·6	19·4	19·12	16·64	26·93	”	”	”	”	0·336
51·0	19·5	19·19	16·82	26·965	”	”	”	”	0·319
51·3	19·6	19·31	16·92	26·93	”	1·805*	”	”	0·321
						Mean	. . .		0·324

---

* After drying the stopper.

† Ann. der Chem. und Pharm. vol. xli. p. 155.

‡ I also worked with mannite which was crystallized in slender prisms and dried at 100°.

Experiments with Naphtha B. Glass 3. Temperature of the Air 17°·4.

T.	T′.	t′.	t.	M.	m	f.	y	x.	sp. H.
				grms.	grms.	grms.		grms.	
49°·5	19°·2	18°·85	16°·61	26·95	2·13	2·14	0·419	0·453	0·302
51·3	19·3	19·03	16·64	26·94	”	”	”	”	0·311
50·5	19·3	19·04	16·74	26·98	”	2·13*	”	”	0·302

I consider the somewhat larger numbers obtained by using the compact pieces which had been melted to be more correct.

79. *Tartaric Acid*, $C_4H_6O_6$. Dried fragments of larger crystals.

Experiments with Naphtha A. Glass 1. Temperature of the Air 20°·6.

T.	T′.	t′.	t.	M. grms.	m. grms.	f. grm.	y.	x. grm.	sp. H.
51°·3	22°·4	22°·12	19°·74	26·985	3·16	1·53	0·431	0·651	0·289
50·5	22·5	22·23	19·94	26·96	,,	,,	,,	,,	0·283
50·7	22·6	22·32	20·03	26·97	,,	1·52*	,,	,,	0·282
							Mean . . .		0·285

Small crystals dried at 100°.

Experiments with Naphtha B. Glass 3. Temperature of the Air 18°·0–18°·4.

T.	T′.	t′.	t.	M. grms.	m. grms.	f. grm.	y.	x. grm.	sp. H.
51°·1	20°·0	19°·68	17°·15	26·97	3·57	1·69	0·419	0·453	0·289
50·9	20·0	19·72	17·20	26·99	,,	,,	,,	,,	0·291
51·3	20·0	19·73	17·18	26·97	,,	,,	,,	,,	0·290
50·5	19·9	19·63	17·13	26·97	,,	1·68*	,,	,,	0·293
							Mean . . .		0·291

The average of the means of both series of experiments gives 0·288 as the specific heat of crystallized tartaric acid between 21° and 51°.

Crystallized Racemic Acid, $C_4H_6O_6+H_2O$. Fragments of air-dried transparent crystals, which remained clear in the experiments made with them.

Experiments with Naphtha B. Glass 1. Temperature of the Air 16°·4–16°·9.

T.	T′.	t′.	t.	M. grms.	m. grms.	f. grm.	y.	x. grm.	sp. H.
50°·5	18°·6	18°·33	15°·63	26·945	3·17	1·495	0·419	0·651	0·317
50·3	18·6	18·33	15·64	26·965	,,	,,	,,	,,	0·319
50·6	18·7	18·43	15·73	26·965	,,	,,	,,	,,	0·317
50·0	18·8	18·52	15·86	26·975	,,	1·48*	,,	,,	0·324
							Mean . . .		0·319

Succinic Acid, $C_4H_6O_4$. Small crystals dried at 100°.

Experiments with Naphtha B. Glass 1. Temperature of the Air 17°·3–17°·7.

T.	T′.	t′.	t.	M. grms.	m. grms.	f. grm.	y.	x. grm.	sp. H.
51°·4	19°·4	19°·05	16°·54	26·985	2·455	1·64	0·419	0·651	0·317
50·5	19·4	19·13	16·70	26·95	,,	,,	,,	,,	0·313
50·8	19·5	19·24	16·80	26·965	,,	,,	,,	,,	0·311
50·9	19·6	19·26	16·82	26·935	,,	1·625*	,,	,,	0·313
							Mean . .		0·313

---

* After drying the stopper.

80. *Formiate of Baryta*, $\text{Є}_2\mathrm{H}_2\mathrm{Ba}\Theta_4$. Beautiful clear crystals dried at 100°.

Experiments with Naphtha B. Glass 3. Temperature of the Air 18°·5–18°·8.

T.	T′.	t′.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
51·0	20·6	20·31	17·93	26·98	6·91	1·615	0·419	0·453	0·142
53·1	20·7	20·40	17·85	26·94	„	„	„	„	0·143
51·8	20·7	20·41	17·95	26·97	„	„	„	„	0·145
52·4	20·7	20·38	17·93	26·99	„	1·58*	„	„	0·141
							Mean	. . .	0·143

Crystallized Neutral Oxalate of Potass, $\text{Є}_2\mathrm{K}_2\Theta_4+\mathrm{H}_2\Theta$. Air-dried transparent crystals, which remained clear in the experiments made with them.

T.	T′.	t′.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
49·4	19·3	19·00	16·52	26·995	3·57	1·765	0·419	0·651	0·233
49·3	19·4	19·12	16·62	26·95	„	„	„	„	0·241
49·0	19·5	19·15	16·72	26·945	„	„	„	„	0·232
50·0	19·6	19·26	16·73	26·97	„	1·755*	„	„	0·240
							Mean	. . .	0·236

Crystallized Oxalate of Potass (quadroxalate), $\text{Є}_2\mathrm{H}\,\mathrm{K}\,\Theta_4+\text{Є}_2\mathrm{H}_2\Theta_4+2\,\mathrm{H}_2\Theta$. Crystals dried in the air, which were also clear after the experiments.

Experiments with Naphtha B. Glass 3. Temperature of the Air 16°·7–16°·9.

T.	T′.	t′.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
50·1	18·6	18·34	15·77	26·965	3·375	1·76	0·419	0·453	0·283
49·8	18·7	18·42	15·86	26·98	„	„	„	„	0·288
50·2	18·8	18·45	15·91	26·98	„	„	„	„	0·278
50·3	18·7	18·43	15·86	26·95	„	1·745*	„	„	0·282
							Mean	. . .	0·283

Acid Tartrate of Potass, $\text{Є}_4\mathrm{H}_5\mathrm{K}\,\Theta_6$. Crystals dried at 100°.

Experiments with Naphtha B. Glass 3. Temperature of the Air 16°·6–16°·8.

T.	T′.	t′.	t.	M.	m.	f.	y.	x.	sp. H.
°	°	°	°	grms.	grms.	grm.		grm.	
50·8	18·6	18·32	15·73	26·965	3·89	1·69	0·419	0·453	0·259
51·0	18·6	18·34	15·72	26·95	„	„	„	„	0·262
50·6	18·7	18·41	15·85	26·935	„	„	„	„	0·257
50·3	18·6	18·34	15·84	26·965	„	1·675*	„	„	0·250
							Mean	. . .	0·257

---

* After drying the stopper.

Crystallized Tartrate of Soda and Potass, $\mathrm{C_4H_4NaKO_6+4H_2O}$. Fragments of transparent air-dried Seignette salt, which remained clear in the experiments made with them.

Experiments with Naphtha B. Glass 1. Temperature of the Air 16°·7–16°·9.

T.	T′.	t′.	t.	M.	m.	f.	y.	x.	sp. H.
				grms.	grms.	grm.		grm.	
50°·0	19°·0	18°·72	16°·03	26·99	3·385	1·415	0·419	0·651	0·324
50·5	18·8	18·47	15·68	26·93	„	„	„	„	0·333
50·5	18·9	18·57	15·82	26·95	„	„	„	„	0·325
50·4	18·9	18·61	15·84	26·965	„	„	„	„	0·333
50·5	18·9	18·57	15·83	26·965	„	1·40*	„	„	0·325
							Mean	. . .	0·328

Crystallized Acid Malate of Lime, $\mathrm{C_4H_4CaO_5+C_4H_6O_5+8H_2O}$. Small crystals dried over sulphuric acid, which remained clear in the following experiments:

T.	T′.	t′.	t.	M.	m.	f.	y.	x.	sp. H.
				grms.	grms.	grm.		grm.	
50°·8	19°·4	19°·11	16°·55	26·985	2·76	1·89	0·419	0·453	0·346
50·1	19·5	19·20	16·73	26·965	„	„	„	„	0·337
50·5	19·6	19·34	16·84	26·94	„	„	„	„	0·339
50·4	19·6	19·27	16·82	26·97	„	1·865*	„	„	0·330
							Mean	. . .	0·338

IV.—TABLE OF THE SUBSTANCES WHOSE SPECIFIC HEAT HAS BEEN EXPERIMENTALLY DETERMINED.

81. In the following I give a summary of those solid substances of known composition for which there are trustworthy determinations of the specific heat. I have endeavoured to make this summary complete; yet I have not thought it necessary to include all known determinations; for instance, all those referring to the metals most frequently investigated. But it appeared to me desirable to include completely the determinations of experimenters who have investigated a greater number of substances, in order to see how far the results obtained by different inquirers are comparable; in inserting the numbers which I found for many substances of which the specific heats had been already determined by others, I had no other intention than that of offering criteria for judging how far these determinations are comparable, and may be used for the considerations which are given in the fifth Division.

The determinations given in the following summary are principally due to Dulong and Petit (D. P.), Neumann (N.), Regnault (R.), and myself (Kp.). There are besides some of Person (Pr.), of Alluard (A.), and the recent investigations of Pape (Pp.) are also included. By far the largest number of these determinations have been made by the method of mixture. A few only of the elements investigated by Dulong and Petit, and some of the chemical compounds by NEUMANN have been determined by the method of cooling. Where it is not otherwise stated in reference to the temperature, all determinations refer to temperatures between 0° and 100°. Where the determination has been made beyond these limits, or where a more accurate statement of temperature is important, it is noticed. Where the same substance has been repeatedly investigated by the same observer, the result obtained for the purer preparation, and in general the most certain result, is taken.

---

* After drying the stopper.

In the following the chemical formula is given for each substance, the symbols used both here and subsequently, when not otherwise mentioned, refer to the numbers given in the last column of § 2 as the most recent assumptions for the atomic weights, the corresponding atomic weight, and the atomic heat, viz. the product of the specific heat and the atomic weight.

82. *Elements and Alloys.*

	Atomic weight.		Specific heat.		Atomic heat.
Ag	108		0·0557	D. P.	6·02
			0·0570	R.	6·16
			0·0560	Kp.	6·05
Al	27·4		0·2143	R.	5·87
			0·202	Kp.	5·53
As	75		0·0814	R.	6·11
Au	197		0·0298	D. P.	5·88
			0·0324	R.	6·38
B	10·9	Amorphous	0·254	Kp.	2·77
		Graphitoidal	0·235	R.	2·56
		Crystalline	0·230	Kp.	2·51
		,,	0·225–0·262	R.	2·45–2·86
Bi	210		0·0288	D. P.	6·05
			0·0308	R.	6·47
			0·0305	Kp.	6·41
Br	80	Between −78° and 20°	0·0843	R.	6·74
C	12	Wood charcoal	0·241	R.	2·89
		Gas carbon	0·204	R.	2·45
		,,	0·185	Kp.	2·22
		Natural graphite	0·202	R.	2·42
		,,	0·174	Kp.	2·09
		Iron graphite	0·197	R.	2·36
		,,	0·166	Kp.	1·99
		Diamond	0·1469	R.	1·76
Cd	112		0·0567	R.	6·35
			0·0542	Kp.	6·07
Co	58·8		0·1067	R.	6·27
Cu	63·4		0·0949	D. P.	6·02
		Hammered	0·0935	R.	5·93
		Heated	0·0952	R.	6·04
			0·0930	Kp.	5·90
Fe	56		0·1100	D. P.	6·16
			0·1138	R.	6·37
			0·112	Kp.	6·27
Hg	200	Between −78° and −40°	0·0319	R.	6·38

	Atomic weight.		Specific heat.		Atomic heat.
I	127		0·0541	R.	6·87
Ir	198		0·0326	R.	6·45
K	39·1	Between −78° and ?	0·1655	R.	6·47
Li	7		0·9408	R.	6·59
Mg	24		0·2499	R.	6·00
			0·245	Kp.	5·88
Mn	55		0·1217	R.	6·69
Mo	96		0·0722	R.	6·93
Na	23	Between −34° and 7°	0·2934	R.	6·75
Ni	58·8		0·1092	R.	6·42
Os	199·2		0·0311	R.	6·20
P	31	Yellow, between 13° and 36°	0·202	Kp.	6·26
		„ „ 7° „ 30°	0·1895	R.	5·87
		„ „ −21° „ 7°	0·1788	Pr.	5·54
		„ „ −78° „ 10°	0·1740	R.	5·39
		Red „ 15° „ 98°	0·1698	R.	5·26
Pb	207		0·0293	D. P.	6·06
			0·0314	R.	6·50
			0·0315	Kp.	6·52
Pd	106·6		0·0593	R.	6·32
Pt	197·4		0·0314	D. P.	6·20
			0·0324	R.	6·40
			0·0325	Kp.	6·42
Rh	104·4		0·0580	R.	6·06
S	32		0·1880	D. P.	6·02
		Rhombic, between 14° and 99°	0·1776	R.	5·68
		„ „ 17° „ 45°	0·163	Kp.	5·22
Sb	122		0·0507	D. P.	6·20
			0·0508	R.	6·20
			0·0523	Kp.	6·38
Se	79·4	Amorphous, bet. −27° and 8°	0·0746	R.	5·92
		Crystalline, „ 98° „ 20°	0·0762	R.	6·05
		„ „ −18° „ 7°	0·0745	R.	5·92
Si	28	Graphitoidal	0·181	Kp.	5·07
		Crystallized	0·165	Kp.	4·62
		„	0·167–0·179	R.	4·68–5·01
		Fused	0·138	Kp.	3·86
		„	0·156–0·175	R.	4· –4·90
Sn	118		0·0514	D. P.	6·06
			0·0562	R.	6·63
			0·0548	Kp.	6·46
Te	128		0·0474	R.	6·07
			0·0475	Kp.	6·08
Tl	204		0·0336	R.	6·85
W	184		0·0334	R.	6·15
Zn	65·2		0·0927	D. P.	6·04
			0·0956	R.	6·23
			0·0932	Kp.	6·08

Alloys which only melt far above 100°.

	Atomic weight.	Specific heat.		Atomic heat.
$Bi\,Sn$	328	0·0400	R.	13.1
$Bi\,Sn_2$	446	0·0450	R.	20·1
$Bi\,Sn_2Sb$	568	0·0462	R.	26·2
$Bi\,Sn_2\,Sb\,Zn_2$	698·4	0·0566	R.	39·5
$Pb\,Sb$	329	0·0388	R.	12·8
$Pb\,Sn$	325	0·0407	R.	13·2
$Pb\,Sn_2$	443	0·0451	R.	20·0

83. *Arsenides and Sulphides.*

$Co\,As_2$	208·8	Speis cobalt	0·0920	N.	19·2

As the locality of this mineral is not given, the formula and atomic weight are not certain. Metals replacing the cobalt can, however, have little influence on the atomic weight and the product.

$Ag_2\,S$	248	Fused	0·0746	R.	18·5
$Co\,As\,S$	166	Cobalt glance	0·1070	N.	17·8
$Cu_2\,S$	158·8	Fused	0·1212	R.	19·2
		Copper glance	0·120	Kp.	19·1
$Fe\,As\,S$	163	Mispickel	0·1012	N.	16·5
$As\,S$	107	Commercial	0·1111	N.	11·9
$Co\,S$	90·8	Fused	0·1251	R.	11·4
$Cu_{\frac{1}{2}}\,Fe_{\frac{1}{2}}\,S$	91·7	Copper pyrites	0·1289	N.	11·8
		,,	0·131	Kp.	12·1
$Fe\,S$	88	Fused	0·1357	R.	11·9
$Hg\,S$	232	Cinnabar	0·052	N.	12·1
		,,	0·0512	R.	11·9
		,,	0·0517	Kp.	12·0
$Ni\,S$	90·8	Fused	0·1281	R.	11·6
$Pb\,S$	239	Galena	0·053	N.	12·7
		,,	0·0509	R.	12·2
		,,	0·0490	Kp.	11·7
$Sn\,S$	150	Fused	0·0837	R.	12·6
$Zn\,S$	97·2	Zinc-blende	0·1145	N.	11·1
		,,	0·1230	R.	12·0
		,,	0·120	Kp.	11·7
$Fe_7\,S_8$	648	Magnetic pyrites	0·1533	N.	99·3
			0·1602	R.	103·8
$As_2\,S_3$	246	Natural	0·1132	N.	27·8
$Bi_2\,S_3$	516	Artificial	0·0600	R.	31·0
$Sb_2\,S_3$	340	Natural	0·0907	N.	30·8
		Artificial	0·0840	R.	28·6
$Fe\,S_2$	120	Marcasite	0·1332	N.	16·0
		Iron pyrites	0·1275	N.	15·3
		,,	0·1301	R.	15·6
		,,	0·126	Kp.	15·1
$Mo\,S_2$	160	Natural	0·1067	N.	17·1
		,,	0·1233	R.	19·7
$Sn\,S_2$	182	Aurum musivum	0·1193	R.	21·7

84. *Chlorine, Bromine, Iodine, and Fluorine compounds.*

	Atomic weight.		Specific heat.		Atomic heat.
$AgCl$	143·5	Fused	0·0911	R.	13·1
$CuCl$	98·9	"	0·1383	R.	13·7
$HgCl$	235·5	Sublimed	0·0521	R.	12·3
KCl	74·6	Fused	0·1730	R.	12·9
		"	0·171	Kp.	12·8
$LiCl$	42·5	"	0·2821	R.	12·0
$NaCl$	58·5	"	0·2140	R.	12·5
		"	0·213	Kp.	12·5
		Rock-salt	0·219	Kp.	12·8
$RbCl$	120·9	Fused	0·112	Kp.	13·5
NH_4Cl	53·5	Crystallized	0·373	Kp.	20·0
$BaCl_2$	208	Fused	0·0896	R.	18·6
		"	0·0902	Kp.	18·8
$CaCl_2$	111	"	0·1642	R.	18·2
$HgCl_2$	271	Sublimed	0·0689	R.	18·7
		Crystallized	0·0640	Kp.	17·3
$MgCl_2$	95	Fused	0·1946	R.	18·5
		"	0·191	Kp.	18·2
$MnCl_2$	126	"	0·1425	R.	18·0
$PbCl_2$	278	"	0·0664	R.	18·5
$SnCl_2$	189	"	0·1016	R.	19·2
$SrCl_2$	158·6	"	0·1199	R.	19·0
$ZnCl_2$	136·2	"	0·1362	R.	18·6
$BaCl_2+2H_2O$	244	Crystallized	0·171	Kp.	41·7
$CaCl_2+6H_2O$	219	Between −21° and 0°	0·345	Pr.	75·6
ZnK_2Cl_4	285·4	Crystallized	0·152	Kp.	43·4
PtK_2Cl_6	488·6	"	0·113	Kp.	55·2
SnK_2Cl_6	409·2	"	0·133	Kp.	54·4
Cr_2Cl_6	317·4	"	0·143	Kp.	45·4
$AgBr$	188	Fused	0·0739	R.	13·9
KBr	119·1	"	0·1132	R.	13·5
$NaBr$	103	" *	0·1384	R.	14·3
$PbBr_2$	367	"	0·0533	R.	19·6
AgI	235	"	0·0616	R.	14·5
CuI	190·4	"	0·0687	R.	13·1
HgI	327	Powder	0·0395	R.	12·9
KI	166·1	Fused	0·0819	R.	13·6
NaI	150	"	0·0868	R.	13·0
HgI_2	454	"	0·0420	R.	19·1
PbI_2	461	"	0·0427	R.	19·7
$CaFl_2$	78	Fluor-spar	0·2082	N.	16·2
		"	0·2149	R.	16·8
		"	0·209	Kp.	16·3
$AlNa_3Fl_6$	210·4	Cryolite	0·238	Kp.	50·1

* The preparation contained carbonate of soda.

	Atomic weight.	85. *Oxides.*	Specific heat.		Atomic heat.
$\mathrm{Cu}_2\Theta$	142·8	Red copper ore	0·1073	N.	15·3
		„	0·111	Kp.	15·9
$\mathrm{H}_2\Theta$	18	Ice between −21° and −2°	0·480	Pr.	8·6
		„ 78° „ 0°	0·474	R.	8·5

Desains found the specific heat of ice between −20° and 0° to be 0·513; Person, between −20° and 0° = 0·504; Hess, between −14° and 0° = 0·533. Person is of opinion that ice, even somewhat below its melting-point, between −2° and 0°, absorbs heat of fusion.

$\mathrm{Cu}\Theta$	79·4		0·137	N.	10·9
			0·1420	R.	11·3
			0·128	Kp.	10·2
$\mathrm{Hg}\Theta$	216	Commercial	0·049	N.	10·6
		Crystalline	0·0518	R.	11·2
		„	0·0530	Kp.	11·4
$\mathrm{Mg}\Theta$	40		0·276	N.	11·0
			0·2439	R.	9·8
$\mathrm{Mn}\Theta$	71		0·1570	R.	11·1
$\mathrm{Ni}\Theta$	74·8	Feebly ignited	0·1623	R.	12·1
		Strongly ignited	0·1588	R.	11·9
$\mathrm{Pb}\Theta$	223	Fused	0·0509	R.	11·4
		Crystalline powder	0·0512	R.	11·4
		„ „	0·0553	Kp.	12·3
$\mathrm{Zn}\Theta$	81·2		0·132	N.	10·7
			0·1248	R.	10·1
$\mathrm{Mg}\Theta + \mathrm{H}_2\Theta$	58	Brucite	0·312	Kp.	18·1
$\mathrm{Fe}_3\Theta_4$	232	Magnetic iron ore	0·1641	N.	38·1
		„ „	0·1678	R.	38·9
		„ „	0·156	Kp.	36·2
$\mathrm{Mg}\,\mathrm{Al}_2\Theta_4$	142·8	Spinelle	0·194	Kp.	27·7
$\mathrm{Mg}_{\frac{1}{2}}\mathrm{Fe}_{\frac{1}{2}}\mathrm{Cr}_{\frac{3}{2}}\mathrm{Al}_{\frac{1}{2}}\Theta_4$	196	Chrome iron ore	0·159	Kp.	31·2
$\mathrm{Al}_2\Theta_3$	102·8	Sapphire	0·1972	N.	20·3
			0·2173	R.	22·3
$\mathrm{As}_2\Theta_3$	198	Opaque	0·1279	R.	25·3
$\mathrm{B}_2\Theta_3$	69·8	Fused	0·2374	R.	16·6
$\mathrm{Bi}_2\Theta_3$	468		0·0605	R.	28·3
$\mathrm{Cr}_2\Theta_3$	152·4		0·196	N.	29·9
			0·1796	R.	27·4
		Crystalline	0·177	Kp.	27·0
$\mathrm{Fe}_2\Theta_3$	160	Artificial, feebly ignited	0·1757	R.	28·1
		„ strongly ignited	0·1681	R.	26·9
		Specular iron	0·1692	N.	27·1
		„	0·1670	R.	26·7
		„	0·154	Kp.	25·1
$\mathrm{Fe}_5\,\mathrm{Ti}_{\frac{3}{4}}\Theta_3$	155·5	Iserine	0·1762	N.	27·4
		„	0·177	Kp.	27·5
$\mathrm{Sb}_2\Theta_3$	292	Fused	0·0901	R.	26·3
$\mathrm{Mn}_2\Theta_3 + \mathrm{H}_2\Theta$	176	Manganite	0·176	Kp.	31·0

	Atomic weight.		Specific heat.		Atomic heat.
$\text{Mn}\Theta_2$	87	Pyrolusite	0·159	Kp.	13·8
$\text{Si}\Theta_2$	60	Quartz	0·1883	N.	11·3
		,,	0·1913	R.	11·5
		,,	0·186	Kp.	11·2
$\text{Si}_{\frac{1}{2}}\text{Zr}_{\frac{1}{2}}\Theta_2$	90·8	Zircon	0·1456	R.	13·2
		,,	0·132	Kp.	12·0
$\text{Sn}\Theta_2$	150	Cassiterite	0·0931	N.	14·0
		,,	0·0933	R.	14·0
		,,	0·0894	Kp.	13·4
$\text{Ti}\Theta_2$	82	Artificial	0·1716	R.	14·1
		Rutile	0·1724	N.	14·1
		,,	0·1703	R.	14·0
		,,	0·157	Kp.	12·9
		Brookite	0·161	Kp.	13·2
$\text{Mo}\Theta_3$	144	Fused	0·1324	R.	19·1
		Pulverulent	0·154 ?	Kp.	22·2
$\text{W}\Theta_3$	232	,,	0·0798	R.	18·5
		,,	0·0894 ?	Kp.	20·7

86. *Carbonates and Silicates.*

$K_2 C\Theta_3$	138·2	Fused	0·2162	R.	29·9
		,,	0·206	Kp.	28·5
$\text{Na}_2 C\Theta_3$	106	,,	0·2728	R.	28·9
		,,	0·246	Kp.	26·1
$\text{Rb}_2 C\Theta_3$	230·8	,,	0·123	Kp.	28·4
$\text{Ba}C\Theta_3$	197	Witherite	0·1078	N.	21·2
			0·1104	R.	21·7
$\text{Ca}C\Theta_3$	100	Calc-spar	0·2046	N.	20·5
		,,	0·2086	R.	20·9
		,,	0·206	Kp.	20·6
		Arragonite	0·2018	N.	20·2
		,,	0·2085	R.	20·9
		,,	0·203	Kp.	20·3
$\text{Ca}_{\frac{1}{2}}\text{Mg}_{\frac{1}{2}}C\Theta_3$	92	Bitter spar	0·2161	N.	19·9
		,,	0·2179	R.	20·0
		,,	0·206	Kp.	19·0
$\text{Fe}C\Theta_3$	116	Spathic iron	0·182	N.	21·1
		,,	0·1934	R.	22·4

The minerals investigated doubtless contained part of the iron replaced by metals of lower atomic weight. The atomic weight and the product assumed above are somewhat too great.

$\text{Fe}_{\frac{8}{11}}\text{Mn}_{\frac{2}{11}}\text{Mg}_{\frac{1}{11}}C\Theta_3$	112·9	Spathic iron	0·166	Kp.	18·7
$\text{Mg}_{\frac{9}{7}}\text{Fe}_{\frac{2}{9}}C\Theta_3$	91·1	Magnesite	0·227	N.	20·7
$\text{Pb}C\Theta_3$	267	Cerussite	0·0814	N.	21·7
		,,	0·0791	Kp.	21·1

Regnault found for precipitated carbonate of lead still containing water, the specific heat 0·0860.

	Atomic weight.		Specific heat.		Atomic heat.
$SrCO_3$	147·6	Strontianite	0·1445	N.	21·3
		Artificial	0·1448	R.	21·4
$CaSiO_3$	116	Wollastonite	0·178	Kp.	20·7
$Ca_{\frac{1}{2}}Mg_{\frac{1}{2}}SiO_3$	108	Diopside from Tyrol	0·1906	N.	20·6
		" "	0·186	Kp.	20·1
$CuSiO_3+H_2O$	157·4	Dioptas	0·182	Kp.	28·7
$Mg_{\frac{20}{11}}Fe_{\frac{2}{11}}SiO_4$	145·8	Olivine	0·189	Kp.	27·6
		Crysolite	0·189	Kp.	27·6
		"	0·2056	N.	30·0
$Al_2K_2Si_6O_{16}$	557	Adularia	0·1861	N.	103·7
		Orthoclase	0·1911	N.	106·4
		"	0·183	Kp.	101·9
$Al_2Na_2Si_6O_{16}$	524·8	Albite	0·1961	N.	102·9
		"	0·190	Kp.	99·7

Borates, Molybdates, Tungstates, Chromates, and Sulphates.

KBO_2	82	Fused	0·2048	R.	16·8
$NaBO_2$	65·9	"	0·2571	R.	16·9
PbB_2O_4	292·8	"	0·0905	R.	26·5
PbB_4O_7	362·6	"	0·1141	R.	41·4
$K_2B_4O_7$	233·8	"	0·2198	R.	51·4
$Na_2B_4O_7$	201·6	"	0·2382	R.	48·0
		"	0·229	Kp.	46·2
$Na_2B_4O_7+10H_2O$	381·6	Crystallized borax	0·385	Kp.	146·9
$PbMoO_4$	367	Yellow lead ore	0·0827	Kp.	30·4
$CaWO_4$	288	Scheelite	0·0967	Kp.	27·9
$Fe_{\frac{2}{5}}Mn_{\frac{3}{5}}WO_4$	303·4	Tungsten	0·0930	Kp.	28·2
		"	0·0978	R.	29·7

The locality of the wolfram investigated by REGNAULT is not known, and the composition uncertain. But the change in the ratio in which iron and manganese are present in the mineral alters little in the atomic weight.

$PbCrO_4$	323.4	Fused	0·0900	Kp.	29·0
K_2CrO_4	194·4	Crystallized	0·1851	R.	36·0
		"	0·189	Kp.	36·7
$K_2Cr_2O_7$	294·6	"	0·1894	R.	55·8
		"	0·186	Kp.	54·8
$KHSO_4$	136·1	"	0·244	Kp.	33·2
K_2SO_4	174·2	Fused	0·1901	R.	33·1
		Crystallized	0·196	Kp.	34·1
Na_2SO_4	142	Fused	0·2312	R.	32·8
		Crystallized	0·227	Kp.	32·2
$N_2H_8SO_4$	132	"	0·350	Kp.	46·2
$BaSO_4$	233	Heavy spar	0·1088	N.	25·4
		"	0·1128	R.	26·3
		"	0·108	Kp.	25·2
$CaSO_4$	136.	Calcined gypsum	0·1966	R.	26·7
		Anhydrite	0·1854	N.	25·2
		"	0·178	Kp.	24·2

	Atomic weight.		Specific heat.		Atomic heat.
$Cu S\Theta_4$	159·4	Solid pieces	0·184	Pp.	29·3
$Mg S\Theta_4$	120	Dehydrated salt	0·2216	R.	26·6
		Solid pieces	0·225	Pp.	27·0
$Mn S\Theta_4$	151	,,	0·182	Pp.	27·5
$Pb S\Theta_4$	303	Artificial	0·0872	R.	26·4
		Lead vitriol	0·0848	N.	25·7
		,,	0·0827	Kp.	25·1
$Sr S\Theta_4$	183·6	Artificial	0·1428	R.	26·2
		Celestine	0·1356	N.	24·9
		,,	0·135	Kp.	24·8
$Zn S\Theta_4$	161·2	Coarse powder	0·174	Pp.	28·0
$Cu S\Theta_4 + H_2 \Theta$	177·4	Pulverulent	0·202	Pp.	35·8
$Mg S\Theta_4 + H_2 \Theta$	138	Coarse powder	0·264	Pp.	36·4
$Zn S\Theta_4 + H_2 \Theta$	179·2	Solid pieces	0·202	Pp.	36·2
$Ca S\Theta_4 + 2 H_2 \Theta$	172	Gypsum	0·2728	N.	46·9
		,,	0·259	Kp.	44·6
$Cu S\Theta_4 + 2 H_2 \Theta$	195·4	Pulverulent	0·212	Pp.	41·4
$Zn S\Theta_4 + 2 H_2 \Theta$	197·2	Solid pieces	0·224	Pp.	44·2
$Fe S\Theta_4 + 3 H_2 \Theta$	206	,,	0·247	Pp.	50·9
$Cu S\Theta_4 + 5 H_2 \Theta$	249·4	Crystallized	0·285	Kp.	71·1
		,,	0·316	Pp.	78·8
$Mn S\Theta_4 + 5 H_2 \Theta$	241	,,	0·323	Kp.	77·8
		,,	0·338	Pp.	81·5
$Ni S\Theta_4 + 6 H_2 \Theta$	262·8	,,	0·313	Kp.	82·3
$Co S\Theta_4 + 7 H_2 \Theta$	280·8	,,	0·343	Kp.	96·4
$Fe S\Theta_4 + 7 H_2 \Theta$	278	,,	0·346	Kp.	96·2
		,,	0·356	Pp.	99·0
$Mg S\Theta_4 + 7 H_2 \Theta$	246	,,	0·362	Kp.	89·1
		,,	0·407	Pp.	100·1
$Zn S\Theta_4 + 7 H_2 \Theta$	287·2	,,	0·347	Kp.	99·7
		,,	0·328	Pp.	94·2
$Mg K_2 S_2 \Theta_8 + 6 H_2 \Theta$	402·2	,,	0·264	Kp.	106·2
$Ni K_2 S_2 \Theta_8 + 6 H_2 \Theta$	437	,,	0·245	Kp.	107·1
$Zn K_2 S_2 \Theta_8 + 6 H_2 \Theta$	443·4	,,	0·270	Kp.	119·7
$Al_2 K_2 S_4 \Theta_{16} + 24 H_2 \Theta$	949	,, alum	0·371	Kp.	352·1
$Cr_2 K_2 S_4 \Theta_{16} + 24 H_2 \Theta$	998·6	,, chrome alum	0·324	Kp.	323·6

88. *Arseniates, Phosphates, Pyrophosphates and Metaphosphates, Nitrates, Chlorates, Perchlorates, and Permanganates.*

$K As \Theta_3$	162·1	Fused	0·1563	R.	25·3
$K H_2 As \Theta_4$	180·1	Crystallized	0·175	Kp.	31·5
$Pb_3 As_2 \Theta_8$	899	Fused	0·0728	R.	65·4
$Ag_3 P \Theta_4$	419	Pulverulent	0·0896 ?	Kp.	37·5
$K H_2 P \Theta_4$	136·1	Crystallized	0·280	Kp.	28·3
$Na_2 H P \Theta_4 + 12 H_2 \Theta$	358	Between −21° and 2°	0·408	Pr.	146·1

The determination of the specific heat refers to the crystallized salt. For the fused and afterwards solidified salt PERSON found the specific heat between the same range of temperature considerably greater, =0·68 to 0·78; but the mass obtained by solidifying the fused salt gradually alters (it becomes crystallized again) with increase of volume, which is very considerable when the fused salt is allowed to cool very rapidly.

	Atomic weight.		Specific heat.		Atomic heat.
$Pb_3P_2\Theta_8$	811		0·0798	R.	64·7
$K_4P_2\Theta_7$	330·4	Fused	0·1910	R.	63·1
$Na_4P_2\Theta_7$	266	"	0·2283	R.	60·7
$Pb_2P_2\Theta_7$	588	"	0·0821	R.	48·3
$NaP\Theta_3$	102	"	0·217	Kp.	22·1
$CaP_2\Theta_6$	198	"	0·1992	R.	39·4
$AgN\Theta_3$	170	"	0·1435	R.	24·4
$KN\Theta_3$	101·1	"	0·2388	R.	24·1
		"	0·227	Kp.	22·9
		Crystallized	0·232	Kp.	23·5
$K_{\frac{1}{2}}Na_{\frac{1}{2}}NO_3$	93	Fused*	0·235	Pr.	21·9
$NaNO_3$	85	"	0·2782	R.	23·6
		"	0·256	Kp.	21·8
		Crystallized	0·257	Kp.	21·8
$N_2H_4O_3$	80	"	0·455	Kp.	36·4
$BaN_2\Theta_6$	261	"	0·1523	R.	39·8
		"	0·145	Kp.	37·9
$PbN_2\Theta_6$	331	"	0·110	Kp.	36·4
$SrN_2\Theta_6$	211·6	"	0·181	Kp.	38·3
$KCl\Theta_3$	122·6	Fused	0·2096	R.	25·7
		Crystallized	0·194	Kp.	23·8
$BaCl_2O_6+H_2O$	322	"	0·157	Kp.	50·6
$KCl\Theta_4$	138·6	"	0·190	Kp.	26·3
$KMn\Theta_4$	158·1	"	0·179	Kp.	28·3

89. *So-called Organic Compounds.*

HgC_2N_2	252	Crystallized cyanide of mercury	0·100	Kp.	25·2
$ZnK_2C_4N_4$	247·4	" cyanide of zinc and potassium	0·241	Kp.	59·6
$FeK_3C_6N_6$	329·3	Crystallized ferricyanide of potassium	0·233	Kp.	76·7
$FeK_4C_6N_6+3H_2\Theta$	422·4	Crystallized ferrocyanide of potassium	0·280	Kp.	118·3
C_2Cl_6	237	Between 18° and 37°	0·178	Kp.	42·2

The specific heat between 18° and 43° was found =0·194; between 18° and 50° =0·277.

$C_{10}H_8$	128	Between −26° and 18°	0·3096	A.	39·6

The specific heat of naphthaline was found to be 0·3208 between 0° and 20°, and 0·3208 between 20° and 65°.

$C_{27}H_{54}\Theta_2$	410	Between −21° and 3°	0·4287	Pr.	175·8
$C_{46}H_{92}\Theta_2$	676				289·8

---

* Obtained as mass of constant melting-point (219°·8) by fusing equivalent quantities of nitrate of potass and nitrate of soda.

The first formula is that of one constituent of bees' wax, cerotic acid; the second is that of the other, palmitate of melissyle. In reference to the numbers found for the specific heat of bees' wax at higher temperatures, compare the last remark in § 77.

	Atomic weight.		Specific heat.		Atomic heat.
$C_{12}H_{22}O_{11}$	342	Crystallized cane-sugar	0·301	Kp.	102·9
		Amorphous cane-sugar	0·342	Kp.	117·0
$C_6H_{14}O_6$	182	Mannite	0·324	Kp.	59·1
$C_4H_6O_4$	118	Succinic acid	0·313	Kp.	36·9
$C_4H_6O_6$	150	Tartaric acid	0·288	Kp.	43·2
$C_4H_6O_6+H_2O$	168	Racemic acid	0·319	Kp.	53·6
$C_2H_2BaO_4$	227	Formate of baryta	0·143	Kp.	32·5
$C_2K_2O_4+H_2O$	184·2	Neutral oxalate of potass	0·236	Kp.	43·5
$C_4H_3KO_8+2H_2O$	254·1	Quadroxalate of potass	0·283	Kp.	71·9
$C_4H_5KO_6$	188·1	Acid tartrate of potass	0·257	Kp.	48·3
$C_4H_4NaKO_6+4H_2O$	282·1	Seignette salt	0·328	Kp.	92·5
$C_8H_{10}CaO_{10}+8H_2O$	450	Acid malate of lime	0·338	Kp.	152·1

The preceding Tables contain the material, obtained experimentally, which serves as subject and basis for the subsequent considerations on the relations of the specific heat of solid bodies to their atomic weight and composition.

PART V.—ON THE RELATIONS BETWEEN ATOMIC HEAT AND ATOMIC WEIGHT OR COMPOSITION.

90. I discuss in the sequel the regularities exhibited by the atomic heats of solid bodies, the exceptions to these regularities, and the most probable explanation of these exceptions. In regard to the views which I here develope, much has been already expressed or indicated in former speculations; in this respect I refer to the first part of this paper, in which I have given the views of earlier inquirers as completely as I know them, and as fully as was necessary to bring out the peculiar value of each. It is unnecessary, then, to refer again to what was there given; but I will complete for individual special points what is to be remarked from an historical point of view.

But before discussing these regularities, the question must be discussed whether the atomic heat of a given solid substance is essentially constant, or materially varies with its physical condition. It depends on the result of this investigation, how far it may with certainty be settled whether the general results already obtained are of universal validity, or whether exceptions to them exist.

The specific heat of a solid body varies somewhat with its temperature; but the variation of the specific heat with the temperature is very small, provided the latter does not rise so high that the body begins to soften. Taking the numbers obtained by REGNAULT for lead, by DULONG and PETIT, and by BEDE and by BYSTRÖM, for the specific heats of several metals at different temperatures, the conviction follows that the changes of specific heat, if not of themselves inconsiderable, are yet scarcely to be regarded in comparison with the discrepancies in the numbers which different observers have found for the specific heat of the same body at the same temperature. At temperatures at which a body softens, the specific heat does indeed vary considerably with the temperature (compare for example § 77); but these numbers, as containing already part of the latent heat of fusion, give no accurate expression for the specific heat, and are altogether useless for recognizing the relations between this property and the atomic weight or composition.

Just as little need the small differences be considered which REGNAULT found for a few metallic substances according as they were hammered or annealed, hard or soft.

For dimorphous varieties of the same substance, even where there are considerable differences in the specific gravity, the specific heats have not been found to be materially different (compare FeS_2, § 83; TiO_2, § 85; $CaCO_3$, § 86). The results obtained with these substances appear to me more trustworthy than those with graphite and the various modifications of boron and silicium, which moreover have given partly the same specific heat for the graphitoidal and adamantine modification of the same element. What trustworthy observations we now possess decidedly favour the view that the dimorphic varieties of the same substance have essentially the same specific heat.

91. The view has been expressed that the same substance might have an essentially different specific heat, in the amorphous and crystalline conditions. I believe that the differences of specific heat found for these different conditions depend, to by far the greatest extent, upon other circumstances.

The Tables in § 83 to § 89 contain a tolerable number of substances which have been investigated both after being melted, and also crystallized; there are no such differences in the numbers as to lead to the supposition that the amorphous solidified substance had a different specific heat to what it had in the crystallized state. No such influence of the condition has been with any certainty shown to affect the validity of DULONG and PETIT'S, or of NEUMANN'S law. I may here again neglect what the determinations of carbon, boron, or silicium appear to say for or against the assumption of a considerable influence of the amorphous or crystalline condition on the specific heat. REGNAULT found (§ 85) that the specific heat of artificially prepared (uncrystalline?) and crystallized titanic acid did not differ. According to my investigations (§ 48) silicic acid has almost the same specific heat in the crystallized and in the amorphous condition.

In individual cases, where the specific heat of the same substance for the amorphous and crystallized modification has been found to be materially different*, it may be shown that foreign influences affected the determination for the one condition. Such influences are especially: 1. That one modification absorbed heat of softening at the temperature of the experiment; that is doubtless the reason why the specific heat of yellow phosphorus was found to be considerably greater at higher temperatures than that of red phosphorus, but not at low ones (compare § 82), that the specific heat of amorphous cane-sugar was found to be decidedly greater than that of crystallized (§ 78), and, according to REGNAULT's opinion, also that the specific heat of amorphous selenium between 80° and 18° was found much greater (=0·103) than that of the crystalline, while for lower temperatures there was no difference in the specific heats of the two substances (§ 82). 2. That in heating one modification its transition into the other is induced, and the heat liberated in this transition makes the numbers for the specific heat incorrect; in § 33 I have discussed the probability that this circumstance, in REGNAULT's first experiments with sulphur, gave the specific heat much too high, and it is possible that it was also perceptible in the above-mentioned experiments with amorphous selenium. 3. That in immersing heated porous bodies in the water of the calorimeter heat becomes free (compare § 19); I consider this as the reason why REGNAULT found the specific heat of the more porous forms of carbon so much greater than that of the more compact (compare § 36); and REGNAULT himself sees in this the reason why he found the specific heat of the feebly ignited and porous oxides of nickel and of iron greater than that of the same oxides after stronger heating (compare § 85).

---

* DE LA RIVE and MARCET (Ann. de Chim. et de Phys. [2] vol. lxxv. p. 118) found the specific heat of vitreous to be different from that of opaque arsenious acid, and considered the fact to be essential; but their method was not fitted to establish such a difference. PAPE'S view, too (POGGENDORFF'S Annalen, vol. cxx. pp. 341 and 342), that it is of essential importance for the specific heat of hydrated sulphates whether the salts are crystallized or not, does not appear to me to be proved by what he has adduced.

From the importance of this subject for the considerations to be afterwards adduced, I have here had to discuss more fully what differences are real and what are only apparent in the numbers found for the specific heat of one and the same substance. Even if the apparent differences are often considerable, their importance diminishes, if allowance is made for the foreign influence which may have prevailed. In many cases, on the other hand, a body in totally different modifications has almost exactly the same specific heat if these foreign influences are excluded. It may, then, be said that, from our present knowledge, one and the same body may exhibit small differences with certain physical circumstances (temperature, different degree of density), but never so great that they may be taken as an explanation why a body decidedly and undoubtedly forms an exception to a regularity which might have perhaps been expected for it—provided that the determination of the specific heat, according to which the body in question forms an exception, is trustworthy, and kept free from foreign influences.

92. Among the regularities in the atomic heat of solid bodies, that found by DULONG and PETIT for the elements stands foremost. A glance at the atomic heats of the so-called elements collated in § 82, shows that for by far the greater number the atomic heats are in fact approximately equal. But the differences in the atomic heats, even of those elements which are usually regarded as coming under DULONG and PETIT's law, are often very considerable, even when the comparison is limited to those which are most easily obtained in a pure state, and even if numbers are taken for the specific heats which give the most closely agreeing atomic heats. REGNAULT * sought an explanation of the differences of the atomic heats of the elements in the circumstance that the latter could not be investigated in comparable conditions of temperature and density; further, that the numbers for the specific heat, as determined for solid bodies, contain, besides

* Annal. de Chim. et de Phys. [2] vol. lxxiii. p. 66, and [3] vol. xlvi. p. 257.

the true specific heat (for constant volume), also the heat of expansion. As specific heat we can indeed only take the sum of the heats necessary for heating and for expansion. But it is not yet proved that the products of the first quantity (the specific heat for constant volume) and the atomic weights would agree better than the atomic heats now do; it is only a supposition, and even the very contrary may be possible with individual substances. Temperature has an influence on the specific heat of solid bodies, and to a different extent with different bodies. Even in this respect, also, all means are wanting by which the different temperatures at which bodies are really comparable can be known, and a comparison made of their atomic heats. The utmost possible is to determine the specific heat at such a distance from the melting-point that latent heat of softening can have no influence. It is impossible to say with certainty whether the atomic heats of bodies compared at other temperatures than those which are nearly identical (ranging about 90° on each side of 10°) will show a closer agreement. It is not probable. Changes in the specific heat of solid bodies, so long as they are unaffected by heat of softening, are small in comparison with the differences which the atomic heats of individual elements show. And it is well worth consideration that individual elements (phosphorus and sulphur, *e. g.*) at temperatures relatively near their melting-points, have not materially greater specific heats than other elements (iron and platinum, for example) at temperatures relatively distant from their melting-points, but, on the contrary, considerably smaller. As regards the influence of density on the specific heat, it is undoubtedly certain that the latter may somewhat vary with the former; but it is equally so that, in all cases in which substances of undoubted purity were examined and the sources of error mentioned (§ 91) excluded, this variation is too inconsiderable to give an adequate explanation of the differences of the atomic heats found for the various solid elements.

I need not here revert to the considerations developed in §§ 90 and 91, as to how far a difference in the physical condition of a solid substance exercises an essential influence on its specific heat; for whatever view may be held in reference to this influence, and generally in reference to the circumstances which alter the specific heat of a substance, and therewith the atomic heat, this is certain, that there are individual elements whose atomic heat is distinctly and decidedly different from that of most other elements. Such elements are, from § 82, first of all boron, carbon, and silicium.

The decision of the question whether these elements really form exceptions to Dulong and Petit's law presupposes, besides a knowledge of their specific heat, a knowledge of their atomic weight also. There can be no exceptions to Dulong and Petit's law, if, regardless of anything which may be in opposition to it, the principle is held to, that the atomic weights of the elements must be so taken as to agree with this law. As a trial whether this law is universally applicable, the atomic weights ought rather to be taken as established in another manner. It may be confessed that the determination of the true atomic weights by chemical and physico-chemical investigations and considerations is still uncertain, and many questions are still unanswered the settlement of which may influence that determination. But there seems now to be no more trustworthy basis for fixing the atomic weights of the elements than that of taking, as the atomic weights of the elements, the relatively smallest quantities which are contained in equal volumes of their gaseous or vaporous compounds, or of which the quantities contained in such volumes are multiples in the smallest numbers; and no better means appear to exist for determining the atomic weights of those elements the vapour-densities of whose compounds could not be determined, than the assumption that in isomorphous compounds the quantities of the corresponding elements are as the atomic weights of the latter. On this basis, and using this means, the numbers for the atomic weights have been determined which are contained in the last column of the Table in § 2, and are used in § 82 *et seq.* The atomic weights $\mathrm{B}=10{\cdot}9$, $\text{Ȼ}=12$, $\text{S̶i}=28$, cannot be changed for others. That the atomic weights of tin and silicium are as 118 to 28, is further proved by the isomorphism of the double fluorides. But to these atomic weights correspond atomic heats which are far smaller than those found for most other elements. From the chemical point of view it is inadmissible to take the atomic weights of boron, carbon, and silicium * in such a manner as to make their atomic heats agree with DULONG and PETIT's law. In any case these three elements form exceptions to DULONG and PETIT's law. The sequel will show that this is the case with many other elements.

93. In many compounds the regularity is observed, that by dividing their atomic heat by the number of elementary atoms contained in one molecule of the compound, a quotient is obtained which comes very near the atomic heat of most of the elements—that is, 6·4. This is found in the alloys enumerated in § 82, and also in a great number of compounds of definite proportions. A few of the most important cases may be given here. For speiscobalt, $\mathrm{Co\,As_2}$ (compare § 83), this quotient is $\frac{19{\cdot}2}{3}=6{\cdot}4$; for the chlorine compounds, $\mathrm{R\,Cl}$ and $\text{Ɍ}\,\mathrm{Cl}$†, the mean of the atomic heats given in § 84 is 12·8, and the quotient $\frac{12{\cdot}8}{2}=6{\cdot}4$. Of the chlorine compounds, $\text{Ɍ}\,\mathrm{Cl_2}$, the mean atomic heat of all the determinations in § 84 is 18·5, and the quotient $\frac{18{\cdot}5}{3}=6{\cdot}2$. It is also very near this value in the double chlorides; in $\text{Ƶn}\,\mathrm{K_2\,Cl_4}$ it is $\frac{43{\cdot}4}{7}=6{\cdot}2$, for $\text{Ɍ}\,\mathrm{K_2\,Cl_6}$ (the mean of the determinations of $\text{Ᵽb}\,\mathrm{K_2\,Cl_6}$ and $\text{S̶n}\,\mathrm{K_2\,Cl_6}$) it is $\frac{54{\cdot}8}{9}=6{\cdot}1$. For bromine compounds, $\mathrm{R\,Br}$ (both here and in the following examples the means are taken of the determinations in § 84), $\frac{13{\cdot}9}{2}=6{\cdot}9$; for $\text{Ᵽb}\,\mathrm{Br_2}$ $\frac{19{\cdot}6}{3}=6{\cdot}5$; for iodine compounds, RI and $\text{Ɍ}\mathrm{I}$, $\frac{13{\cdot}4}{2}=6{\cdot}7$, and for the iodine compounds, $\text{Ɍ}\,\mathrm{I_2}$, $\frac{19{\cdot}4}{3}=6{\cdot}5$.

But this regularity, though met with in many compounds, is by no means quite universal. The oxygen compounds of the metals correspond to it in general the less the greater the number of oxygen atoms they contain as compared with that of metal. The mean atomic heat of the oxides $R\Theta$ in § 85 is 11·1, and the quotient $\frac{11\cdot1}{2}=5\cdot6$. The quotient for the oxides $R_2\Theta_3$ and $R_2\Theta_3$ (even excluding the determinations of alumina and boracic acid) is only $\frac{27\cdot2}{5}=5\cdot4$; for the oxides $R\Theta_2$ (even excluding the determinations for silicic acid and zircon) only $\frac{13\cdot7}{3}=4\cdot6$; for the oxides $R\Theta_3$, the mean of **Regnault**'s determinations only $\frac{18\cdot8}{4}=4\cdot7$. Still smaller is the quotient for compounds which contain boron in addition to oxygen (*e. g.* for the compounds $RB\Theta_2$ (compare § 87) it is only $\frac{16\cdot8}{4}=4\cdot2$; for boracic acid, $B_2\Theta_3$, it is only $\frac{16\cdot6}{5}=3\cdot3$), and also for compounds which contain silicium in addition to oxygen (it is $\frac{11\cdot3}{3}=3\cdot8$ for silicic acid, $Si\Theta_2$, compare § 85), or which contain oxygen as well as hydrogen (for ice, $H_2\Theta$, it is only $\frac{8\cdot6}{3}=2\cdot9$*, compare § 85), or which contain hydrogen and carbon besides oxygen (*e. g.* it is only $\frac{36\cdot9}{14}=2\cdot6$ for succinic acid, $C_4H_6\Theta_4$, compare § 89). It may be said in a few words what are the cases in which this quotient approximates to the atomic heat of most elements, and what the cases in which it is smaller. It is near 6·4 in those compounds which only contain elements whose atomic heats, corresponding to **Dulong** and **Petit**'s law, are nearly $=6\cdot4$; it is smaller in compounds which contain elements not coming under **Dulong** and **Petit**'s law and having a much smaller atomic heat than 6·4, and which are recognized as exceptions to this law, either directly, if their specific heat has been determined for the solid condition (compare § 92), or indirectly, if it be determined in the manner to be subsequently described.

---

* For REGNAULT's observation, whether, considering the specific heat which he found for silicium, its atomic weight is to be so taken that silicic acid contains 2 atoms of silicium to 5 of oxygen, compare Ann. de Chim. et de Phys. [3] vol. lxiii. p. 30. For SCHEERER's remark, that according to the most probable specific heat of silicium its atomic weight must be taken so that for 1 atom of silicium there are 3 atoms of oxygen, compare POGGENDORFF's 'Annalen,' vol. cxviii. p. 182.

† In the sequel R stands for a uni-equivalental and Ɍ a polyequivalental atom of a metal.

94. The determinations of specific heat given in §§ 83 to 89 contain the proofs hitherto recognized for the law that chemically-similar bodies of analogous atomic constitution have approximately the same atomic heat; and a considerable number of new examples of the prevalence of this regularity are given by my determinations. The groups of analogous compounds need not again be collated, as **Neumann** has done for a smaller and **Regnault** for a larger number of groups and for individual elements contained in them. What I will here discuss is the prevalence, beyond the limits of our previous

---

* Considering the atomic heat of liquid water to be 18, **Garnier** (Compt. Rendus, vol. xxxv. p. 278) thought that the quotient obtained by dividing the atomic weight by the number of elementary atoms in one atom of the compound, $\frac{18}{3}=6$, came near the atomic heat of the elements. But it requires no explanation that, in a comparison with the atomic heats of solid elements and solid compounds, that atomic heat must be taken for the compound $H_2\Theta$ which is obtained from the specific heat of ice, and not from that of water. **Garnier** is not alone in his error, which is rather to be ascribed to the circumstance that formerly both solids and liquids were compared, as regards their specific heat, in considerations how this property is influenced by the composition. **Hermann** more especially (Nouveaux Mémoires de la Société des Naturalistes de Moscou, vol. iii. p. 137) compared liquid water with solid compounds, as did also **Schröder** (**Poggendorff**'s 'Annalen,' vol. lii. p. 279) and L. **Gmelin** in an early discussion of this subject (**Gehler**'s 'Physicalische Wörterbuch, neue Bearbeitung,' vol. ix. p. 1942), while he subsequently (Handbuch der Chemie, 4. Aufl., vol. i. p. 220) more correctly compared the specific and the atomic heat of ice with that of other solid compounds.

knowledge, of the regularity, that compounds of analogous atomic constitution have approximately the same atomic heat.

To this belongs, first, the existence of this regularity in the case of chemically similar bodies, which exhibit an analogy of atomic constitution, when their formulæ are written with the atomic weights admitted in recent times for the elements, but which could not be recognized so long as the equivalents of the elements were taken as a basis, or the formula written, as by Regnault, with the use of the so-called thermal atomic weights.

The approximate equality of the atomic heats of analogous nitrates and chlorates, of the alkalies for example, had been already observed. The same character, the haloid, is ascribed both to carbonates and to silicates, but as these formulæ were formerly written, an analogy in the composition of chlorates and nitrates, or carbonates and silicates, could not be assumed. But salts of these four different classes, as well as arseniates and metaphosphates, have analogous atomic constitutions if we assume the recent atomic weights. The same salts have then also approximately equal atomic heats. We get the atomic heat

Of chlorate of potass, $\mathrm{K\,Cl\,\Theta_3}$, § 88	M*	24·8
„ the nitrates, $\mathrm{R\,N\Theta_3}$, in § 88.	M	23·0
„ metaphosphate of soda, $\mathrm{Na\,P\Theta_3}$, § 88		22·1
„ arseniate of potass, $\mathrm{K\,As\,\Theta_3}$, § 88		25·3
„ the carbonates, $\mathrm{\bar{R}\bar{C}\Theta_3}$, § 86	M	20·7
„ the silicates, $\mathrm{R\,Si\,\Theta_3}$, § 86	M	20·5

The differences in these approximately concordant atomic heats are partly essential and explainable. I come to this again (§ 95).

According to the more recent assumptions for the atomic weights, certain perchlorates, permanganates, and sulphates have analogous atomic composition, and these salts have also approximately equal atomic heats; this has been found to be

For perchlorate of potass, $\mathrm{K\,Cl\,\Theta_4}$, § 88		26·3
„ permanganate of potass, $\mathrm{K\,Mn\,\Theta_4}$, § 88		28·3
„ the sulphates, $\mathrm{\bar{R}S\Theta_4}$, named in § 88	M	26·1

But approximate equality in the atomic heat is not only found in such compounds of analogous chemical composition as have similar chemical character, but also in such as have totally dissimilar chemical character.

The chemical character of protosesquioxide of iron (magnetic iron ore) is quite different from that of neutral chromate of potass. Sesquioxide of iron, or arsenious acid, have a chemical character totally different from nitrates or arseniates, or bodies of similar constitution But for the first-named compounds and for the last-named compounds, as respectively compared with each other, there is analogy in chemical composition and approximate equality of atomic heat. The atomic heat has been found to be

---

* M signifies the mean of all determinations.

For magnetic iron ore, $Fe_3\Theta_4$, § 85	M	37·7
„ chromate of potass, $K_2 Cr\Theta_4$, § 87	M	36·4
„ sesquioxide of iron, $Fe_2\Theta_3$, § 85	M	26·8
„ arsenious acid, $As_2\Theta_3$, § 85		25·3
„ the nitrates, $RN\Theta_3$, named in § 88		23·0
„ arseniate of potass, $KAs\Theta_3$, § 88		25·3

But there is even in a more extended sense approximate equality of atomic heat in bodies of analogous atomic composition. If the formulæ of the oxides, $R\Theta_2$ (oxide of tin for instance) are doubled, they become $R_2\Theta_4$, and are then analogous to those of the sulphates, $RS\Theta_4$, or of tungstate of lime or of perchlorate of potass and other salts. To the formulæ thus made analogous equal atomic heats correspond. The following atomic heats have been found:—

Oxide of tin, $Sn_2\Theta_4$, compare § 85	M	27·6
Titanic acid, $Ti_2\Theta_4$, „	M	27·3
The sulphates, $RS\Theta_4$, in § 87	M	26·1
Tungstate of lime, $CaW\Theta_4$, compare § 87		27·9
Perchlorate of potass, $KCl\Theta_4$, compare § 88		26·3
Permanganate of potass, $KMn\Theta_4$, compare § 88		28·3

If the formulæ of the oxides, $R\Theta_2$, are trebled they become $R_3\Theta_6$, analogous to those of the nitrates $RN_2\Theta_6$ (nitrate of baryta, *e.g.*), and similar salts. Here also approximately equal atomic heats correspond to the formulæ thus made analogous. The atomic heats are as follows:—

Oxide of tin, $Sn_3\Theta_6$, compare § 85	M	41·4
Titanic acid, $Ti_3\Theta_6$, „	M	41·0
The nitrates, $RN_2\Theta_6$, in § 88	M	38·1
Metaphosphate of lime, $CaP_2\Theta_6$, compare § 88		39·4

How little the atomic heat of compounds depends on their chemical character may be proved from a greater series of examples than those adduced in the preceding. It is, however, unnecessary to dwell upon this. The comparisons and considerations contained in the sequel complete what has here been developed as a proof of the principle that the atomic heat of bodies is independent of their chemical character.

95. The foregoing comparisons give examples of cases in which bodies of analogous atomic structure, with a totally different chemical character, have approximately the same atomic heat; they show that with reference to the atomic heat, monoequivalent and polyequivalent elementary atoms have the same influence, which, indeed, followed already from Regnault's comparisons; that the atomic heat of a substance for its polyfold atomic formula may be compared with that of another substance for a simple atomic formula.

The preceding contains a generalization of NEUMANN'S law; but as certainly as this law is recognized in the preceding in a more general manner than was formerly assumed, as little is it universally applicable.

REGNAULT'S investigations have shown that NEUMANN'S law is not rigidly valid. Even for those compounds which contain the same element as electronegative constituent, and have similar atomic constitution, he found the atomic heats as much as $\frac{1}{10}$ to $\frac{1}{9}$ different from each other*. The reason of this he seeks in the same circumstances, which in his view prevent a closer agreement in the atomic weights of the elements (compare § 92).

Differences of this kind, and even still more considerable, occur in the atomic heats of compounds for which greater agreement in these numbers might be expected—of such compounds, that is, as contain elements of the same, or almost the same atomic heat combined with the same other element in the same atomic proportion. To this belongs the fact that the atomic heat has been found so different (§ 85) for the isomorphous compounds, magnetic iron ore (37·7), chrome iron ore (31·2), and spinelle (27·7), and for alumina (21·3) and for sesquioxide of iron (26·8). In the atomic heats of such analogous compounds there are differences for which, or rather for the magnitude of which, as furnished by our present observations, I know at present no adequate explanation.

But there is another kind of difference in the atomic heats of analogous compounds, which exhibits a regularity, and for which an explanation can be given. Certain elements impress on all their compounds the common characteristic, that their atomic heat is much smaller than that of most analogous compounds. The atomic heat of boracic acid, $B_2\Theta_3$, is only 16·6, while that of most other compounds, $R_2\Theta_3$ and $R_2\Theta_3$, is between 25 and 28 (§ 85). The atomic heat of the borates, $RB\Theta_2$, is (§ 87) only 16·8, while that of $R_2\Theta_2$, as the mean of the determinations in § 85, is 22·2. The atomic heat of $PbB_2\Theta_4$ is (§ 87) only 26·5, while that of $Fe_3\Theta_4$ (§ 85) in the mean is 37·7. Similar results have been obtained for compounds of certain other elements, of carbon and of silicium for instance, that is, of those elements which in the free state have a smaller atomic heat than that of most other elements.

This observation leads to the question whether the elements enter into compounds with the atomic heats which they have in the free state, and in connexion with this, how far is it permissible to make an indirect determination of the atomic heat of the elements (in their solid state) from the atomic heats of their (solid) compounds.

96. The assumption that elements enter into compounds with the atomic heats they have in the free state would be inadmissible, if not only the atomic structure as expressed by the empirical formula, but also the grouping of the elements to proximate constituents, as is endeavoured to be expressed by the rational formula, influenced the atomic heat of the compounds. That the latter is not the case is very probable from the comparisons made in § 94, where approximately equal atomic heats were obtained for compounds of analogous empirical formulæ, even with the greatest dissi-

---

* Ann. de Chim. et de Phys. [3] vol. i. p. 196.

milarity of chemical character. That that, which may be supposed and expressed by the so-called rational formula in reference to the internal constitution of compounds, does not affect the atomic heat, becomes more probable from the fact that chemically similar, and even isomorphous compounds, one of which contains an atomic group in the place of an individual atom in the other, exhibit dissimilar atomic heats. This is seen, for instance, in comparing analogous chlorine and cyanogen compounds ($\text{Cy}=\text{C N}$); the latter have far greater atomic heats. Thus the atomic heat

Of chloride of mercury, Hg Cl_2, § 84, is	18·0
„ cyanide of mercury, Hg Cy_2, § 89	25·2
„ chloride of zinc and potassium, $\text{Zn K}_2\text{ Cl}_4$, § 84	43·4
„ cyanide of zinc and potassium, $\text{Zn K}_2\text{ Cy}_4$, § 89	59·6

In like manner ammonium compounds ($\text{Am}=\text{N H}_4$) have atomic heats considerably greater than the corresponding potassium compounds. This is seen from the following Table:—

Chloride of potassium, K Cl, § 84	M	12·9
„ ammonium, Am Cl, § 84		20·0
Nitrate of potass, $\text{K N }\Theta_3$, § 88	M	23·5
„ ammonia, $\text{Am N }\Theta_3$, § 88		36·4
Sulphate of potass, $\text{K}_2\text{ S}\Theta_4$, § 87	M	33·6
„ ammonia, $\text{Am}_2\text{ S}\Theta_4$, § 87		46·2

97. That undecomposable atoms and atomic groups are contained in compounds with the atomic heats they have in the free state is further probable from the fact that the sum of the atomic heats of such atoms, or atomic groups, as when united form a certain compound, is equal or approximately equal to the atomic heat of this compound. For many compounds whose elements obey Dulong and Petit's law, what has been stated in § 93 contains the proof that the atomic heat of these compounds is equal to the sum of the atomic heats of the elementary atoms contained in one atom of the compounds. That this is also observed when atomic groups are supposed to be united, forming more complicated compounds, will be seen by bringing forward a few examples. The atomic heat has been found

For the oxides, $\text{R }\Theta$, enumerated in § 85	M	11·1
„ sesquioxide of iron, $\text{Fe}_2\text{ }\Theta_3$, § 85	M	26·8
Sum for $\text{Fe}_2\text{ R}\Theta_4$		37·9
„ magnetic iron ore, $\text{Fe}_3\text{ }\Theta_4$, § 85	M	37·7
„ the oxides, $\text{R }\Theta$, in § 85	M	11·1
„ the acids, $\text{R }\Theta_3$, in § 85, according to Regnault	M	18·8
Sum for $\text{R R }\Theta_4$		29·9
„ chromate of lead, $\text{Pb Cr }\Theta_4$, § 87		29·0

For the oxides named in § 85, $\text{Ɍ}\,\Theta$	M	11·1
„ binoxide of tin, $\text{Sn}\,\Theta_2$, § 85	M	13·8
Sum for $\text{Ɍ}\,\text{Ɍ}\,\Theta_3$		24·9
„ sesquioxide of iron, $\text{Fe}_2\,\Theta_3$, § 85	M	26·8
„ chromate of potass, $\text{K}_2\,\text{Ȼr}\,\Theta_4$, § 87	M	36·4
„ the acids, $\text{Ɍ}\,\Theta_3$, in § 85 (Regnault)		18·8
Sum for $\text{K}_2\,\text{Ȼr}\,\text{Ɍ}\,\Theta_7$		55·2
„ acid chromate of potass, $\text{K}_2\,\text{Ȼr}_2\,\Theta_7$, § 87	M	55·3
„ binoxide of tin, $\text{Sn}_3\,\Theta_6$, § 85	M	41·4
„ base, $\text{Ɍ}_2\,\Theta_2$, mean of determinations, § 85	M	22·2
Sum for $\text{Ɍ}_5\,\Theta_8$		63·6
„ arseniate of lead, $\text{Pb}_3\,\text{As}_2\,\Theta_8$, § 88		65·4

To this belongs the fact that water is contained in solid compounds with the atomic heat of ice[*]. The different determinations of the specific heat of this substance (§ 85) gave the atomic heat for greater distances from 0°, 8·6, and for temperatures nearer 0°, 9·1 to 9·2. The atomic heats have been found

			For H_2O.
For $\text{Ƀa}\,\text{Cl}_2+2\,H_2\,\Theta$, § 84		41·7	
„ the chlorides, $\text{Ɍ}\,\text{Cl}_2$, § 84	M	18·5	
Remains for $2\,H_2\,\Theta$		23·2	11·6
„ $\text{Ȼa}\,\text{Cl}_2+6\,H_2\,\Theta$, § 84		75·6	
„ the chlorides, $\text{Ɍ}\,\text{Cl}_2$, § 84	M	18·5	
Remains for $6\,H_2\,\Theta$		57·1	9·5
„ Brucite, $\text{Mg}\,\Theta+H_2\,\Theta$, § 85		18·1	
„ the oxides, $\text{Ɍ}\,\text{O}$, § 85	M	11·1	
Remains for $H_2\,\Theta$		7·0	7·0
„ dioptase, $\text{Ȼu}\,\text{Si}\,\Theta_3+H_2\,\Theta$, § 86		28·7	
„ the silicates, $\text{Ɍ}\,\text{Si}\,\Theta_3$, § 86	M	20·5	
Remains for $H_2\,\Theta$		8·2	8·2
„ $\text{Na}_2\,\text{B}_4\,\Theta_7+10\,H_2\,\Theta$, § 87		146·9	
„ $\text{Na}_2\,\text{B}_4\,\Theta_7$, § 87		47·1	
Remains for $10\,H_2\,\Theta$		99·8	10·0
„ gypsum, $\text{Ȼa}\,\text{S}\,\Theta_4+2\,H_2\,\Theta$, § 87	M	45·8	
„ the sulphates, $\text{Ɍ}\,\text{S}\,\Theta_4$, § 87	M	26·1	
Remains for $2\,H_2\,\Theta$		19·7	9·9

---

[*] Even before Person (compare § 14) L. Gmelin had speculated (Handbuch der Chemie, [4] Aufl. vol. i. p. 223) whether from the atomic heats of anhydrous sulphate of lime and of ice that of gypsum could be calculated. The results of calculation deviated considerably from the atomic heat as deduced from the observed specific heat of gypsum; the specific heat, and therewith the atomic heat of ice, were at that time incorrectly known.

The Tables in § 84 to 89 contain data for several such comparisons, which lead to the same result as the preceding—that the atomic heat of water contained in solid compounds may, by subtracting the atomic heat of the anhydrous solid from that of the hydrated solid compound, be obtained in sufficient approximation to the atomic heat deduced from the direct determination of the specific heat of ice. The deviations from each other and from the atomic heat of ice as directly determined, which these indirect determinations exhibit, are not to be wondered at when it is considered that all uncertainties in the atomic heats, from whose difference the atomic heat of solid water is deduced, are concentrated upon this difference.

98. The view already expressed and defended (compare especially § 12 and 13), that atoms and atomic groups are contained in solid compounds with the same atomic heat which they have in the free state, is opposed to the view which has also been frequently expressed and defended—that the atomic heat of an element may in certain compounds differ from what it is in the free state, and may be different in different compounds. This view, and the reasons which may possibly be urged in its favour, must here be discussed.

The first statement of this view (compare § 6) simply goes to assert that the atomic heats of compounds may be calculated in accordance with the values resulting from the determinations of the specific heat, assuming that one constituent of the compound has the same atomic heat as in the free state, the other an altered one. What alteration is to be assumed depends merely on what assumption adequately satisfies the observed specific heat of the compound. The accuracy of the assumption is susceptible of no further control; the assumption itself cannot be regarded as an explanation of the observed atomic heat of the compound. And nothing is altered in this by assuming (compare § 6 and 11) that the changes in the atomic heat of a substance on entering into chemical compounds take place in more or less simple ratios.

A greater degree of probability must be granted to the view (compare § 10) that the atomic heats of the constituents of compounds, and the differences in the atomic heats of these bodies, according as they are combined or in the free state, depend upon the state of condensation in which these bodies are contained. If, for instance, from a consideration of the specific gravities or specific volumes (the quotient of the specific weights into the atomic weights) of compounds and of their constitutents, a conclusion could be drawn with some degree of certainty as to the state of condensation in which the latter are present in the former, and if definite rules could be given for the variations of the atomic heats with the state of condensation, the result of such an investigation, if it agreed with the observed results for the atomic heats of compounds, might be called an explanation of these observations. But what is here presupposed is partially not attained and partially not attempted. And, moreover, as far as can be judged from individual cases, the same element, when contained in different states of condensation, appears to have the same atomic heat. It has been attempted to deduce the state of condensation, or the specific volume of oxygen in its compounds with heavy metals, by subtracting from the specific volume of the oxide that of the metal in it, and considering the remainder as the volume of oxygen. It would follow from this that the specific volume of oxygen in suboxide of copper is much greater (about four times as great) than in oxide of tin. But if the atomic heat of oxygen be deduced by subtracting from the atomic heat of the oxide that of the metal in it, it is found that the atomic heat of oxygen in suboxide of copper and in oxide of tin gives almost exactly the same number. Hence it does not seem that the state of condensation in which a constituent may be contained in a compound has any material influence on the atomic heat of this constituent.

99. From all that has been said in the foregoing paragraphs the following must be adhered to. (1) Each element in the solid state, and at a sufficient distance from its melting-point, has *one* specific or atomic heat, which may, indeed, somewhat vary with physical conditions, different temperature, or density for instance, but not so considerably as to be regarded in considering in what relations the specific heat stands to the atomic weight or composition; and (2) that each element has essentially the same specific or atomic heat in compounds as it has in the free state. On the basis of these two fundamental laws it may now be investigated what atomic heats individual elements have in the solid free state and in compounds. Indirect deductions of the atomic heats of such elements as could not be investigated in the solid free state are from these propositions admissible: that from the atomic heat of a compound containing such an element the atomic heat of everything else in the compound is subtracted, and the remainder considered as the expression for the atomic heat of that element. Such indirect determinations of the atomic heat of elements may be uncertain, partly because the atomic heat of the compounds is frequently not known with certainty, as is seen from the circumstance that analogous compounds, for which there is every reason to expect the same atomic heat, are found by experiment to have atomic heats not at all agreeing; but more especially because the entire relative uncertainty in the atomic heats for a compound, and for that which is to be subtracted from its composition, is concentrated upon a small number, the residue remaining in the deduction. But when such deductions are made, not merely for individual cases, but for different compounds, and for entire series of corresponding compounds, they may be considered sufficiently trustworthy to make the speculations based upon them worthy of attention. Of course in indirectly deducing the atomic heat of an element, its simpler compounds, and those containing it in greatest quantity (measured by the number of atoms), promise the most trustworthy results.

100. For *Silver, Aluminium, Arsenic, Gold, Bismuth, Bromine, Cadmium, Cobalt, Copper, Iron, Mercury, Iodine, Iridium, Potassium, Lithium, Magnesium, Manganese, Molybdenum, Sodium, Nickel, Osmium, Lead, Palladium, Platinum, Rhodium, Antimony, Selenium, Tin, Tellurium, Thallium, Tungsten, and Zinc*, it may be assumed, from the determinations of their specific heat in the solid state (§ 82), that their atomic heats, in

accordance with Dulong and Petit's law, are approximately equal, the average being 6·4. I do not think that all these elements have really the same atomic heat, but think that some of them will subsequently be considered as exceptions to the above-mentioned law, as it will in the sequel be proved that several elements have an atomic heat differing from 6·4. But for none of the previously mentioned elements are the present data, and the presumed deviation of the atomic heat from that of other elements, sufficient to justify their being separated from them.

To the elements just mentioned *chlorine* must be associated from the close agreement of the corresponding chlorine, bromine, and iodine compounds (§ 84), and of the compounds $\mathrm{K\,Cl\,\Theta_3}$, 24·8, and $\mathrm{K\,As\,\Theta_3}$, 25·3 (§ 88). To the atomic heats of these latter compounds those of individual salts $\mathrm{K\,N\,\Theta_3}$ approximate closely; the latter gave (§ 88) 21·8–24·4, mean 23·0, which on the whole agrees sufficiently closely with those found for the metallic oxides, $\bar{\mathrm{R}}_2\,\Theta_3$ (§ 85). I count *nitrogen* also among the elements whose atomic heat may be assumed at 6·4, like that of most other elements; without, however, considering the determination of the atomic heat of this element as very trustworthy. To deduce the atomic heat of this element with certainty, compounds are wanting which contain, besides nitrogen, elements whose atomic heat has been directly determined. The fact that the atomic heat of the nitrates, $\bar{\mathrm{R}}_2\,\mathrm{N}_2\,\Theta_6$, was found (§ 88) in the mean to be 38·1, a third of which, 12·7, is somewhat less than the average atomic heat found for the oxides of heavy metals of the formula $\bar{\mathrm{R}}\,\Theta_2$, might be a reason for assigning to nitrogen a smaller atomic heat; while, on the other hand, the atomic heats of other nitrogen compounds, in which it is true other elements enter whose atomic heat is only indirectly determined, do not favour this view.

In the class of elements with the atomic heat about 6·4, *barium*, *calcium*, and *strontium* may be placed from the agreement in the atomic heats of their compounds with the atomic heats of corresponding compounds of such elements as have been found by the direct determination of their specific heat in the free solid state to belong to that class (compare the atomic heats of the compounds $\bar{\mathrm{R}}\,\mathrm{Cl}_2$ in § 84, $\bar{\mathrm{R}}\,\bar{\mathrm{C}}\,\Theta_3$ in § 86, $\bar{\mathrm{R}}\,\bar{\mathrm{S}}\,\Theta_4$ in § 87, and $\bar{\mathrm{R}}\,\mathrm{N}_2\,\Theta_6$ in § 88); further, *rubidium* (compare the atomic heats of the compounds $\bar{\mathrm{R}}\,\mathrm{Cl}$ in § 84, and $\mathrm{R}_2\,\bar{\mathrm{C}}\,\Theta_3$ in § 86); then also *chromium* (from the agreement in the atomic heats of $\bar{\mathrm{Cr}}_2\,\Theta_3$ and $\bar{\mathrm{Fe}}_2\,\Theta_3$, § 84), and *titanium* (from the agreement in the atomic heats of $\bar{\mathrm{Ti}}\,\Theta_2$ and $\mathrm{Sr}\,\Theta_2$, § 84). To place *zirconium* in the same class has no other justification than that on this assumption the atomic heat of zircon may be calculated in accordance with that deduced from the observed specific heat of this mineral.

101. According to direct determinations of the specific heat, *sulphur* and *phosphorus* do not belong to this class. The more trustworthy determinations (for sulphur the last two, for phosphorus the last three of the numbers in § 82) assign to these elements the atomic heat 5·4. That sulphur has a smaller atomic heat than the elements discussed in the last paragraphs follows from the atomic heats of sulphur compounds, compared

with those of the corresponding compounds of such elements as have an atomic heat $=6{\cdot}4$. The average atomic heat of compounds R S and Ꞧ S is 11·9, according to the determinations in § 83, while those of chlorine compounds R Cl and Ꞧ Cl (§ 84) $=12{\cdot}8$, that of the corresponding bromine compounds $=13{\cdot}9$, and of the corresponding iodine compounds $=13{\cdot}4$. In comparing more complicated sulphur compounds, sulphates, for instance, with other compounds of analogous composition, the same is met with; although such complicated compounds are of little value in giving data for deciding on such small differences. The specific heat of the simpler phosphorus compounds has not been investigated; for more complicated compounds, although they point to a smaller atomic heat for P than 6·4, the above remark also applies.

The determinations of the specific heat of *silicium* give for this element also a smaller atomic heat than 6·4 (compare § 82), and the same conclusion results from a comparison of the atomic heats of $\text{Si}\,\Theta_2$, and the oxides, $\text{Ꞧ}\,\Theta_2$, of the silicates $\text{Ꞧ Si}\,\Theta_3$, and the oxides $\text{Ꞧ}_2\,\Theta_3$. The atomic heat to be assigned to silicium cannot as yet be settled with any degree of certainty. Direct determinations, varying considerably from each other, give a specific heat mostly greater than 4; while the numbers obtained indirectly, and themselves also not closely agreeing, are partly considerably smaller. If in the sequel I put the atomic heat of silicium at 3·8, corresponding to the lowest number found for the specific heat of this element, I do so for want of other and more certain data. I consider this number as quite uncertain.

The atomic heat of *boron*, from the direct determinations of the specific heat, is considerably smaller than 6·4; and the atomic heats of boron compounds confirm this, as was discussed in §§ 93 and 95. By comparing the atomic heats of such boron and sulphur compounds as contain along with boron and sulphur the same elements in the same proportions, the atomic heat of boron is found to be half that of sulphur. The atomic heat of $\text{K B}\,\Theta_2=16{\cdot}8$ is exactly half that found for $\text{K}_2\,\text{S}\,\Theta_4=33{\cdot}6$; the atomic heat of $\text{Ᵽb B}_2\,\Theta_4=26{\cdot}5$ is almost exactly equal to that for $\text{Ᵽb S}\Theta_4=25{\cdot}7$. Taking the atomic heat of S, in accordance with the above discussion, at 5·4, that of B would be 2·7; the numbers obtained directly for the atomic heat of boron (§ 82) from the experiments on the specific heat of this element agree with sufficient accuracy. In the sequel I take the atomic heat of B at 2·7. A smaller number is obtained in other comparisons; for instance, of the atomic heats of $\text{B}_2\,\Theta_3$ and of the oxides $\text{Ꞧ}_2\,\Theta_3$, or of the salts $\text{R B}\,\Theta_2$ and the oxides $\text{Ꞧ}_2\,\text{O}_2$; but in such indirect determinations of the atomic heat, where such small numbers are to be determined, as is here the case with the atomic heat of boron, the results are very uncertain, owing to the fact that the entire uncertainty in the atomic heats of the compounds, and in the assumption that the elements corresponding to boron in compounds of analogous composition have really the atomic heat $=6{\cdot}4$, is thrown on the final result.

Lastly, *carbon* also, from the direct determinations of its specific heat (§ 82), has a much smaller atomic heat than 6·4. The same result follows from a comparison of the atomic heats of carbon compounds: the atomic heat of the carbonates, $\text{Ꞧ}_2\,\text{Ꞓ}\,\Theta_3=28{\cdot}4$ as the mean of the determinations in § 86, is much smaller than that of $\mathrm{Ɍ}_3\Theta_3(=3\mathrm{Ɍ}\Theta)$, which is the mean of the numbers in § 85 $=33{\cdot}3$; the atomic heat of the carbonates $\mathrm{ꞒƟ}_3$ $=20{\cdot}7$, as the mean of the determinations in § 86, is much smaller than 27·1, the number found for $As_2\Theta_3$, $Bi_2\Theta_3$, $\mathrm{Ꞓr}_2\Theta_3$, $\mathrm{Ꞙe}_2\Theta_3$, and $Sb_2\Theta_3$ as the atomic heat of oxides $\mathrm{Ɍ}_2\Theta_3$. I put the atomic heat of carbon at 1·8 for Ꞓ, as deduced from the determination of the specific heat of its purest variety, diamond.

102. In the preceding paragraphs I have discussed the elements which, from the determinations of their specific heat in the solid free state, have a smaller atomic heat than about 6·4. There remain to be discussed a few elements whose atomic heats are also less than those of most other elements, but can only be deduced from those of their compounds.

To this category belongs *hydrogen**, even if the indirect determination of its atomic heat in the solid state is liable to the uncertainty just discussed. The atomic heat of water, $H_2\Theta$, is (§ 85) $=8{\cdot}6$, and smaller by 7 than that of suboxide of copper, $\mathrm{Ꞓu}_2\Theta$, which was found in the mean to be 15·6; the atomic heat of hydrogen would thus be $\frac{7}{2}=3{\cdot}5$ less than that of the elements to which copper belongs, as regards its atomic heat; hence the former would be $6{\cdot}4-3{\cdot}5=2{\cdot}9$. The atomic heat of chloride of ammonium, NH_4Cl, has been found to be 20·0 (§ 84); the subtraction of the atomic heats for $N+Cl=6{\cdot}4+6{\cdot}4=12{\cdot}8$, leaves 7·2 as the atomic heat of 4H, and therefore 1·7 for that of H. The atomic heat of nitrate of ammonia, $N_2H_4\Theta_3$, is 36·4 (§ 88); subtracting therefrom as the atomic heat of $N_2+\Theta_3$, the number 27·1, which has previously been frequently mentioned as the atomic heat of oxides $\mathrm{Ɍ}_2\Theta_3$, we have 9·3 as the atomic heat of 4H, that is 2·3 for that of H. I put in the sequel the atomic heat of *hydrogen* at 2·3.

That *oxygen* has a smaller atomic heat than 6·4, follows from the fact that the oxygen compounds of the metals have a considerably smaller atomic heat than the corresponding chlorides, iodides, or bromides. For instance, the atomic heat of the oxides $\mathrm{Ɍ}\Theta$ is as the mean of the determinations in § 85 $=11{\cdot}1$, while that of the chlorides R Cl and Ɍ Cl (§ 84), is 12·8, that of the corresponding bromides 13·9, and of the corresponding iodides 13·4. That of the oxides, $\mathrm{Ɍ}\Theta_2$, as the mean of the determinations in § 85, of $Mn\Theta_2$, $\mathrm{Ꞩn}\Theta_2$, and $\mathrm{Ŧi}\Theta_2$ is 13·7, while that of the chlorides $\mathrm{Ɍ}Cl_2$ (§ 85) is 18·5, and of the iodides $\mathrm{Ɍ}I_2=19{\cdot}4$. Taking the atomic heat of the other elements, which are contained in the following compounds, at 6·4, the atomic heat of oxygen, as deduced from the atomic heat of the oxides $\mathrm{Ɍ}\Theta$ (11·1 in the mean), is $=4{\cdot}7$; as deduced from the oxides $\mathrm{Ɍ}_2\Theta_3$ (27·1 as the mean of the oxides of this formula previously frequently mentioned), it is $=4{\cdot}8$; from the above oxides, $\mathrm{Ɍ}\Theta_2$ (13·7 in the mean), it is $=3{\cdot}7$; it is found (compare § 88) from $KAs\Theta_3$ (25·3) to be 4·1; from $\mathrm{Ᵽb}_3As_2\Theta_8$ (65·4) to be 4·2; from $KCl\Theta_3$ (24·8) to be 4·0; from $KCl\Theta_4$ (26·3) to be 3·4; from $KMn\Theta_4$ (28·3) to be 3·9. In the sequel I take the round number 4 for the atomic heat of Θ.

---

* L. Gmelin (Handbuch der Chemie, 4 Aufl. vol. i. pp. 216 and 222) ascribed to hydrogen the same capacity for heat as that of an equivalent quantity of lead or mercury ($H=1$, $Cu=31{\cdot}7$, $Hg=100$); Schröder (Poggend. Ann. vol. lii. p. 279) and Cannizzaro (Il Nuovo Cimento, vol. vii. p. 342) ascribed to hydrogen the same atomic heat as that of most other elements ($H=1$, $Cl=35{\cdot}5$, $\mathrm{Ꞓu}=63{\cdot}4$, $\mathrm{Ħg}=200$).

Fluorine appears, lastly, to have a considerably smaller atomic heat than 6·4. The atomic heat of fluoride of calcium, $Ca\,Fl_2$, has been found to be (§ 84) only 16·4, considerably smaller than the corresponding chlorides, bromides, and iodides. I put the atomic heat of fluorine at $\frac{16{\cdot}4-6{\cdot}4}{2}=5$.

103. Taking, in accordance with what has just been said, the atomic heat which an element has in a solid compound,

At 6·4 for Ag, Al, As, Au, Ba, Bi, Br, Ca, Cd, Cl, Co, Cr, Cu, Fe, Hg, I, Fr, K, Li, Mg, Mn, Mo, N, Na, Ni, Os, Pb, Pd, Pt, Rb, Rh, Sb, Se, Sn, Sr, Te, Ti, Tl, W, Zn, and Zr,

At 5·4 for S and P, at 5 for Fl, 4 for O, 3·8 for Si, 2·7 for B, 2·3 for H, and 1·8 for C; and assuming that the atomic heat of a solid is given by the sum of the atomic heats of the elements in it, we obtain the atomic heats; and dividing them by the atomic weights, we obtain the specific heats, in sufficiently close agreement with the specific heats as obtained by direct determinations of this property.

In the following Table I give for all compounds for which the specific heat has been determined in a trustworthy manner, the specific heat calculated on these assumptions, compared with the numbers found experimentally. I give this calculation and this comparison in the same order which was followed in the synopsis § 82 to 89, and I refer to the latter as regards special remarks on the determinations. To distinguish the observers, N. again stands for Neumann, R. Regnault, Kp. Kopp, Pr. Person, A. Alluard, and Pp. Pape.

Alloys. (Compare § 82.)

	Atomic weight.	Atomic heat. Calculated.	Specific heat. Calculated.	Specific heat. Observed.	
$Bi\,Sn$	328	12·8	0·0390	0·0400	R.
$Bi\,Sn_2$	446	19·2	0·0430	0·0450	R.
$Bi\,Sn_2\,Sb$	568	25·6	0·0451	0·0462	R.
$Bi\,Sn_2,\,Sb\,Zn_2$ . .	698·4	38·4	0·0550	0·0566	R.
$Pb\,Sb$	329	12·8	0·0389	0·0388	R.
$Pb\,Sn$	325	12·8	0·0394	0·0407	R.
$Pb\,Sn_2$	443	19·2	0·0433	0·0451	R.

104. *Arsenides and Sulphides.* (Compare § 83.)

$Co\,As_2$	208·8	19·2	0·0919	0·0920	N.				
$Ag_2\,S$	248	18·2	0·0734	0·0746	R.				
$Co\,As\,S$	166	18·2	0·110	0·107	N.				
$Cu_2\,S$	158·8	18·2	0·115	0·121	R.	0·120	Kp.		
$Fe\,As\,S$	163	18·2	0·112	0·101	N.				
$As\,S$	107	11·8	0·110	0·111	N.				
$Co\,S$	90·8	11·8	0·130	0·125	R.				
$Cu_{\frac{1}{2}}\,Fe_{\frac{1}{2}}\,S$	91·7	11·8	0·129	0·129	N.	0·131	Kp.		
$Fe\,S$	88	11·8	0·134	0·136	R.				
$Hg\,S$	232	11·8	0·0509	0·052	N.	0·0512	R.	0·0517	Kp.
$Ni\,S$	90·8	11·8	0·130	0·128	R.				

	Atomic weight.	Atomic heat. Calculated.	Specific heat. Calculated.	Specific heat. Observed.					
PbS	239	11·8	0·0494	0·053	N.	0·0509	R.	0·0490	Kp.
SnS	150	11·8	0·0787	0·0837	R.				
ZnS	97·2	11·8	0·121	0·115	N.	0·123	R.	0·120	Kp.
Fe_7S_8	648	88·0	0·136	0·153	N.	0·160	R.		
As_2S_3	246	29·0	0·118	0·113	N.				
Bi_2S_3	516	29·0	0·0562	0·060	R.				
Sb_2S_3	340	29·0	0·0853	0·0907	N.	0·0840	R.		
FeS_2	120	17·2	0·143	0·128–0·133	N.	0·130	R.	0·126	Kp.
MoS_2	160	17·2	0·107	0·107	N.	0·123	R.		
SnS_2	182	17·2	0·0945	0·119	R.				

105. *Chlorides, Bromides, Iodides, and Fluorides.* (Compare § 84.)

$AgCl$	143·5	12·8	0·0892	0·0911	R.				
$CuCl$	98·9	12·8	0·129	0·138	R.				
$HgCl$	235·5	12·8	0·0543	0·0521	R.				
KCl	74·6	12·8	0·172	0·173	R.	0·171	Kp.		
$LiCl$	42·5	12·8	0·301	0·282	R.				
$NaCl$	58·5	12·8	0·219	0·214	R.	0·213–0·219	Kp.		
$RbCl$	120·9	12·8	0·106	0·112	Kp.				
NH_4Cl	53·5	22·0	0·411	0·373	Kp.				
$BaCl_2$	208	19·2	0·0923	0·0896	R.	0·0902	Kp.		
$CaCl_2$	111	19·2	0·173	0·164	R.				
$HgCl_2$	271	19·2	0·0708	0·0689	R.	0·640	Kp.		
$MgCl_2$	95	19·2	0·202	0·195	R.	0·191	Kp.		
$MnCl_2$	126	19·2	0·152	0·143	R.				
$PbCl_2$	278	19·2	0·0691	0·0664	R.				
$SnCl_2$	189	19·2	0·102	0·102	R.				
$SrCl_2$	158·6	19·2	0·121	0·120	R.				
$ZnCl_2$	136·2	19·2	0·141	0·136	R.				
$BaCl_2+2H_2O$	244	36·4	0·149	0·171	Kp.				
$CaCl_2+6H_2O$	219	70·8	0·323	0·345	Pr.				
ZnK_2Cl_4	285·4	44·8	0·157	0·152	Kp.				
PtK_2Cl_6	488·6	57·6	0·118	0·113	Kp.				
SnK_2Cl_6	409·2	57·6	0·141	0·133	Kp.				
Cr_2Cl_6	317·4	51·2	0·161	0·143	Kp.				
$AgBr$	188	12·8	0·0681	0·0739	R.				
KBr	119·1	12·8	0·107	0·113	R.				
$NaBr$	103	12·8	0·124	0·138	R.				
$PbBr_2$	367	19·2	0·0523	0·0533	R.				
AgI	235	12·8	0·0545	0·0616	R.				
CuI	190·4	12·8	0·0672	0·0687	R.				
HgI	327	12·8	0·0391	0·0395	R.				
KI	166·1	12·8	0·0771	0·0819	R.				
NaI	150	12·8	0·0853	0·0868	R.				
HgI_2	454	19·2	0·0423	0·0420	R.				
PbI_2	461	19·2	0·0416	0·0427	R.				
$CaFl_2$	78	16·4	0·210	0·208	N.	0·215	R.	0·209	Kp.
$AlNa_3Fl_6$	210·4	55·6	0·264	0·238	Kp.				

106. *Oxides.* (Compare § 85.)

	Atomic weight.	Atomic heat. Calculated.	Specific heat. Calculated.	Specific heat. Observed.					
Cu_2O	142·8	16·8	0·118	0·107	N.	0·111	Kp.		
H_2O	18	8·6	0·478	0·480	Pr.	0·474	R.		
CuO	79·4	10·4	0·131	0·137	N.	0·142	R.	0·128	Kp.
HgO	216	10·4	0·0481	0·049	N.	0·052	R.	0·053	Kp.
MgO	40	10·4	0·260	0·276	N.	0·244	R.		
MnO	71	10·4	0·146	0·157	R.				
NiO	74·8	10·4	0·139	0·159	R.				
PbO	22·3	10·4	0·0466	0·0512	R.	0·0553	Kp.		
ZnO	81·2	10·4	0·128	0·132	N.	0·125	R.		
$MgO+H_2O$	58	19·0	0·328	0·312	Kp.				
Fe_3O_4	232	35·2	0·152	0·164	N.	0·168	R.	0·156	Kp.
$MgAl_2O_4$	142·8	35·2	0·246	0·194	Kp.				
$Mg_{\frac{1}{2}}Fe_{\frac{1}{2}}Cr_{\frac{3}{2}}Al_{\frac{1}{2}}O_4$	196	35·2	0·179	0·159	Kp.				
Al_2O_3	102·8	24·8	0·241	0·197	N.	0·217	R.		
As_2O_3	198	24·8	0·125	0·128	R.				
B_2O_3	69·8	17·4	0·249	0·237	R.				
Bi_2O_3	468	24·8	0·0530	0·0605	R.				
Cr_2O_3	152·4	24·8	0·163	0·196	N.	0·180	R.	0·177	Kp.
Fe_2O_3	160	24·8	0·155	0·169	N.	0·167	R.	0·154	Kp.
$Fe_{\frac{5}{4}}Ti_{\frac{3}{4}}O_3$	155·5	24·8	0·160	0·176	N.	0·177	Kp.		
Sb_2O_3	292	24·8	0·0849	0·0901	R.				
$Mn_2O_3+H_2O$	176	33·4	0·189	0·176	Kp.				
MnO_2	87	14·4	0·166	0·159	Kp.				
SiO_2	60	11·8	0·197	0·188	N.	0·191	R.	0·186	Kp.
$Si_{\frac{1}{2}}Zr_{\frac{1}{2}}O_2$	90·8	13·1	0·144	0·146	R.	0·132	Kp.		
SnO_2	150	14·4	0·096	0·093	N.	0·093	R.	0·089	Kp.
TiO_2	82	14·4	0·176	0·172	N.	0·171	R.	0·159	Kp.
MoO_3	144	18·4	0·128	0·132	R.	0·154?	Kp.		
WO_3	232	18·4	0·0793	0·0798	R.	0·0894?	Kp.		

107. *Carbonates and Silicates.* (Compare § 86.)

K_2CO_3	138·2	26·6	0·192	0·216	R.	0·206	Kp.		
Na_2CO_3	106	26·6	0·251	0·273	R.	0·246	Kp.		
Rb_2CO_3	230·8	26·6	0·115	0·123	Kp.				
$BaCO_3$	197	20·2	0·103	0·108	N.	0·110	R.		
$CaCO_3$	100	20·2	0·202	0·203	N.	0·209	R.	0·205	Kp.
$Ca_{\frac{1}{2}}Mg_{\frac{1}{2}}CO_3$	92	20·2	0·220	0·216	N.	0·218	R.	0·206	Kp.
$Fe_{\frac{8}{11}}Mn_{\frac{2}{11}}Mg_{\frac{1}{11}}CO_3$	112·9	20·2	0·179	0·166	Kp.				
$Mg_{\frac{7}{9}}Fe_{\frac{2}{9}}CO_3$	91·1	20·2	0·222	0·227	N.				
$PbCO_3$	267	20·2	0·0757	0·0814	N.	0·0791	Kp.		
$SrCO_3$	147·6	20.2	0·137	0·145	N.	0·145	R.		
$CaSiO_3$	116	22·2	0·191	0·178	Kp.				
$Ca_{\frac{1}{2}}Mg_{\frac{1}{2}}SiO_3$	108	22·2	0·205	0·191	N.	0·186	Kp.		
$CuSiO_3+H_2O$	157·4	30·8	0·195	0·182	Kp.				
$Mg_{\frac{20}{11}}Fe_{\frac{2}{11}}SiO_4$	145·8	32·6	0·223	0·206	N.	0·189	Kp.		
$Al_2K_2Si_6O_{16}$	557	112·4	0·202	0·191	N.	0·183	Kp.		
$Al_2Na_2Si_6O_{16}$	524·8	112·4	0·214	0·196	N.	0·190	Kp.		

108. *Borates, Molybdates, Tungstates, Chromates, and Sulphates.* (Compare § 87.)

	Atomic weight.	Atomic heat. Calculated.	Specific heat. Calculated.	Specific heat. Observed.					
$KB\Theta_2$	82	17·1	0·209	0·205	R.				
$NaB\Theta_2$	65·9	17·1	0·260	0·257	R.				
$PbB_2\Theta_4$	292·8	27·8	0·0949	0·0905	R.				
$PbB_4\Theta_7$	362·6	45·2	0·124	0·114	R.				
$K_2B_4\Theta_7$	233·8	51·6	0·221	0·220	R.				
$Na_2B_4\Theta_7$ . . .	201·6	51·6	0·256	0·238	R.	0·229	Kp.		
$Na_2B_4\Theta_7+10H_2\Theta$	381·6	137·6	0·366	0·385	Kp.				
$PbMo\Theta_4$	367	28·8	0·0785	0·0827	Kp.				
$CaW\Theta_4$	288	28·8	0·100	0·0967	Kp.				
$Fe_{\frac{2}{5}}Mn_{\frac{3}{5}}W\Theta_4$ . .	303·4	28·8	0·0949	0·0978	R.	0·0930	Kp.		
$PbCr\Theta_4$	323·2	28·8	0·0891	0·0900	Kp.				
$K_2Cr\Theta_4$	194·4	35·2	0·181	0·185	R.	0·189	Kp.		
$K_2Cr_2\Theta_7$	294·6	53·6	0·182	0·189	R.	0·186	Kp.		
$KHS\Theta_4$	136·1	30·1	0·221	0·244	Kp.				
$K_2S\Theta_4$	174·2	34·2	0·196	0·190	R.	0·196	Kp.		
$Na_2S\Theta_4$	142	34·2	0·241	0·231	R.	0·227	Kp.		
$N_2H_8S\Theta_4$ . . .	132	52·6	0·398	0·350	Kp.				
$BaS\Theta_4$	233	27·8	0·119	0·109	N.	0·113	R.	0·108	Kp.
$CaS\Theta_4$	136	27·8	0·204	0·197	R.	0·185	N.	0·178	Kp.
$CuS\Theta_4$	159·4	27·8	0·174	0·184	Pp.				
$MgS\Theta_4$	120	27·8	0·232	0·222	R.	0·225	Pp.		
$MnS\Theta_4$	151	27·8	0·184	0·182	Pp.				
$PbS\Theta_4$	303	27·8	0·0917	0·0872	R.	0·0848	N.	0·0827	Kp.
$SrS\Theta_4$	183·6	27·8	0·151	0·143	R.	0·136	N.	0·135	Kp.
$ZnS\Theta_4$	161·2	27·8	0·172	0·174	Pp.				
$CuS\Theta_4+H_2\Theta$ . .	177·4	36·4	0·205	0·202	Pp.				
$MgS\Theta_4+H_2\Theta$ . .	138	36·4	0·264	0·264	Pp.				
$ZnS\Theta_4+H_2\Theta$ . .	179·2	36·4	0·203	0·202	Pp.				
$CaS\Theta_4+2H_2\Theta$.	172	45·0	0·262	0·273	N.	0·259	Kp.		
$CuS\Theta_4+2H_2\Theta$.	195·4	45·0	0·230	0·212	Pp.				
$ZnS\Theta_4+2H_2\Theta$.	197·2	45·0	0·228	0·224	Pp.				
$FeS\Theta_4+3H_2\Theta$.	206	53·6	0·260	0·247	Pp.				
$CuS\Theta_4+5H_2\Theta$.	249·4	70·8	0·284	0·285	Kp.	0·316	Pp.		
$MnS\Theta_4+5H_2\Theta$.	241	70·8	0·294	0·323	Kp.	0·338	Pp.		
$NiS\Theta_4+6H_2\Theta$.	262·8	79·4	0·302	0·313	Kp.				
$CoS\Theta_4+7H_2\Theta$.	280·8	88·0	0·313	0·343	Kp.				
$FeS\Theta_4+7H_2\Theta$.	278	88·0	0·317	0·346	Kp.	0·356	Pp.		
$MgS\Theta_4+7H_2\Theta$.	246	88·0	0·358	0·362	Kp.	0·407	Pp.		
$NiS\Theta_4+7H_2\Theta$.	280·8	88·0	0·313	0·341	Pp.				
$ZnS\Theta_4+7H_2\Theta$.	287·2	88·0	0·306	0·347	Kp.	0·328	Pp.		
$MgK_2S_2\Theta_8+6H_2\Theta$	402·2	113·6	0·282	0·264	Kp.				
$NiK_2S_2\Theta_8+6H_2\Theta$	437	113·6	0·260	0·245	Kp.				
$ZnK_2S_2\Theta_8+6H_2\Theta$	443·4	113·6	0·256	0·270	Kp.				
$Al_2K_2S_4\Theta_{16}+24H_2\Theta$	949	317·6	0·335	0·371	Kp.				
$Cr_2K_2S_4\Theta_{16}+24H_2\Theta$	998·6	317·6	0·318	0·324	Kp.				

109. *Arseniates, Phosphates, Pyrophosphates and Metaphosphates, Nitrates, Chlorates, Perchlorates, and Permanganates.* (Compare § 88).

	Atomic weight.	Atomic heat. Calculated.	Specific heat. Calculated.	Specific heat. Observed.			
$\mathrm{KAs}\Theta_3$	162·1	24·8	0·153	0·156	R.		
$\mathrm{KH_2As}\Theta_4$ . . .	180·1	33·4	0·185	0·175	Kp.		
$\text{Ᵽb}_3\mathrm{As}_2\Theta_8$ . . .	899	64·0	0·0712	0·0728	R.		
$\mathrm{Ag_3P}\Theta_4$	419	40·6	0·0969	0·0896 ?	Kp.		
$\mathrm{KH_2}\text{Ᵽ}\Theta_4$ . . .	136·1	32·4	0·238	0·208	Kp.		
$\mathrm{Na_2HP}\Theta_4+12\mathrm{H_2}\Theta$	358	139·7	0·390	0·408	Pr.		
$\text{Ᵽb}_3\mathrm{P}_2\Theta_8$	811	62·0	0·0764	0·0798	R.		
$\mathrm{K_4P_2}\Theta_7$	330·4	64·4	0·195	0·191	R.		
$\mathrm{Na_4P_2}\Theta_7$	266	64·4	0·242	0·228	R.		
$\text{Ᵽb}_2\mathrm{P}_2\Theta_7$	588	51·6	0·0878	0·0821	R.		
$\mathrm{NaP}\Theta_3$	102	23·8	0·233	0·217	Kp.		
$\text{Ȼa}\mathrm{P}_2\Theta_6$	198	41·2	0·208	0·199	R.		
$\mathrm{AgN}\Theta_3$	170	24·8	0·146	0·144	R.		
$\mathrm{KN}\Theta_3$	101·1	24·8	0·245	0·239	R.	0·230	Kp.
$\mathrm{K_{\frac{1}{2}}Na_{\frac{1}{2}}NO_3}$ . . .	93	24·8	0·267	0·235	Pr.		
$\mathrm{NaNO_3}$	85	24·8	0·292	0·278	R.	0·257	Kp.
$\mathrm{N_2H_4O_3}$	80	34·0	0·425	0·455	Kp.		
$\text{Ƀa}\mathrm{N}_2\Theta_6$	261	43·2	0·166	0·152	R.	0·145	Kp.
$\text{Ᵽb}\mathrm{N}_2\Theta_6$	331	43·2	0·130	0·110	Kp.		
$\mathrm{SrN_2}\Theta_6$	211·6	43·2	0·204	0·181	Kp.		
$\mathrm{KClO_3}$	122·6	24·8	0·202	0·210	R.	0·194	Kp.
$\text{Ƀa}\mathrm{Cl_2O_6+H_2O}$.	322	51·8	0·161	0·157	Kp.		
$\mathrm{KClO_4}$	138·6	28·8	0·208	0·190	Kp.		
$\mathrm{KMnO_4}$	158·1	28·8	0·182	0·179	Kp.		

110. *Organic Compounds.* (Compare § 89).

		Atomic weight.	Atomic heat. Calculated.	Specific heat. Calculated.	Specific heat. Observed.	
Cyanide of mercury	$\text{Ħg}\,\text{Ȼ}_2\mathrm{N}_2$	252	22·8	0·091	0·100	Kp.
,, zinc and potassium . .	$\text{Ƶn}\,\mathrm{K}_2\text{Ȼ}_4\mathrm{N}_4$	247·4	52·0	0·210	0·241	Kp.
Ferrocyanide of potassium . . .	$\text{Ꞙe}\,\mathrm{K}_3\text{Ȼ}_6\mathrm{N}_6$	329·3	74·8	0·227	0·233	Kp.
Ferricyanide of potassium . . .	$\text{Ꞙe}_4\mathrm{K}_4\text{Ȼ}_6\mathrm{N}_6+3\mathrm{H}_2\Theta$.	422·4	107·0	0·253	0·280	Kp.
Chloride of carbon	$\text{Ȼ}_2\mathrm{Cl}_6$	237	42·0	0·177	0·178	Kp.
Napthaline . .	$\text{Ȼ}_{10}\mathrm{H}_8$	128	36·4	0·284	0·310	A.
Cerotic acid . . .	$\text{Ȼ}_{27}\mathrm{H}_{54}\Theta_2$	410	108·8	0·441	0·429	Pr.
Palmitate of melissyle	$\text{Ȼ}_{46}\mathrm{H}_{92}\Theta_2$	676	302·4	0·447		
Cane-sugar . . .	$\text{Ȼ}_{12}\mathrm{H}_{22}\Theta_{11}$	342	116·2	0·340	0·301	Kp.
Mannite	$\text{Ȼ}_6\mathrm{H}_{14}\Theta_6$	182	67·0	0·368	0·324	Kp.
Succinic acid . .	$\text{Ȼ}_4\mathrm{H}_6\Theta_2$	118	37·0	0·314	0·313	Kp.
Tartaric acid . .	$\text{Ȼ}_4\mathrm{H}_6\Theta_6$	150	45·0	0·300	0·288	Kp.
Racemic acid . .	$\text{Ȼ}_2\mathrm{H}_6\Theta_6+\mathrm{H}_2\Theta$	168	53·6	0·319	0·319	Kp.
Formiate of baryta	$\text{Ȼ}_2\mathrm{H}_2\text{Ƀa}\Theta_4$	227	30·6	0·135	0·143	Kp.
Oxalate of potass	$\text{Ȼ}_2\mathrm{K}_2\Theta_4+\mathrm{H}_2\Theta$ . . .	184·2	41·0	0·223	0·236	Kp.

		Atomic weight.	Atomic heat. Calculated.	Specific heat. Calculated.	Specific heat. Observed.	
Quadroxalate of potass	$Ȼ_2 H_3 K Ɵ_8 + 2 H Ɵ$ . .	254·1	69·7	0·274	0·283	Kp.
Bitartrate of potass	$Ȼ_4 H_5 K Ɵ_6$	188·1	49·1	0·261	0·257	Kp.
Seignette salt . .	$Ȼ_4 H_4 Na K Ɵ_6 + 4 H_2 Ɵ$	282·1	87·6	0·311	0·328	Kp.
Bimalate of potass .	$Ȼ_8 H_{10} Ȼa Ɵ_{10} + 8 H_2 Ɵ$.	450	152·6	0·339	0·338	Kp.

111. The preceding synopsis shows, for the great majority of substances contained in it, an adequate agreement between the observed specific heats and those calculated on such simple assumptions. In estimating the differences, the extent must be remembered to which various observers differ for the same substance. It must be considered that the present better determinations of the specific heat, even those made by the same experimenter, for substances where it may be expected that NEUMANN'S law applies, do not exactly agree with it, not more nearly than within $\frac{1}{10}$ or $\frac{1}{9}$ of the value; and that for those elements which are considered here as obeying DULONG and PETIT'S law, even greater deviations occur between the numbers found experimentally and those to be expected on the assumption of the universal validity of this law. (These deviations, *i. e.* the differences between the atomic heats found for these elements, are seen from § 82.) The extent to which the experimentally determined specific heats deviate from such a law, NEUMANN'S for instance, in bodies for which calculation takes it as applying, gives of course the means of judging what differences may occur between the observed and calculated numbers without invalidating the admissibility of the calculation attempted. And it is as much a matter of course that, in those bodies in which a marked deviation from NEUMANN'S law has been already mentioned (compare § 95), a greater difference is found in the present synopsis between calculation and observation.

I consider the agreement between calculation and observation, as shown in the synopsis § 103 to 110, as in general sufficient for a first attempt of that kind. But it need scarcely be mentioned that I by no means consider the calculated as more accurate than the observed numbers, or among several numbers consider that the most accurate which is nearest the calculated; for that, the bases of calculation are much too uncertain. The list of atomic heats given at the commencement of § 103 is scarcely much more accurate than were the first tables of atomic weights; but just as the latter have experienced continual improvements, and thus what was at first only an approximate agreement between the calculated and observed composition of bodies has been brought within considerably narrower limits, and apparent exceptions been explained, so, in like manner, will this be the case for ascertaining what atomic heats are to be assigned to the elements, and how the atomic heats of compounds may be deduced therefrom. This much, however, may even now be said, that while formerly for many solid substances a statement of the specific heat could in no way be controlled, a concealed source of error for the determination of this property was not indicated, and an error which materially altered the number for this property could not be recognized, at present, even if only roughly, such a control is possible. Compare § 77.

PART VI.—CONSIDERATIONS ON THE NATURE OF THE CHEMICAL ELEMENTS.

112. The proof given in the preceding that DULONG and PETIT'S law is not universally valid, justifies certain conclusions, in reference to the nature of the so-called chemical elements, which may here be developed.

What bodies are to be regarded as chemical elements? Does the mere fact of indecomposability determine this? or may a body be indecomposable in point of fact and yet from reasons of analogy be regarded not as an element but as a compound? The history of chemistry furnishes numerous examples of cases in which sometimes one and sometimes another mode of view led to results which at present are regarded as accurate. The earths were in 1789 indecomposable in point of fact, when LAVOISIER expressed the opinion that they were compounds, oxides of unknown metals. LAVOISIER'S argumentation was based on the fact that the earths enter as bases into salts, and that it was to be assumed in regard to all salts, that they contained an oxygen acid and an oxygen base. But the view, founded on the same basis, that common salt contains oxygen, and the subsequent view that what is now called chlorine contained a further quantity of oxygen besides the elements of an oxygen acid, did not find an equally permanent recognition. On the basis of the actual indecomposability of chlorine, DAVY maintained from about 1810 its elementary character; and this view has become general, especially since BERZELIUS, after a long struggle against it, adopted it, more I think because he was outvoted than because he was convinced.

Almost all chemists of the present time consider chlorine, and in conformity therewith bromine and iodine, as elementary bodies; but the persistence is known with which SCHÖNBEIN attacks this view, and adheres to the opinion that these bodies are oxygen compounds, peroxides of unknown elements. Is there anything which enables us to decide with more certainty on the elementary nature of chlorine and the analogous bodies than has hitherto been the case?

No one can maintain that the bodies which chemists regard as elements are absolutely simple substances. The possibility must be confessed that they may be decomposed into still simpler bodies; how far a body is to be regarded as an element is so far relative, that it depends on the development of the means of decomposition which practical chemistry has at its disposal, and on the trustworthiness of the conclusions which theoretical chemistry can deduce. A discussion as to whether chlorine or iodine is an elementary body can only be taken in the sense whether chlorine is as simple a body as oxygen or manganese, or nitrogen; or whether it is a compound body, as peroxide of manganese or peroxide of hydrogen for example.

If DULONG and PETIT'S law were universally valid, it would not merely indicate for chemical elements a relation between the atomic weight and the specific heat in the solid state, but it could be used as a test for the elementary nature of a body whose atomic weight is known. That iodine, from a direct determination of specific heat, and chlorine by an indirect determination had atomic heats agreeing with DULONG and

PETIT'S law, would be a proof that iodine and chlorine, if compounds at all, are not more so than other so-called elements for which this law is regarded as valid.

According to NEUMANN'S law, compounds of analogous atomic composition have approximately the same atomic heats. In general, bodies, whose atom consists of a greater number of indecomposable atoms, or is of more complicated composition, have greater atomic heats. In these compounds, more especially those whose elements all follow DULONG and PETIT'S law, magnitude of atomic heat is exactly a measure of the complexity or of the degree of composition (compare § 93). If DULONG and PETIT'S law were valid, it could be concluded with great positiveness that the so-called elements, if they are compounds of unknown and simpler substances, are compounds of the same order. It would be a remarkable result that the act of chemical decomposition had everywhere found its limit at such bodies as those which, if compound at all, have with every difference of chemical deportment the same degree of composition. Imagine the simplest bodies, probably as yet unknown to us, the true chemical elements, forming a horizontal spreading layer, and piled above them, the simpler and then the more complicated compounds; the universal validity of DULONG and PETIT'S law would include the proof, that all elements at present assumed by chemists lay in the same layer, and that chemistry in recognizing hydrogen, oxygen, sulphur, chlorine, and the different metals as indecomposable bodies, had penetrated to the same depth in that field of inquiry, and had found at the same depth the limit to its penetration.

This result I formerly propounded[*] when I still believed in the validity of DULONG and PETIT'S law. But with the proof that this law is not universally true, the conclusion to which this result leads loses its justification. Starting now from the elements recognized in chemistry, we must rather admit that the magnitude of the atomic heat of a body depends not only on the number of elementary atoms contained in one atom of it, or on the complexity of the composition, but also on the atomic heat of the elementary atoms entering into its composition; it appears now possible that a decomposable body may have the same atomic heat as an indecomposable one.

To assume in chlorine the presence of oxygen, and to consider it as analogous to peroxide of manganese, or in general to the peroxide of a biatomic element[†], is less in accordance with what is at present considered true in chemistry, than to consider it as the peroxide of a monoequivalent element, analogous to peroxide of hydrogen. It is remarkable that peroxide of hydrogen, in the solid state or in solid compounds, must have almost as great an atomic heat (for $H\Theta$ $2{\cdot}3+4=6{\cdot}3$) as those elements which obey DULONG and PETIT'S law, and especially as iodine, bromine, and chlorine, according to the direct and to the indirect determination of their atomic heat; the same must be the case for the analogous peroxides of such still unknown elements as have an atomic heat

---

[*] "On the Difference of Matter from the Empirical point of view," an Academical Discourse. Giessen, 1860.

[†] I will not omit to mention that equivalent weights of iodine and peroxide of manganese have almost equal capacity for heat. As regards oxidizing action, 127 of iodine corresponds to 43·5 peroxide of manganese; REGNAULT found the specific heat of the former $=0{\cdot}0541$; I found that of the latter $=0{\cdot}159$;

$$127\times0{\cdot}0541=6{\cdot}87;\ 43{\cdot}5\times0{\cdot}159=6{\cdot}92.$$

as great as that of hydrogen. As far as may be judged from its specific heat, chlorine *may* be such a peroxide; but this consideration shows no necessity for assuming that it actually is so.

In a great number of cases the atomic heat of compounds gives more or less accurately a measure for the degree of complexity of their composition*. And this is the case also with such compounds as are comparable in their chemical deportment to undecomposed bodies. If cyanogen or ammonium had not been decomposed, or could not be so with the means at present offered by chemistry, the greater atomic heats of their compounds, compared with those of analogous chlorine or potassium compounds (compare § 96), and of cyanogen and ammonium as compared with chlorine and potassium, would indicate the more complex nature of those so-called compound radicals. The conclusion appears admissible that for the so-called elements the directly or indirectly ascertained atomic heats are a measure for the complexity of their composition. Carbon and hydrogen, for example, if not themselves simple bodies, are more so than silicium or oxygen; and still more complex compounds are the elements which are now considered as following DULONG and PETIT's law; with the restriction, however, that for these also the atomic heats may be more accurately determined and differences proved in them which justify similar conclusions†. One might be tempted, by comparing atomic heats, to form an idea how the more complex of the present indecomposable bodies might be composed of more simple ones, just as such a comparison has been shown to be possible for chlorine; but it is at once seen that to carry out such an attempt the atomic heats of the elements, especially those which can only be indirectly determined, are not settled with adequate certainty.

It may appear surprising, or even improbable, that so-called elements which can replace each other in compounds, as, for instance, hydrogen and the metals, or which enter into compounds as isomorphous constituents, like silicium and tin, should possess unequal atomic heats and unequal complexity of composition. But this is not more surprising than that indecomposable bodies, and those which can be proved to be compound, as, for example, hydrogen and hyponitric acid, or potassium and ammonium, should replace one another, preserving the chemical character of the compounds, and even be contained as corresponding constituents in isomorphous compounds.

I have here expressed suppositions in reference to the nature of the so-called elements which appear to me based on trustworthy conclusions from well-proved principles. It is in the nature of the case that the certain basis of fact and of what can be empirically demonstrated must be left. It must also not be forgotten that these conclusions only allow something to be supposed as to which of the present indecomposable bodies are more complex and which of simpler composition, and nothing as to the question what simpler substances may be contained in the more complex ones. The consideration of the atomic heats may say something as to the structure of a compound atom, but in general gives no clue as to the qualitative nature of the simpler substances used in the construction of the more complex atoms. But even if these suppositions are not free from uncertainty and imperfection, they appear worthy of attention in a subject which, for science, is still so much in darkness, as is the nature of the indecomposable bodies.

---

* The differences in the atomic heats of the elements are of course most distinctly seen in their free state, but in their analogous compounds these differences are the less prominent the more complex the compounds, that is, the greater the number of atoms of the same kind and the same atomic heat which are united to those elementary atoms whose atomic heat is assumed to be unequal. The difference in the atomic heats of C and As, for instance (1·8 and 6·4), is relatively far greater than for $\mathrm{Ca\,C\,O_3}$ and $\mathrm{K\,As\,O_3}$ (20·2 and 24·8).

† It is possible, for example, that certain indecomposable bodies which only approximately obey DULONG and PETIT's law, are analogous compounds of simpler substances of essentially different atomic heat: the approximate agreement of the atomic heats of such indecomposable bodies would then depend on a similar reason to that for the atomic heats of $\mathrm{Ca\,C\,O_3}$ and $\mathrm{K\,As\,O_3}$. Compare the previous note.

IV. *On the Composition of Sea-water in the different parts of the Ocean.* By GEORG FORCHHAMMER, *Professor at the University, and Director of the Polytechnic Institution at Copenhagen. Communicated by the President.*

Received July 28,—Read November 17, 1864.

IN the year 1843 a friend of mine, Mr. ENNIS of Falmouth, sent me some bottles of sea-water from the Mediterranean, which I subjected to a chemical examination, a work which induced me to collect what other chemists had determined about the constitution of the water of the great Ocean. This labour convinced me that our knowledge of the composition of sea-water was very deficient, and that we knew very little about the differences in composition which occur in different parts of the sea.

I entered into this labour more as a geologist than as a chemist, wishing principally to find facts which could serve as a basis for the explanation of those effects that have taken place at the formation of those voluminous beds which once were deposited at the bottom of the ocean. I thought that it was absolutely necessary to know with precision the composition of the water of the present ocean, in order to form an opinion about the action of that ocean from which the mountain limestone, the oolite and the chalk with its flint have been deposited, in the same way as it has been of the most material influence upon science to know the chemical actions of the present volcanos, in order to determine the causes which have acted in forming the older plutonic and many of the metamorphic rocks. Thus I determined to undertake a series of investigations upon the composition of the water of the ocean, and of its large inlets and bays, and ever since that time I have assiduously collected and analyzed water from the different parts of the sea. It is evident that it was impossible to collect this material in a short time, and without the assistance of many friends of science, and I most gratefully acknowledge how much I am indebted to many distinguished officers of the Danish and British Navy, as well as to many private men, who were all willing to undertake the trouble carefully to collect samples of sea-water from different parts of the ocean, both from the surface and from different depths. I shall afterwards, when giving the particular analyses, find an opportunity to mention the name of each of those to whom I am indebted for my material.

While I was thus occupied for a space of about twenty years, another series of experiments closely allied to my work was commenced in England, and has partly been published under the able and scientific superintendence of Rear-Admiral FITZROY. This most important series of observations regards the specific gravity of sea-water from the most different parts of the globe; it comprehends a much more numerous series than my observations, but I trust that it will not make my work superfluous, but that both these investigations will supplement each other. By the kindness of Admiral FITZROY I am able to compare the instruments which are used by the British Navy with my chemical analyses, and thus to obtain a comparison between both series.

I have at different times found an opportunity to publish several parts of my observations, and in 1859 I collected what had been done up to that time in an academical treatise in the Danish language*. Since that time I have obtained numerous samples of sea-water, principally from places which my previous examination had not reached. In this new form, and greatly augmented by new facts, I permit myself to lay it before the illustrious scientific society of a nation to whose navigators I owe so great a part of the material for my inquiries. This part contains an enumeration of the elements which hitherto have been ascertained to exist in the water of the ocean, and an explanation of the methods used to show their presence and to determine their quantity. It contains a determination as complete as possible of the distribution of the saline substances at the surface of the different parts of the sea, and in the different depths at the same place.

On the Elements which occur in the Water of the Ocean.

The elements which occur in greatest quantity in sea-water have been long known, and chlorine, sulphuric acid, soda, magnesia, and lime have for more than a century past been considered as its essential parts. In our century iodine, bromine, potash, silica, phosphoric acid, and iron have been discovered in sea-water, and the latest inquiries, my own included, have brought the number of elements occurring in sea-water up to twenty-seven.

Next to direct analyses of sea-water, the analysis of sea-weeds, and of animals living in the sea, offers us precious means of determining those elements which occur in so small a quantity in sea-water, that it hitherto has been impossible to ascertain their presence in the water by chemical tests. It is now well known that the organic beings collect substances which are necessary for their existence, and thus offer the means to the chemist of ascertaining that these substances were present in the medium in which the organisms lived, and from which they collected their food. As to the plants of the sea, the whole fucoid tribe derive the substances of which they consist from the surrounding sea-water and from the air with which they are in contact, but not from the soil on the bottom of the sea, since that part of them which generally is called their root is no root at all, and is not qualified to extract food from the soil and stones to which it adheres. Even those marine plants which do not belong to the fucoid tribe, as, for instance, the *Zostera marina*, and which have a real root, that may extract food from the soil, will most probably extract the great quantity of mineral elements which they contain mostly from the surrounding sea-water. As to the animals that live in the sea, they derive their substance either from the sea-water itself, or from plants that are nourished by sea-water, or from other animals that live upon sea-weeds, thus deriving their whole mineral substance either directly or indirectly from the sea. I have availed myself of the means which the organisms of the sea furnish, to determine a great number of elements that thus must exist in solution in sea-water.

---

* Om Söevandets bestanddele og deras Fordeling: Havet. af G. FORCHHAMMER, Professor ved Kjøbenhavns Universitet.

As to this great number of elements contained in the sea-water, we might ask one question, which is of great importance for the history of the earth, viz. how all these elements got into the sea, whether they were in the original sea, or subsequently got into the sea, where they are now slowly accumulating. When we consider that the sea constantly loses a great quantity of pure water by evaporation, and that a large part of this water falls on the land, dissolves a number of substances from it, and carries them at last into the sea, where they constantly would increase in quantity if it were not for its organisms which deprive it again of them, we may well suppose that these two effects, of which the one acts to increase, and the other to diminish the quantity of mineral substances in sea-water, are pretty equal, and leave the sea unchanged. I will, however, not dwell upon these mutual chemical decompositions and combinations, which, partly depending upon organic life, partly upon inorganic mechanical and chemical forces, play such a great part in the changes of the earth, but I hope at some future time to find leisure to publish my investigations in this branch of the history of the earth.

The elements which hitherto have been found in sea-water are,—

1. *Oxygen.*—Besides that oxygen which is a constituent part of water, and other compounds that occur in the sea, such as the sulphates, phosphates, carbonates, and silicates, it occurs in a free uncombined state, absorbed by the water itself. It plays a very material part in the small but constant changes which take place in the sea-water, and whose general effects are that the organic substances dissolved in it are changed into carbonic acid and water. This effect takes place principally near the surface, and decreases with increasing depth; and water from the deeper parts of the sea is able to destroy the colour of a greater quantity of the hypermanganate of potash than that from the surface, which again shows that there is more organic matter undestroyed in the deep sea.

2. *Hydrogen.*—Besides the hydrogen which belongs to the composition of water, it occurs in the organic substances and in the ammonia which are dissolved in sea-water.

3. *Chlorine.*—Next to the elements of water chlorine is the element which occurs in greatest quantity in sea-water, and has from the earliest times been recognized as such.

4. *Bromine* has been long known as an essential part of the sea, easily recognized in the residue from the evaporation of sea-water after the crystallization of the greater part of the chloride of sodium.

5. *Iodine.*—This substance is well known to have been the first element in sea-water discovered not directly, but by the analysis of the ashes of fucoidal plants, which by organic power had collected and concentrated it from sea-water.

6. *Fluorine.*—Dana long ago showed that fluorine occurs in the lime of corals, where

its presence may be ascertained with great facility. To prove directly its existence in sea-water, I evaporated 100 lbs. of it taken in the Sound near Copenhagen, and when it was so much condensed that the salt began to crystallize, I precipitated the whole by an excess of ammonia, washed the precipitate, and dissolved in muriatic acid. It was now again precipitated by ammonia, and the precipitate boiled with a solution of muriate of ammonia. The washed precipitate weighed now 3·104 English grains, and was divided into two parts, of which one was heated in a small platinum crucible with sulphuric acid. The vapours etched glass. The other part was distilled in a bent glass tube with sulphuric acid, and the vapour condensed in a solution of ammonia. The vapours etched the glass tube, and when the ammoniacal liquor was evaporated and the salt dissolved, silica remained. With much greater facility the fluorine was shown in the stony matter deposited at the bottom of the boilers of the Transatlantic steamers, of which I owe samples to the late Dr. G. WILSON of Edinburgh, who likewise discovered fluorine in sea-water.

7. *Sulphur.*—This element occurs in considerable quantity in sea-water combined with oxygen as sulphuric acid, forming salts with baryta, strontia, lime, and magnesia. In pure sea-water, or in such sea-water as only contains a very small quantity of organic matter, no decomposition of the sulphates takes place, and I have kept sea-water for many years in well-corked bottles without the least alteration. Near the shores and at the mouth of great rivers, where considerable quantities of organic matter are washed into the sea, it is easily decomposed, particularly if it is kept in bottles. This decomposition shows itself always by the production of sulphuretted hydrogen. Water from the polar regions is very subject to decomposition, probably on account of a greater quantity of organic matter than in water from lower latitudes. It is, however, very difficult to assign all the different causes which may produce decomposition of sea-water. All the water which was brought by the Swedish Spitzbergen Expedition in bottles from the polar sea was decomposed, and emitted sulphuretted hydrogen when the bottles were opened, while all the water brought from the same sea by the same Expedition in tubes of glass, hermetically closed by melting, was undecomposed. Hypermanganate of potash is the best test for the sulphuretted hydrogen of such water, its colour is instantaneously destroyed by the water, and sulphuric acid is formed again. The quantity of sulphuretted hydrogen formed in such water differs greatly, and depends, at least partly, upon the quantity of organic matter contained in it. Water from the Mediterranean is very subject to this kind of decomposition; but the greatest quantity of sulphuretted hydrogen which I have met with in any sample was found in water which I owe to Admiral WASHINGTON, and which had been taken by Captain PREVOST of the 'Satellite', under 35° 46′ S. lat. and 52° 57′ W. long., off the east coast of South America, and not very far from the mouth of the Rio de la Plata; 3000 grains of this water destroyed the colour of 455 drops of a solution of hypermanganate of potash, of which the same quantity of ordinary sea-water only bleaches four to six drops[*].

---

[*] This test has only a relative value in comparing different kinds of water, the quantity of oxygen required for complete oxidation being proportional to the quantity of hypermanganate destroyed.

In this kind of decomposition, where sulphuretted hydrogen is formed, the organic matter is changed into carbonic acid and water, while the oxygen which this change requires is taken from the sulphates, and the sulphuret thus formed takes its oxygen again from the hypermanganate. Thus the result of the series of decompositions is the revival of the same sulphate with which it began, and the formation of carbonic acid and water from the organic matter which was present. In the second case, where the hypermanganate directly oxidizes the organic matter, the same quantity of oxygen must be used, and the same products are obtained. In both cases the oxygen is ultimately derived from the hypermanganate. This reasoning supposes that no oxygen from the atmosphere is absorbed, and no sulphuretted hydrogen has escaped during the operations. The absorption of oxygen is prevented by the cork of the bottle, but when it is opened some sulphuretted hydrogen certainly will escape, and we may conclude that in the cases where sulphuretted hydrogen is formed, there has been a little more organic matter than the hypermanganate indicates.

This fermentation of the sea-water occasions of course a loss of sulphuric acid, and makes the analysis in some degree inaccurate. The greatest loss of sulphuric acid which I have observed was in the case of the water from the 'Satellite' above mentioned, where the proportion to chlorine was found to be 9·13 : 100, while the mean proportion is 11·94 : 100, thus about one-seventh of the sulphuric acid was decomposed. It is very probable that this great quantity of organic matter is owing to the water of the Rio de la Plata, because the water contained only 17·721 chlorine, while the mean number for that region is 19·376, which seems to prove a considerable admixture of river-water. I may here also mention a curious instance where no decomposition had taken place, although the circumstances seemed to be very favourable for it. The sample had been taken by the late Sir JAMES ROSS in 1841, at 77° 32′ S. lat., in the neighbourhood of the great ice-barrier, and it was marked "Sea-water containing animalculæ." It was very muddy when I opened the bottle, but had not the least smell of sulphuretted hydrogen. Tested without being filtered, 1000 grains bleached 180 drops of the hypermanganate; when filtered the same quantity bleached 39 drops. It contained thus a great quantity of organic matter. The quantity of chlorine was 15·748, which proves that it was much diluted, probably by the melted ice from the barrier; the proportion of sulphuric acid to chlorine was 11·65 : 100, which approaches pretty near to the normal proportion. It had been about twenty years in the bottle when I analyzed it, and the cork was sound. It is difficult to conceive why this water had not suffered any decomposition.

8. *Phosphorus.*—This element, in combination with oxygen, is a never failing part of sea-water, which remains as phosphate of lime when the water is evaporated to dryness and the salts remaining dissolved in boiling water. The small quantity of insoluble matter which remains consists of phosphate of lime, sulphates of baryta and of strontia, fluoride of calcium, carbonate of lime, and silica. When this mixed substance is heated with muriatic acid, filtered, and tested with molybdate of ammonia, phosphoric acid will

always be found; or when the insoluble remainder from evaporation is heated in a glass tube with potassium, it will, when breathed upon, emit the smell of phosphuretted hydrogen.

9. *Nitrogen* occurs in sea-water combined with hydrogen as ammonia, and its presence may be shown by mixing sea-water with a solution of baryta, and distilling the mixture in a glass retort. In the distilled portion ammonia may be shown by adding some drops of nitrate of protoxide of mercury, which will form grey clouds, or by muriatic acid and chloride of platinum, which, when carefully evaporated, will leave the well-known yellow salt insoluble in alcohol. It can hardly be doubted that this ammonia is partly formed by the living animals of the sea, which exhale ammonia, and partly by the putrefaction of their dead bodies. We might ask why we find so small a quantity of ammonia, the causes for its formation being so general; but it is well known that plants will absorb it, and that the circulation of nitrogen in the sea is between sea-water, plants, and animals, as it is on the dry land between soil, plants, and animals.

10. *Carbon* occurs always in the water of the sea, partly as free carbonic acid, partly, but in very small quantities, as carbonate of lime, partly in combination with oxygen, hydrogen, and nitrogen as organic matter, derived from the destruction of the numerous organic beings that live in the sea. It is by the oxidation of these substances that the sulphates of sea-water are decomposed, and that the hypermanganate of potash is bleached when boiled with sea-water; and it is owing to these substances that all sea-water disoxidizes the peroxide of iron either to protoxide or to sulphuret, and that all ferruginous clay or sand deposited in deep sea has a dark colour.

11. *Silicium.*—Silica is found in the insoluble remainder from the evaporation of sea-water when the salts are dissolved in water. It can be separated from the phosphates and fluorides by dissolving in weak muriatic acid, when it remains undissolved along with small quantities of sulphate of baryta and strontia. In this state it is easily recognized by the blowpipe. In the Sponges it is collected in great quantity; and when the large cyathiform sponge from Singapore is calcined, it leaves a skeleton which retains the original form and size of the sponge, and consists almost entirely of silica, the large pores of it being lined with oxide of iron, which evidently has belonged to some part of the animal itself. It is found also in other animals of the sea, and it occurs in the ashes of sea-weeds of the fucoid family, though it is not yet ascertained whether it belongs to the fucus itself, or to the infusoria which usually cover its surface.

12. *Boron.*—I have long tried to find boracic acid in sea-water, but for a long time all my endeavours were vain. Notwithstanding I felt convinced that it must be there, since both boracic acid and borates are not very rare, and a great part of its salts with lime and magnesia are more or less soluble in water. Thus I thought that water from the land must have carried boracic acid into the sea, where it still must be accumulating, since we do not know any combination by which it could be separated again from the water. An additional proof of the correctness of this idea I found in the occurrence of Stassfurthite (mostly consisting of borate of magnesia), together with all

other salts that occur in sea-water, in the beds of rock-salt at Stassfurth in Germany. The lower part of this bed of rock-salt, which by a boring was not penetrated through at a depth of 800 feet, consists of pure chloride of sodium. Upon this rest the other salts of sea-water, consisting of magnesia, lime, and potash combined with muriatic and sulphuric acids in numerous combinations, among which we also find the Stassfurthite (borate of magnesia with chloride of magnesium). Boracite, a similar combination of boracic acid, occurs at Luneburg and at Segeberg, associated with gypsum and chloride of sodium, which latter at Luneburg forms a spring of saturated brine, and at Segeberg occurs in separate crystals imbedded in the gypsum.

I thought I might be able to form a borate insoluble in water, and with such characteristic properties that it might be possible to determine the boracic acid in it. It is well known that HEINTZ, by melting chloride of magnesium, chloride of sodium, magnesia, and boracic acid, obtained octohedral crystals, which were boracite, and another set of crystals, of hemiprismatic form, which also contained boracic acid and magnesia. The crystals were microscopic, but could easily be recognized by their different form of crystallization. To make myself acquainted with these different artificial combinations, I melted borax, common salt, and sulphate of magnesia in a crucible, allowed it to cool slowly, and dissolved it in water. There remained a heavy crystalline powder, which under the microscope proved to consist of six-sided hemiprismatic prisms, containing both magnesia and boracic acid. I could not discover any octohedral crystal, and no boracite seemed to have been formed. In another experiment I fused common salt, magnesia, and borax; after solution I obtained the same hemiprismatic crystals, but no octohedrons; and felt now convinced that I hardly should obtain boracite by fusing salt of sea-water, but that I might obtain the hemiprismatic borate if sea-water contained boracic acid.

The experiment was made in the following way:—I evaporated 6 lbs. of sea-water taken from the Sound near Copenhagen, transferred the salt into a perfectly clean platinum crucible, which was placed upon magnesia in a common Hessian crucible, exposed it to a white heat, and cooled slowly. After solution of the salt, the powder remaining was placed under the microscope, where it was found to consist almost entirely of hemiprismatic crystals which frequently formed twins, and by their whole exterior showed themselves to be essentially different from the hemiprismatic borate. Many of them were corroded at the sides and ends, as if they had partly been dissolved. I supposed them to be gypsum, which of course must be formed by the evaporation of sea-water; and although the gypsum by melting would be changed into anhydrite, they afterwards, during washing with water, would again form a hydrate. I thought even several times to have seen square prisms (anhydrite?) change into the hemiprismatic form under my observation in the microscope, and get oblique cracks like one cleavage of gypsum. The powder was again washed with hot water, and the solution was found to contain both sulphuric acid and lime. When the wash-water contained only traces of sulphuric acid, the powder, greatly diminished in quantity, was again observed under the microscope, and showed very few half-dissolved prisms of gypsum, but numerous very small octohedrons, which had been hidden by the gypsum. Besides these octohedrons, some hemiprismatic crystals were found, precisely similar to those which I formerly had obtained when forming a borate of magnesia. The powder contained, further, some prisms which were striated parallel to the axis, and had a face perpendicular to this axis; they resembled precisely the crystals which I several years ago described as artificial apatite, and which were obtained by fusing calcined bones with chloride of sodium; and they were in fact apatite, formed of the phosphoric acid, fluorine, chlorine, and lime of the sea-water. Of the powder in question, which essentially consisted of octohedrons, I dissolved 7·184 grains in nitric acid, which left 0·160 grain of a reddish powder consisting mostly of oxide of iron, but showing also under the microscope hemiprismatic crystals like the borate of magnesia. The nitric solution gave with ammonia a precipitate which weighed 0·633, and contained phosphoric acid. At last the remaining solution gave with phosphate of soda and an excess of ammonia 16·667 ignited phosphate of magnesia=6·074 pure magnesia. The sum of all these substances thus determined was 6·867, so that only a quantity amounting to 0·317 grain which was wanting could be boracic acid.

It was thus clear that the octohedrons analyzed could not be boracite, and there could hardly be any doubt but that the substance was essentially pure magnesia, mixed with small quantities of oxide of iron, phosphate of lime, and other substances which were still to be determined. Pure magnesia occurs among the Vesuvian minerals crystallized in regular octohedrons, and has obtained the name of Periclase. In this case the periclase was formed by the decomposition of the hydrate of chloride of magnesium contained in the salt of sea-water, and decomposed in the melting heat. As a further proof of its nature as pure magnesia, it may be mentioned that, when boiled with a solution of sal-ammoniac, it was dissolved with a strong smell of ammonia. The solution contained magnesia, and nothing else besides salts of ammonia could be discovered.

When the octohedral crystals were removed by boiling with a solution of sal-ammoniac, the remaining powder contained only hemiprismatic prisms of the supposed borate of magnesia, crystals of apatite, and very acute six-sided pyramids, which in their form had some similarity to crystals of sapphire, and a considerable quantity of amorphous red oxide of iron, probably mixed with silica. A portion of this powder was moistened with sulphuric acid, and during twenty-four hours left to spontaneous evaporation. I could now observe crystals of sulphate of magnesia and needles of sulphate of lime. The substance, nearly dry, was mixed with diluted alcohol, which, when inflamed, showed the green margin of the flame characteristic of boracic acid, and gave a brown colour to curcuma paper, although the solution was acid. It is thus proved that this salt contained boracic acid, which in this case could only be derived from sea-water. When this powder was boiled with muriatic acid, apatite, borate of magnesia, and silicate of peroxide of iron were dissolved, and a very small quantity of the six-sided pyramids remained, which resisted the action of acids, but were made soluble by fusing with carbonate of soda. When the soda was washed away, the remaining substance dissolved in muriatic acid, and it could now be proved that *alumina* was present. The quantity of these six-sided pyramids obtained from 6 lbs. of sea-water was, however, so small, that no experiments could be made to ascertain whether it contained other substances besides alumina.

I have been somewhat more explicit in relating my experiments to ascertain the existence of boracic acid and alumina in sea-water, partly because I found it very difficult to find unequivocal proofs of their presence, and partly because it interested me highly to find how useful the microscope may be in inorganic analysis, when used in combination with chemical tests.

When I had convinced myself that boracic acid occurred in sea-water, it appeared to me in the highest degree probable that the organisms of the sea would collect it, and that it might be found in their ashes. I was so fortunate as to begin my experiments with a plant that contained it in a rather large quantity, viz. the *Zostera marina.* The plant was collected in the month of December, at the sea-shore near Copenhagen, dried, and burnt. The ashes were washed with water, and the solution, which contained mostly chloride of potassium and sulphate of potash, contained also a small quantity of boracic acid, probably combined with soda. The insoluble part of the ashes was moistened with sulphuric acid until it had a sour taste, evaporated in a moderate heat to dryness, and washed with water. When this solution was mixed with strong alcohol and filtered, it burned with a green flame, and gave to curcuma paper a brown, and to litmus paper a red colour. To separate the boracic acid from the other substances I chose superheated steam, a method to which I was led by a consideration of the way in which boracic acid reaches the lagoons of Tuscany. It is well known that this acid comes with steam from the interior of the earth, and is condensed when escaping from the fumaroles. An experiment in which I mixed dry borax with sulphuric acid, and exposed it to the action of superheated steam at 300° to 400° Centigrade, volatilized not only boracic acid in form of a solution, but gave even the well-known scales of its hydrate. The experiment with the distillation of the ashes of *Zostera marina* with sulphuric acid and superheated steam succeeded completely. The water contained boracic acid, which by a slow evaporation was obtained in crystalline scales; and another portion of it was converted into borax, which was obtained in its regular form. Even *Fucus vesiculosus* contains the same acid, but in a much smaller quantity.

13. *Silver.*—Malaguti first showed that silver occurs in the organisms of the sea; I have subsequently proved it to exist in a coral, a *Pocillopora*, and several chemists have since tried to prove that silver is precipitated by the galvanic current between the copper coating of a vessel and sea-water. If the last determination is confirmed, the existence of silver in sea-water is proved by direct experiment. From the *Pocillopora alcicornis* I have separated it in the following manner:—I dissolved the coral in muriatic acid, precipitated the solution by hydrosulphate of ammonia, and dissolved the precipitate, which consisted of sulphurets, of phosphate of lime, and fluoride of calcium, in

very weak cold muriatic acid, which left the sulphurets of silver, lead, and copper probably mixed with those of cobalt and nickel. These sulphurets were separated from the solution, evaporated to dryness with a little nitric acid, to which were added a few drops of muriatic acid, and dissolved in water, which leaves sulphate of lead and chloride of silver undissolved. When the filter which contained the latter substances is burnt, the silver is reduced to metal; a solution of pure soda will dissolve the sulphate of lead and leave the silver, which, when dissolved in nitric acid, can be tested with muriatic acid. I obtained from *Pocillopora alcicornis* about $\frac{1}{3,000,000}$, or from a solid cubic foot of the coral about half a grain of silver.

14. *Copper* has not been discovered in sea-water itself, but occurs so frequently in the lime-salts of the animals of the sea, and in the ashes of the sea-weeds, that it can be discovered with great facility by its well-known tests. In the *Pocillopora* I found about six times more copper than silver, in the coral *Heteropora abrotanoides* about $\frac{1}{350,000}$ copper, and in the yellowish-green substance which remained after the filtration of the muddy sea-water which Sir JAMES ROSS had taken in 77° 33′ S. lat., it could be shown with great facility. Also the ash of *Fucus vesiculosus* contained copper.

15. *Lead* occurs, like copper, in the shells of the animals of the sea and in the ashes of sea-weeds, but in greater quantity. In the *Pocillopora alcicornis* there was found about eight times as much lead as silver, and in *Heteropora abrotanoides* about $\frac{1}{50,000}$ of the coral. It occurs likewise in *Fucus vesiculosus*.

16. *Zinc.*—It has not been shown directly in sea-water, nor could I find it in the lime-salts of shells and corals, but it occurs in considerable quantities in the ashes of sea-weeds; 400 grains of the ashes of *Zostera marina* contained 0·139 oxide of zinc $=\frac{1}{3000}$. It occurs also in the ashes of *Fucus vesiculosus*.

17. *Cobalt.*—I have discovered this metal in the ashes of *Zostera marina*, and in the fossil sponges of the chalk, but not in the large cyathiform sponge of the present sea from Singapore.

18. *Nickel.*—We have no such delicate test for nickel as the blowpipe is for cobalt, but I have several times observed the well-known brown colour of the solution on precipitating the sulphurets of the ashes of sea-weed by hydrosulphate of ammonia, and I think we are fairly entitled to suppose that these two metals occur together in sea-water as they occur in company in the mineral kingdom.

19. *Iron* can be discovered directly in sea-water by evaporating it to dryness and dissolving the salts again in water, when it remains insoluble and combined with silica. It remains mixed with all the other combinations that are insoluble or difficultly soluble in water, but in the solution of these residues in muriatic acid can easily be indicated by the common prussiate of potash. It occurs in great quantity in the ashes of sea-weeds and the lime-salts of sea animals.

20. *Manganese* can be determined directly in sea-water, accompanying the oxide of iron separated from a rather large quantity of sea-water, by the application of the well-known test for manganese before the blowpipe with carbonate of soda and nitrate

of soda or potash. In some sea-weeds it occurs in considerable quantity, particularly in the ashes of *Zostera marina* when it is in full growth. This ash contains about 4 per cent. of it, enough, when muriatic acid is poured upon the ash, to cause an effervescence of chlorine. Manganese is found in a much smaller quantity in the animals of the sea.

21. *Aluminium.*—I have often tried to find alumina in sea-water which had been filtered, but always without result, until at last, in my experiments to find boracic acid, I found alumina also, as is mentioned under boron. Aluminium must thus be enumerated as one of the elements that occur in the water of the sea. It occurs in greater quantity than most metals, iron, and perhaps manganese, excepted.

22. *Magnesium.*—This element occurs, as is well known, in large quantity in sea-water, in about the same quantity as sulphuric acid, and only sodium and chlorine are found in greater quantity. Sea-weeds contain it likewise in considerable quantity, and it is a constant companion of the carbonate of lime which the shell-fishes and corals deposit. In *Serpula filigrana* it amounts to 13·49 per cent. carbonate of magnesia. Its average quantity is, however, only 1 per cent.

23. *Calcium.*—Lime occurs in sea-water in a small quantity combined with carbonic acid, and dissolved in an excess of it; in a greater quantity combined with phosphoric acid, and as fluoride of calcium; but the greatest quantity is combined with sulphuric acid. Among all the bases which river-water carries into the sea, lime is the most frequent; and it is only owing to the organic beings of the sea, and principally to its lower animals, that so small a quantity remains, lime being constantly separated by the organo-chemical action of these animals.

24. *Strontium.*—I have discovered this element in the sea-water, and also in the deposit of the boilers of the Transatlantic steamers. It occurs likewise in the ashes of the fucoid plants, and specially in the *Fucus vesiculosus.* I shall here explain how I have convinced myself that this plant contains both strontia and baryta. When the ash was successively extracted, first with water, and then with muriatic acid, a rather considerable quantity of insoluble substances remained, which was fused with carbonate of soda, and again extracted by water containing some pure soda to dissolve the silica, while the sulphuric acid from the sulphate of strontia and baryta had combined with the soda of the carbonate. To remove the lime from the remainder, I dissolved it in muriatic acid which contained a little sulphuric acid. What remained undissolved was again fused with carbonate of soda and extracted with water. The remaining carbonates were now dissolved in muriatic acid, and afterwards precipitated by a solution of sulphate of lime. The mixed sulphates of strontia and baryta were separated by fluosilicic acid, and the salt of strontia dissolved in alcohol, which then burned with the beautiful red colour of strontia.

25. *Baryta* occurs both in sea-weeds and in sea-animals, but the ashes of sea-weeds contain more of it than the corals and shells. It can even be determined directly in sea-water, and in the deposits of the boilers of the Transatlantic steamers.

26. *Sodium.*—It is well known that sodium in combination with chlorine forms the most important salt in sea-water; next to chlorine, oxygen, and hydrogen, sodium is the most abundant element in sea-water.

27. *Potassium* is the alkaline element which, next to sodium, occurs most frequently in sea-water, and it may easily be shown in the sea-water itself.

On the Quantitative Analysis of Sea-water.

It is evident that an analysis which should determine the quantity of every one of the substances now enumerated would be a very laborious task, and that the number of analyses required to ascertain the composition of sea-water in different parts of the ocean would be a work exceeding the power of a single observer. Besides this there is another difficulty, which makes a series of such analyses quite impossible; 100 lbs. of sea-water would be the least quantity that could be used, but such a quantity could but with difficulty be procured, and could not be kept unaltered by evaporation and fermentation. Fortunately such analyses are not required, and of the numerous elements discovered in sea-water, only a few occur in such a quantity that their quantitative determination can be of any consequence. It is besides a result of my analyses of sea-water, that the differences which occur in water from different parts of the ocean essentially regard the proportion between all salts and water, the strength of sea-water, or, to use another expression, its *salinity*, and not the proportion of the different elements of the salts *invicem*; in other words, the difference in the proportion between chlorine and water may be very variable, but the proportion between chlorine and sulphuric acid, or lime or magnesia will be found almost invariable. The substances which, in respect of quantity, play the principal part in the constitution of sea-water, are chlorine, sulphuric acid, soda, potash, lime, and magnesia; those which occur in less, but still determinable quantity are silica, phosphoric acid, carbonic acid, and oxide of iron. All the numerous other elements occur in so small a proportion, that they have no influence whatever on the analytical determination of the salinity of sea-water, though, on account of the immense quantity of sea-water, they are by no means indifferent, when we consider the chemical changes of the surface of the earth which the ocean has occasioned, or is still producing.

In my complete quantitative analyses I have always determined the quantity of chlorine, sulphuric acid, magnesia, lime, and potash. The sodium or soda is calculated under the supposition that there were no other metalloids or acids than chlorine or sulphuric acid, and no other bases or oxides of metals than lime, magnesia, potash, and soda; it was supposed, besides, that the sea-water was neutral. These suppositions are not quite correct: of metalloids we find, besides chlorine, bromine, iodine, and fluorine; of acids we find, besides sulphuric acid, also carbonic, boracic, silicic, and phosphoric acids; and of bases we find, besides those that have been enumerated, a great number; but all these substances occur in very small quantities, and may be neglected. I have, however, in most cases determined the quantity of insoluble remainder left when sea-water is evaporated to dryness, dissolved in water, and washed until all sulphate of lime is removed. This remainder contains silica, phosphate of lime, carbonate of lime, sulphate of baryta and strontia, oxide of iron, and probably borate of magnesia or lime, and is in my memorandum of the analysis mentioned under one head, with the designation *Silica, &c.* In those cases where this small remainder was not determined, it was calculated proportionally to the quantity of chlorine. Thus, for instance, water taken in 44° 33′ N. lat. and 42° 54′ W. long. contained, in 1000 parts, chlorine 18·842, and *silica, &c.* 0·069. In water taken in 47° 50′ N. lat. and 33° 50′ W. long., the quantity of chlorine was found to be 19·740, and silica is, according to the former proportion, calculated as 0·072. In this case the silica, &c. was $\frac{1}{273}$ of the quantity of the chlorine, and in general it is less than $\frac{1}{200}$; thus the possible error is utterly unimportant.

I rejected a method often used, which consists in evaporating sea-water to dryness, because it is inaccurate, and the result depends partly upon trifling circumstances. If evaporated by steam of 100° C. there will remain a very notable quantity of water, which quantity can only be ascertained with great difficulty. If it is dried at a higher temperature, muriatic acid from the chloride of magnesium will be driven out together with the water. I preferred thus, as I have already mentioned, to determine the quantity of the five above-named substances, to ascertain under one head all the small quantities of the different substances that remain insoluble in water, such as silica, phosphate of lime, &c., and to calculate the soda. At first I tried to separate the quantity of all the different substances in one portion of sea-water, but soon found that this method was neither so exact nor so easy as that which I shall now explain.

1. Of one portion of 1000 grains, I separated the chlorine by nitrate of oxide of silver after I had poured a few drops of nitric acid into the water. In those cases where the water had fermented, I allowed it to stand in an open glass jar, in a warm place, until all smell of sulphuretted hydrogen had disappeared. To try how exact a result this method could give, I took a larger portion of sea-water, and weighed three different portions, each of 3000 grains, and precipitated the chlorine. The result was—

	Chloride of silver.
	145·451
	145·544
	145·642
Mean . .	145·541

The greatest difference is

−0·090=0·022 chlorine.
+0·083=0·020 chlorine.

These small differences are probably due to the small irregularities occasioned by the evaporation of very small quantities of water during weighing. The dried chloride of silver was as much as possible removed from the filter, melted in a porcelain crucible, weighed, and calculated as pure chloride of silver. The filter was burnt in a platinum crucible, by which the small quantity of chloride of silver was reduced to metallic silver, from which the chlorine which had been combined with it was calculated. This supposition is correct if the quantity of chloride of silver adhering to the filter is very small.

2. The determination of the *sulphuric acid* was likewise made with 1000 grains of sea-water, which, after addition of some few drops of nitric acid, was precipitated with nitrate of baryta. To try the exactness of the method three portions of sea-water were weighed, each of 3000 grains. The result was—

	Sulphate of baryta.
	12·417
	12·316
	12·250
Mean . . .	12·328

The greatest difference was

−0·078=0·027 sulphuric acid.
+0·089=0·030 sulphuric acid.

3. To determine *lime* and *magnesia* 2000 grains (in the latter experiments only 1000 grains) were weighed, and mixed with so much of a solution of sal-ammoniac that pure ammonia did not produce any precipitate, then ammonia was added until the liquid had a strong smell thereof. It was now precipitated with a solution of the common phosphate of soda and ammonia, and filtered when the precipitate had collected into a granular powder. The precipitate thus obtained consists of tribasic phosphate of lime, and tribasic phosphate of magnesia and ammonia, which was washed with a weak solution of ammonia. All the filtered solution and the wash-water was evaporated in a steam-bath to dryness, and afterwards digested in a tolerably strong solution of pure ammonia, by which means there is further obtained a small quantity of the phosphates. The dry phosphates of lime and magnesia are heated, and if they are not completely white, they are moistened with a few drops of nitric acid, and again heated and afterwards weighed. The mass was now dissolved in muriatic acid mixed with alcohol until the whole contained 60 per cent. (volume) thereof, mixed with a few drops of sulphuric acid, and allowed to stand for twelve hours, when the sulphate of lime is collected on a filter, heated and weighed. It contains, besides the sulphate of lime, silica, oxide of iron, phosphate of alumina, and sulphate of baryta and strontia, from which substances the sulphate of lime is separated by boiling it with a solution containing 10 per cent. of chloride of sodium, which dissolves the sulphate of lime and leaves the other combinations undissolved. The remainder is washed, heated, and its weight deducted from that of the sulphate of lime. To try how exact the determination of the lime was, I have taken three times 3000 grains of the same water, separated the lime, and obtained the following results:—

Sulphate of lime.
2·761
2·753
2·684
Mean . . . 2·733

The greatest differences are—

−0·049=0·020 lime.
+0·028=0·012 lime.

To find the quantity of magnesia contained in the weighed mixture of the phosphates of magnesia and lime, the lime, whose quantity has been determined, must, by calculation, be converted into tribasic phosphate of lime, and deducted from the whole quantity of phosphates; the other small quantities of different salts, which had been precipitated with the sulphate of lime, must likewise be deducted; the remainder is bibasic phosphate of magnesia, from which the pure magnesia is calculated. The sea-water tried in this way gave, after deduction of lime, silica, &c., the following result:—

Pure magnesia.
3·913
3·970
3·942
Mean . . . 3·942

The differences from the mean are—

−0·029
+0·028

4. The determination of potash or potassium in sea-water was tried by different methods, but gave no satisfactory results, so that I must consider the quantity of potash in the analyses as far less exact than any of the other substances whose quantity has been determined in sea-water. Happily there is so small a quantity of potash in sea-water, that any error in the determination of that substance has only an insensible influence on the whole result. For a number of the analyses I have used the following method. The weighed sea-water was evaporated to dryness, the dry mass again dissolved in water, and the undissolved residue washed with warm water until all sulphate of lime is dissolved, and the wash-water does not contain any sulphuric acid. The remaining powder consists of the different after-named salts and oxides insoluble in water; it is generally weighed and noted under one head.

To this solution I add so much carbonate of lime that the sulphuric acid finds lime enough to combine with, and as much muriatic acid as would dissolve the lime of the carbonate. The quantity of carbonate of lime is determined in the following way. The equivalent of sulphate of baryta being 1456, and that of carbonate of lime being 625, there will be an excess of lime if I take carbonate of lime in such a quantity that its weight is one-half of the quantity of sulphate of baryta, obtained from an equal quantity of the same sea-water in a previous experiment for the determination of sulphuric acid. All is now evaporated to dryness and dissolved in alcohol of 60 per cent., which leaves the sulphate of lime and dissolves all the chlorides; so that the solution is quite free from sulphuric acid. It is now a third time evaporated with a sufficient quantity of chloride of platinum. Alcohol of 60 per cent. leaves the chloride of platinum and potassium, which might be weighed, and the quantity of chloride of potassium calculated from it; but as it is most difficult in a laboratory where there is constantly work going on to avoid the absorption of the vapours of ammonia by evaporating liquors, I prefer heating the double chloride to a dull red heat, and assisting the decomposition of the chloride of platinum by throwing small pieces of carbonate of ammonia in the crucible. When all the chloride of platinum is decomposed, the crucible is weighed, the chloride of potassium is extracted by alcohol of 60 per cent., and the remainder weighed again. This method has the advantage, that even if a small quantity of gypsum should have accompanied the double chloride, it will have no influence upon the determination of the chloride of potassium. When I do not want to determine the insoluble remainder, I evaporate the sea-water with a sufficient quantity of chloride of calcium, and thus leave out one evaporation and solution.

In the few cases where I have tried to determine the different substances which in this chapter I have called silica, &c., I have used the following method. The filter upon which the remainder is collected and washed is burnt in a platinum crucible, evaporated with some drops of muriatic acid, and dissolved in water. What remains is silica, often coloured by a little oxide of iron, and mixed with a small quantity of sulphates of baryta and strontia. It is evaporated with fluoric acid and a drop of sulphuric acid to get rid of the silica. What remains after evaporation and heating is sulphate of baryta, of strontia, and oxide of iron. The solution in muriatic acid is precipitated by ammonia, and the precipitate is noted as phosphate of lime, but contains besides a little fluoride of calcium. The remaining liquid contains a little lime, which I precipitate with oxalate of ammonia, and suppose to have been in the sea-water as carbonate of lime dissolved by carbonic acid. In the water of the great ocean there occurs only a very small quantity of carbonate of lime, but near the shores, in the bays and inlets, and principally in the mouth of the great rivers, its quantity increases with the quantity of fresh water from the land. If the sulphates of the sea-water are decomposed to sulphurets, there is always precipitated a larger quantity of carbonate of lime, but that is the result of the decomposition, and its carbonic acid is owing to the organic substances which are oxidized by the oxygen of the sulphates.

I have never tried to ascertain the nature and quantity of the gases which occur in sea-water, because the collection of sea-water for that purpose would require quite different precautions from those which were necessary for the water intended for the analysis of its solid contents.

It might seem that the relative quantity of salt might be inexact, because water might have evaporated through the cork during the long time which often elapsed between the time when it was taken up from the sea, and the time when it was analyzed. It is, however, easy to see whether the quantity of water in the bottle has diminished, or whether the cork has been corroded; in both cases the sample has been rejected, but I must remark that these cases have been rare. In the last three or four years all the samples which have been taken according to my direction have been marked on the neck of the bottle with a file, on that place to which the water reached when the bottle was filled.

As to the calculation of the combinations of the different substances that have been found by the analysis, I have chosen the following method:—

The whole quantity of lime was supposed to be united with sulphuric acid.

What remained of sulphuric acid after the saturation of lime, was supposed to be combined with magnesia.

What remained of magnesia after the saturation of sulphuric acid, was supposed as magnesium to enter into combination with chlorine, and form chloride of magnesium.

The potash was supposed to form chloride of potassium.

That portion of chlorine which was not combined with magnesium or potassium, was supposed to form a neutral combination with sodium.

Lastly, that small quantity of different substances, "silica, &c.," was added, and the sum of all these combinations thus calculated forms the number which in the Tables is called "All Salts." It is hardly necessary to remark, that it is quite indifferent how we suppose the acids and bases to be combined in sea-water, the sum must always be the same, provided the salts are neutral, and all the acids (chlorine included) are determined, as well as all the bases, with the exception of soda.

On the Distribution of the Salts in the different parts of the Sea.

The next question to be considered refers to the proportion between all the salts together and the water; or to express it in one word, I may allow myself to call it the *salinity* of the sea-water, and in connexion with this salinity or strength, the proportion of the different solid constituent parts among themselves. On comparing the older chemical analyses of sea-water, we should be led to suppose that the water in the different seas had, besides its salinity, its own peculiar character expressed by the different proportions of its most prevalent acids and bases, but the following researches will show that this difference is very trifling in the ocean, and has a more decided character only near the shores, in the bays of the sea, and at the mouth of great rivers, wherever the influence of the land is prevailing.

In the Tables which are annexed to this paper I have always calculated the single substances and the whole quantity of salt for 1000 parts of sea-water, but besides this I have calculated the proportion between the different substances determined, referred to chlorine $=100$, and of all the salts likewise referred to chlorine. This last number is found if we divide the sum of all the salts found in 1000 parts of any sea-water by the quantity of chlorine found in it, and I call it the *coefficient* of that sample of sea-water. The following remarks, and the Tables which belong to them, will show that there is a very small difference in the coefficient of the different parts of the ocean, but that the differences become striking in the neighbourhood of the shores.

A. *On the salinity of the surface of the different parts of the ocean and its inlets.*

In the Tables annexed to this paper I have divided the sea into seventeen regions. My reason for doing so was that by this method I was able to avoid the prevailing influence which those parts of the ocean which are best known, and from which I have most observations, would exert upon the calculations of the mean number for the whole ocean.

First Region. *The Atlantic Ocean between the Equator and* 30° *N. lat.*—The mean of fourteen complete analyses is 36·169 per 1000 salt; the maximum is 37·908 per 1000, the minimum 34·283. The maximum lies in 24° 13′ N. lat. and 23° 11′ W. long., about 5° W. from the coast of Africa, where no rivers of any size carry water from the land, and where the influence of the dry and hot winds of the Sahara is prevailing. The maximum for the region is also the maximum of surface-water for the whole Atlantic; it is equal to the mean salinity of the Mediterranean, and only the maximum of that sea off the Libyan desert and that of the Red Sea are higher. The minimum is from 4° 10′ S. lat. and 5° 36′ W. long. close to the coast of Africa, where the large masses of fresh water which the great rivers of that region pour into the ocean exercise their influence. Its coefficient is 1·810.

Second Region. *The Atlantic Ocean between* 30° *N. lat. and a line from the north point of Scotland to the north point of Newfoundland.*—The mean of twenty-four complete analyses is 35·946 salt, the maximum 36·927, and the minimum 33·854. The maximum is in 38° 18′ N. lat. and 43° 14′ W. long. in the middle of the Atlantic; the minimum occurs in 43° 26′ N. lat. and 44° 19′ W. long., and is evidently owing to the enormous quantity of fresh water which the St. Lawrence, through its southern mouth, pours into the Atlantic. This region is under the influence of the Gulf-stream, and the corresponding South Atlantic region has only a mean salinity of 35·038. Its coefficient is 1·812.

Third Region. *The northern part of the Atlantic, between the northern boundary of the second region, and a line from the south-west cape of Iceland to Sandwich Bay in Labrador.*—The mean salinity deduced from twelve complete analyses is 35·391, its maximum 36·480, its minimum 34·831. The maximum falls in 55° 45′ N. lat. and 20° 30′ W. long., just on the boundary of Region 2, the minimum in 60° 25′ N. lat. and 3° 15′ W. long., near the large northerly opening of the North Sea. This region owes evidently its high salinity to the large northern direct branch of the Gulf-stream. Its coefficient is 1·808.

Fourth Region. *This region comprehends the East Greenland current, which flows along the east coast of Greenland towards the south and west, turns towards the north, when it reaches the south promontory of Greenland, runs along the west coast of that large land into Davis Straits, where it disappears in the polar current from Baffin's Bay.*

—I owe most of the samples from this current to Colonel SCHAFFNER, who took them on his expedition to Iceland and Greenland connected with the Northern Transatlantic Telegraph. The quantities being too small to allow a complete analysis, I have only determined the quantities of chlorine and sulphuric acid. I have, however, analyzed three other samples of water from this current taken by Captain GRAM, who during many years commanded one of the Danish Government's Greenland ships; and from these three complete analyses I have deduced the coefficient 1·813, instead of 1·812, which is the mean coefficient of the whole ocean. Thus I have calculated the mean salinity of the East Greenland current to be 35·278 *, while it is in the third region 35·391, and in the sea between Norway and Spitzbergen 35·347. These observations about the salinity of the current, connected with some other observations which will be afterwards discussed, make it highly probable that the East Greenland current is the returning Gulf-stream. At all events it is no polar current, which will easily be seen in comparing it with the Baffin's Bay current with a salinity 33·281, or the water to the north of Spitzbergen with 33·623, or the Patagonian polar current, which runs along the west coast of South America, and has 33·966. Nor is it probable that it comes from the north shores of Siberia, where such a great number of powerful rivers bring a vast quantity of fresh water into the sea. Its salinity is so great that it even exceeds that of the South Atlantic Region, between 30° S. lat. and the line between the Cape of Good Hope and Cape Horn, whose salinity is only 35·038.

Fifth Region, A. *The Baffin's Bay and Davis Straits Region.*—The mean of eight complete analyses is 33·281, the maximum 34·414, the minimum 32·304. This region shows the very interesting fact that its salinity increases on passing from latitude 64° toward the North, being in 64° 32·926, in 67° 33·187, somewhat further to the North 33·446, and in latitude 69° 33·598. This peculiarity is owing to the powerful current from the Parry Islands, which through different sounds passes into Baffin's Bay, where it is mixed with the great quantity of fresh water that comes into the sea from the West Greenland glaciers. Had this fact been known before the sounds that connect the Parry Archipelago with Baffin's Bay were discovered, it might have proved the existence of these sounds, because bays and inlets show quite the reverse; the further we get into them the less saline the water becomes.

Fifth Region, B. *The Polar Sea between the North Cape in Norway and Spitzbergen.*—I have eleven samples of water taken on the Swedish Spitzbergen Expedition by Professors NORDENSKJÖLD and BLOMSTRAND, of which I have rejected one taken in one of the bays of Spitzbergen, and another belonging to the sea to the north of Spitzbergen. None of these analyses were complete, and I have only determined the quantity of chlorine and of sulphuric acid; and even the latter could in several instances not be determined, since the water had fermented. The mean quantity of chlorine in the nine remaining samples was 19·507; and if we take the mean coefficient of the four North

---

* If we take the general coefficient of the ocean, 1·812, the salinity of the East Greenland current would be 35·258, which of course makes no material difference.

Atlantic regions (the East Greenland current included), 1·810, 1·812, 1·808, 1·813, it will be 1·811; and if we use this coefficient, the mean salinity of that part of the sea will be 35·327, or if we take the mean coefficient of the whole ocean, 1·812, it will be 35·347. The maximum was in 76° 15′ N. lat. and 13° 15′ E. long., with 20·019 chlorine =36·254 salt; the minimum in 70° 30′ N. lat. and 19° 5′ E. long., with 18·993 chlorine =34·396, near the coast of Norway, which evidently has had influence upon the result*.

Fifth Region, C. *The Polar Sea to the North of Spitzbergen.*—I have only one observation, of which I owe the sample to Professor Blomstrand. It is from 80° N. lat. and 12° E. long., containing 18·517 chlorine, which gives, with a coefficient of 1·812, a salinity of 33·623.

Sixth Region. *The German Ocean or the North Sea.*—The mean of six complete analyses is 32·823 per 1000 salt, the maximum is 35·041, the minimum 30·530 per 1000 salt, the maximum is from the mouth of the channel near the Gallopper, and the minimum is from Heligoland, where the water of the Elbe has a considerable influence. The mean coefficient is 1·816, which also shows the influence of the land.

Seventh Region. *The Kattegat and the Sound.*—The quantity of salt in the water of this region is very variable; a northerly current and wind brings water which is richer in salt than that brought by a southerly wind and current. The mean of six complete analyses and 141 observations, in which only the chlorine was determined, gives 16·230 per 1000 salt, the maximum 23·243, and the minimum 10·869. It must further be remarked that the proportion of chlorine and lime, which in the whole ocean are in mean number 100 : 2·96, in this region are 100 : 3·29, which again must be considered as depending upon the influence of the land. The mean coefficient is 1·814.

Eighth Region. *The Baltic.*—The mean numbers are deduced from complete analyses of samples of sea-water taken on board the Frigate 'Bellona,' on a voyage from Copenhagen to St. Petersburg, combined with a complete analysis of water from Svartklubben to the north of Stockholm. Its salinity varies very much in the different localities, and is of course less in the eastern than in the western portions of the Baltic; it varies also in the same place according to wind and current. I found the mean for this region 4·931 per 1000 salt, the maximum 7·481 in the channel between Bornholm and Sweden, the minimum in the merchant harbour of Kronstadt =0·610 per 1000 salt. The mean proportion of chlorine and lime is 100 : 3·64, in the Bay of Finland it is 100 : 7·49. The mean coefficient is 1·835, in the merchant harbour of Kronstadt it is 2·230. The influence of the land is here expressed in these different numbers.

Ninth Region. *The Mediterranean.*—All my observations lie between the Straits of Gibraltar and the Greek Archipelago. It is a general belief that the water of the Mediterranean contains more salt than the water of the ocean in general, and this opinion depends partly upon some analyses, partly upon the observation that at the Straits of Gibraltar there is a constant upper-current, which runs into the Mediterranean, and an under-current which carries its waters into the Atlantic. This opinion of the superior salinity of the Mediterranean has been completely confirmed by eleven complete analyses of water taken between the Straits of Gibraltar and the Greek Archipelago. The mean salinity of this region is 37·936, while the whole ocean contains 34·388 per 1000 salt. Its coefficient is 1·815. Its maximum (39·257) falls between the Island of Candia and the African shore off the Libyan desert, as the maximum of the Atlantic is off the Sahara, but the mean of the Mediterranean is a little higher than the maximum of the Atlantic; the whole Mediterranean is under the influence of Africa, and its hot and dry winds. The minimum for the Mediterranean is at the Straits of Gibraltar with 36·301; the mean salinity of the northern Atlantic Ocean between 30° and 40° N. lat., but more towards the west, is 36·332 (deduced from eight complete analyses); the surface-water from the Straits of Gibraltar is thus corresponding to that from the Atlantic of the same latitude. When entering the Straits the quantity of salt increases rather rapidly, and is at a short distance from them, at 4° 2′ W. long., 37·014; between the Balearic Islands and the Spanish coast it is 38·058, and a little further on 38·321, between the Island of Sardinia and Naples 38·654. Somewhat nearer to the coast of Malta it decreases to 38·541, and further on towards Greece it decreases again to 38·013, and would probably decrease more in the direction of the Bosphorus, but I have no observations from that part of the Mediterranean. From Malta to the coast of Africa it increases to the maximum of 39·257.

---

* That this sea is a branch of the Gulf-stream was acknowledged long ago.

There is another opinion generally reported, that the water of the Mediterranean contains a greater proportion of magnesia than the water of the ocean. This is, however, not the case; the mean proportion between chlorine and magnesia is for the Mediterranean 100 : 10·90, and for the ocean 100 : 11·07; nor is there any remarkable difference in the proportions of the other main substances. The proportion between chlorine and sulphuric acid is for the ocean 100 : 11·89, and for the Mediterranean 100 : 11·82; for lime it is in the ocean 100 : 2·96, and in the Mediterranean 100 : 3·08.

Tenth Region, A. *The Black Sea and the Sea of Assov.*—Like the Baltic, the Black Sea contains sea-water of but little strength, and the mean deduced from three observations, of which one is from myself, the two others by M. Göbel, is 15·894, maximum =18·146, minimum =11·880. In my own analysis of water from the Black Sea, fifty English miles from the Bosphorus, I found the proportion of chlorine 100, to sulphuric acid 11·71, to lime 4·22, to magnesia 12·64, and thus a considerable increase in the lime and magnesia.

Tenth Region, B. *The Caspian Sea.*—This sea being by many geologists considered to have been in former times in connexion with the Black Sea, it might be of some interest to compare its water with that of the Black Sea. I have, however, not had opportunity of making an analysis of it myself, but have calculated other analyses according to my method. Of these five analyses four are by M. Mahner, and published by M. Baer in his 'Caspian Studies' (Caspische Studien). As might be expected, the quantity of saline matter shows great differences, between 56·814 per 1000 in the Bay of Karassu or

Kaidaik, and 6·236 per 1000. The proportion between chlorine, sulphuric acid, lime, and magnesia, is

100 : 44·91 : 9·34 : 21·48.

It is quite evident that the Caspian Sea, if it ever had any connexion with the Black Sea, must have changed its character entirely since that time, and this change might either be occasioned by the different salts which the rivers brought into the lake, and which accumulated there by evaporation of the water, or it might be caused by the deposition of different salts in the basin of the Caspian Sea itself. If we now compare the abnormal proportions in the Caspian Sea,

Chlorine 100, Sulphuric acid 44·91, Lime 9·34, Magnesia 21·48,

with the normal proportions in the ocean,

Chlorine 100, Sulphuric acid 11·89, Lime 2·96, Magnesia 11·07,

we find that the excess of lime and magnesia will nearly neutralize the excess of sulphuric acid, and leave only a small quantity of sulphuric acid (3·72), which may be neutralized by alkalies. Thus rivers which brought sulphate of lime and of magnesia into the Caspian Sea, might in the lapse of 100 and 1000 years certainly change the composition of its water in the direction which it now has. Its mean coefficient is 2·434.

Eleventh Region. *The Atlantic Ocean between the Equator and* 30° *S. lat.*—The mean quantity of salts in this region, deduced from seven observations, is 36·553, the maximum 37·155, the minimum 35·930. The relative quantity of chlorine, sulphuric acid, lime, and magnesia is 100 : 12·03 : 2·91 : 10·96. The water of this region is richer in salt than the corresponding region in the North Atlantic Sea. Its coefficient is 1·814.

Twelfth Region. *The Atlantic Ocean between* 30° *S. lat. and a line from Cape Horn to the Cape of Good Hope.*—Mean salinity 35·038, maximum 35·907, minimum 34·151; the maximum not far from the Cape of Good Hope, the minimum not far from the Falkland Islands. Its salinity is less than the corresponding region in the North Atlantic (Region 2), which is 35·932, even less than the third and fourth regions (the East Greenland current), whose salinity is 35·278. This seems partly to depend upon the Gulf-stream, which causes a considerable evaporation in the northern part of the Atlantic, partly upon the River Plata in the South Atlantic, which carries an enormous quantity of fresh water into the southern sea. I have analyzed four samples of sea-water taken under the influence of that large river. One, taken by Captain PREVOST in 35° 46′ S. lat. and 52° 57′ W. long., almost at the mouth of the Plata, contained so much organic matter that a great part of its sulphuric acid was decomposed, so that the original quantity of salt could not be ascertained, but the quantity of chlorine, which, as far as we know, is not affected by the fermentation of the water, was only 17·721, which, multiplied by 1·808, the coefficient of this region, gives a quantity of salts =32·040; the other three samples, taken between 40° 30′ and 50° 31′ S. lat., and 40° 50′ and 52° 15′ W. long., are all far below the mean salinity of this region. It deserves to be remarked, that all the samples from the western part of this region have a less quantity of sulphuric acid than the normal, and the samples from the eastern part of the region nearer to the African coast have a proportion of sulphuric acid which is considerably greater than the normal quantity. Does this depend upon the more prevailing volcanic character of the west coast of Africa compared to the east coast of America?

Thirteenth Region. *The sea between Africa and the East Indian Islands.*—The mean of this region is 33·868, but it is deduced from observations that have given very different results. The maximum (35·802) is from 31·54 S. lat., 72° 37′ E. long., about midway between the Cape of Good Hope and Australia. Now in the North Atlantic Ocean even the mean salinity between 30° and 55° N. lat. is 35·932, thus greater than the maximum in this region, though this maximum is from near 32° S. lat. The fact is striking. The minimum (25·879) is from a place high up in the Bay of Bengal, and of course highly influenced by the vast quantity of water from the Ganges. It lies, however, about 300 English miles from the mouth of the Ganges; and another specimen from N. lat. 17° 20′, and about sixty miles nearer the mouth of the Ganges, has 32·365 per 1000 salt, so that it seems as if some other cause has also been operating to weaken the sea-water at the minimum place.

Fourteenth Region. *The sea between the south-east coast of Asia, the East Indian Islands, and the Aleutic Islands.*—The mean quantity of salt, deduced from seven complete analyses, is 33·506, the maximum from a place to the south-east of Japan, in 38° 31′ N. lat., is only 34·234, less than the maximum of the German Ocean between 50° 60′ N. lat., and surrounded by land (35·041). The minimum (32·370) between the larger East Indian Islands depends evidently upon the influence of the surrounding land. The mean proportion of chlorine, suphuric acid, lime, magnesia, is 100:11·76: 3·05:10·99, very nearly normal. The mean coefficient is 1·815.

Fifteenth Region. *The sea between the Aleutic Islands and the Society Islands, between 38° N. lat. and 32° S. lat.*—The mean quantity of salt is only 35·219, which is very near the mean of the East Greenland current (35·278), and very much below the mean of the Atlantic between 30° S. and 30° N. lat., which is 36·321. Its maximum is 36·061 near Borabora, about 16° S. lat., while the maximum of the corresponding tropical part of the Atlantic is 37·908; its minimum, under 38° 26′ N. lat., very far from any land, is 34·157. The mean proportion of chlorine, sulphuric acid, lime, and magnesia is 100:11·67:2·93:11·06. The mean coefficient is 1·806.

Sixteenth Region. *The Patagonian cold-water current.*—Mean 33·966 per 1000, maximum 34·152, minimum 33·788. The minimum is in the southernmost part of this current, and the maximum under 35° 22′ S. lat. The mean proportion of chlorine, sulphuric acid, lime, and magnesia is 100:11·78:2·88:11·04. The mean coefficient is 1·806.

Seventeenth Region. *The South Polar Sea.*—I have only three analyses, all on samples taken by the late Sir JAMES ROSS. One was from 77° 32′ S. lat., 188° 21′ E. long., close to the great ice-barrier. The water was full of animalculæ, but, notwithstanding, had not fermented. The quantity of salt which it contained was 28·565 per 1000. The next sample was from 74° 15′ S. lat., 167° E. long.; the water was muddy, probably from animalculæ and diatomaceæ. The place was not far from Victoria Land, at some distance from Coulman Island. It contained only 15·598 salt. The third, from 65° 57′ S. lat., 164° 37′ E. long., had the surprising quantity of salt 37·513 per 1000. The mean of these three observations is 27·225 per 1000; but this mean number is of very little consequence, being derived from numbers differing so greatly. It is, however, very surprising that water from the neighbourhood of the supposed Antarctic continent should have a salinity higher than any one found in the south equatorial regions of the Atlantic, and only be exceeded by a single one in the North Atlantic regions. I am sure that no material fault exists in the analysis, and this curious fact must thus remain unexplained until repeated observations in that region shall procure us further information. Should the observation be proved to be correct, it would render the existence of a "Gulf-stream" in the Antarctic zone very probable. There is still another peculiarity in these observations which deserves attention, viz. the great proportion of sulphuric acid to chlorine. In the water in the neighbourhood of Coulman's Island it is 12·47 : 100, and in that from 65° 57′ S. lat. 12·55 : 100, while in the whole ocean it is as 11·89 : 100. This might depend upon the very pronounced volcanic character of the Antarctic continent. There is still one question to be discussed with respect to the Antarctic Sea, how it is to enter into the mean numbers of the whole ocean. The observation from the neighbourhood of Coulman's Island must be rejected, because it is too near the land, and we have no corresponding observations from the open Antarctic Ocean. Its high coefficient (1·861) shows the great influence of the neighbouring land. The observation from 65° 57′ S. lat. must also be rejected as doubtful; there remains only the observation from the neighbourhood of the great ice-barrier, and I have taken that for the mean of the Antarctic region.

General Results of the preceding investigation.

If we except the North Sea, the Kattegat, Sound, and Baltic, the Mediterranean and Black Sea, the Caribbean Sea and the Red Sea, which have all the characters of bays of the great ocean, the mean numbers are the following:—

Sea-water.	Chlorine.	Sulphuric acid.	Lime.	Magnesia.	All salts.	Coefficient.
1000	18·999	2·258	0·556	2·096	34·404	1·812
	100	11·88	2·93	11·03		
Equivalents	429	45	16	82		

Thus it is evident that sea-water in its totality is as little a chemical compound as the atmospheric air; that it is composed of solutions of different chemical compounds; that it is neutral, because it everywhere in the atmosphere finds carbonic acid to neutralize its bases, and everywhere on its bottom and shores finds carbonate of lime to neutralize any prevailing strong acid; that, lastly, the great stability of its composition depends upon its enormous mass and its constant motion, which occasions that any local variation is evanescent compared to the whole quantity of salt.

If we take the mean numbers for the five regions of the Atlantic between the southernmost point of Greenland and that of South America, we find the mean quantity of salt for the whole Atlantic 35·833, while the sea between Africa and the East Indies has only 33·850, the sea between the East Indies and the Aleutic Islands 33·569, and the South Sea, between the Aleutic Islands and the Society Islands, 35·219 per 1000 salt. The Atlantic is thus that part of the ocean which contains the greatest proportion of salt, which result is rather surprising if we consider the vast quantity of fresh water which the rivers of Africa, America, and Europe pour into it: of Africa four-fifths are drained into the Atlantic either directly or through the Mediterranean; it is most probably nine-tenths of America which is drained into the Atlantic, since the Cordilleras run close to the western shore of the continent; and of Europe, also, about nine-tenths of the surface sends its superfluous water to the Atlantic. This greater quantity of fresh water from the land, and the greater quantity of salts in the corresponding sea, seem to contradict each other, but can be explained by a higher temperature, and, as the result of this higher temperature, a greater evaporation.

Some of the large bays of the ocean have in the tropical or subtropical zone a greater mean than the Atlantic: such are the Mediterranean, with 37·936 per 1000 salt (mean of eleven observations); the Caribbean Sea, with 36·104 per 1000 (one observation); the Red Sea, 43·067 per 1000 (mean of two but little differing observations), which is the greatest salinity of the sea I know of.

In approaching the shores the sea-water becomes less rich in salts, a fact which finds its explanation in the more or less great quantity of fresh water which runs into the sea. On such shores where only small rivers flow out, the effect produced is but very trifling, as, for instance, on the western shores of South America. The effect of large rivers in diluting the sea-water is much greater than is generally supposed; thus the effect of the La Plata river, whose mouth lies in about 35° of S. latitude, was still observable in a sample of sea-water taken at 50° 31′ S. lat., at a distance of 15° of latitude, or 900 English miles from the mouth of the river; at about the same distance, the water of the North-Atlantic Sea suffered a considerable depression in salinity, probably owing to the water of the St. Lawrence. This influence is of a double kind, partly in diluting the sea-water, partly in mixing it up with organic substances that will occasion its decomposition by putrefaction.

The polar currents contain less salt than the equatorial. I have determined the quantity and nature of the salts in two very well-defined polar currents,—the West-Greenland polar current, with 33·176 per 1000 salt, and the Antarctic polar or Patagonian current, on the west side of South America, which contains 33·966. It is highly interesting to observe that the East Greenland current, which according to its geographical relations might be considered as a polar current, which in fact has been considered in that way, has a very high mean quantity of salt, viz. 35·278 per 1000, while the sea to the north of Spitzbergen, according to one analysis, contains 33·623 per 1000 salt. I think I shall afterwards, from other phenomena also, prove that the East

Greenland current is a returning branch of the Gulf-stream; but I may here remark that the great quantity of salt which it contains almost by itself proves the more equatorial nature of this current.

As to the chemical substances which constitute the salts of the sea-water, it must be remarked that the polar current of West Greenland contains a larger quantity of sulphuric acid than any other region, with the exception of the south polar region and the East Greenland current.

The proportion between chlorine and sulphuric acid is—

For the West Greenland current. . .	100 : 12·27
For the East Greenland current . . .	100 : 12·34
Near Coulman's Island, Victoria Land .	100 : 12·47
From 65° 57′ S. lat.	100 : 12·55
The mean proportion for the ocean is .	100 : 11·89

This excess of sulphuric acid in the Antarctic Sea might be explained by the decided volcanic character of its islands and shores; even for the East Greenland current, the neighbourhood of Iceland and its volcanos might account for the excess of sulphuric acid; but the West Greenland polar current is under no such influence, and the surface-water of the Mediterranean, where so many volcanos exist, has 11·82 sulphuric acid, which is even a little below the mean proportion, 11·89. Only the water from the depth of the Mediterranean has an increased proportion of sulphuric acid, viz. 12·07. Thus it appears improbable that the excess of sulphuric acid in these polar regions should be owing only to volcanic action. It might depend upon the want of fucoidal plants. I have formerly, in a paper printed in the Report of the British Association for 1844, shown that the fucus tribe has a great attraction for sulphuric acid, and that the sulphuric acid, by the putrefaction of the plant, is reduced to soluble sulphurets and to sulphuretted hydrogen, which with the oxide of iron, which is partly dissolved, partly suspended in water, will form sulphuret of iron. Thus the sulphur will disappear from sea-water, and a great quantity of sea-weeds will diminish the quantity of sulphuric acid in the sea-water. Now it is well known that the polar regions have few or no sea-weeds, and Sir James Ross, when returning from the Antarctic polar region, remarks expressly that he observed the first sea-weed very far from the southernmost port of his voyage. An unusually small quantity of sulphuric acid seems to exist in the first of my regions, that part of the Atlantic which lies between the Equator and 30° N. lat., its relative quantity being 11·75. Does that depend upon the Sargassum Sea?

The greatest proportion of lime in the ocean occurs in its second region, the middle part of the northern Atlantic, where its proportion is 3·07, the mean proportion being 2·96; the least quantity of lime is found in the West Greenland polar current, with a proportion of 2·77; and next to that in the Patagonian polar current, with a proportion of 2·88. Wherever in other regions the influence of land is prevailing, the lime is likewise prevailing. In the Baltic I found its proportion 3·59, in the Kattegat 3·29, in that part of the German Ocean which lies close to the Kattegat 3·15, and in the whole German Ocean 2·87. In a sample from the Black Sea which I analyzed I found it 4·22.

B. *On the difference of the contents of Sea-water at the surface and in different*

It would be natural to suppose that the quantity of salts in sea-water would increase with the depth, as it seems quite reasonable that the specific gravity of sea-water would cause such an arrangement. But this difference in specific gravity relative to the increase in the quantity of salts is counteracted by the decreasing temperature from the surface to the bottom. We have parts of the sea where the quantity of solid salts increases with the depth; in other parts it decreases with the increasing depth; in other places hardly any difference can be found between surface and depth; and, lastly, I have found one instance where water of a certain depth contained more salt than both that aboveand below. These differences are to a great extent dependent upon currents both on the surface and in different depths. The phenomenon of double currents at the Straits of Gibraltar has been long known, and in close connexion with these double currents the saline contents of the water of the Mediterranean increase in quantity with the depth. There is, however, one exception in the Mediterranean, under interesting circumstances, which I shall afterwards discuss more at length. I have made eleven complete analyses of the surface-water of the Mediterranean, and calculated another quoted in VIOLETTE et ARCHAMBAULT, 'Dictionnaire des analyses chimiques,' vol. i. p. 358, without a more exact reference to the place where it was taken. Of my own analyses, one must be rejected on account of the great quantity of sulphuretted hydrogen that had been formed, and of course caused a loss of sulphuric acid; but it causes also a loss of lime, because the formation of sulphuretted hydrogen is contemporaneous with the formation of carbonic acid, which will precipitate the lime when deprived of its sulphuric acid. The mean number of the remaining analyses of surface-water is 20·889 per 1000 for the chlorine, and 37·936 for all salts. The mean number for chlorine of eight analyses of water taken from a depth of between 300 to 600 feet is 21·138. In each case the deep water was richer in chlorine than that from the surface, except in one instance, where the chlorine of the surface-water was 21·718, and all salts, calculated from a complete analysis, were 39·257 per 1000, while the chlorine of water taken from a depth of 522 feet was 21·521 per 1000. This curious exception occurred between Candia and the African coast, where the dry and hot winds from the neighbouring Libyan desert evidently cause a strong evaporation and a considerable elevation of temperature, which counteract each other as to specific gravity. The difference between the upper and lower current in the Straits of Gibraltar is, in the surface-water, chlorine 20·160 per 1000, all salts 36·391, and in the depth of 540 feet, chlorine 20·330.

The cause why the surface-current is Atlantic water flowing into the Mediterranean, and the under-current Mediterranean water flowing into the Atlantic, has long since been assigned to depend upon the comparatively small quantity of water that flows from the land into the Mediterranean, and the hot and dry African winds that cause more water to evaporate than the rivers bring into the sea. My analyses have not given me any reason to alter anything in our views of the cause of this difference, nor do I regard the single instance of water that is more rich in salts at the surface than in the depth as more than a local exception.

As to the difference between surface and deep water for other substances, I shall only remark that the deep water of the Mediterranean contains a remarkable excess of sulphuric acid. The proportion between chlorine and sulphuric acid is

For the whole ocean	100 : 11·89
Mediterranean surface	100 : 11·82
Mediterranean depth	100 : 12·07

Already in the Straits of Gibraltar the difference has the same character. The proportion is

For the surface	100 : 11·42
For the deep water	100 : 11·93

In some places, however, in the Mediterranean the surface-water is richer in sulphuric acid than water from the depth; thus, for instance, the sea between Sardinia and Naples had a proportion of 12·55 sulphuric acid in surface-water.

In the Baltic we have the same phenomenon; the water from the depth contains likewise more salt than that from the surface, but the direction of the currents is the reverse. The upper-current goes generally (not always) out of the Baltic, and the under-current goes, as it would appear, always into the Baltic. The cause of this great difference between the Baltic and the Mediterranean is evident; the Baltic receives the excess of atmospheric water from a great part of Europe. The greater part of Sweden, the greater part of European Russia, and a great part of North Germany send their water into the Baltic, and the evaporation is comparatively small. Thus the excess must find its way through the Sound and the Belts. With the assistance of Captain Prosilius, who in the year 1846 commanded the vessel at the station of Elsinore, the surface-current was observed on 134 days, from the 27th of April to the 11th of September; of which on 24 days it ran from the north, on 86 days from the south, and on 24 days there was no surface-current at all. The quantity of chlorine was determined for every sample by titration, and from that the quantity of salt deduced by multiplication with the determined coefficient 1·812. The mean quantity of salt for the current from the North was 15·994 per 1000; that for the current from the South 11·801; that for the period when there was no current at all was 13·342. Once a week a sample was taken from the bottom, by sending a reversed bottle down to the bottom, turning it there, and after having allowed it to stand some time, taking it slowly up. The mean of nineteen observations was 19·002 per 1000 salt, which, according to the manner in which the samples were taken, is rather under than above the real mean, and proves clearly that it is water from the Kattegat which runs at the bottom of the Sound. But we have also direct observations of the same fact. Some years ago a steamer was, close to Elsinore, struck by another steamer, and sunk a very short time after the collision.

When afterwards, in quiet sea, without current, a diver went down to save the passengers' goods, he found a violent current from the North. To the same class of phenomena belongs also the observation that large deep-going vessels not unfrequently go on in the Sound against surface-current, where smaller vessels do not succeed.

This under-current of Elsinore reaches often, and perhaps always, the harbour of Copenhagen, which I ascertained by a series of observations for which the laying of gas- and water-pipes offered me a good opportunity. To carry these pipes under the harbour, from Copenhagen to Christianshaven, on the Island Amager, a tunnel was projected through a solid hard limestone of the chalk formation, which lies under Copenhagen, its harbour, and its neighbourhood. When the tunnel was completed, it was found that the sea-water slowly filtered through the limestone, and fell down in drops from the roof of the tunnel. Comparative analyses would show how the water of the bottom of the harbour differed from that of the surface, and I might at the same time clear up another rather important question. It is generally known that the question of the formation of the dolomites, or the double carbonates of lime and magnesia, has excited a great interest, and many theories have been proposed about their formation. I myself have shown that a solution of carbonate of lime in carbonic acid water, when poured into sea-water, precipitates both carbonate of lime and carbonate of magnesia, but that the quantity of magnesia increases with the increased temperature in which the decomposition takes place. Neutral carbonate of lime thrown into sea-water would however, even at the boiling-point, not precipitate any carbonate of magnesia. It might, however, be a question of time, and it might be possible that such a decomposition would take place if sea-water during a long time was in close connexion with solid carbonate of lime. This would be the case if sea-water slowly filtered through 30 feet of solid limestone, which it does in the tunnel. We cannot, of course, expect to obtain any result by comparative analyses of the limestone; any change in the composition of this great mass of limestone would be so small that no result could be drawn from it, but we might analyze the sea-water filtered through the stone, and determine very small changes in its composition. Thus a series of comparative analyses of the sea-water from the surface of the harbour, of that from the bottom of it, and of the water filtered through the limestone into the tunnel, would show, first, whether the under-current from Elsinore reaches the harbour of Copenhagen; and secondly, whether the limestone roof of the tunnel acts upon the salts of magnesia in the sea-water which filters through it.

The experiments were made in the following way: once a week, from the 3rd of March to the 25th of April, 1852, one sample was taken of sea-water from the surface of the harbour over the tunnel, another sample from the bottom of the harbour at the same place, and a third sample was collected from the filtering water in the tunnel. The mean of these analyses gave,

For the surface	15·845	per 1000 salt
For the bottom of the harbour .	17·546	„
For the tunnel	18·315	„

which seems to prove that the under-current from Elsinore, at least at that season, reached Copenhagen. The difference between the water from the bottom of the harbour and the tunnel might either be occasioned by the slowness with which the water filters through the limestone of 30 feet thickness, so that it was water from another period which at last reaches the tunnel, or it may be explained by the way in which the samples from the bottom were taken, by sending an open bottle reversed down to the bottom, where it was turned and allowed to stand some time, to let the heavier water from the bottom dislodge the lighter water which had entered the bottle. The mean relative quantity of lime and magnesia was—

For the surface . .	1 lime to 4·062 magnesia.
For the bottom . .	1 lime to 4·153 magnesia.
For the tunnel . .	1 lime to 3·485 magnesia.

The proportion between lime and magnesia is therefore pretty much the same in the water from the surface and the bottom of the harbour, but in the water from the tunnel the relative proportion of the lime is increased. This may depend either upon a diminution of the magnesia, or upon an increase of the lime, or upon a combination of both effects; but if these changes took place only according to equivalents, it would prove that there had been formed dolomitic combination by the filtration of the magnesia salts of sea-water through the carbonate of lime in the limestone. To ascertain this point, I have compared the lime and magnesia with a third substance in sea-water, for which I chose chlorine. This mean proportion was—

For the surface . .	100 chlorine :	2·82 lime :	11·07 magnesia.
For the bottom . .	100 chlorine :	2·62 lime :	10·96 magnesia.
For the tunnel . .	100 chlorine :	3·11 lime :	11·08 magnesia.

It follows from these comparisons that the absolute quantity of lime had increased in the water of the tunnel, but that the absolute quantity of the magnesia in the same filtered water had not decreased, but was as nearly the same as an analysis could show. Thus the increase of the lime depended upon the solution of some carbonate of lime from the limestone. It was further found that water from the tunnel, when evaporated to dryness and dissolved, left more carbonate of lime than surface-water. The cause of this solution of the carbonate of lime was evidently to be sought in a bed of black mud which covers the bottom of the harbour, and is slowly converted into carbonic acid by the atmospheric oxygen absorbed by the sea-water. The sea-water impregnated with carbonic acid had dissolved some of the limestone through which it filtered.

Here might also be the place to mention and explain a rather curious phenomenon which is observed all along our coasts of the Sound and the Baltic, at least as far as Kiel. When the ice in spring begins to thaw, it disappears quite suddenly, and all the fishermen along the shore assure you unanimously that it sinks. I have examined a great number of these men, and have not found a single one who did not confirm the sudden disappearance of the ice in spring, and who did not consider it to be quite certain that the ice in spring sinks. I could, however, not find a single one of them who had *in spring* fished the ice up in his nets, while they very often in autumn and the beginning of the winter find it at the bottom, and see it rise to the surface*. It was evident that the sudden disappearance of the ice in spring was the fact which they had observed, and that the sinking of the ice was the popular explanation of the fact.

The natural philosopher will not allow ice to sink in sea-water, and it seems necessary to find another explanation. In order to give that I must first mention another peculiarity with the under-current of Elsinore. I observed on the 2nd of March, 1850, the temperature of the under-current with a maximum thermometer to be +2·6 C. (36°·8 F.) at the depth of 108 feet, while the temperature at the surface was +1·6 C. (34°·9 F.). Early in the next spring a friend of mine repeated the observation, and found likewise the higher temperature in the under-current, the difference being about 2° C. A third observation made in summer gave no difference. To explain this, I must observe that the water of the Kattegat, at least in its depth, is a branch of that great part of the Gulf-stream that passes along the western shores of Norway, and that the under-current at Elsinore necessarily must be less affected by the cold which reigns over the Baltic in winter time. Thus the under-current has in spring a higher temperature than the water of the surface, and at the same time contains a greater quantity of salt. Suppose, now, that the ice towards spring has begun to thaw and has become porous, as is generally the case, the warmer and more saline water will come in contact with it from below, and will melt it, partly on account of its temperature above freezing-point, partly on account of the greater quantity of salt which it contains. Thus without any apparent greater changes on the surface the ice will melt quickly and almost imperceptibly, and disappear. This effect of the under-current will be increased by the peculiarity of sea-water, that its point of greatest density lies below the freezing-point of pure water, and a constant series of small vertical currents will be formed where the warmer water rises, and that which is refrigerated by the contact with the ice sinks, which motion always will increase the melting of the surface-ice.

Besides at Elsinore and at Copenhagen, it has been observed at Kiel, near Stockholm, and in the Bay of Finland, that the deeper water is more saline than that of the surface. At Svartklubben, near Stockholm, water from the surface contained 3·256 chlorine =5·919 salt, and from a depth of 720 feet 3·912 chlorine =7·182 salt (coefficient 1·836); in the Bay of Finland, between the islands Nervoe and Sukjeld, the surface-water contained 3·552 per 1000 salt, while in a depth of 180 feet it contained 4·921.

It was only for the two larger salt-water basins of Europe, the Mediterranean and the Baltic, that I was able to determine the quantity of salt near the surface and in the depth, but it is very probable that similar differences also may occur in other large inlets of the ocean. I want, however, direct observations in sufficient number, and shall here only mention an observation from the Caribbean Sea, where surface-water contained 19·936 chlorine, and water from a depth of 1170 feet contained 19·823 per 1000 chlorine. This difference in which the deeper water is less saline may be another instance of the effect of hot winds, like the water from the Mediterranean between Africa and the Island of Candia.

---

* This formation of the bottom ice is very frequently observed on our shores. There is a fishing bank a little to the north of Elsinore, where the fishermen often in the beginning of the winter find themselves suddenly surrounded by ice, which they see rise through the water, containing numerous pieces of Fucus inclosed in its mass. The same fact has also been observed not far from Copenhagen, and off Nyborg in the Great Belt. It seems, in fact, a phenomenon peculiar to such places where a strong current runs over a place that is not very deep.

Going on now to the main section of the ocean, we will begin with the Atlantic, about which we have the best information, and which seems to show the most interesting facts. I will state the results of my investigations in moving from Baffin's Bay towards the south. In Baffin's Bay itself the water of the surface contains the same quantity of salt as that of the depth, but as soon as we pass the southernmost point of Greenland, the water of the surface contains more salt than that from the depth. This difference increases in going towards the Equator, and is indeed very considerable near that line. About the Equator, and a little to the south of it, many irregularities appear, as, for instance, in one case where the strongest water was found between two weaker portions above and below. In other cases the quantity of salt decreased with the depth, and in some instances it increased with it. I shall now state the observations themselves. Dr. Rink sent me water from the surface in Baffin's Bay to the west of Disco Island, which contained 33·594 per 1000 salt, and at the same place from a depth of 420 feet, which contained 33·607. The difference is so small that it signifies nothing. At the southernmost point of Greenland a small difference is observed, viz. in 59° 45′ N. lat. and 39° 4′ W. long., where surface-water contains 35·067, and that from a depth of 270 feet 34·963; but in about the same latitude and about 13° further towards W., at 59° 42′ N. lat. and 51° 20′ long., the proportion was reversed, the surface-water contained 34·876 per 1000 salt, while that from the depth contained 34·975 per 1000. From the sea between Iceland and Greenland (in which it appears that a returning branch of the Gulf-stream, the East Greenland current, runs towards the S.W.) I have obtained eight specimens from a depth between 1200–1800 feet. Unfortunately no specimens of water from the surface were taken at the same time, but we have a sufficient number of other surface observations, and thus we may compare the mean numbers, which are 35·356 for the surface, and 35·057 for a depth between 1200–1800 feet. In comparing the single observations of the deep water, we find that it contains the greatest quantity of salt in the eastern part at 35° 1′ W. long., with 35·179 per 1000 salt, decreasing regularly towards the westernmost part of this region in 55° 40′ W. long., with a quantity of salt =34·858 per 1000. Specimens taken by Captain Gram in 59° 50′ N. lat. and 7° 52′ W. long., contained for surface-water 35·576 per 1000, and for water from 270 feet depth 35·462.

I have two other comparative analyses of water from the East Greenland current, of which I owe the specimens to Colonel Schaffner. The analyses were not made com-

plete, but only chlorine and sulphuric acid were determined, which gives at 64° 30′ N. lat. and 26° 24′ W. long.,
for the surface,
19·616 chlorine, which with a coefficient 1·812 is =35·544 salt;
for a depth of 1020 feet,
19·504 chlorine, which with a coefficient 1·812 is =35·341 salt.

The next analysis of water from 62° 47′ N. lat. and 37° 31′·5 W. long., gave
for the surface,
19·491 chlorine =35·318 per 1000 salt;
for a depth of 1200 feet,
19·466 chlorine =35·272 per 1000 salt.

Further to the S.W., near the bank of Newfoundland, specimens taken by Captain von Dockum gave,
for the surface,
36·360 per 1000 salt;
for a depth of 240 feet,
36·598 per 1000 salt,

which is an increasing quantity of salt for the deep water, and coincides with other observations which show that this curious decreasing of the quantity of salt, with the increasing depth, belongs only to the deep part of the Atlantic far from the shores. On the European side of that ocean three samples, taken by Captain Schulz at 47° 15′ N. lat. and 9° 30′ W. long., gave the following quantities of salt:—
from the surface,
35·922 per 1000;
from a depth of 390 feet,
35·925 per 1000;
from a depth of 510 feet,
36·033 per 1000;
thus showing a trifling increase of salt with the depth.

The most complete set of experiments showing this influence of the shores, I have made on twelve samples taken by the 'Porcupine' in 1862, which I owe to the obliging kindness of Rear-Admiral FitzRoy. The samples are taken between 50° 56′ and 55° 22′ N. lat., and 12° 6′ and 15° 59′ W. long., about four degrees to eight degrees of longitude to the west of Ireland, and five of them were from the surface, while seven were from deep water, between 1200 and 10,500 feet.

The mean of the five surface observations is—

Chlorine.	Sulphuric acid.	Lime.	Potash.	Magnesia.	All salts.
19·662	2·342	0·566	0·367	2·205	35·613.

The mean of the seven observations from the deep sea is—

Chlorine.	Sulphuric acid.	Lime.	Potash.	Magnesia.	All salts.
19·677	2·357	0·583	0·363	2·193	35·687

Chlorine =100, the proportions are—

	Chlorine.	Sulphuric acid.	Lime.	Potash.	Magnesia.	All salts.
For surface . . .	100	11·91	2·88	1·87	11·21	181·1
For deep water . .	100	11·98	2·96	1·84	11·14	181·4.

The difference is very trifling, and the quantities of salts increase in a very slight degree with the depth.

I owe all the other samples from the North Atlantic Ocean which have been used for my analyses, of which I am now going to give the results, to the late Sir James Ross, through the assistance of the most honourable and learned President of the Royal Society, General Sabine, who always is most willing to assist scientific labours with his powerful influence and his prudent advice, and to whose intercession I am indebted for several of the most interesting results I have obtained in this investigation.

At 18° 16′ N. lat. and 29° 56′ W. long.,
from the surface,
20·429 chlorine =36·833 per 1000 salt {(coefficient deduced from five complete analyses of water from Sir J. Ross=1·803);
from 3600 feet,
19·666 chlorine =35·448 per 1000 salt.

At 16° 27′ N. lat. and 29° W. long.,
from the surface,
20·186 chlorine =36·395 per 1000 salt (coefficient 1·803);
from 900 feet,
20·029 chlorine =36·112 per 1000 salt (coefficient 1·803);
from 2700 feet,
19·602 chlorine =35·342 per 1000 salt (coefficient 1·803).

At 15° 38′ N. lat. and 28° 10′ W. long.
from the surface,
20·081 chlorine =36·206 per 1000 salt (coefficient 1·803);
from 3360 feet,
19·744 chlorine =35·598 per 1000 salt (coefficient 1·803).

At 14° 18′ N. lat. and 27° 15′ W. long., surface observation wanting;
from 900 feet,
19·934 chlorine =35·941 per 1000 salt (coefficient 1·803);
from 2700 feet,
19·580 chlorine =35·303 per 1000 salt (coefficient 1·803);
from 3600 feet,
19·705 chlorine =35·528 per 1000 salt (coefficient 1·803).

At 12° 36′ N. lat. and 25° 35′ W. long.,
from the surface,
20·114 chlorine =36·195 per 1000 salt (direct observation);
from 11,100 feet,
19·517 chlorine =35·170 per 1000 salt (direct observation).

At 11° 43′ N. lat. and 25° 6′ W. long.,
from the surface,
20·035 chlorine =36·123 per 1000 salt;
from 3600 feet,
19·855 chlorine =35·799 per 1000 salt;
from 4500 feet,
19·723 chlorine =35·561 per 1000 salt.

At 1° 10′ N. lat. and 25° 54′ W. long.,
from the surface,
19·757 chlorine =35·622 per 1000 salt;
from 1800 feet,
19·715 chlorine =35·546 per 1000 salt;
from 3600 feet,
19·548 chlorine =35·245 per 1000 salt.

For the South Atlantic Ocean, the relation between the salts of the upper and lower parts of the sea is variable and difficult to explain. In 0° 15′ S. lat. and 25° 54′ W. long. the quantity of salts found in different depths was as follows:—
from the surface, wanting;
from 900 feet,
19·763 chlorine =35·820 (coefficient 1·814);
from 1800 feet,
19·991 chlorine =36·264 (coefficient 1·814);
from 4500 feet,
19·786 chlorine =35·892 (coefficient 1·814);
from 5400 feet,
20·007 chlorine =36·293 (coefficient 1·814).

Most deviating is a series of observations from 22° 37′ S. lat. and 34° 57′ W. long.:—
from the surface,
20·397 chlorine =37·000 (coefficient 1·814);
from 900 feet,
20·323 chlorine =36·866 (coefficient 1·814);
from 1800 feet,
23·189 chlorine =42·165 (coefficient 1·814);
from 2700 feet,
20·331 chlorine =36·880 (coefficient 1·814);
from 3600 feet,
20·405 chlorine =37·015 (coefficient 1·814).

Already in the water from different depths immediately on the south side of the Equator there is a curious variation; at 1800 feet it is about one-half per 1000 richer in salt than at 900 feet, and in 4500 feet the quantity of salt has diminished as much as it had increased before. At 5400 feet it has a greater quantity of salt than any of the upper specimens has shown. In the series from 22° 37′ S. lat. the surface has a high number, higher than any corresponding sample from the North Atlantic, it sinks a little at 900 feet, but rises at 1800 feet to a quantity of salt which does not occur in any other place in the whole Atlantic, not even the maximum of the Mediterranean, and we know only the Red Sea which exceeds it; it is as if the water of the Red Sea were transported to this submarine current. I thought there might be a fault in the determination of the chlorine, and repeated it; but the difference was very insignificant, being in the one case 23·187, in the other 23·191, the mean being 23·189. I thought that by some accident some salt might have come into the instrument by which the water was taken, and I made a complete analysis of the water, but the different substances which were determined showed but slight differences from the normal proportions, viz.—

	Chlorine.	Sulphuric acid.	Lime.	Potash.	Magnesia.
22° 37′ S. lat., 1800 feet . .	100	11·59	2·77	2·14	11·29
South Atlantic	100	12·03	2·91	——	10·96

It might perhaps be owing to an evaporation in the bottle, but then the bottle was full, and cork and sealing-wax were sound, while about one-seventh of its whole contents must have been evaporated to explain the difference. If there is any mistake in this curious observation, it must probably have been caused by a negligence which left the instrument for taking the water from the deeper part of the sea partly filled with sea-water, exposed to evaporation in tropical heat, and sent it down without being cleaned. I should hardly think that such a fault could have been committed, and we must hope that new experiments will confirm the fact. The series of observations from 0° 15′ S. lat. belong in fact to the same kind, by the alternation of stronger and weaker sea-water in different depths; but the curious and surprising fact in the observation from 22° 37′ S. lat. is, that in the whole Atlantic Ocean we do not know a single place where water with that quantity of salt occurs. The next specimen, from 22° 37′ S. lat. and a depth of 2700 feet, is very nearly the same as that from 900 feet, and that from 5400 feet very near that from the surface of the same place.

It appears thus that the water of the North Atlantic Ocean, between the southernmost part of Greenland and the equator, decreases in salinity with the depth, but that this curious fact is observed only in the middle bed of the Atlantic, and disappears when we approach the shores on both sides of the ocean. As to the cause of this rather surprising state, I am still of the same opinion which I expressed when I first observed it, that it depends upon a polar under-current. The hypothesis has been published, that it depended upon fresh-water springs at the bottom of the ocean, and such an opinion might have some chance as long as we only had few observations; but now we have such a number of observations spread over a vast extent of the ocean, that it appears to be quite impossible to explain it by springs of fresh water, which of course must be more frequent and more powerful near the land, from which they have their origin. Observation, however, shows the reverse; near the shores the water is either uniform throughout its whole depth, or the quantity of salt increases with the depth.

The next question is whether we can find a similar distribution in the other parts of the ocean. As to the southern portion of the Atlantic, there occurs such a confused distribution of the quantity of salt in the different depths at the same place, that we are not able as yet to draw any conclusions from it, but must wait for more copious observations.

As to the other parts of the ocean, I have only very few observations from the sea between Africa and the Aleutic Islands; but these few observations do not show any regularity, or at all events seem more to incline to an increase of the quantity of salt with the increasing depth. The geographical distribution between land and sea is, however, quite different in this large part of the ocean. While a strong polar current from Baffin's Bay pours its cold and less saline waters into the North Atlantic Sea, the large mass of Asia prevents any north polar current from reaching the south Asiatic sea, into which the numerous great rivers of Asia send vast quantities of warm fresh water.

As to the south polar currents, we know very little about their influence upon the salinity of the southern ocean; but in Sir JAMES ROSS's 'Voyage' (vol. ii. p. 133) there is an observation upon the different specific gravity in different depths, which indicates a state of things similar to that in the North Atlantic Ocean. His observations are these:—"At 39° 16′ S. lat., 177° 2′ W. long., the specific gravity of the surface-water 1·0274, at 150 fathoms 1·0272, and at 450 fathoms 1·0268, all tried at the temperature of 60° F., and showing that the water beneath was specifically lighter than that of the surface when brought to the same temperature; our almost daily experiments confirmed these results"*.

The principal currents of the Atlantic, the Equatorial current, the Gulf-stream, and the East Greenland current.

These three currents are in fact only the same; they begin, as is well known, in the Bay of Benin, under the Equator, and the main current runs straight to the west over the Atlantic to Cape Roque, on the east coast of South America. I certainly shall not try to lessen the weight of the arguments which assign the cause of this equatorial current to the rotatory motion of the earth, but I will only give some remarks as to other influences that act to the same effect.

If we compare the quantity of salt which is found in sea-water in the region between

---

* To compare these observations of specific gravity with the quantity of salt in different depths, which I have mentioned in the former part of this paper, I shall here refer to some experiments which I have made to obtain a ratio by which I could compute the quantity of salts in the sea-water from the specific gravity, and *vice versâ*. I have compared, in thirteen specimens of sea-water, the specific gravity with the quantity of chlorine which the water contained, between 13°·75 C. (56°·75 F.) and 18°·8 C. (65°·8 F.). It was found that a unit in the fourth decimal place of the specific gravity of sea-water, measured by the hydrometer, is equal to $\frac{71}{1,000,000}$ of chlorine, the minimum being 66, the maximum 76. To find what quantity of salt corresponds to the specific gravity of the surface-water, as determined by Sir JAMES Ross to be 1·0274, we must multiply 274 by 71, which gives 19·454 chlorine in the sea-water, and that number being multiplied by the general coefficient 1·812, gives 35·251 per 1000 salt for the water from the surface. According to the same computation the sea-water from 150 fathoms contained 34·993 per 1000 salt, and that from 450 fathoms 34·478 per 1000 salt.

5° N. lat. and 5° S. lat. with those between 5° and 20° to the North and of 5° to 30° to the South, we find the interesting fact that the water flowing in the vicinity of the Equator contains less salt than that which flows both to the north and to the south of it. For the equatorial region (5° S. to 5° N.) the mean of six observations is 35·575 per 1000; or if we leave out a sample from Sir JAMES ROSS, from 150 fathoms' depth (that from the surface is wanting), it is 35·520. From 5° to 20° N. the mean of eight analyses is 36·279, and from 5° to 30° S. the mean of six analyses is 36·631 per 1000. This difference is still more striking on comparing the salinity of the equatorial region with that of the northern Atlantic region (second region), whose mean is 35·932 per 1000 salt. It deserves further attention, that the maximum of the equatorial region is below the mean of its neighbours both to the south and to the north. It appears to me that this curious fact can be explained only by the vast quantity of fresh water which the Niger, the Ogaway, and a number of other West African rivers carry in this region into the sea, which all gets into the equatorial current, and moves to the westward. It is evident that this warm water must increase its relative quantity of salt by evaporation during its motion across the Atlantic, and a comparison of the analyses of the single samples of the water from the equatorial current shows that this effect really takes place. The easternmost sample contains the minimum, with 34·238 per 1000, and the two westernmost samples contain the greatest quantity of salt, with 36·084. Thus the equatorial current appears as a continuation of the large West African rivers of the equatorial zone, which dilute the sea-water of the equatorial region with about 8 per cent. of fresh water, and thus counteract the great evaporation. While the equatorial current continues its course along the north-east coast of South America, it receives and carries with it the waters of the Paranahyba, the Araguai, the Amazon river, the Essequibo, the Orinoco, and numerous smaller rivers of the north coast of South America; but though I have no observations from this part of the current*, the fact is shown by three observations from the sea in the neighbourhood of the Danish islands of St. Croix and St. Thomas, whose mean salinity is 35·7 per 1000; while two degrees more to the north the mean of two observations is 36·7, which seems to be the normal salinity of the West Indian Sea. In the Caribbean Sea, where the Magdalene river gives a new quantity of fresh water, the sea contains on the surface, according to one observation, 36·104 per 1000 salt. I have unfortunately no observation from the Mexican Gulf, nor from the beginning of the Gulf-stream, where it leaves the Mexican Gulf, but to the north of the Bermudas it contains only 35·883 per 1000 salt, about the same quantity which the equatorial current contains between 20° and 30° W. longitude. From that place the salt of the Gulf-stream increases constantly during its course towards the north-east, viz. 36·105 per 1000, 36·283 per 1000. In 43° 26′ N. lat. and 44° 19′ W. long., about 16° of longitude to the east of the southern mouth of the St. Lawrence, between Nova Scotia and Newfoundland, it sinks suddenly to 33·854 per 1000, and rises from thence slowly in its course towards the east to 34·102 and 35·597, until, midway between Newfoundland and the south-western cape of Great Britain, it has risen to 35·896 per 1000, a quantity of salt which diminishes very little in the whole North Atlantic Ocean between Scotland and Iceland. During this whole long course, from the Bay of Benin to Spitzbergen, this remarkable current shows a constant oscillation between the diluting influence of the large rivers and the evaporation occasioned by the high temperature of the current.

---

* [When my remarks on the equatorial current between Cape Roque and the West Indian islands were written, I was not aware of the very interesting observations which General SABINE made in 1822, on the influence of the water of the Amazon river on that of the Equatorial current. I shall now insert them here, their bearing being in the same way as my deficient observations.

In 5° 8′ N. lat. and 50° 28′ W. long. a distinct line of separation was observed between the pure blue water of the ocean and the discoloured water mixed with that from the Amazon river, the mouth of which was about 300 miles distant. The blue water had a specific gravity of 1·0262, which according to my calculation (p. 37) is =33·672 per 1000 salt, while the water on the other side of the line of separation was 1·0204=26·345 per 1000 salt; further on, under the influence of the river, it was 1·0185=23·800 per 1000 salt. But the river water kept on the surface and in a depth of 126 feet, the specific gravity was 1·0262(=33·672 per 1000 salt).

In 7° 1′ N. lat. and 52° 38′·5 W. long. the specific gravity was 1·0248=31·905 per 1000 salt, and in 120 feet depth again 1·0262 specific gravity.

In 7° 5′ N. lat. and 53° 30′ W. long. it was 1·0253=32·549 per 1000 salt.

In the Gulf of Paria, off the mouth of the Orinoco, the specific gravity was 1·0204=26·345 per 1000 salt, and in crossing one of the branches of the river itself the specific gravity was found to be only 1·0064=8·234 per 1000 salt. See 'An Account of Experiments to determine the Figure of the Earth, by EDWARD SABINE. London, 1825.'—G. F., April, 1865.]

Now we shall try to trace its further progress. I have always thought that the East Greenland current was of polar origin, and that it carried the waters from the large opening between Spitzbergen and the northernmost coast of Greenland into Davis's Straits, where it turns and mixes its waters with the polar current that comes from the North American polar sea through Lancaster Sound, and the numerous other sounds that connect Baffin's Bay with the American polar sea, but I never had an opportunity of making comparative analyses of the water from that but seldom visited part of the ocean. Colonel SCHAFFNER had the kindness on his voyage between the eastern part of Iceland and the south part of Greenland to take a number of samples, which I have analyzed, and the result of which will be found in my fourth region, the East Greenland current. The mean of twelve observations of water, taken for the greatest part by Colonel SCHAFFNER (three by Captain GRAM), is 35·278 per 1000 salt, where one analysis of water taken in the ice-pack is left out, being no fair sample of sea-water from that region. In comparing this mean number with that of the North Atlantic Ocean (35·391), there will hardly be found any difference in the quantity of salt the two contain; while there is a great difference between these and the real polar current of Baffin's Bay, which is 33·281 per 1000, or of the Patagonian polar current (33·966). I think we may infer from this fact, that the East Greenland current is a returning branch of the Gulf-stream, and that the east coast of Greenland proportionally gives very few icebergs and very little glacial water to the sea. For comparison's sake I shall mention here that the sea about midway between Norway and Spitzbergen contains 35·222 per 1000. I found the water taken on the south side of that island to contain 35·416 per 1000, while that on the north side of Spitzbergen contained 33·623 per 1000. The last-mentioned sample seems to be real polar water, while all the water that flows between Norway, Spitzbergen, Iceland, and the east coast of Greenland partakes of the nature of the Gulf-stream.

Besides the reasons just mentioned for considering the East Greenland current to be a returning branch of the Gulf-stream, reasons which are deduced from the quantity of salt which the water contains, there are other reasons which lead to the same result. It is well known that the Gulf-stream brings tropical fruits from America to the coast of Norway, and it has once brought a river-vessel loaded with mahogany to the coast of the Faroe Islands. It is likewise known that similar fruits to those which are found on the Norwegian shores are carried by the sea to the coast of Iceland, and principally to its north and east coasts, where they only could get if the Gulf-stream turns between Spitzbergen and Iceland, and thus runs between Iceland and Greenland towards the south-west. It would be difficult to explain how a polar current could bring tropical fruit to the north coast of Iceland.

On the west coast of Greenland the south-easterly wind brings in winter a mild temperature, and this fact is so generally known in the Danish colonies of Greenland, that many of the colonists are convinced that there are volcanos in the interior of that snow-clad land. The temperature which this current, that in winter and spring is full of drifting ice (not icebergs), communicates, can of course not be above freezing-point, but that temperature is mild, when the general temperature in winter is 8°, 10°, or 12° R. below the freezing-point. All these facts together leave hardly any doubt in my mind that it is the Gulf-stream which runs along the east coast of Greenland, and at last in Davis's Straits mixes its waters with the polar current from Baffin's Bay. In its course towards the south it meets the main part of the Gulf-stream at Newfoundland, where it partly mixes with it to begin its circulation anew, partly dives under it, and runs as a ground stream as far as the Equator. In a similar way the southern branch of the Gulf-stream, which goes parallel to the western shores of South Europe and North Africa, joins the equatorial current at its beginning in the Bay of Benin, and begins also its circulation anew.

Chemical Decomposition in Sea-water.

If we consider the almost uniform composition of sea-water in the different parts of the ocean, such as they are represented by comparing the salts with the quantity of chlorine as unity, and thus avoiding the influence of the different quantities of water in which they are dissolved, we might be inclined to suppose the salts of sea-water to be in chemical combination with each other, and to form a compound salt with definite proportions. This is however not the case, and sea-water is not more a chemical compound than the atmospheric air, and the steadiness of the quantity of the different substances depends partly upon the enormous mass of the water of the ocean, compared to which all changes disappear, partly upon the constant motion which current and wind occasion. In the bays and those parts of the sea which only have narrow sounds that connect them with the main ocean, where therefore the general motion of the sea cannot have that influence it has in the open ocean, we observe differences which show the influence of the land upon the constituent parts of the sea-water. This want of chemical combination between different salts will become more evident when, instead of comparing their different quantities, we compare the relative number of their equivalents. The mean quantity of the different substances in the whole ocean, as deduced from the mean of regions I., II., III., IV., V., XI., XII., XIII., XIV., XV., XVI., XVII., is in 1000 parts of sea-water,—

Chlorine.	Sulphuric acid.	Lime.	Potash.	Magnesia.	All salts.	Coefficient.
18·999	2·258	0·556	0·367*	11·03	34·404	1·811

Chlorine =100.

Sulphuric acid.	Lime.	Potash.	Magnesia.	All salts.
11·88	2·93	1·87*	11·03	181·1

Proportion of Equivalents.

Chlorine.	Sulphuric acid.	Lime.	Potash.	Magnesia.
429	45	16	6	82.

There is one question which deserves a closer examination, viz. how the salts that now constitute the water of the sea came into it? Is it the land that forms the sea, or is it the sea that makes the land? Are the salts that now are found in sea-water washed out of the land by the atmospheric water? Has the sea existed from the beginning of the earth? and has it slowly but continually given its elements to form the land?

To try to give an answer to these most important questions, let us suppose that any river, for instance the Rhine, had its outlet into a valley with no communication with the sea, it would be filled with water until its surface was so great, that the annual evaporation was equal to the quantity of water which the river carried into it; then there would be a physical equilibrium but no chemical, because all the water that was carried into the lake would contain different mineral substances, which the rain-water had dissolved from the country which the river drains, while the loss by evaporation would be pure water. The quantity of saline substances in the lake would constantly go on increasing until chemical changes would occasion the precipitation of different salts. By comparing the chemical constitution of the water of the Rhine, we might form an idea of the different elements contained in the water of this lake. We should find that among the bases the lime was prevailing, and next to it the magnesia, next to it the soda, the iron, the manganese, the alumina, and potash. Of acids the carbonic would be prevailing, and next to it the sulphuric, the muriatic (chlorine), and the silicic. Now all these substances are found in sea-water, but the proportions are quite different.

---

* The potash which I have mentioned here represents in fact not the mean of all the observations in the great ocean, but only the mean of a number of determinations for the northern part of the Atlantic, my older observations on the quantity of potash in the other parts of the ocean being not exact enough. This quantity of potash differs most probably very little from the real mean.

The ocean is in fact such a lake, into which all the rivers carry what they have dissolved from the land, and from which pure water evaporates; and whatever we think about the constitution of the primitive ocean, this effect of the rivers, which has lasted for thousands of years, must have had an influence upon the sea. Why do we not observe a greater influence of the rivers? Why does not lime, the prevailing base of river-water, occur in a greater proportion in the water of the ocean? In all river-water the number of equivalents of sulphuric acid is much smaller than that of lime, and yet we find in sea-water about three equivalents sulphuric acid to one of lime. There must thus be in sea-water a constantly acting cause that deprives it again of the lime which the rivers furnish, and we find it in the shell fishes, the corals, the bryozoa, and all the other animals which deposit carbonate of lime. From the proportion between sulphuric acid and lime in river-water and in sea-water, it is evident that these animals are able not only to deprive the water of its carbonate of lime, of which sea-water contains very little, but that they also must decompose the sulphate of lime, a decomposition which probably depends upon the carbonate of ammonia which is formed by the vital process of these animals. I have shown that a salt of ammonia occurs in sea-water, certainly in small quantities, which however does not signify much, since the ammonia is constantly absorbed by the sea-weeds. Thus it is a chemical action of small animals which constantly deprives the sea of its excess of lime.

Next to the lime we must consider the silica, which is a constant constituent of river-water, and the immense quantity of diatomaceæ, of infusoria, and sponges will account for the small quantity of it at any given time in sea-water. I shall name next the sulphuric acid. All the shells of shell fishes, all the carbonate of lime in the corals and bryozoa contain some sulphate of lime, about one per cent. or less, but all the sea-weeds attract a great quantity of sulphates, which by the putrefaction of the plants are changed into sulphurets; and the sulphurets give again their sulphur to the iron, both that which is dissolved in sea-water, and that which in the form of oxide, combined with clay and other earths, is mechanically suspended in the water of the sea, principally near the shores. Thus the sulphur is made insoluble and disappears from the brine. The magnesia enters in a small quantity into the shells and corals, but only a small quantity is thus abstracted from sea-water, and at last the soda and muriatic acid or chlorine form, as far as we know, by the pure chemical or organico-chemical action that takes place in the sea, no insoluble compound. Thus the quantity of the different elements in sea-water is not proportional to the quantity of elements which river-water pours into the sea, but inversely proportional to the facility with which the elements in sea-water are made insoluble by general chemical or organo-chemical actions in the sea; and we may well infer that the chemical composition of the water of the ocean in a great part is owing to the influence which general and organo-chemical decomposition has upon it, whatever may have been the composition of the primitive ocean. I shall, however, not dwell any longer on this side of the question, which deserves a much more detailed representation than I can give it here.

There is a more special decomposition of sea-water, which takes place exceptionally, but these exceptions are very frequent. They depend upon the organic beings that live in the sea, die, and decay in the sea, and are finally dissolved. Of these substances that have their origin from organic beings, I have already named ammonia; but there are other substances of organic origin, probably of a more complicated nature, which I have observed in the following way. If we pour one or two drops of a solution of hypermanganate of potash into fresh sea-water, which has no smell of sulphuretted hydrogen, we shall after a short time observe a change in the colour of the liquid, but it is hardly more than the first drop that is decomposed so soon after it has been mixed with sea-water. The next decomposition goes slower, and is only finished after the liquid has been boiled for some time. Now if we pour hypermanganate of potash into a very diluted solution of ammonia, it will be completely decomposed by warming the mixture to a slight degree. I suppose that the first action upon the hypermanganate depends upon the ammonia in sea-water, and the next, which is slower and requires boiling and a longer time of action, depends probably upon the other products of spontaneous decomposition of organic matter. Coinciding with these observations is the experience that sea-water taken near the surface decomposes a smaller quantity of hypermanganate than that which is taken from the depth. If it was ammonia that produced the decomposition, there is no reason why there should be less of it near the surface than in deep water, since it being combined with a strong acid (either sulphuric or muriatic) neither could be volatilized nor oxidized. If it was organic matter, it would be oxidized near the surface, on account of the absorbed oxygen of the atmosphere.

When this organic matter increases in sea-water near the shores, or at the mouth of rivers, it will cause a real putrefaction, and attack the sulphates, converting them into sulphurets, which again are decomposed by the carbonic acid formed from the organic substances at the expense of the oxygen of the sulphuric acid. This sulphuretted hydrogen gets free, the carbonic acid will precipitate lime, and a loss of sulphuric acid by fermentation will always occasion a loss of lime in sea-water. Putrefaction seldom decomposes more than a small quantity of the sulphuric acid present in sea-water, and even where it seems to have been very powerful, not one-third part of the sulphuric acid has been destroyed. While thus a portion of the sulphates always remains undecomposed, there also seems always to remain a portion of the organic matter unoxidized. The sulphuretted hydrogen acts instantaneously upon hypermanganates, but when all smell of sulphuretted hydrogen has disappeared, there still remains some substance in putrefied sea-water which bleaches the hypermanganates when the water is boiled. It may be one of the lower oxides of sulphur, or it may be that the organic substance was not fully oxidized.

There is still one general effect of the organic substances dissolved in sea-water, that all iron is reduced from peroxide to protoxide, all mud from the deeper parts of the sea is dark coloured, either grey, bluish, or green. All Sir JAMES ROSS's deep soundings brought blue or green mud or sand to the surface.

In the following Tables the sulphuric acid, lime, magnesia, and potash are given both in parts per 1000 sea-water, and referred to chlorine as 100. The latter numbers are distinguished by being enclosed in parentheses.

First Region.—From the Equator to 30° N. lat.

	Chlorine.	Sulphuric acid.	Lime.	Magnesia.	All salts together.	Coefficient.
1. Sir James Ross, June 11, 1843. N. lat. 1° 10′, W. long. 25° 54′....................	19·757	2·303 (11·66)	0·584 (2·96)	2·333 (11·81)	35·737	1·801
2. Captain Irminger, September 9, 1847. Tocorady Bay, Guinea, 1 mile from the land...	19·584	2·315 (11·66)	0·765 (3·85)	2·179 (10·99)	35·803	1·803
3. Captain Irminger, September 7, 1847. N. lat. 4° 10′, W. long. 5° 36′	19·014	2·224 (11·64)	0·660 (3·47)	2·163 (11·37)	24·283	1·803
4. Sir James Ross, July 6, 1843. N. lat. 6° 43′, W. long. 27° 4′	20·070	......	......	......	36·327	......
5. Valkyrie, February 3, 1848. N. lat. 10°, W. long. 24° 19½′	19·766	2·415 (12·22)	0·568 (2·87)	2·117 (10·71)	35·941	1·818
6. Sir James Ross, July 11, 1843. N. lat. 11° 43′, W. long. 25° 6′	20·035	......	... ..	......	36·263	......
7. Sir James Ross, July 22, 1843. N. lat. 12° 36′, W. long. 25° 33′	20·114	2·343 (11·39)	0·619 (3·08)	2·315 (11·21)	36·195	1·800
8. Sir James Ross, July 25, 1843. N. lat. 15° 38′, W. long. 27° 15′..................	20·081	......	......	......	36·347	......
9. Sir James Ross, July 26, 1843. N. lat. 16° 57′, W. long. 29°	20·186	......	......	......	36·537	......
10. Sir James Ross, July 27, 1843. N. lat. 18° 6′, W. long. 29° 56′	20·429	......	......	......	36·976	......
11. Ornen, October 19, 1846. N. lat. 19° 20′, W. long. 65° 28′..................	19·818	2·376 (11·99)	0·567 (2·86)	2·123 (10·76)	35·775	1·805
12. Valkyrie, January 28, 1848. N. lat. 24° 13′, W. long. 23° 11′..................	20·898	2·446 (11·70)	0·595 (2·85)	2·280 (10·91)	37·908	1·814
13. Captain von Dockum, July 17, 1845. Between the Islands St. Croix and St. Thomas	19·650	2·309 (11·75)	0·567 (2·89)	2·236 (11·36)	35·732	1·819
14. Captain von Dockum, July 18, 1845. Likewise between the two islands................	17·798	2·304 (11·64)	0·426 (2·15)	2·195 (11·69)	35·769	1·807
15. Ornen, October 23, 1846. N. lat. 22° 43′, W. long. 65° 12′	20·320	2·423 (11·92)	0·602 (2·96)	2·208 (10·87)	36·784	1·810
16. Captain von Dockum, July 29, 1845. N. lat. 22° 30′, W. long. 69° 10′	20·145	2·344 (11·64)	0·554 (2·75)	2·164 (10·74)	36·508	1·812
17. Captain Irminger, March 17, 1849. N. lat. 25° 4′, W. long. 65° 40′	20·302	2·450 (12·07)	0·620 (3·05)	2·302 (11·34)	36·736	1·809
18. Captain von Dockum, July 30, 1845. N. lat. 23° 26′, W. long. 64° 8′	20·291	2·207 (10·88)	0·606 (2·99)	2·251 (11·09)	36·352	1·792
19. Ornen, October 28, 1846. N. lat. 29° 27′, W. long. 60° 1′	20·389	2·418 (11·86)	0·600 (2·94)	2·217 (10·87)	36·838	1·807
Mean	20·034	2·348 (11·75)	0·595 (2·98)	2·220 (11·11)	36·253	1·810
Maximum	20·898	2·450 (12·22)	0·765 (3·85)	2·333 (11·81)	37·908	1·819
Minimum	19·014	2·207 (10·88)	0·426 (2·15)	2·117 (10·71)	34·283	1·792

Second Region.—The Atlantic between 30° N. lat. and a line from the northernmost point of Scotland to the north point of Newfoundland.

*	Chlorine.	Sulphuric acid.	Lime.	Magnesia.	All salts together.	Coefficient.
1. Captain von Dockum, August 3, 1843. N. lat. 31° 51′, W. long. 67° 23′	20·159	2·449 (12·15)	0·607 (3·01)	2·460 (12·20)	36·480	1·810
2. Captain von Dockum, August 3, 1843. N. lat. 32° 52′, W. long. 68°. To the west of the Bermudas	20·064	2·489 (12·15)	0·566 (2·82)	2·062 (10·28)	36·635	1·826
3. Captain Schulz, September 28, 1860. Straits of Gibraltar	20·160	2·302 (11·42)	0·610 (3·03)	2·134 (10·59)	36·391	1·805
4. Ornen, November 5, 1846. N. lat. 36° 13′, W. long. 55° 7′	20·080	2·398 (11·94)	0·600 (2·98)	2·250 (11·20)	36·304	1·808
5. Captain von Dockum, August 6, 1843. N. lat. 36° 52′, W. long. 66° 38′. North from Bermudas in the Gulf-stream	19·890	2·336 (11·74)	0·595 (2·99)	2·299 (11·66)	35·883	1·804
6. Ornen, November 7, 1846. N. lat. 37° 5′, W. long. 48° 24′	20·103	2·518 (12·52)	0·643 (3·15)	2·177 (10·83)	36·643	1·823
7. Captain von Dockum, August 7, 1843. N. lat. 37° 24′, W. long. 61° 8′	19·943	2·374 (11·90)	0·595 (2·98)	2·284 (11·45)	36·105	1·810
8. Ornen, November 9, 1846. N. lat. 38° 18′, W. long. 43° 2′	20·247	2·557 (12·63)	0·689 (3·40)	2·260 (11·16)	36·928	1·824
9. Captain von Dockum, August 13, 1843. N. lat. 39° 39′, W. long. 55° 16′	20·063	2·432 (12·12)	0·588 (2·93)	2·208 (11·01)	36·283	1·808
10. Captain von Dockum, August 13, 1843. N. lat. 40° 21′, W. long. 54° 15′	20·098	2·425 (12·07)	0·606 (3·02)	2·391 (11·90)	36·360	1·809
11. Ornen, November 11, 1846. N. lat. 40° 53′, W. long. 36° 23′. S.W. from the Newfoundland Bank	20·062	2·427 (12·10)	0·718 (3·58)	3·123 (10·58)	36·389	1·814
12. Captain von Dockum, August 17, 1843. N. lat. 43° 26′, W. long. 44° 19′	18·685	2·208 (11·82)	0·534 (2·86)	2·081 (11·14)	33·854	1·812
13. Captain von Dockum, August 18, 1843. N. lat. 44° 33′, W. long. 42° 34′. E. from the Newfoundland Bank	18·842	2·236 (11·87)	0·560 (2·97)	2·079 (11·03)	34·102	1·810
14. Ornen, November 13, 1846. N. lat. 44° 39′, W. long. 30° 20′	19·890	2·376 (11·95)	0·650 (3·27)	2·154 (10·83)	36·032	1·812
15. Ornen, November 15, 1846. N. lat. 46° 22′, W. long. 22° 55′	19·857	2·400 (12·09)	0·582 (2·93)	2·185 (11·01)	36·010	1·813
16. Ornen. N. lat. 47° 10′, W. long. 18° 45′	19·892	2·400 (12·09)	0·586 (2·94)	2·175 (10·94)	36·090	1·814
17. Ornen. N. lat. 47° 17′, W. long. 14° 24′	19·722	2·441 (12·38)	0·590 (2·99)	2·166 (10·98)	35·872	1·819
18. Captain von Dockum. N. lat. 47° 17½′, W. long. 19° 9′	19·656	2·346 (11·94)	0·580 (2·95)	2·170 (11·04)	35·625	1·812
19. Captain von Dockum. N. lat. 47° 18′, W. long. 21° 6½′	19·915	2·413 (12·12)	0·587 (2·95)	2·172 (10·91)	36·119	1·814
20. Captain von Dockum. N. lat. 47° 40′, W. long. 32° 7′	19·860	2·327 (11·72)	0·583 (2·94)	2·265 (11·40)	35·896	1·808
21. Captain Schulz. N. lat. 47° 45′, W. long. 9° 30′	19·664	2·556 (13·01)	0·589 (2·99)	2·273 (11·57)	35·922	1·823
22. Captain von Dockum. N. lat. 47° 50′, W. long. 33° 50′	19·749	2·320 (11·75)	0·601 (3·04)	2·183 (11·06)	35·597	1·803
23. Ornen. N. lat. 48° 10′, W. long. 9° 35′	19·882	2·393 (12·03)	0·726 (3·65)	2·077 (10·45)	36·093	1·815
24. Captain von Dockum. N. lat. 50° 3′, W. long. 11° 6′	19·691	2·336 (11·86)	0·572 (2·90)	2·208 (11·21)	35·570	1·806
25. Porcupine, mean of 5 analyses of surface-water taken between 51° 9′ and 55° 32′ N. lat., and 12° 11′ and 13° 59′ W. long.	19·662	2·342	0·566	2·205	35·613	1·811
Mean	19·828	2·389 (12·05)	0·607 (3·07)	2·201 (11·10)	35·932	1·812
Maximum	20·247	2·557 (13·01)	0·726 (3·65)	2·460 (12·20)	36·927	1·826
Minimum	18·685	2·208 (11·08)	0·534 (2·82)	2·062 (10·28)	33·854	1·791

Third Region.—The northern part of the Atlantic, between the northern boundary of the second region, and a line from the south-west point of Iceland to Sandwich Bay, Labrador.

	Chlorine.	Sulphuric acid.	Lime.	Magnesia.	All salts.	Coefficient.
1. Lieutenant Skibsted, 1844. W. long. 3° 15′, N. lat. 60° 25′	19·287	2·254 (11·68)	0·488 (2·51)	2·136 (11·07)	34·831	1·806
2. Captain Paludan, May 8, 1845. W. long. 5° 19′, N. lat. 60° 9½′	19·485	2·289 (11·75)	0·568 (2·92)	2·146 (11·01)	35·223	1·808
3. Captain Gram, May 5, 1845. W. long. 7° 52′, N. lat. 59° 50′	19·671	2·342 (11·91)	0·592 (3·01)	2·210 (11·23)	35·576	1·809
4. Captain Gram, 1845. W. long. 7° 20′, N. lat. 60° 20′	19·619	2·296 (11·70)	0·587 (2·99)	1·820 (9·28)	35·387	1·814
5. Captain Gram, May 7, 1845. W. long. 14° 7′, N. lat. 60° 9′....................	19·620	2·306 (11·75)	0·581 (2·96)	2·189 (11·16)	35·493	1·809
6. Captain Gram, 1845. W. long. 16° 32′, N. lat. 61°	19·558	2·285 (11·68)	0·581 (2·97)	2·330 (11·91)	35·281	1·804
7. Taken by an Unknown. W. long. 20½°, N. lat. 55¾°........................	20·185	2·336 (11·59)	0·699 (3·31)	2·241 (11·10)	36·480	1·807
8. Captain Gram, May 10, 1845. W. long. 20° 30′, N. lat. 59° 58′..................	19·560	2·294 (11·73)	0·584 (2·99)	2·214 (11·32)	35·291	1·804
9. Captain Paludan, May 10, 1845. W. long. 23° 3′, N. lat. 62° 15′	19·466	2·343 (12·04)	0·576 (2·96)	2·117 (10·88)	35·348	1·816
10. Captain Gram, May 15, 1845. W. long. 26° 23′, N. lat. 59° 50′..................	19·545	2·330 (11·92)	0·583 (2·98)	2·190 (11·20)	35·397	1·811
11. Captain Gram. W. long. 26° 37′, N. lat. 60° 30′	19·579	2·277 (11·63)	0·570 (2·91)	2·196 (11·22)	35·399	1·808
12. Captain Gram, September 1, 1845. W. long. 36°, N. lat. 58° 58′	19·386	2·365 (12·20)	0·578 (2·98)	2·135 (11·01)	34·990	1·805
Mean	19·581	2·310 (11·80)	0·528 (2·97)	2·160 (11·03)	35·391	1·808
Maximum	20·185	2·385 (12·50)	0·669 (3·31)	2·330 (11·98)	36·480	1·811
Minimum	19·287	2·254 (11·59)	0·488 (2·51)	1·820 (9·28)	34·831	1·804

Fourth Region.—The East Greenland Current.

	Chlorine.	Sulphuric acid.	Sulphuric acid. Chlorine =100.	All salts. Coefficient 1·813.
1. Colonel Schaffner, September 2, 1860. Faxefjord, Iceland. W. long. 24° 1′ 30″, N. lat. 64° 16′ 11′	19·517	2·360	12·09	35·385
2. Colonel Schaffner, September 3, 1860. W. long. 26° 24′, N. lat. 64° 30′	19·616	2·420	12·34	35·563
3. Colonel Schaffner, September 6, 1860. W. long. 27° 8′, N. lat. 64° 15′	19·579	2·382	12·17	35·495
4. Colonel Schaffner, September 8, 1860. W. long. 29° 36′, N. lat. 63° 25′	19·518	2·293	11·75	35·386
5. Colonel Schaffner, September 9, 1860. W. long. 27° 34′ 35″, N. lat. 63° 34′ 30″	19·545	2·300	11·77	35·435
6. Colonel Schaffner, September 9, 1860. W. long. 33° 22′ 45″, N. lat. 63° 24′............	19·442	2·341	12·04	35·248
7. Colonel Schaffner, September 10, 1860. W. long. 37° 31′ 30″, N. lat. 62° 47′............	19·491	2·291	11·75	35·337
8. Colonel Schaffner, September 11, 1860. W. long. 38° 18′, N. lat. 62° 16′ 34″............	19·469	2·309	11·86	35·297
9. Colonel Schaffner, September 13, 1860. In ice pack. W. long. 41° 45′, N. lat. 60° 48′ 40″*	16·831	1·995	11·85	30·515
10. Colonel Schaffner, September 14, 1860. W. long. 40° 56′, N. lat. 59° 49′	19·136	2·252	11·75	34·694
11. Captain Gram, May 18, 1845. W. long. 33° 32′, N. lat. 60° 23′*...............	19·512	2·385	12·22	35·390
12. Captain Gram, May 20, 1845. W. long. 39° 4′, N. lat. 59° 26′*	19·306	2·310	11·97	35·067
13. Captain Gram, May 22, 1845. W. long. 46° 1′, N. lat. 57° 57′*	19·365	2·305	11·90	35·038
Mean	19·458	2·329	11·97	35·278
Maximum	19·616	2·420	12·34	35·563
Minimum	19·136	2·252	11·75	34·694

* This observation in the pack is not used for determining the means. Observations 11, 12, 13 are complete analyses with a coefficient 1·814, 1·816, and 1·809; mean 1·813. This mean coefficient is used for calculating the quantity of all salts in Colonel SCHAFFNER's samples, where there was not enough for complete analysis.

Fifth Region.—Davis Straits and Baffin's Bay.

	Chlorine.	Sulphuric acid.	Lime.	Magnesia.	All salts.	Coefficient.
1. Captain Gram, May 26, 1845. N. lat. 60° 32′, W. long. 53° 11′	19·010	2·283 (12·01)	0·550 (2·89)	2·115 (11·13)	34·414	1·810
2. Captain Gram, June 2, 1845. N. lat. 62° 8′, close to the island ved Frederickshaab	18·317	2·161 (11·80)	0·551 (3·01)	2·036 (11·12)	33·109	1·808
3. Captain Gram, June 12, 1845. Close to the Killiksut Islands near Nanarsuit (about N. lat. 60°)............................	18·386	2·144 (11·66)	0·546 (2·97)	2·018 (10·98)	33·190	1·806
4. Dr. Kaiser, September 5, 1845. N. lat. 64° 41′, Davis Straits........................	18·251	2·131 (11·68)	0·455 (2·49)	2·140 (11·73)	32·926	1·804
5. Dr. Kaiser, September 4, 1845. N. lat. 66° 58′, about 30 English sea-miles from Greenland	17·818	2·187 (12·27)	0·496 (2·78)	2·005 (11·25)	32·304	1·813
6. Dr. Kaiser, August 30, 1845. N. lat. 68° 43′, W. long. 52° 45′, harbour of Egedesminde..................................	18·325	2·238 (12·21)	0·495 (2·70)	2·080 (11·35)	33·187	1·811
7. Dr. Kaiser, September 3, 1845. 8 sea-miles from Godhavn, Disco (about N. lat. 69° 50′)	18·401	2·255 (12·25)	0·455 (2·47)	2·008 (10·91)	33·446	1·818
8. Dr. Rink, July 5, 1849. N. lat. 69° 45′, 24 English sea-miles W. from Disco	18·524	2·268 (12·24)	0·530 (2·86)	2·109 (11·39)	33·595	1·814
Mean	18·379	2·208 (12·01)	0·510 (2·77)	2·064 (11·23)	32·281	1·811
Maximum	19·010	12·27	3·01	11·73	34·414	1·818
Minimum	17·818	11·66	2·47	10·91	32·304	1·804

Sixth Region.—The North Sea.

	Chlorine.	Sulphuric acid.	Lime.	Magnesia.	All salts.	Coefficient.
1. 1844. Between the Orkneys and Stavanger, in Norway	18·772	2·312 (12·31)	0·488 (2·59)	2·128 (11·33)	34·302	1·827
2. 1844. S.W. of Egernsund, Norway.....................	18·278	2·223 (12·14)	0·455 (2·49)	2·192 (11·98)	33·294	1·822
3. Captain von Dockum, September 16, 1845. In the Hooft in the deep channel near the Galloppers	19·282	2·351 (12·19)	0·560 (2·90)	2·166 (11·23)	35·041	1·817
4. Captain von Dockum, September 18, 1845. About forty-five English sea-miles W. from the lighthouse of Hanstholm.................	17·127	2·079 (12·09)	0·548 (3·19)	1·929 (11·26)	31·095	1·815
5. Captain von Dockum, September 18, 1845. Skagerack, between Hirtshals and the Skau.	18·131	2·141 (11·81)	0·565 (3·12)	2·037 (11·23)	32·674	1·802
6. Back, S. Heligoland. Analysis from Erdmann's Journal, Bd. 34, p. 185	16·830	2·008 (11·93)	0·485 (2·88)	1·866 (11·09)	30·530	1·814
Mean	18·070	2·185 (12·09)	0·517 (2·86)	2·053 (11·25)	32·823	1·816
Maximum	19·295	2·351 (12·31)	0·565 (3·19)	2·192 (11·98)	35·041	1·827
Minimum	17·127	2·008 (11·77)	0·455 (2·49)	1·866 (11·09)	30·530	1·808

Seventh Region.—The Kattegat and the Sound.

	Chlorine.	Sulphuric acid.	Lime.	Magnesia.	All salts.	Coefficient.
1845, April. North of Kullen. Current from the South	6·227	0·776 (12·46)	0·195 (3·13)	0·712 (11·43)	11·341	1·821
1845, April. North of the island of Anhalt. Current from the South	8·429	1·028 (12·09)	0·257 (3·02)	.....	......	......
1845, June. North of Kullen. Current from the North	9·376	1·178 (12·57)	0·393 (4·19)	0·986 (10·51)	17·254	1·840
1845, June. North of Anhalt. Current from the North	9·632	1·099 (11·41)	0·298 (3·10)	1·059 (10·99)	17·355	1·802
1844. Captain Skibsted. Kattegat	10·077	1·208 (11·54)	0·319 (2·78)	1·253 (11·31)	19·940	1·801
Elsinore. Mean of 134 observations between April 17 and September 11, 1846	12·827	......	......	......	23·243	......
1846, October 4. Copenhagen. Current from the South	5·966	0·750 (12·57)	0·196 (3·28)	0·620 (10·39)	10·869	1·822
Copenhagen. Mean of 7 observations between March 3 and April 21, 1852	8·742	......	......	......	15·841	......
Sandefjord, on the south-east coast of Norway. Analyzed by Professor Strecker	7·740	0·875 (11·30)	0·266 (3·44)	0·818 (10·59)	13·996	1·808
Mean	8·780	0·998 (11·94)	0·275 (3·29)	0·908 (10·86)	16·230	1·814
Maximum	12·827	1·278 (12·57)	0·393 (4·19)	1·253 (11·43)	23·243	1·840
Minimum	5·966	0·750 (11·30)	0·195 (2·78)	0·620 (10·39)	10·869	1·801

Eighth Region.—The Baltic.

	Chlorine.	Sulphuric acid.	Lime.	Potash.	Magnesia.	All salts.	Coefficient.
1. Bellona. N. lat. 58° 27′, E. long. 20°	3·863	0·489 (12·65)	0·136 (3·52)	0·066 (1·71)	0·447 (11·57)	7·061	1·828
2. Bellona. Between Hammershuus, on the Island of Bornholm, and Sandhammer in Sweden	4·079	0·514 (12·60)	0·126 (3·09)	0·094 (1·99)	0·436 (10·69)	7·481	1·834
3. Bellona. Between Oland and Gothland	3·991	0·527 (13·19)	0·137 (3·43)	0·075 (1·88)	0·480 (12·03)	7·319	1·834
4. Bellona. Entrance of the Bay of Finnland	3·833	0·472 (12·33)	0·145 (3·78)	0·068 (1·77)	0·508 (13·25)	6·933	1·809
5. Bellona. Bay of Finnland, between Hogland and Tysters	2·596	0·346 (13·31)	0·092 (3·54)	0·044 (1·69)	0·299 (11·52)	4·763	1·835
6. Bellona. Bay of Finnland, between Nervoe and Seskjeld	1·931	0·239 (12·38)	0·076 (3·91)	0·047 (2·43)	0·226 (11·70)	3·552	1·839
7. Bellona. Bay of Finnland, W. from Kronstadt	0·331	0·040 (11·95)	0·019 (5·81)	0·023 (0·69)	0·046 (13·90)	0·738	2·230
8. Bellona. Bay of Finnland. Merchant-harbour of Kronstadt	0·294	0·044 (14·97)	0·022 (7·49)	0·006 (0·21)	0·046 (15·65)	0·610	2·075
9. Svartklubben, to the North of Stockholm	3·265	0·407 (12·50)	0·132 (4·05)	0·056 (1·72)	0·403 (12·38)	5·919	1·836
Mean	2·687 (100·00)	0·342 (12·73)	0·098 (3·64)	0·053 (1·97)	0·321 (11·94)	4·931	1·835
Maximum	4·079 (100·00)	0·527 (14·97)	0·145 (7·49)	0·094 (2·43)	0·508 (15·65)	7·481	2·230
Minimum	0·294 (100·00)	0·040 (11·95)	0·019 (3·09)	0·006 (0·21)	0·046 (10·69)	0·610	1·809

Ninth Region.—The Mediterranean.

	Chlorine.	Sulphuric acid.	Lime.	Potash.	Magnesia.	All salts.	Coefficient.
1. **Heimdal**, Captain Schulz, Sept. 28, 1860. Straits of Gibraltar	20·160	2·302 (11·42)	0·610 (3·03)	0·415 (2·06)	2·134 (10·59)	36·391	1·805
2. Heimdal, Captain Schulz, Sept. 29, 1860. N. lat. 36° 9′, W. long. 4° 2′	20·235	2·583 (12·8)	0·613 (3·03)	0·345 (1·70)	2·305 (11·39)	37·014	1·829
3. Heimdal, Captain Schulz, Oct. 8, 1860. N. lat. 40° 28′, E. long. 1° 48′. Between the Balearic island and the Spanish coast	21·085	2·444 (11·59)	0·641 (3·04)	0·474 (2·25)	2·402 (11·39)	38·058	1·805
4. Heimdal, Captain Schulz, Oct. 10, 1860. N. lat. 41° 12′, E. long. 2° 23′	21·056	2·542 (12·07)	0·635 (3·02)	0·336 (1·60)	2·356 (11·19)	38·321	1·819
5. Heimdal, Captain Schulz, Oct. 12, 1860. N. lat. 42° 25′, E. long. 6° 0′. Between Barcelona and Corsica	21·217	2·458 (11·59)	0·629 (2·96)	0·428 (2·03)	2·379 (11·21)	38·290	1·805
6. Heimdal, Captain Schulz, Oct. 20, 1860. N. lat. 40° 25′, E. long. 11° 43′. Between Sardinia and Naples	21·139	2·652 (12·55)	0·660 (3·12)	0·492 (2·33)	2·322 (10·98)	38·654	1·828
7. Mr. Ennis, 1837. Malta	20·497	2·471 (12·06)	0·640 (3·12)	0·174	2·074 (10·12)	37·177	1·814
8. Heimdal, Captain Schulz, Nov. 13, 1860. N. lat. 36° 10′, E. long. 16° 10′. To the East of Malta	21·297	2·514 (11·8)	0·686 (3·22)	0·417 (1·96)	2·403 (11·28)	38·541	1·809
9. Heimdal, Captain Schulz, Oct. 23, 1860. N. lat. 37° 20′, E. long. 16° 32′. Between Malta and Greece	21·180	2·390 (11·29) Sulphuretted hydrogen.	0·597 (2·82)	0·304 (1·41)	2·392 (11·29)	38·013	1·795
10. Heimdal, Captain Schulz, Oct. 28, 1860. N. lat. 33° 34′, E. long. 24° 34′. Between Candia and the coast of Africa	21·718	2·517 (11·60)	0·677 (3·12)	0·392 (1·80)	2·447 (11·27)	39·257	1·808
11. The Mediterranean; exact place unknown. Calculated after an analysis in Violette and Archambault's 'Analyses chimiques'	20·900 Brom. 432 21·332	2·433 (11·64)	0·621 (2·97)	0·32	2·223 (10·64)	37·655	......
Mean	20·889	2·470 (11·82)	0·642 (3·08)	0·372 (1·78)	2·277 (10·90)	37·936	1·815
Maximum	21·718	2·652 (12·59)	0·622 (3·22)	0·492 (2·33)	2·447 (11·39)	39·257	1·829
Minimum	20·160	2·302 (11·42)	0·597 (2·82)	0·174	2·074 (10·12)	36·391	1·805

Remarks.—No. 9 is not taken in the calculation of the mean coefficient, on account of the decomposition of the sulphuric acid, which always lowers the coefficient; the small quantity of lime in No. 9 depends probably upon the same decomposition, the sulphate of lime being changed into sulphuret of calcium, which again, by carbonic acid and water, is decomposed into sulphuretted hydrogen and carbonate of lime, which is precipitated.

Tenth Region, A.—The Black Sea and the Sea of Assou.

	Chlorine.	Sulphuric acid.	Lime.	Magnesia.	All salts.	Coefficient.
1. Water from the Black Sea, 50 English miles from the Bosphorus. F.	9·963 (100·00)	1·167 (11·71)	0·420 (4·22)	1·259 (12·64)	18·146	1·821
2. Water from the Black Sea. Gobel	9·869 (100·00)	1·032 (10·46)	0·182 (1·84)	1·126 (11·41)	17·666	1·790
3. Water from the Sea of Assou. Gobel ...	6·569 (100·00)	0·674 (10·26)	0·128 (1·95)	0·672 (10·23)	11·880	1·808
Mean	8·800 (100·00)	0·958 (10·89)	0·243 (2·76)	1·019 (11·58)	15·897	1·806
Maximum	9·963 (100·00)	1·167 (11·71)	0·420 (4·22)	1·259 (12·64)	18·146	1·821
Minimum...............	6·569 (100·00)	0·674 (10·26)	0·128 (1·84)	0·672 (10·23)	11·880	1·790

B.—From the Caspian Sea.

1.	2·731 (100·00)	1·106 (40·50)	0·268 (9·81)	0·700 (25·63)	6·236	2·283
2. Baer. From Tuik Karaga. Analysis by Mehner, Baer (Caspische Studien)	5·741 (100·00)	2·316 (40·34)	0·373 (6·50)	1·240 (21·60)	14·000	2·439
3. Baer. Bay of Kaidak or Karassi. Analysis by Mehner, Baer (Caspische Studien)	23·976 (100·00)	10·112 (42·11)	1·432 (5·91)	4·657 (19·42)	56·814	2·370
4. Baer. Bay of Mertuyi Kultak. Analysis by Mehner, Baer (Caspische Studien)......	12·504 (100·00)	5·613 (44·89)	1·733 (13·86)	2·096 (16·76)	31·000	2·480
5. Baer. Bay of Krasnowood. Analysis by Mehner, Baer (Caspische Studien)	6·182 (100·00)	3·494 (56·52)	0·760 (12·29)	1·471 (23·80)	16·410	2·654
Mean	10·227	4·528 (44·27)	0·913 (8·93)	2·033 (19·88)	24·892	2·434
Maximum	23·976	10·112 (56·52)	1·733 (13·86)	4·657 (25·63)	56·814	2·654
Minimum	2·731	1·106 (40·34)	0·268 (5·91)	0·700 (16·76)	6·236	2·283

Eleventh Region.—The Atlantic, between the Equator and 30^{c} S. latitude.

	Chlorine.	Sulphuric acid.	Lime.	Magnesia.	All salts.	Coefficient.
1. Valkyrie, February 11, 1848. S. lat. 3° 19′, W. long. 25° 34′	20·003	2·312 (11·56)	0·596 (2·98)	2·235 (11·17)	36·084	1·804
2. Valkyrie, February 16, 1848. S. lat. 17° 9′, W. long. 33° 29′	20·491	2·465 (12·03)	0·598 (2·92)	2·218 (10·82)	37·155	1·813
3. Sir James Ross. S. lat. 22° 37′, W. long. 34° 57′.................	20·397	......	......	......	37·001	1·814
4. Dr. Fischer, 1846. S. lat. 23°′ 5′, W. long. 37° 15′	20·115	2·428 (12·07)	0·580 (2·88)	2·233 (11·10)	36·442	1·812
5. Dr. Fischer, 1846. S. lat. 28° 15′, W. long. 38° 26′.................	19·831	2·393 (12·07)	0·596 (3·01)	2·254 (11·37)	35·930	1·812
6. Captain Prevost, February 4, 1857. S. lat. 29° 14′, W. long. 47° 37′.................	20·049	2·379 (11·87)	0·563 (2·81)	2·253 (11·24)	36·261	1·809
7. Valkyrie, March 15, 1848. S. lat. 29° 13½′, W. long. 38° 26′	20·166	2·537 (12·58)	0·585 (2·90)	2·022 (10·03)	36·997	1·835
Mean	20·150	2·419 (12·03)	0·586 (2·91)	2·203 (10·96)	36·553	1·814
Maximum	20·491	2·537 (12·58)	0·598 (3·01)	2·254 (11·37)	37·155	1·835
Minimum	19·831	2·312 (11·56)	0·563 (2·81)	2·022 (10·03)	35·930	1·804

Twelfth Region.—The Atlantic between S. lat. 30° and the southernmost points of America and Africa.

	Chlorine.	Sulphuric acid.	Lime.	Magnesia.	All salts.	Coefficient.
Dr. Fischer, 1846. S. lat. 30° 45′, W. long. 42° 30′	19·809	2·329 (11·76)	0·583 (2·94)	2·234 (11·28)	35·807	1·808
Dr. Fischer, 1846. S. lat. 40° 30′, W. long. 40° 50′	19·237	2·253 (11·71)	0·582 (3·03)	2·156 (11·21)	34·774	1·808
Dr. Fischer, 1846. S. lat. 45° 20′, W. long. 48° 40′	19·154	2·194 (11·45)	0·557 (2·91)	2·135 (11·15)	34·526	1·803
Dr. Fischer, 1846. S. lat. 50° 31′, W. long. 52° 15′	18·909	2·245 (11·87)	0·518 (2·74)	2·190 (11·58)	34·151	1·806
Fregat Valkyrie, 1848. S. lat. 36° 11½′, W. long. 6° 39′	19·431	2·451 (12·61)	0·541 (2·78)	2·091 (10·76)	35·065	1·805
Fregat Valkyrie, 1848. S. lat. 37° 11½′, E. long. 12° 25½′..............	19·713	2·404 (12·19)	0·553 (2·81)	2·156 (11·04)	35·907	1·821
Mean	19·376	2·313 (11·94)	0·556 (2·87)	2·160 (11·15)	35·038	1·809
Maximum	19·809	2·451 (12·61)	0·583 (3·03)	2·234 (11·58)	35·907	1·821
Minimum	18·909	2·194 (11·45)	0·518 (2·74)	2·091 (10·76)	34·151	1·803
Captain Prevost*. S. lat. 35° 46′, W. long. 52° 57′	17·721	1·615 (9·10) Sulphuretted hydrogen.	0·448 (2·49)	1·899 (10·72)	34·489	1·946

---

* This sample has been left out in the calculation of the mean numbers because the quantity of sulphuric acid was greatly diminished by putrefaction.

Thirteenth Region.—The sea between Africa and the East Indian Islands.

	Chlorine.	Sulphuric acid.	Lime.	Magnesia.	All salts.	Coefficient.
1. Galathea, September 24, 1845. S. lat. 31° 54′, E. long. 72° 27′	19·753	2·361 (11·98)	0·600 (3·04)	2·207 (11·17)	35·802	1·812
2. Galathea, October 1, 1845. S. lat. 14° 14′, E. long. 83° 38′	19·498	2·341 (12·01)	0·569 (2·92)	2·105 (10·80)	35·381	1·814
3. Galathea, October 6, 1845. N. lat. 0° 19′, E. long. 84° 51′	19·381	2·334 (12·04)	0·591 (3·05)	2·005 (10·35)	35·169	1·815
4. Galathea, October 28, 1845. N. lat. 17° 20′, E. long. 88° 12′..................	14·289	1·724 (12·06)	0·446 (3·12)	1·699 (11·89)	25·879	1·818
5. Galathea, December 31, 1845. N. lat. 18° 17′, E. long. 90° 13′..................	17·838	2·131 (11·94)	0·543 (3·04)	1·944 (10·90)	32·365	1·814
6. Galathea. Nancovri on the Nicobar Islands...............	18·246	2·156 (11·81)	0·547 (3·00)	1·997 (10·94)	33·036	1·817
7. Galathea, May 13, 1846. S. lat. 4° 54′, E. long. 107° 15′, Sea of Java...	17·970	2·132 (11·88)	0·547 (3·07)	1·979 (11·01)	32·766	1·823
8. Valkyrie, April 14, 1848. S. lat. 38° 52′, E. long. 30° 31′	19·413	2·470 (12·72)	0·543 (2·80)	2·134 (10·99)	35·583	1·833
9. Valkyrie, April 19, 1848. S. lat. 36° 59′, E. long. 47° 23′	19·710	2·349 (11·92)	0·572 (2·90)	2·193 (11·13)	35·701	1·816
10. Valkyrie, April 26, 1848. S. lat. 35° 2′, E. long. 62° 52′	19·548	2·380 (12·17)	0·588 (3·01)	2·101 (10·75)	35·415	1·817
11. Valkyrie, May 14, 1848. S. lat. 1° 56′, E. long. 81° 5′......................	19·626	2·330 (11·87)	0·567 (2·89)	2·207 (11·24)	35·512	1·809
12. Valkyrie, May 21, 1848. N. lat. 12° 3′, E. long. 80° 8′......................	18·763	2·250 (11·99)	0·567 (3·02)	2·086 (11·12)	33·809	1·802
Mean	18·670	2·247 (12·04)	0·557 (2·98)	2·055 (11·01)	33·868	1·814
Maximum	19·753	2·470 (12·72)	0·600 (3·12)	2·207 (11·89)	35·802	1·833
Minimum	14·289	1·724 (11·81)	0·446 (2·80)	1·699 (10·35)	25·879	1·802

Fourteenth Region.—The sea between the S.E. shore of Asia, the East Indian and the Aleutic Islands.

	Chlorine.	Sulphuric acid.	Lime.	Magnesia.	All salts.	Coefficient.
1. Galathea, May 18, 1846. The Chinese Sea. S. lat. 0° 33′, E. long. 107° 22′..................	17·757	2·104 (11·85)	0·516 (2·90)	1·958 (11·03)	32·370	1·823
2. Galathea. N. lat. 4° 30′, E. long. 107° 16′..................	18·486	2·258 (12·21)	0·572 (3·03)	2·067 (11·19)	33·680	1·822
3. Galathea. N. lat. 25° 40′, E. long. 120° 50′	17·923	2·160 (12·05)	0·533 (2·97)	1·961 (10·94)	32·533	1·815
4. Galathea. N. lat. 30° 56′, E. long. 127° 30′	18·564	2·209 (11·90)	0·552 (2·97)	2·022 (10·89)	33·580	1·809
5. Galathea. N. lat. 30° 56′, E. long. 139° 39′	18·847	2·257 (11·98)	0·575 (3·05)	2·089 (10·08)	34·153	1·812
6. Galathea. N. lat. 38° 31′, E. long. 148° 27′	18·873	2·247 (11·90)	0·613 (3·25)	2·046 (10·84)	34·234	1·814
7. Galathea. N. lat. 38° 35′, E. long. 148° 44′	18·788	2·213 (11·78)	0·580 (3·09)	2·048 (10·90)	33·990	1·809
Mean	18·462	2·207 (11·95)	0·563 (3·05)	2·027 (10·93)	33·506	1·815
Maximum	18·873	2·258 (12·05)	0·613 (3·25)	2·089 (11·19)	34·234	1·823
Minimum	17·757	2·104 (11·78)	0·516 (2·90)	1·958 (10·84)	32·370	1·809

Fifteenth Region.—The sea between the Aleutic Islands and the Society Islands.

	Chlorine.	Sulphuric acid.	Lime.	Magnesia.	All salts.	Coefficient.
1. Galathea, September 11, 1846. N. lat. 38° 26', E. long. 172° 11'	18·908	2·195 (11·61)	0·545 (2·88)	2·066 (10·93)	34·157	1·806
2. Galathea, September 17, 1846. N. lat. 38° 42', W. long. 176° 53'	19·006	2·220 (11·68)	0·535 (2·82)	2·078 (10·93)	34·274	1·803
3. Galathea, September 21, 1846. N. lat. 37° 3', W. long. 160° 5'	19·244	2·243 (11·65)	0·555 (2·88)	2·110 (10·69)	34·715	1·804
4. Galathea, September 24, 1846. N. lat. 32° 8', W. long. 150° 17'	19·824	2·316 (11·68)	0·549 (2·83)	2·209 (11·14)	35·877	1·809
5. Galathea, October 5, 1846. Off Honolulu, Sandwich Islands	19·625	2·283 (11·63)	0·580 (2·95)	2·152 (10·96)	35·395	1·804
6. Galashea. Off Matuiti	19·943	2·326 (11·66)	0·610 (3·06)	2·224 (11·15)	36·051	1·808
7. Galathea. Off Borabora	19·917	2·347 (11·78)	0·623 (3·13)	2·252 (11·31)	36·061	1·805
Mean	19·495	2·276 (11·67)	0·571 (2·93)	2·156 (11·06)	35·219	1·807
Maximum	19·943	0·347 (11·78)	0·623 (3·13)	2·252 (11·31)	36·061	1·809
Minimum	18·908	2·195 (11·61)	0·535 (2·82)	2·066 (10·69)	34·157	1·803

Sixteenth Region.—The Patagonian current of cold Water.

	Chlorine.	Sulphuric acid.	Lime.	Magnesia.	All salts.	Coefficient.
1. Dr. Fischer. S. lat. 57° 27', W. long. 66° 57'	18·769	2·133 (11·37)	0·507 (2·70)	2·116 (11·27)	33·788	1·800
2. Dr. Fischer. S. lat. 52° 38', W. long. 76° 20'	18·796	2·210 (11·76)	0·546 (2·91)	2·048 (10·90)	33·969	1·807
3. Dr. Fischer. S. lat. 47° 40', W. long. 78° 25'	18·760	2·238 (11·89)	0·560 (2·98)	2·036 (10·85)	33·980	1·811
4. Dr. Fischer. S. lat. 38° 10', W. long. 78° 14'	18·768	2·226 (11·86)	0·563 (3·00)	2·100 (11·19)	23·932	1·808
5. Dr. Fischer. S. lat. 33° 54', W. long. 74° 23'	18·754	2·224 (11·86)	0·537 (2·86)	2·079 (11·09)	33·976	1·812
6. Captain Prevost. S. lat. 35° 22', W. long. 73° 49'	18·976	2·257 (11·89)	0·531 (2·80)	2·076 (10·94)	34·152	1·800
Mean	18·804	2·215 (11·78)	0·541 (2·88)	2·076 (11·04)	33·966	1·806
Maximum	18·976	2·257 (11·93)	0·563 (3·00)	2·116 (11·27)	34·152	1·812
Minimum	18·754	2·133 (11·37)	0·507 (2·70)	2·036 (10·85)	33·788	1·800

Seventeenth Region.—The South Polar Region.

	Chlorine.	Sulphuric acid.	Lime.	Magnesia.	All salts.	Coefficient.
1. Sir James Ross, January 30, 1841. S. lat. 77° 32', E. long. 188° 21'. Near the ice-barrier	15·748	1·834 (11·65)	0·498 (3·16)	1·731 (10·99)	28·565	1·814
2. Sir James Ross, February 25, 1841. S. lat. 74° 15', E. long. 167° 0'. Near Caulmans Island	8·477	1·053 (12·42)	0·251 (2·96)	·887 (10·46)	15·776	1·861
3. Sir James Ross, March 6, 1841. S. lat. 65° 57', E. long. 164° 34'	20·601	2·586 (12·55)	0·623 (3·02)	2·231 (10·83)	37·513	1·821
Mean*	14·942	1·824 (12·21)	4·57 (3·06)	1·616 (10·81)	27·285	1·826

---

* These mean numbers are uncertain, the number of observations being very limited, and so very different. I should think that the first observation will be a fair sample of South Polar water, and have preferred it to the mean of the three observations in the calculation of the means of the whole ocean.

Comparison of the Means of all the Regions of the Ocean (German Ocean, Kattegat, Baltic, Mediterranean, and Black Sea excepted).

	Chlorine.	Sulphuric acid.	Lime.	Magnesia.	All salts.	Coefficient.
I. The Atlantic between the equator and N. lat. 30°	20·034	2·348 (11·75)	0·595 (2·98)	2·220 (11·11)	36·253	1·810
II. The Atlantic between N. lat. 30° and a line from the north point of Scotland to Newfoundland	19·828	2·389 (12·05)	0·607 (3·07)	2·201 (11·10)	35·932	1·812
III. The northernmost part of the Atlantic...	19·581	2·310 (11·80)	0·528 (2·97)	2·160 (11·03)	35·391	1·808
IV. The East Greenland Current	19·458	2·329 (11·97)	....	....	35·278	1·813
V. Davis Straits and Baffin's Bay	18·379	2·208 (12·01)	0·510 (2·77)	2·064 (11·23)	33·281	1·811
XI. The Atlantic between the equator and S. lat. 30°	20·150	2·419 (12·03)	0·586 (2·91)	2·203 (10·96)	36·553	1·814
XII. The Atlantic between S. lat. 30° and a line from Cape Horn to the Cape of Good Hope	19·376	2·313 (11·94)	0·556 (2·87)	2·160 (11·15)	35·038	1·809
XIII. The Ocean between Africa, Borneo, and Malacca	18·670	2·247 (12·04)	0·557 (2·98)	2·055 (11·01)	33·868	1·814
XIV. The Ocean between the S.E. coast of Asia, the East Indian, and the Aleutic Islands	18·462	2·207 (11·95)	0·563 (3·05)	2·027 (11·98)	33·506	1·815
XV. The Ocean between the Aleutic and the Society Islands	19·495	2·276 (11·67)	0·571 (2·93)	2·156 (11·06)	35·219	1·807
XVI. The Patagonian cold-water current ...	18·804	2·215 (11·78)	0·541 (2·88)	2·076 (11·04)	33·966	1·806
XVII. The South Polar Sea	15·748	1·834 (11·65)	0·498 (3·16)	1·731 (10·99)	28·565	1·814
Mean	18·999	2·258	0·556	2·096	34·404	1·811
Mean proportion of the most important substances in sea-water, chlorine=100	....	11·88	2·93	11·03		
Equivalents	429	45	16	82		

Comparison between the quantity of Salt in the water of the surface and the depth of the Sea, between Africa and the East Indies.

	Depth.	Chlorine.	Sulphuric acid.	Lime.	Magnesia.	All salts.	Coefficient.
Valkyrie, May 14, 1848. S. lat. 1° 56′, E. long. 81° 5′	Surface	19·626	2·330 (11·87)	0·567 (2·89)	2·207 (11·25)	35·512	1·809
	215 feet	19·606	2·451 (12·50)	0·558 (2·85)	2·147 (10·75)	35·819	1·827
Valkyrie, April 28, 1848. S. lat. 35° 2′, E. long. 62° 52′	Surface	19·548	2·349 (12·02)	0·588 (3·01)	2·101 (10·75)	35·415	1·817
	300 feet	19·786	2·380 (1 203)	0·572 (2·89)	2·218 (11·21)	35·671	1·803

Comparison between the quantity of Salt in the water of the surface and the depth of the Sea, between the East Indian and Aleutic Islands.

	Depth.	Chlorine.	Sulphuric acid.	Lime.	Magnesia.	All salts.	Coefficient.
Galathea, August 27, 1846. N. lat. 38° 31′, E. long. 148° 27′ ...	Surface	18·873	2·178 (11·54)	0·615 (3·26)	2·046 (10·84)	34·052	1·804
	300 feet	19·075	2·249 (11·79)	0·543 (2·85)	2·132 (11·18)	34·426	1·805
Galathea, May 23, 1846. N. lat. 4° 30′, E. long. 107° 16′ ...	Surface	18·846	2·258 (11·98)	0·572 (3·04)	2·067 (10·97)	34·132	1·811
	360 feet	18·885	2·195 (11·62)	0·567 (3·00)	2·147 (11·38)	34·033	1·802

Comparison between the quantity of Salt in Sea-water from different depths in the South Atlantic Ocean.

Samples taken by Sir James Ross.	Depth.	Chlorine.	Sulphuric acid.	Lime.	Magnesia.	All salts.	Coefficient.
Sir James Ross, June 10, 1844. S. lat. 0° 15′, W. long. 25° 54′	900 feet	19·763	2·584 (13·07)	0·657 (3·32)	2·249 (11·38)	36·165	1·830
	1800 feet	19·991	2·456 (12·29)	0·566 (2·83)	2·191 (10·96)	36·358	1·819
	4500 feet	19·786	2·398 (12·12)	0·554 (2·80)	2·320 (11·73)	35·689	1·814
	5400 feet	20·007	2·418 (12·09)	0·574 (2·87)	2·187 (10·93)	36·313	1·815
Sir James Ross, June 2, 1843. S. lat. 14° 22′, W. long. 22° 35′	3600 feet	19·743	......	......	......	......	......
Sir James Ross, June 4, 1843. S. lat. 15° 23′, W. long. 23° 40′	2700 feet	19·346	......	......	......	......	......
Sir James Ross, June 8, 1843. S. lat. 21° 48′, W. long. 31° 24′	900 feet	19·604	......	......	......	......	......
Sir James Ross, June 9, 1843. S. lat. 22° 24′, W. long. 32° 53′	3600 feet	19·627	......	......	......	......	......
Sir James Ross, June 10, 1843. S. lat. 22° 37′, W. long. 34° 57′	Surface	20·397	......	......	......	......	......
	900 feet	20·323	......	......	......	......	......
	1800 feet	23·189	......	......	......	......	......
	2700 feet	20·331	......	......	......	......	......
	3600 feet	20·405	......	......	......	......	......
Valkyrie, March 15, 1848. S. lat. 29° 15′·5, W. long. 38° 26′ ...	Surface	20·166	2·537 (12·58)	0·585 (2·90)	2·022 (10·03)	36·997	1·835
	480 feet	19·736	2·448 (12·40)	0·573 (2·90)	2·023 (10·25)	36·227	1·835
Sir James Ross, March 28, 1843. S. lat. 43° 10′, Long. 14° 44′ φ	6300 feet	19·635	2·346 (11·95)	0·631 (3·21)	2·140 (10·90)	35·607	1·813
Sir James Ross, Dec. 21, 1840. S. lat. 57° 52′, Long. 170° 30′ φ ...	Surface	19·396	2·293 (11·82)	0·624 (3·22)	2·108 (10·87)	35·131	1·811
Sir James Ross, March 6, 1841. S. lat. 65° 57′, Long. 164° 37′ φ ...	Surface	20·600	2·586 (12·55)	0·623 (3·02)	2·231 (10·83)	37·513	1·821
Sir James Ross, January 25, 1841. S. lat. 74° 15′, Long. 167° 0′ φ	Surface	8·477	1·053 (1·242)	0·251 (2·96)	0·887 (10·46)	15·776	1·861

Comparison between the quantity of Salt in Sea-water from the surface and different depths in the North Atlantic Ocean.

Samples taken by Sir James Ross, Dr. Rink, Mr. Gram, Captain Schulz, and Admiral von Dockum.	Depth.	Chlorine.	Sulphuric acid.	Lime.	Magnesia.	All salts.	Coefficient.
Dr. Rink, July 5, 1849. W. from Disco. N. lat. 69° 45′......	Surface	18·524	2·268 (12·24)	0·530 (2·86)	2·119 (11·39)	35·595	1·814
	420 feet	18·532	......	0·542 (2·92)	2·098 (11·32)	......	......
Merchant-Capt. Gram, May 20, 1845 W. long. 39° 4′, N. lat. 59° 45′ ...	Surface	19·306	2·310 (11·97)	0·575 (2·98)	2·119 (10·98)	35·067	1·816
	270 feet	19·364	2·337 (12·07)	0·579 (2·99)	2·186 (11·28)	34·963	1·806
Merchant-Capt. Gram, May 5, 1845. W. long. 7° 52′, N. lat. 59° 50′	Surface	19·671	2·342 (11·91)	0·592 (3·01)	2·210 (11·23)	35·576	1·809
	270 feet	19·638	2·338 (11·91)	0·598 (3·05)	2·210 (11·25)	35·462	1·806
Between Iceland and Greenland. Mean	Surface	......	......	......	......	35·356	......
Ditto, Mean of eight samples from	1200 to 1800 feet	......	......	......	......	35·057	......
Captain Schulz, R.D.N., 1845. W. long. 9° 30′, N. lat. 47° 45′......	Surface	19·644	2·556 (13·01)	0·589 (3·00)	2·273 (11·57)	35·925	1·829
	390 feet	19·640	2·595 (13·21)	0·623 (3·17)	2·357 (12·00)	35·925	1·829
	510 feet	19·699	2·594 (13·17)	0·628 (3·19)	2·296 (11·66)	36·033	1·829
Admiral von Dockum, Aug. 13, 1845. W. long. 54° 15′, N. lat. 40° 21′ ...	Surface	20·098	2·425 (12·07)	0·606 (3·02)	2·391 (11·90)	36·360	1·809
	210 to 270 feet	20·172	2·425 (12·02)	0·605 (3·00)	2·261 (11·21)	36·598	1·814
Captain Irminger, March 17, 1849. W. long. 64°, N. lat. 25° 40′.........	Surface	20·302	2·450 (12·07)	0·620 (3·05)	2·301 (11·33)	36·705	1·808
	2880 feet	20·222	2·380 (11·77)	0·581 (2·87)	2·274 (11·26)	36·485	1·804
Sir James Ross July 29, 1843. W. long. 32° 10′, N. lat. 20° 54′ ...	2700 feet	20·238	......	......	......	......	......
	3600 feet	19·703	......	......	......	......	......
Sir James Ross, July 27, 1843. W. long. 29° 56′, N. lat. 18° 16′ ...	Surface	20·429	......	......	......	......	......
	3600 feet	19·666	......	......	......	......	......
Sir James Ross, July 26, 1843. W. long. 29° 0′, N. lat. 16° 57′......	Surface	20·186	......	......	......	......	......
	900 feet	20·029	......	......	......	......	......
	2700 feet	19·602	......	......	......	......	......
Sir James Ross, July 25, 1843. W. long. 28° 10′, N. lat. 15° 38′ ...	Surface	20·081	......	......	......	......	......
	6360 feet	19·747	......	......	......	......	......
Sir James Ross, July 24, 1843. W. long. 27° 15′, N. lat. 14° 18′ ...	900 feet	19·934	......	......	......	......	......
	2700 feet	19·580	......	......	......	......	......
	3600 feet	19·705	......	......	......	......	......
Sir James Ross, July 22, 1843. W. long. 25° 35′, N. lat. 12° 36′ ...	Surface	20·114	2·343 (11·65)	0·619 (3·08)	2·315 (11·51)	36·195	1·800
	1850 feet	19·517	2·271 (11·64)	0·598 (3·06)	2·128 (10·90)	35·170	1·802
Sir James Ross, July 11, 1843. W. long. 25° 6′, N. lat. 11° 43′......	Surface	20·035	......	......	......	......	......
	3600 feet	19·855	......	......	......	......	......
	4500 feet	19·723	......	......	......	......	......
Sir James Ross, July 6, 1843. W. long. 27° 4′, N. lat. 6° 55′	Surface	20·070	......	......	......	......	......
	900 feet	19·956	......	......	......	......	......
	3600 feet	19·885	......	......	......	......	......
Sir James Ross, 1843. W. long. 25° 54′, N. lat. 1° 10′......	Surface	19·757	2·303 (11·66)	0·584 (2·96)	2·333 (11·81)	35·737	1·809
	1800 feet	19·715	2·265 (11·49)	0·547 (2·77)	2·253 (11·43)	35·520	1·802
	3600 feet	19·548	2·322 (11·88)	0·545 (2·79)	2·239 (11·45)	35·365	1·809

Comparison of water from the surface and the depth of the North Atlantic

	Depth.	Chlorine.	Sulphuric acid.	Lime.	Potash.
Porcupine. N. lat. 51° 1½′, W. long. 14° 21′ ... Sp. gr. 1·0270.	2370 feet	19·677	2·343 (11·91)	0·556 (2·83)	0·442 (2·24)
Porcupine, June 25, 1862. N. lat. 50° 56′, W. long. 12° 6′ ... Sp. gr. 1·0282.	6000 feet	19·776	2·376 (12·01)	0·610 (3·08)	0·381 (1·93)
Porcupine, June 27, 1862. N. lat. 51° 9′, W. long. 15° 59′ ... Sp. gr. 1·0280.	Surface	19·690	2·285 (11·60)	0·577 (2·93)	0·433 (2·20)
Porcupine, July 3, 1862. N. lat. 52° 9′, W. long. 15° 10′ ... Sp. gr. 1·0265.	Surface	19·706	2·381 (12·08)	0·570 (2·89)	0·367 (1·86)
Porcupine, July 3, 1862. N. lat. 52° 9′, W. long. 15° 10′ ... Sp. gr. 1·0280.	5100 feet	19·752	2·297 (11·73)	0·580 (2·94)	0·433 (2·19)
Porcupine, Aug. 29, 1862. N. lat. 51° 58′, W. long. 12° 47′ ... Sp. gr. 1·0280.	2400 feet	19·666	2·323 (11·81)	0·611 (3·11)	0·364 (1·85)
Porcupine, August 28, 1862. N. lat. 52° 40′, W. long. 15° 58′ ...	Surface	19·645	2·339 (11·91)	0·583 (29·7)	0·335 (1·71)
	10,500 feet	19·758	2·423 (12·26)	0·563 (2·90)	0·325 (1·64)
Porcupine. N. lat. 53° 1½′, W. long. 12° 55′ ... Sp. gr. 1·0280.	Surface	19·651	2·352 (11·97)	0·557 (2·83)	0·374 (1·90)
	1200 feet	19·424	2·405 (12·38)	0·559 (2·83)	0·351 (1·81)
Porcupine, August 16, 1862. N. lat. 55° 32′, W. long. 12° 11′ ... Sp. gr. 1·0255.	Surface	19·616	2·359 (11·99)	0·545 (2·78)	0·325 (1·65)
	9780 feet	19·686	2·330 (11·84)	0·599 (3·04)	0·323 (1·64)
Mean of surface observations	....	19·662	2·342 (11·91)	0·566 (2·88)	0·367 (1·87)
Mean of observations from the depth ...	....	19·677	2·357 (11·98)	0·583 (2·96)	0·374 (1·90)

Water from the Red Sea, and from different depths in the Baltic.

	Depth.	Chlorine.	Sulphuric acid.	Lime.	Potash.
Water from the Red Sea. Procured by Mr. Polack of Alexandria	....	23·730	2·889 (12·17)	0·689 (2·90)	0·387 (1·63)
From Wady Rarandel, upon the Sanai peninsula, taken by Mr. Neergaard ...	....	23·171	2·761 (11·92)		
Baltic. Water from Svartklubleen, taken by Messrs. Widegreen and Nyström	Surface	3·256	0·407 (12·50)	0·132 (4·05)	0·056 (1·71)
	108 feet	3·663			
	240 feet	3·881			
	300 feet	3·912			
	510 feet	3·969			
	600 feet	3·958	0·565 (14·27)	0·137 (3·46)	0·058 (1·47)
	720 feet	3·960			
	948 feet	3·977			

Sea between lat. N. 51° 1′½ and 55° 32′; and long. W. 12° 6′ and 15° 59′.

Magnesia.	Silica, &c.	Chloride of sodium.	Sulphate of magnesia.	Sulphate of lime.	Chloride of potassium.	Chloride of magnesium.	All salts.	Coefficient.
2·211 (11·24)	0·110	27·977	2·376	1·353	0·700	3·212	35·728	1·816
2·211 (11·18)	0·100	28·056	2·279	1·483	0·603	3·344	35·865	1·814
2·235 (11·35)	0·074	27·735	2·213	1·402	0·686	3·438	35·548	1·805
2·226 (11·30)	0·105	28·005	2·373	1·385	0·581	3·305	35·754	1·814
2·179 (11·03)	0·071	28·119	2·298	1·409	0·685	3·206	35·788	1·812
2·175 (11·06)	0·071	27·914	2·193	1·487	0·575	3·330	35·570	1·809
2·128 (10·83)	0·071	28·139	2·279	1·418	0·531	3·145	35·583	1·811
2·209 (11·18)	0·078	28·188	2·451	1·369	0·517	3·203	35·806	1·812
2·145 (10·92)	0·113	28·119	2·355	1·354	0·592	3·131	35·664	1·815
2·183 (11·24)	0·104	27·740	2·432	1·359	0·555	3·158	35·348	1·820
2·225 (11·34)	0·088	27·916	2·379	1·326	0·517	3·298	35·524	1·811
2·182 (11·08)	0·069	28·081	2·253	1·457	0·511	3·261	35·632	1·810
2·192 (11·15)	0·090	27·983	2·320	1·377	0·581	3·263	35·615	1·811
2·193 (11·14)	0·086	28·011	2·326	1·417	0·592	3·245	35·677	1·813

Water from the Red Sea, and from different depths in the Baltic.

Magnesia.	Silica, &c.	Chloride of sodium.	Sulphate of magnesia.	Sulphate of lime.	Chloride of potassium.	Chloride of magnesium.	All salts.	Coefficient.
2·685 (11·31)	0·136	33·871	2·882	1·676	0·612	3·971	43·148	1·818
0·403 (12·38)	0·027	4·474	0·329	0·322	0·089	0·678	5·919	1·818
0·441 (11·14)	0·072	5·810	0·632	0·333	0·092	0·526	7·465	1·886

Water from the Mediterranean.—Comparison between water from the surface and from different depths.

	Depth.	Chlorine.	Sulphuric acid.	Lime.	Potash.	Magnesia.	Silica,*&c.	All salts.	Coefficient.
1. Straits of Gibraltar, procured by Mr. Ennis, Falmouth, 1837	Surface.	20·046	2·221 (11·08)	0·676 (3·37)	0·155 (0·77)	2·133 (10·64)	0·057	36·147	1·803
2. Straits of Gibraltar, taken by Captain Schulz, September 28, 1860..........	Surface.	20·160	2·302 (11·42)	0·610 (3·03)	0·415 (2·06)	2·134 (10·59)	0·073	36·391	1·805
3. Straits of Gibraltar, taken by Captain Schulz, September 28, 1860, from 540 feet depth	540 feet	20·330	2·425 (11·93)	....	....	....	....	....	....
4, 5*. A little on the Mediterranean side of the Straits, N. lat. 36° 9′, W. long. 4° 2′, September, 29, 1860..........	Surface.	20·235	2·583 (12·77)	0·613 (3·03)	0·345 (1·70)	2·305 (11·39)	0·093	37·014	1·829
	Depth *	21·119	2·493 (11·80)	....	....	....	....	....	....
6, 7*. Between the Balear island and the Spanish coast, N. lat. 40° 28′, E. long. 1° 48′, October 8, 1860..........	Surface.	21·085	2·444 (11·59)	0·641 (3·04)	0·474 (2·25)	2·402 (11·39)	0·083	38·058	1·805
	Depth *	21·207	2·746 (12·95)	0·664 (3·13)	0·354 (1·67)	2·317 (10·97)	0·138	38·946	1·836
8, 9*. Between the Balear island and the Spanish coast, N. lat. 41° 12′, E. long. 2° 23′, October 10, 1860	Surface.	21·056	2·542 (12·07)	0·635 (3·02)	0·336 (1·60)	2·356 (11·19)	0·087	38·321	1·820
	Depth *	21·211	2·513 (11·85)	....	....	....	....	....	....
10, 11. About midway between Corsica and Barcelona, N. lat. 42° 25′, E. long. 6° 0′, October 12, 1860..........	Surface.	21·217	2·458 (11·59)	0·629 (2·96)	0·428 (2·02)	2·379 (11·21)	0·075	38·290	1·805
	420 feet	21·304	2·500* (11·73)	....	....	....	....	....	....
12, 13. Between Sardinia and Naples, N. lat. 40° 25′, E. long. 11° 43′, October 20, 1860..........	Surface.	21·139	2·652 (12·55)	0·660 (3·12)	0·492 (2·33)	2·322 (10·98)	0·080	38·654	1·829
	300 feet	21·100	2·610 (12·37)	....	....	....	....	....	....
14. Malta, procured by Mr. Ennis, 1837	Surface.	20·497	2·471 (12·06)	0·640 (3·12)	0·174	2·074 (10·12)	0·080	37·177	1·814
15, 16. Somewhat to the east of Malta, N. lat. 36° 10′, E. long. 16° 10′, November 13, 1860..........	Surface.	21·297	2·514 (11·85)	0·686 (3·22)	0·417 (1·96)	2·403 (11·29)	0·118	38·541	1·810
	390 feet	21·311	2·515 (11·80)	....	....	....	....	....	....
17, 18. Between Malta and Greece, N. lat. 37° 20′, E. long. 16° 32′, October 23, 1860..........	Surface*.	21·180	2·390 (11·28)	0·597 (2·82)	0·304 (1·44)	2·392 (11·29)	0·029	38·013	1·795
	522 feet	21·290	2·510 (11·79)	....	....	....	....	....	....
19, 20. Between Candia and the coast of Africa, N. lat. 33° 34′, E. long. 24° 34′, October 28, 1860	Surface.	21·718	2·517 (11·59)	0·677 (3·12)	0·392 (1·80)	2·447 (11·27)	0·098	39·257	1·808
	522 feet	21·521	2·524 (11·73)	....	....	....	....	....	....
Mean of surface observations..........	....	20·845	2·470 (11·85)	0·647 (3·10)	....	2·296 (11·01)	....	37·785	1·813
Mean of observations of deep water	....	21·155	2·537 (11·99)	....	....	....	....	....	....
Mean of the surface of the ocean	....	18·999	2·258 (11·88)	0·556 (2·93)	....	2·096 (11·03)		34·404	1·811

* The depth in samples 5, 7, 9 is not exactly noticed, but it must have been between 300 and 540 feet.

V. *On the Magnetic Character of the Armour-plated Ships of the Royal Navy, and on the Effect on the Compass of particular arrangements of Iron in a Ship.* *By* FREDERICK JOHN EVANS, *Esq., Staff Commander R.N., F.R.S., Superintendent of the Compass Department of Her Majesty's Navy; and* ARCHIBALD SMITH, *Esq., M.A., F.R.S., late Fellow of Trinity College, Cambridge, Corresponding Member of the Scientific Committee of the Imperial Russian Navy.*

Received March 9,—Read March 16, 1865.

THE present paper may be considered as a sequel to a paper published in the Philosophical Transactions for 1860, page 337, under the title "Reduction and Discussion of the Deviations of the Compass observed on board of all the Iron-built Ships, and a selection of the Wood-built Steam-ships in Her Majesty's Navy, and the Iron Steam-ship 'Great Eastern'; being a Report to the Hydrographer of the Admiralty. By F. J. EVANS, Master R.N." Like the former, the present paper is presented to the Royal Society, with the sanction of the Lords Commissioners of the Admiralty.

In the brief interval which has elapsed since the publication of that paper, changes of the greatest importance have taken place in the construction of vessels of war, which have been accompanied by corresponding changes in the magnetic disturbance of their compasses. Not only has there been a great increase in the surface and mass of iron used in the construction of those parts of the ship in which iron was formerly used, but iron has been adopted for many purposes for which it was not then used, and much of the iron thus added far exceeds in thickness any that was formerly in use. Among the masses thus added we may specially mention iron masts and yards, armour-plating, and gun-turrets.

These changes have materially affected the problem of the correction of the deviation of the compass. They have not only greatly increased those errors which were formerly taken into account, but they have given importance to errors and causes of error which it was formerly considered might be safely neglected. These changes led to, if they did not necessitate, a complete revision of the mathematical theory of the deviations of the compass, and of the practical methods of ascertaining and applying the deviation.

This revision was undertaken by us at the request of the Admiralty, and the results are contained in the 'Admiralty Manual for ascertaining and applying the Deviations of the Compass caused by the Iron in a Ship,' published by the order of the Lords Commissioners of the Admiralty. London: POTTER, 1862. Second edition, 1863. It is gratifying to us to be able to state, as an indication that this work has been found useful by others engaged in the like investigations, that it has been already translated into Russian, French, and German.

The methods of reduction previously in use, and which are those made use of in the paper already referred to, as well as in the valuable Reports of the Liverpool Compass Committee, are those deduced from the approximate formula for the deviation,

$$\delta = A + B \sin \zeta' + C \cos \zeta' + D \sin 2\zeta' + E \cos 2\zeta',$$

as given in the Supplement to the 'Practical Rules for ascertaining the Deviations of the Compass which are caused by the Ship's Iron,' published by the Admiralty in 1855.

In connexion with this formula use was made of the invaluable graphic method known as NAPIER'S curve.

At that time observations of horizontal and vertical force did not enter into the usual routine of observations made on board ship, although many very valuable observations of these forces had been made by the Liverpool Compass Committee; and no formulæ had been published for the deduction from such observations of any of the parts of the deviation. This will explain why, in the paper of 1860, the discussion was confined to the coefficients which are derived from observations of deviation only, viz. A, B, C, D, E.

The new modes of construction brought into prominence the diminution of mean directive force which a compass-needle suffers in an iron ship, particularly when placed between two iron decks. It is well known that in the interior of a thick iron shell the effect of the earth's magnetic force is nearly insensible. This is not caused by the iron of the shell *intercepting* the earth's magnetism, but by an opposite magnetism being induced which nearly neutralizes the earth's magnetism whatever be the inductive capacity of the shell, and whatever be the thickness of the shell, provided only that the thickness bears a considerable proportion to the diameter of the shell. When the shell is thin, the diminution of force is still considerable, but it then depends in a very much greater degree on the inductive capacity and the thickness of the shell. The destruction of force is total in the case of a spherical shell whatever be its thickness, if the inductive capacity be infinite.

An iron ship, as regards a compass-needle between decks, may be compared to a thin iron shell. Before the ship is launched, and when every particle of iron in her structure has by continued hammering become saturated with magnetism, she may be compared to a thin shell of high inductive capacity, and the directive force on a needle in the interior is consequently greatly diminished. When the ship is launched and placed successively on every azimuth, she may be compared to a thin shell of low inductive capacity. The mean directive force on a needle in her interior will be considerably diminished, but the diminution will depend much more on the thickness of the surrounding iron.

This diminution has been found so considerable in the case of iron-built and particularly iron-plated ships, as to have become a matter of serious consideration in selecting a place for the compasses.

Observations of horizontal force, for the purpose of ascertaining the diminution of the mean directive force, have now become part of the regular series of observations made in ships in which its determination is of importance, and formulæ and graphic methods, for the purpose of deducing from them the proportion of the mean value of the directive force to North to the earth's horizontal force, are given in the 'Admiralty Manual.'

Another error of the greatest importance, which has been brought into prominence in the modern class of iron-built ships, is the "heeling error."

The deviations obtained by the usual process of swinging are for a vessel in an upright position. It is found by experience that, as the vessel heels over, the north end of the compass-needle is drawn either to the weather or lee side, generally in the northern hemisphere to the former, and the deviation so produced when the ship's head is near North or South, often exceeds the angle of heel. This not only produces a deviation which may cause a serious error in the ship's course, but if the ship is rolling, and particularly if the period of each roll approximates to the period of oscillation of the compass, it produces a swinging of the compass-needle which may make the compass for the time useless for steering.

This error had been known to exist, and its amount had even been measured in the case of Her Majesty's ships Recruit (1846), Bloodhound (1847), Sharpshooter (1848), and in various cases recorded by the Liverpool Compass Committee (1855–61); but no method had been proposed for determining this error by observations made with the ship upright, and considerable obscurity was even supposed to rest on the causes and law of this deviation. The application of Poisson's formulæ has entirely removed the obscurity, and furnishes an easy method of determining the heeling error by observations of vertical force made on one or more directions of the ship's head. These observations have likewise now become a regular part of the complete series of magnetic observations made in the principal iron ships of Her Majesty's Navy.

Fortunately the mechanical correction of this error, when its amount is ascertained, is not difficult, and as the correction does not affect the deviation when the ship is upright, its application is free from some of the objections which exist to the mechanical correction of the ordinary deviation.

The importance of being thus able to detect the heeling error by observations of a simple kind made with the ship upright is great, and this is perhaps one of the most practically useful of the immediate results of the application of mathematical formulæ to this subject.

Besides these, which may be called the direct results of the additional observations now made, and of the application to them of the mathematical formulæ, there are some other results of the use of the formulæ which have a practical value as well as a theoretical interest.

Among these is the separation into their constituent parts of the several coefficients, so as to indicate the particular arrangements of the iron from which each arises. This is not only of great theoretical interest, but is of considerable practical importance in indicating the place which should be selected for the compass, and also in enabling us to anticipate or account for the subsequent changes which take place in the deviation.

Another and perhaps even more important result is that we are enabled by observations made with the ship's head in one direction, and therefore when she is in dock or even on the stocks, to determine the coefficients and construct a table of deviations, including the heeling error, without swinging the ship. To explain this, we may observe that for the complete determination of the deviations of the compass when the ship is upright and in one geographical position, six coefficients are required. But of these two vanish when the iron is symmetrically arranged, two more are so nearly the same in ships of the same class that they can be estimated with a near approximation to the truth; we have therefore only two coefficients left, and these can be determined by an observation of deviation, and an observation of horizontal force made without altering the direction of the ship's head.

So as regards the heeling error, to determine this three additional quantities are generally necessary, but of these one is zero when the iron is symmetrically arranged; another may be estimated, and the third may then be determined by a single observation of vertical force.

The quantities so estimated change little after the ship is completed, so that any assumption made as to their value may be checked by subsequent observations.

These considerations will show the importance of not only making the observations we have mentioned, but of reducing the observations made, and of tabulating, discussing, and publishing the results of the observations. In the Tables it will be seen that the original observations are not given; they, as well as the curves and computations by which the coefficients are derived, are carefully preserved among the records of the Admiralty Hydrographic Office, and may at any time be referred to; but the coefficients, at least so far as regards the deviation of the horizontal needle, represent so exactly the observations made, that to give them here at length would be an unnecessary waste of space.

The observations, the results of which are tabulated, were made in the following manner. The deviations of the Standard Compass were observed by reciprocal simultaneous bearings of the Standard Compass and an azimuth compass on shore, in the manner described in the 'Admiralty Manual.' The admirable construction of the Admiralty Standard Compass, as regards design and workmanship, accuracy of adjustment and magnetic power, leaves nothing further to be desired for such observations. The arrangement of its four needles obviates, as we have shown in a former paper*, the sextantal error caused by the length of the needle when acted on by iron placed near it.

The deviations of the steering and maindeck compasses were obtained by observations of the direction of the ship's head by those compasses, made simultaneously with the observations of the Standard Compass. These compasses in the Royal Navy are of simpler construction than the Standard, not being fitted with the azimuth circle, and generally having only two needles, but they are of little inferior accuracy, magnetic power and delicacy. The two needles are arranged so as to obviate the sextantal error above alluded to.

---

* Philosophical Transactions, Part II. 1862.

The Tables of deviations of these compasses have in all cases been most satisfactory, and on those points on which the directive force is very much diminished, they continue to give satisfactory indications which compasses of inferior workmanship would wholly fail to do.

The observations of horizontal force were made by vibrating a small flat lenticular needle $2\frac{3}{4}$ inches long and $\frac{1}{3}$ inch broad, fitted with a sapphire cap, on a pivot of its own, made to screw into the socket of the pivot of the Standard Compass, and comparing the time of vibration with that of the same needle vibrated on shore.

The observations of vertical force were made by vibrating a dipping-needle of $2\frac{3}{4}$ inches, placed in the position of the compass, the needle being made to vibrate in a vertical plane at right angles to the magnetic meridian. The observation might of course be made by vibrating the needle in the plane of the meridian and observing the dip; and in low dips that method is probably the best. In so high a dip as that of England, vibrations in the east and west plane are sufficiently accurate, and enable us to dispense with observations of dip.

In the selection of these instruments it has been found of great importance that they should be light, portable, easily and quickly fixed in position, capable of being placed in the exact position of the compass, should admit of observations being made quickly and in rough and boisterous weather, and should be such that each separate observation should give a useful result.

When the observer can command favourable circumstances of observation, as in the case of observations made in a ship on the stocks, it is possible that instruments of greater nicety may give more exact results, but for the ordinary observations which can be made in the process of swinging a ship, we have every reason to be satisfied with the results obtained from the instruments we have described.

As the formulæ made use of in the reductions are nowhere published except in the 'Admiralty Manual,' it seems necessary here to give them with a brief indication of the manner in which they are obtained.

The effect of the iron of a ship on the compass-needle is assumed to be due partly to the transient magnetism induced in the soft iron by the magnetism of the earth, and partly to the permanent magnetism of the hard iron. Simple physical considerations show that the components of the first in any three directions in the ship are linear functions of the components of the earth's magnetism in the same directions, the last is expressed by constant forces acting in the same three directions.

If, therefore, the components of the earth's force on the compass be X in the direction of the ship's head, Y to starboard, Z vertically downwards or to nadir, and if the components of the ship's permanent magnetism in the same directions be P, Q, and R, and of the total force of earth and ship in the same three directions X′, Y′, Z′, then

$$\text{Ship's force to head} \quad = X' - X = aX + bY + cZ + P, \quad \ldots \quad (1)$$

$$\text{Ship's force to starboard} = Y' - Y = dX + eY + fZ + Q, \quad \ldots \quad (2)$$

$$\text{Ship's force to nadir} \quad = Z' - Z = gX + hY + kZ + R, \quad \ldots \quad (3)$$

$a, b, c, d, e, f, g, h, k$ being coefficients depending on the amount and arrangement of the soft iron of the ship. These are POISSON'S fundamental equations, first given in the Mémoires de l'Institut, tom. v. p. 533.

To adapt these formulæ to observation, let

H be the earth's horizontal force,
ζ the easterly azimuth of the ship's head measured from the correct magnetic north;
θ the dip.

Then $X = H \cos \zeta$, $Y = -H \sin \zeta$, $Z = H \tan \theta$.

Substituting these values, and dividing (1) and (2) by H, *i. e.* taking the earth's horizontal force at the place as unit, equations (1) and (2) become

$$\text{Ship's force to head} \quad = \frac{X' - X}{H} = a \cos \zeta - b \sin \zeta + c \tan \theta + \frac{P}{H}. \quad \ldots \quad (4)$$

$$\text{Ship's force to starboard} = \frac{Y' - Y}{H} = d \cos \zeta - e \sin \zeta + f \tan \theta + \frac{Q}{H}. \quad \ldots \quad (5)$$

Dividing (3) by Z, *i. e.* taking the earth's vertical force as unit, we have

$$\text{Force of earth and ship to nadir} = \frac{Z'}{Z} = \frac{g}{\tan \theta} \cos \zeta - \frac{h}{\tan \theta} \sin \zeta + 1 + k + \frac{R}{Z}. \quad \ldots \quad (6)$$

If we resolve the forces (4) and (5) in the direction of the magnetic north, we shall find, besides periodical terms, one non-periodical term $\frac{a+e}{2}$, which therefore represents the mean force of the ship to North, and therefore $\left(1 + \frac{a+e}{2}\right) H = \lambda H$, is the "mean force to North," or the mean value of the northern component of the force of earth and ship.

If we take the "mean force to North,' or λH for unit, or, in other words, divide by λH, we derive from (4) and (5) the following expressions for the force of earth and ship to North and to East respectively, viz.

$$\text{To North} = \frac{H' \cos \delta}{\lambda H} = 1 + \mathfrak{B} \cos \zeta - \mathfrak{C} \sin \zeta + \mathfrak{D} \cos 2\zeta - \mathfrak{E} \sin 2\zeta, \quad \ldots \quad (7)$$

$$\text{To East} = \frac{H' \sin \delta}{\lambda H} = \mathfrak{A} + \mathfrak{B} \sin \zeta + \mathfrak{C} \cos \zeta + \mathfrak{D} \sin 2\zeta + \mathfrak{E} \cos 2\zeta, \quad \ldots \quad (8)$$

in which H′ is the directive force of earth and ship on the needle, δ the deviation.

$$\lambda=1+\frac{a+e}{2},\quad \mathfrak{A}=\frac{d-b}{2\lambda},\quad \mathfrak{B}=\frac{1}{\lambda}\left(c\tan\theta+\frac{P}{H}\right),$$

$$\mathfrak{D}=\frac{a-e}{2\lambda},\quad \mathfrak{E}=\frac{d+b}{2\lambda},\quad \mathfrak{C}=\frac{1}{\lambda}\left(f\tan\theta+\frac{Q}{H}\right).$$

From equations (7) and (8) we obtain

$$\tan\delta=\frac{\mathfrak{A}+\mathfrak{B}\sin\zeta+\mathfrak{C}\cos\zeta+\mathfrak{D}\sin 2\zeta+\mathfrak{E}\cos 2\zeta}{1+\mathfrak{B}\cos\zeta-\mathfrak{C}\sin\zeta+\mathfrak{D}\cos 2\zeta-\mathfrak{E}\sin 2\zeta}, \quad\ldots\ldots\quad (9)$$

whence if ζ' be the azimuth of the ship's head measured from the direction of the disturbed needle so that $\zeta'=\zeta-\delta$,

$$\sin\delta=\mathfrak{A}\cos\delta+\mathfrak{B}\sin\zeta'+\mathfrak{C}\cos\zeta'+\mathfrak{D}\sin(2\zeta'+\delta)+\mathfrak{E}\cos(2\zeta'+\delta). \quad\ldots\quad (10)$$

If the deviations are small, we have approximately

$$\delta=A+B\sin\zeta'+C\cos\zeta'+D\sin 2\zeta'+E\cos 2\zeta', \quad\ldots\ldots\quad (11)$$

in which A, B, C, D, E are (nearly) the arcs of which $\mathfrak{A}$, $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, $\mathfrak{E}$ are the sines.

The term $\mathfrak{B}\sin\zeta'+\mathfrak{C}\cos\zeta'$ may be put under the form $\sqrt{\mathfrak{B}^2+\mathfrak{C}^2}\sin(\zeta'+\alpha)$, in which α, called the starboard angle, is an auxiliary angle such that $\tan\alpha=\frac{\mathfrak{C}}{\mathfrak{B}}$.

If the soft iron of the ship be symmetrically arranged on each side of the fore-and-aft line of the ship through the compass, then

$$\begin{array}{lll} b=0, & d=0, & f=0, \\ \mathfrak{A}=0, & \mathfrak{E}=0, & \\ A=0, & E=0. & \end{array}$$

If we put $\mu=1+k+\frac{R}{Z}$, the expression of the nadir force of earth and ship in terms of earth's vertical force as unit, is

$$\text{Nadir force}=\frac{Z'}{Z}=\frac{g}{\tan\theta}\cos\zeta-\frac{h}{\tan\theta}\sin\zeta+\mu, \quad\ldots\ldots\quad (12)$$

If the ship heels over to starboard an angle i, $\mathfrak{B}$ and $\mathfrak{D}$ (or B and D) remain unaltered; and representing the altered values of $\mathfrak{A}$, $\mathfrak{C}$ and $\mathfrak{E}$ by $\mathfrak{A}_i$, $\mathfrak{C}_i$, and $\mathfrak{E}_i$, we have

$$\mathfrak{A}_i=\mathfrak{A}-\frac{g-c}{2\lambda}i,$$

$$\mathfrak{E}_i=\mathfrak{E}-\frac{g+c}{2\lambda}i,$$

$$\begin{aligned}\mathfrak{C}_i&=\mathfrak{C}-\left(\mathfrak{D}+\frac{\mu}{\lambda}-1\right)\tan\theta\, i\\ &=\mathfrak{C}-\chi i.\end{aligned}$$

The alteration in $\mathfrak{A}$ and $\mathfrak{E}$ may generally be neglected; that in $\mathfrak{C}$ is often of great importance. The quantity $\chi=\left(\mathfrak{D}+\frac{\mu}{\lambda}-1\right)\tan\theta$ is called the heeling coefficient, and represents the degrees of deviation to windward, or the high side of the ship, produced by a heel of one degree when the ship's head is North or South by the disturbed compass.

The effect of the coefficients on the deviation is most easily seen by considering the effect of the derivative coefficients λ, $\mathfrak{A}$, $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, $\mathfrak{E}$, and of the heeling coefficients, which, for convenience of reference, are here arranged in a tabular form. These are as follows:—

$\lambda=1+\frac{a+e}{2}$ is a factor generally less than 1, giving the northern component of the mean directive force on the needle, or "mean force to North."

$\mathfrak{A}=\frac{d-b}{2\lambda}$ (approximate value in degrees $=$A) is the constant term of the deviation; its real value is 0 when the iron is symmetrically placed on each side of the compass, and it is not in general distinguishable from an index error of the compass, or an error in the assumed variation of the compass (declination).

$\mathfrak{B}=\frac{1}{\lambda}\left(c\tan\theta+\frac{\mathrm{P}}{\mathrm{H}}\right)$ (approximate value in degrees $=$B) is the maximum of semicircular deviation from fore-and-aft forces; $\frac{c}{\lambda}\tan\theta$ arises from soft iron; $\frac{\mathrm{P}}{\lambda\mathrm{H}}$ from hard iron.

$\mathfrak{C}=\frac{1}{\lambda}\left(f\tan\theta+\frac{\mathrm{Q}}{\mathrm{H}}\right)$ (approximate value in degrees $=$C) is the maximum of semicircular deviation from transverse forces; $\frac{f}{\lambda}\tan\theta$ arises from soft iron, and is zero if the iron is symmetrically arranged; $\frac{\mathrm{Q}}{\lambda\mathrm{H}}$ from hard iron.

$\sqrt{\mathfrak{B}^2+\mathfrak{C}^2}$ (approximate value in degrees $=\sqrt{\mathrm{B}^2+\mathrm{C}^2}$) is the maximum of semicircular deviation.

$\frac{\mathfrak{C}}{\mathfrak{B}}$ is the tangent starboard angle, or of angle measured to right of fore and aft of line of ship, in which the force causing the semicircular deviation acts.

$\mathfrak{D}=\frac{a-e}{2\lambda}$ (approximate value in degrees $=$D) is the maximum of quadrantal deviation from soft iron symmetrically placed.

$\frac{\mathfrak{D}}{2}-\frac{1}{2}\left(\frac{1}{\lambda}-1\right)=\frac{a}{2\lambda}$ is the part of $\mathfrak{D}$ arising from fore-and-aft soft iron.

$\frac{\mathfrak{D}}{2}+\frac{1}{2}\left(\frac{1}{\lambda}-1\right)=-\frac{e}{2\lambda}$ is the part of $\mathfrak{D}$ arising from transverse soft iron.

$\mathfrak{E}=\frac{d+b}{2\lambda}$ (approximate value in degrees $=$E) is the maximum of quadrantal deviation from soft iron unsymmetrically placed.

$\left(\mathfrak{D}+\frac{\mu}{\lambda}-1\right)\tan\theta=\chi$ is the heeling coefficient, or the deviation to windward in degrees for one degree of heel when ship's head North or South by disturbed compass.

$\left(\mathfrak{D}+\frac{1}{\lambda}-1\right)\tan\theta$ is the part of heeling coefficient from transverse soft iron.

$\left(\frac{\mu}{\lambda}-\frac{1}{\lambda}\right)\tan\theta$ is the part of heeling coefficient from vertical soft iron, and vertical force of hard iron.

$\frac{g}{\tan\theta}$ is the increase or decrease of vertical force above or below mean when ship's head is North or South.

$$\frac{P}{\lambda}+\frac{c}{\lambda}H\tan\theta=\mathfrak{B}H,$$

$$\frac{P}{\lambda}+\frac{c}{\lambda}H'\tan\theta'=\mathfrak{B}'H'$$

are the equations for determining c and P separately when $\mathfrak{B}$ has been determined in two different latitudes;

$$\frac{c}{\lambda}=\frac{\mathfrak{A}_i-\mathfrak{A}_{i'}}{i-i'}-\frac{\mathfrak{E}_i-\mathfrak{E}_{i'}}{i-i'},$$

$$\frac{P}{\lambda}=\mathfrak{B}H-\frac{c}{\lambda}H\tan\theta$$

are equations for determining c and P separately when observations have been made in one geographical position, but on two different angles of heel;

$$\mathfrak{B}=\frac{1}{\lambda}\frac{H'}{H}\cos\zeta'-(1+\mathfrak{D})\cos\zeta,$$

$$\mathfrak{C}=-\frac{1}{\lambda}\frac{H'}{H}\sin\zeta'+(1-\mathfrak{D})\sin\zeta$$

are equations for determining $\mathfrak{B}$ and $\mathfrak{C}$ by observations of deviation and horizontal force on one azimuth of the ship's head, λ and $\mathfrak{D}$ being known or estimated.

There is a physical representation of POISSON'S fundamental equations so simple, and which gives us so great a power of estimating the effect on the compass of different arrangements of iron in a ship, as well as of tracing to their cause any peculiarities in the observed deviation, that it seems desirable, before entering on the peculiarities of structure and deviation in armour-plated ships, to explain this representation, and to show how it explains the phenomena of deviation.

If an infinitely thin straight rod of soft iron be magnetized by the induction of the earth, the effect will be the same as if each end became a pole having an intensity proportional to the component of the earth's force resolved in the direction of the rod, and to the section and capacity for induction of the rod.

Let us now suppose nine soft iron rods placed as Plate X. It will be seen that for each rod we must distinguish the two cases, that in which its coefficient is +, and that in which it is −. It will also be seen that in the three cases, viz. $-a$, $-e$, $-k$, in which the rod passes through the compass, we may consider both ends as acting, but that in other cases it is convenient to consider only the action of the near end, and that the far end is at an infinite distance.

The rod a, it will be observed, can only be magnetized by the component X, b only by Y, and c only by Z; and if we call aX, bY, and cZ the force with which these rods attract the north end of the needle, and if we suppose, as we are at liberty to do, the rods being imaginary, that they exercise no action on one another, a, b, and c will produce a force to head

$$=aX+bY+cZ;$$

so d, e, and f will produce a force to starboard

$$=dX+eY+fZ,$$

and g, h, and k will produce a force to nadir

$$=gX+hY+kZ.$$

By comparing these results with POISSON'S formulæ, we see that for the effect of the soft iron of the ship, however complicated its arrangement may be, we may substitute the nine soft iron rods.

The quantities P, Q, R in the general equations may be conveniently represented by three bar-magnets, placed in fixed positions in the ship; P attracting the north end of the compass-needle to the head, Q to starboard, and R to nadir.

Very simple considerations will show us that the two rods a and e will increase the directive power on the needle in the proportion of $1+\frac{a+e}{2}:1$, and that the other seven rods, as well as the permanent forces P, Q, R, will not affect the mean directive force.

Simple considerations will also show that a and e will produce a deviation,

$$\frac{a-e}{2\lambda}\sin 2\zeta'=\mathrm{D}\sin 2\zeta'$$

nearly. Like considerations will show that c and P will produce a deviation,

$$\frac{c\mathrm{Z}+\mathrm{P}}{\lambda\mathrm{H}}\sin\zeta'=\left(\frac{c}{\lambda}\tan\theta+\frac{\mathrm{P}}{\lambda\mathrm{H}}\right)\sin\zeta'=\mathrm{B}\sin\zeta'.$$

Also that f and Q will produce a deviation,

$$\frac{f\mathrm{Z}+\mathrm{Q}}{\lambda\mathrm{H}}\cos\zeta'=\left(\frac{f}{\lambda}\tan\theta+\frac{\mathrm{Q}}{\mathrm{H}}\right)\cos\zeta'=\mathrm{C}\cos\zeta'.$$

The other less important terms, as well as the heeling error, may be obtained in the same manner.

DISCUSSION OF THE TABLES.

At the risk of some repetition it may be convenient to give here a brief explanation of the quantities tabulated.

The first five quantities, A, B, C, D, E, are the "approximate coefficients" which give the deviation of the compass on every course by means of the expression

$$\delta=\mathrm{A}+\mathrm{B}\sin\zeta'+\mathrm{C}\cos\zeta'+\mathrm{D}\sin 2\zeta'+\mathrm{E}\cos 2\zeta',$$

in which δ is the deviation, ζ' the azimuth of the ship's head measured eastward from the direction of the disturbed needle, A, B, C, D, E being expressed in degrees and minutes.

This expression is sufficiently accurate for deviations not exceeding 20°; for larger deviations, the exact expression for the deviation given in the preceding part of the paper requires the use of the "exact coefficients" $\mathfrak{A}$, $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, $\mathfrak{E}$, which are not expressed in degrees and minutes, but are nearly the sines of the corresponding angles A, B, C, D, E.

For the purpose of this discussion we may confine our attention to A, B, C, D, E.

A is the "constant part of the deviation." A *real* value of A can only be caused by elongated horizontal masses of *soft* iron *unsymmetrically* arranged with reference to the compass, and would be the same in all parts of the globe. An arrangement of horizontal soft iron rods such as that in fig. 1 would give a positive value to A and no other term in the deviation. This, however, is not an arrangement which would occur on shipboard.

Fig. 1.

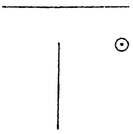
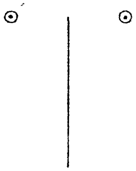


Fig. 2.



A soft iron rod such as that in fig. 2 would give +A to the starboard compass, combined with +E; and —A, combined with —E, to the port compass.

This arrangement is not unfrequent in the relative positions of the spindle of the steering-wheel and the binnacle compasses placed near it for the guidance of the helmsman.

In compasses placed in the middle line of the ship such an arrangement is improbable, and in such case A has probably little or no *real* value. An *apparent* value may, however, be given to A by index-error in the compass on board, index or other error in the shore compass with which it is compared, or error of observations generally.

When the ship heels over, an elongated horizontal mass of iron, which was symmetrically placed from being below the compass, as the screw-shaft or the keel, is thrown to one side, and an A may then be introduced caused by and proportional to the angle of heel; but this has not been found of sufficient amount to require attention in practice.

The terms $B \sin \zeta' + C \cos \zeta'$ make up together what is called the "semicircular deviation;" B depending on fore-and-aft forces, and having its zero when the ship's head is North or South, its maximum when it is East or West; C depending on transverse forces, and having its zero when the ship's head is East or West, its maximum when it is North or South.

B consists of two parts, one a coefficient arising from vertical induction in soft iron before or abaft the compass, and being multiplied by the tangent of the dip and a factor $\frac{1}{\lambda}$ hereafter explained; the other a coefficient arising from permanent magnetism of the hard iron in the ship acting in the fore-and-aft line, and multiplied by the reciprocal of the earth's horizontal force, and also by the factor $\frac{1}{\lambda}$. The last part may be considered as itself consisting of two parts; one, of the subpermanent magnetism induced while the ship was building by the vertical component of the earth's force, and which probably bears some relation to the transient magnetism induced by the same vertical component; another, of the subpermanent magnetism induced while the ship was building by the headward component of the earth's horizontal force.

C theoretically consists of similar parts acting towards the sides of the ship; but as the iron may in general be considered as symmetrically arranged on each side of the compass, the value of C is probably, in all cases when the ship is upright and the compass is amidships, to be attributed to subpermanent magnetism induced while the ship was building by the transverse component of the earth's horizontal force. The part of B consisting of transient induced magnetism varies as the tangent of the dip. The other part of B and C vary inversely as the earth's horizontal force. As regards changes which take place after launching, without a change of geographical position, there are differences between the several parts of B and C which require notice.

When the ship is launched, notwithstanding that her head is no longer kept in one fixed direction, the forces which cause the two first-mentioned parts of B still act in precisely the same direction as before, and these two parts probably undergo little change.

With the third part of B and the whole of C the case is very different. The forces which cause these parts cease to act in the same direction as at first. If the vessel is allowed to swing at her anchors, or is under sail or steam, she will probably on an average be nearly as much on one point as on another; or, which would come to nearly the same thing, if she is lying in a tideway she may be alternately for six hours in one direction and for six hours in the opposite direction. A great portion of the C and of that part of the B which arose from horizontal force thus become dispelled.

The symmetry which gives C its character ceases the moment the ship heels. An addition is then made to C proportional to the angle of heel, and this addition consists in fact of two parts, corresponding to the two parts of B which, as we have seen, do not exist in the original C, viz. a part consisting of transient magnetism induced by the vertical force, and a part consisting of subpermanent magnetism induced by the same force. These will be more conveniently considered when we come to discuss the heeling error.

The semicircular deviation may be put under the form $\sqrt{B^2+C^2}\sin(\zeta'+\alpha)$, in which $\sqrt{B^2+C^2}$ represents the maximum of semicircular deviation, $\alpha\left(\tan\alpha=\frac{C}{B}\right)$ the angle to the right of the ship's head of the force causing this deviation; for convenience, these two quantities are tabulated in the eleventh and thirteenth columns.

The terms $D\sin 2\zeta'+E\cos 2\zeta'$ make up what is called the "quadrantal deviation."

This can only be caused by *horizontal* induction in soft iron. E can only be caused by horizontal induction in soft iron *unsymmetrically* distributed, but of any shape; an E may therefore be caused by the compass being placed out of the midship line and exposed to the influence of spherical or cylindrical masses, such as the iron gun-turrets of modern war-vessels.

D, which in ordinary cases is always +, is caused by horizontal induction in soft iron arranged according to one or other of the following types:—

Fig. 3.

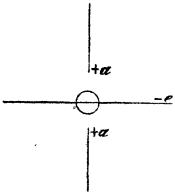
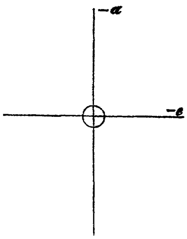


Fig. 4.



In the figures $+a$ represents masses of soft iron entirely before or entirely abaft the compass, as engines, boilers, funnels, iron masts, &c.; $-a$ represents soft iron extending through the position of the compass, as the keel and hull of the ship, the screw-shaft, armour-plating, &c., the effect of the latter in almost all cases exceeding that of the former, so that a is in general negative; $-e$ represents the effect of all the transverse soft iron, as the bottom of the ship, the iron decks (except where interrupted by hatchways near the compass), iron deck beams, and the engines, boilers, &c.; $+e$ represents the masses of iron, comparatively few in number, which lie to one side of the compass, as decks where the compass is in or over a hatchway, occasional guns, davits, &c. In every ship which has been examined, the effect of the transverse iron extending through the position of the compass exceeds that of any masses of iron wholly on one side, and e is negative and greater than a; and as $\mathfrak{D}=\frac{a-e}{2\lambda}$, $\mathfrak{D}$, and consequently D, are in almost all cases +.

D and E do not change with a change of geographical position.

In almost all cases in iron-built ships, not only is the direction of the needle directly affected by the iron of the ship, but a further prejudicial effect is caused by the soft iron diminishing the mean directive force of the needle, and so indirectly increasing the effect of all disturbing forces. This is shown by the factor λ, which gives the mean value of the directive force, or rather of the northern component of the directive force in the ship, and which is almost always less than unity, the force on shore being considered as unity.

The cause of this diminution will be seen by figs. 3 & 4. In fig. 4 a little consideration will show that both $-a$ and $-e$ diminish the directive force. In fig. 3 $+a$ increases the directive force, $-e$ diminishes it; but as $-e$ always exceeds $+a$, the result is a diminution on the whole.

The expression for λ in terms of a and e is

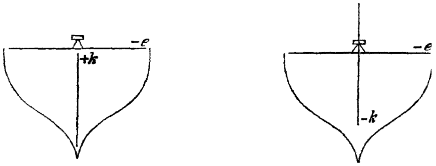
$$\lambda=1+\frac{a+e}{2}.$$

The tabulated values of λ are obtained by comparing the terms of vibration of a horizontal needle vibrated in the position of the compass in the ship and also on shore; λ does not change with a change of geographical position.

The determination of $\mathfrak{D}$ and λ gives us the means of determining the two parts a and e, and also the two parts of which D is composed, separately; and these are accordingly tabulated.

The preceding are the only coefficients which affect the compass when the ship is upright; but when the ship heels over, new disturbing forces are called into play, caused by arrangements of soft or hard iron of one or other of the following types:—

Fig. 5.



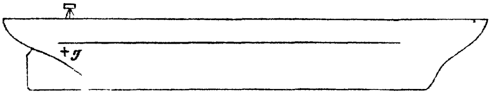
$-e$ represents, as before, the transverse soft iron, which will evidently, as the ship heels over, produce a force to windward, or the high side of the ship, on the north end of the needle. If the rods $+k$ and $-k$ represent soft iron, then $+k$ gives a force acting downwards on the north end of the needle, which, as the ship heels, becomes a force to windward; $-k$ a force acting upwards, which, as the ship heels, becomes a force to leeward. The permanent magnetism of the ship will generally act downwards if the compass is over the end which has been South in building, upwards if over the end which has been North in building. The amount of the two forces may be ascertained by vibrating a dipping-needle on shore and in the ship with her head in certain positions. The proportion of the mean vertical force on board to the vertical force on shore is denoted by the coefficient μ, which is tabulated for those ships in which the observations have been made.

From the values of μ, $\mathfrak{D}$ and λ we obtain by a simple formula, viz. $\left(\mathfrak{D}+\frac{\mu}{\lambda}-1\right)\tan\theta\,1^\circ$, the "heeling coefficient to windward," or the deviation to windward caused, when the ship's head is N. or S. by compass, by an angle of heel of 1°. When this coefficient has a negative sign it indicates a deviation to leeward. The values of the heeling coefficient so deduced are tabulated. The value changes with a change of geographical position.

From the values of μ, $\mathfrak{D}$ and λ we may also determine how much of the heeling error arises from the transverse soft iron represented in the figures 3, 4 & 5, and how much from the vertical soft iron and the hard iron, the first $=\left(\mathfrak{D}+\frac{1}{\lambda}-1\right)\tan\theta\, i^{\circ}$, the second $=\left(\frac{\mu}{\lambda}-\frac{1}{\lambda}\right)\tan\theta\, i^{\circ}$; and these two parts are tabulated in the next columns.

If we have not an opportunity of observing the vertical force on a sufficient number of points to obtain its mean value, the values observed will be affected by soft iron represented by the rod g, in the following figure:—

Fig. 6.



the value of μ on any azimuth ζ being in fact increased by $+\frac{g}{\tan\theta}\cos\zeta$, where θ is the dip. It is therefore convenient to know the values of g or $\frac{g}{\tan\theta}$, and these are also tabulated; g does not change with a change of geographical position.

In comparing the heeling error when the ship's head is North or South, we must beware of falling into the error of confusing the two senses in which these words may be used. It may seem most natural to suppose the ship's head to be North or South when upright, and that she is then heeled over without altering her direction. In that case we should have (nearly)

Heeling error head North : heeling error head South :: $1-\mathfrak{B}:1+\mathfrak{B}$.

In fact the heeling error is nearly inversely proportional to the directive force on the needle.

But this is not the sense in which the term is generally used. In general we suppose the ship swung when heeled to starboard and again when heeled to port, and the deviations tabulated in the usual way, according to the ship's azimuth by disturbed compass. In this case, which is the simplest mode of considering the error for the purpose of correction, the heeling error, head North, will only differ from the heeling error, head South, by reason of the quantity g, i. e. by reason of the difference of the vertical and not of the horizontal forces in the two positions.

The importance of the heeling error, owing to its large amount in certain ships, will be seen in the discussion of the values given in the Tables; and the importance of being able to determine it by observations easily made, and without the necessity of actually heeling over the ship, can hardly be overrated.

We are now in a position to consider the numerical values of the coefficients given in the Tables.

Constant Deviation.

A.

The values of A, when the compass is placed in the middle line of the ship, and when the deviations have been observed with every care, are always so small, that the values which appear in the Tables may be considered rather as errors of adjustment and observation than as real values. In fact it may be inferred that in all cases where the compass is in the middle line of the ship, we may consider A as zero. It results from this, and is important in practice, that we may safely take the mean of the compass bearings of any object, on four or more equidistant compass courses, as the *correct* magnetic bearing; observing, however, that if we observe on four points only, and D be large, these ought to be either the cardinal or the quadrantal points.

Semicircular Deviation,

$$B \sin \zeta' + C \cos \zeta'.$$

The points which require attention are,—

1. Its original value and its connexion with the direction of the ship in building, and the position of the compass in the ship.
2. The changes which take place after launching.
3. The subsequent changes.
4. The changes which take place on a change of geographical position.

1. In wood-built ships, as may be seen by an inspection of the Deviation Tables given in the work of the late Captain E. J. Johnson, R.N., on the deviation of the compass, the direction of the force causing the semicircular deviation is in northern latitudes nearly towards the ship's bow. In iron-built ships it is nearly to that part of the ship which was South in building; or, in other words, the starboard angle as given in the Tables, is nearly the same as the azimuth of the ship's head to the East of South in building; thus,—

	Direction of head in building.	Starboard angle, or direction of semicircular deviation.
Orontes . .	N. 66° W. or S. 246° E.	235°
Tamar . . .	West or S. 270° E.	279°

The case of the armour-plated ships is an interesting exception to this rule. Such ships are generally plated after launching, and in a different position from that of building. In these ships the angle of the semicircular force is generally intermediate between the angle of the ship's head to the East of South in building, and the like angle in being iron plated; thus,—

	Direction of head in building.		Direction of head in plating.		Direction of Semicircular Deviation.
Warrior . . .	N. 3° E.	or S. 177° E.	N.W.	or S. 225° E.	195°
Black Prince .	S. 20 E.	20	South.	0	8
Defence . . .	S. 47 W.	313	S. 19° E.	19	0
Resistance .	West.	270	generally to westward . . .		313
Valiant . . .					282

From these results we may infer that the process of plating an iron ship in the direction opposite to that of building will always produce a diminution, which in some cases may become a reversal of her semicircular deviation; and that by duly taking advantage of this circumstance, the deviations of iron-plated ships may be brought within manageable limits.

The Tables show, as might have been anticipated, the much larger amount of the deviation in the steering and main-deck compasses than in the Standard Compass, and the advantages to be derived from a judicious selection of a place for the compass; unfortunately even in the case of the Standard Compass the choice of position is so limited by the exigencies of the arrangements for working and fighting the ship, that the deviations in these compasses are generally larger than could be wished.

2. After launching, and when the vessel is swinging at anchor, or sailing or steaming in various directions, the values of B and C generally diminish rapidly; and this change would no doubt be accelerated by the vessel being exposed to blows or jars in a position different from that of building.

The following cases show a rapid change of B and C after launching. The most instructive have been selected from the Tables, but the elaborate series of observations made in the Great Eastern (Phil. Trans. 1860) are the most conclusive, as that ship was in every respect prepared for sea, and the observations are strictly comparable throughout.

H.M.S. Achilles, built in dry dock at Chatham, and fully plated there also, head S. 52° E., floated out of dock 24th December 1863, and moored head and stern in the River Medway, head S. 62° E. In March 1864, after taking in steam machinery, the ship made a short trial trip down the river, and then returned to the former moorings, but with her head secured in the opposite direction, or N. 62° W. Equipment and fittings completed by October 11th, when the head was shifted round to S. 55° E., and on the following day steamed to Sheerness and commenced sea service.

			𝔅.	ℭ.
1863.	Dec.	23.—In dock at Chatham	+·464	+·323
1864.	Sept.	26.—Complete for sea, head N. 62° W. . . .	+·377	+·037
	Oct.	11.—Complete for sea, head S. 55° E. . .	+·355	+·062
	Oct.	13.—Swinging at anchor, Sheerness . . .	+·362	+·047
	Dec.	5.—At Plymouth, after 25 days in dock, head S. 79° E.	+·361	+·123

H.M.S. Royal Oak, wood-built ship, iron-plated in dock at Chatham, head S. 49° E.

1863.	Mar.	19.—Floated out of dock	+·253	+·287
	April	11.—Swinging at anchor, River Medway . .	+·231	+·197
	June	2.—Swinging at anchor, River Medway . .	+·248	+·128
1864.	Jan.	8.—Swinging at anchor, Plymouth . . .	+·218	+·172

The example of the Achilles is very instructive. The large value of ℭ+·323 giving a C of 19°, which was caused by the ship having been built, plated, and moored with the starboard side South, is reduced to +·037 or 2° 10′ by lying for six months with the port side South. This amount does not alter materially while the ship is allowed to swing, but when she is twenty-five days in dock with the starboard side South, it suddenly rises to +·123 or 7°.

𝔅, it will be observed, changes much less at first, and hardly changes at all afterwards; this difference must be attributed in part to this, that while the whole of ℭ is to be attributed to subpermanent magnetism arising from horizontal induction in transverse hard iron, a large part of the original 𝔅 was probably caused by the transient magnetism arising from vertical induction in soft iron, and a further part by the subpermanent magnetism arising from vertical induction in hard iron, so that possibly not more than ·100 was caused by the subpermanent magnetism arising from induction from the headward component of the horizontal force, nearly the whole of which may have been removed by six months' reversal of her direction, so as to leave little room for subsequent change of 𝔅.

In connexion with this part of the subject we may observe that the same circumstances which cause the transient magnetism arising from horizontal induction in transverse iron $(-e)$ to be greater than the transient magnetism arising from horizontal induction in fore-and-aft iron $(-a)$, lead us to expect that the subpermanent magnetism arising from horizontal induction in transverse hard iron (ℭ) will be greater than the subpermanent magnetism arising from horizontal induction in fore-and-aft hard iron (changing part of 𝔅), and that consequently we should expect the relative changes of ℭ which take place on a change of direction to be greater than those of 𝔅, and this will be found to be verified in almost all cases, except when the ship has been built nearly North and South.

3. After a certain time, which may be roughly estimated at a year after launching, this process seems to stop, and the values of B and C remain remarkably permanent. The former paper* contains numerous examples of this in ordinary iron-built ships.

This will appear also from the following instances of the iron-plated ships.

			Standard Compass.	
			𝔅.	ℭ.
Warrior.	September	1861	−·449	−·124
	October	1861	−·409	−·092
	July	1862	−·321	−·114
	June	1863	−·317	−·132
	July	1864	−·311	−·054
	October	1864	−·307	−·072
Defence.	February	1862	+·464	+·005
	March	1863	+·379	−·034
	December	1863	+·403	−·016
	April	1864	+·391	−·007
	October	1864	+·379	−·034

---

* Philosophical Transactions, Part II. 1860.

			Standard Compass. 𝔅.	Standard Compass. ℭ.
Black Prince.	November	1861	+·422	+·058
	September	1862	+·383	+·074
	July	1863	+·384	+·067
	April	1864	+·389	+·086
	October	1864	+·349	+·050
Resistance.	August	1862	+·149	−·158
	June	1863	+·152	−·138
	December	1863	+·106	−·120
	December	1864	+·065	−·153

It will be remembered in the foregoing examples that the ships have been frequently subjected to the strains in docking, trials, in gales of wind, and at high rates of speed, and especially to concussions from the drilling and firing their heavy ordnance.

A striking example of the permanency of the magnetism of an "old" iron ship after severe concussion is afforded in the case of the Adventure troop-ship built in 1854. This ship, in the course of foreign service during a fog, struck on a rock with sufficient force to tear away and crush in 20 feet of the stem and bow under water; appended are the coefficients observed *before* proceeding on the foreign service, and *after* the injuries sustained had been repaired in dock.

	𝔅.	ℭ.
1862. April 26th	−·073	+·186
1862. October 28th	−·071	+·186

An equally close agreement will be found, on reference to the Tables, to exist in the other magnetic coefficients of this ship; the exact accordance of the numerical values is of course accidental, but is conclusive as to the great wear and tear and rough usage an old iron ship can undergo without her magnetic conditions being changed.

4. The determination of the proportion of the semicircular deviation, or rather of B, which arises from vertical induction in soft iron, and that which arises from the permanent or subpermanent magnetism of hard iron, is a matter of great interest. Theoretically it may be determined in two modes, either by observing the deviation in two different magnetic latitudes, or by observing the deviation with the ship upright and heeled over. Unfortunately there is a great want of observations under these circumstances. The deviations of the iron-plated ships, given in the Tables, were carefully observed both at Lisbon and Gibraltar, but the difference of latitude between either place and England is too small, and the change in the subpermanent magnetism too great to enable us to derive any very certain results from these observations.

The difficulty of heeling a large ship is so great that few observations except in an upright position can be expected; we owe, however, to the zeal of the officers in command of the Warrior[*], Black Prince, and Defence, that these ships were swung at

---

[*] Magnetic science is indebted to the Honourable Captain Cochrane of Her Majesty's Ship Warrior, for the interest he has evinced, and the assistance he has rendered in obtaining complete records of that ship; and

Lisbon upright, and heeled about 7° to starboard and to port. The agreement of the values of the coefficient $\frac{c}{\lambda}$ derived by the different methods is not very satisfactory, and it can only be considered as a rough approximation to the truth.

From the equation for comparison of semicircular deviation in different latitudes

$$\frac{\mathrm{P}}{\lambda}+\mathrm{H}\tan\theta\frac{c}{\lambda}=\mathfrak{B}\mathrm{H}.$$

	$\frac{\mathrm{P}}{\lambda}$.	$\frac{c}{\lambda}$.
Warrior	−·471	+·058
Black Prince . . .	+·061	+·142
Defence	+·206	+·079
Resistance	−·330	+·190

From heeling-error formulæ.	$\frac{c}{\lambda}$.
Warrior	+·108
Black Prince	+·181
Defence	+·119

Taking the mean of the several values in the ships.

	Original value of B.	$\frac{c}{\lambda}$.	Part of B from soft iron.	Part of B from hard iron.
Warrior	−24¼°	·083	+12°	−36¼°
Black Prince	+23	·161	+23	0
Defence	+25¾	·099	+14¼	+11½

Taking the present values of B in the ships.

	B.	$\frac{c}{\lambda}$.	Part of B from soft iron.	Part of B from hard iron.
Warrior	−17°	·083	+12°	−29°
Black Prince	+19	·161	+23	− 4
Defence	+21	·099	+14¼	+ 6¾

And in any other magnetic latitude for which the horizontal force is H, the horizontal force in England being 1 and the dip θ, we should have

$$\text{Warrior . . . .}\quad \mathrm{B}=-\frac{29^{\circ}}{\mathrm{H}}+4\tfrac{3}{4}^{\circ}\tan\theta.$$

$$\text{Black Prince. . .}\quad \mathrm{B}=-\frac{4^{\circ}}{\mathrm{H}}+9\tfrac{1}{4}^{\circ}\tan\theta.$$

$$\text{Defence . . . .}\quad \mathrm{B}=\frac{6\tfrac{3}{4}^{\circ}}{\mathrm{H}}+5\tfrac{3}{4}^{\circ}\tan\theta.$$

---

also to **WILLIAM MAYES**, Esq., Master of Her Majesty's Ship Defence, for a valuable series of observations made in that ship, and for his exertions in obtaining results in several ships of the Channel Squadron.

Quadrantal Deviation, D sin 2ζ′+E cos 2ζ′.

Mean force to North λH.

The Tables show that the values of E when the ship is upright and the compass in the midship line, give no certain indication of any real value. The more accurate the instrument, and the more careful the observations, the smaller E generally is.

When the compass is not in the midship line the case is different; an E may then have a considerable value. Instances of this will be seen in the deviations of the Royal Sovereign, the peculiar construction and fittings of which ship made it necessary to place compasses considerably out of the midship line, and with gun turrets placed diagonally to them.

		°	′
At the steering wheel on upper deck		E=−9	14
At the steering wheel in captain's cabin . . .		E=−5	10
Forward on lower deck	Port side	E=+4	38
	Starboard side . . .	E=−4	42

It will easily be seen that a +E would be caused by a gun turret in the first and third quadrant relatively to the compass, and a −E by a turret in the second and fourth. The close agreement of the numerical value of E in the two last examples, with the difference in their signs, is striking.

The value of the E introduced by the ship heeling by an angle i to starboard being

$$-\frac{c+g}{2\lambda}i,$$

and both c and g being generally positive, we should expect a −E when the ship heels to starboard, a +E when she heels to port, and this is the case in the few instances we have in the Tables.

		E.	
	°	°	′
Warrior.—Standard Compass . . .	7½ to starboard	−0	45
	7½ to port . .	+0	59
Black Prince.—Standard Compass . .	6½ to starboard	−1	25
	6½ to port . .	+1	50
Defence.—Standard Compass . . .	7½ to starboard	−0	05
	7½ to port . .	+1	50

D.

As regards D, the most important point is its magnitude in different positions in ships of different classes.

The usual or average value of D has greatly increased since the publication of the Paper in 1860. In that paper it was observed that a value for this coefficient not exceeding 4° and ranging between that amount and 2°, might be assumed to represent the average or normal amount in vessels of all sizes, and in only two vessels mentioned in that paper did D exceed 5°.

In the iron-built armour-plated ships its average amount in the Standard Compass is about 7°, in the steering-compass about 10°, and in the main-deck compass about 12°. In the wood-built iron-plated ships the value of D is small.

The following Table gives the value in different ships.

	Warrior.	Black Prince.	Achilles.	Defence.	Resistance.	Hector.	Valiant.	Royal Oak (wood-built).
Standard compass	+ 8° 27′	+ 7° 38′	+ 6° 58′	+ 7° 0′	+ 6° 17′	+5° 24′	+4° 54′	+3° 09′
Starboard steering......	+11 56	+10 32	+ 8 51	+10 16	+ 8 28	+8 24	+6 52	+1 47
Main deck	+11 43	+13 16	+12 13	+14 35	+14 0	+9 47	+8 05	+1 28

The large amount in the Standard and Steering Compass of the Warrior is doubtless owing to the rifle tower which is immediately before them, and which gives a $+a$. The small comparative values in the Hector and Valiant to the iron-plating being extended from end to end in the ship giving a $-a$, and the absence of a complete transverse armour bulkhead, the existence of which in the Defence and Resistance, as well as in the Warrior and Black Prince, give large $-e$, and consequently large deviations in the binnacle and main-deck compasses.

Between the Resistance and the Defence there is a remarkable difference. These are nearly sister ships, but with this difference, that from the different position of the mizenmast in the two ships their standard and steering compasses are very differently placed with reference to the transverse armour bulkhead. In the Resistance the Standard Compass is exactly above the bulkhead at a height of 12 feet. The steering-compass is about 4 feet in front, and the same height above it; while in the Defence these compasses are about 20 feet abaft it.

Such a bulkhead, when magnetized at right angles to its plane, will produce a fore-and-aft force on all points in, or nearly in, the same plane in the opposite direction to the magnetizing force. It will therefore, in the case of the standard and steering-compasses of the Resistance, introduce a $-a$ as well as a $-e$, while it will produce little or no $-a$ in compasses placed as in the Defence, and a much smaller $-e$.

These differences do not show themselves in the value of D, which is in fact less in the Resistance than in the Defence, notwithstanding the much more powerful action of the forces which cause it. In order to see them, we must obtain separately the two parts of the quadrantal deviation D, or the value of a and e. This is done in the following Table:—

		Warrior.	Black Prince.	Achilles.	Defence.	Resistance.	Hector.	Valiant.	Royal Oak (wood-built).
Standard ..	From fore-and-aft induction…	+ 0° 6′	− 4° 4′	− 2° 45′	− 2° 42′	− 5° 55′	− 3° 51′	− 2° 14′	− 1° 19′
	From transverse induction …	+ 8 24	+11 42	+ 9 40	+ 9 44	+12 21	+ 9 15	+ 7 11	+ 4 32
Starboard Steering …	From fore-and-aft induction…	+ 0 14	− 3 47	− 3 47	− 2 17	− 7 53	− 3 23	− 2 59	− 2 7
	From transverse induction …	+11 46	+14 28	+12 43	+12 35	+16 33	+11 49	+ 9 54	+ 3 51
Main Deck	From fore-and-aft induction…	……	− 2 35	− 3 9	− 2 10	− 1 02	− 5 58	− 6 56	− 3 51
	From transverse induction …	……	+15 58	+15 36	+16 58	+15 11	+15 54	+15 14	+ 5 20
Standard ..	*a* ……	+·002	−·112	−·079	−·078	−·158	−·109	−·068	−·043
	e ……	−·256	−·322	−·277	−·278	−·326	−·263	−·214	−·143
Starboard Steering …	*a* ……	+·006	−·100	−·103	−·064	−·193	−·093	−·085	−·066
	e ……	−·340	−·380	−·343	−·348	−·401	−·325	−·251	−·122
Main Deck	*a* ……	……	−·068	−·083	−·048	−·027	−·151	−·176	−·116
	e ……	. …	−·418	−·407	−·434	−·409	−·397	−·380	−·160

The conclusions we have drawn will be seen to be supported by this separation. Thus we see that the Warrior is the only vessel which has a $+a$ and a $+\mathrm{D}$ from fore-and-aft iron. In the Hector and Valiant the D is comparatively small, because the $-a$ is large, the $-e$ small.

In the Resistance the two parts, the difference of which makes up the D, are very much larger than in the Defence, though the resulting value of D is less.

The comparison of the values of D and of a and e in the compasses of the Royal Oak with those in the compasses of the Hector and Valiant is very instructive. These ships are nearly alike in dimension, in the arrangement of the iron-plating, and the position of the compasses. The Royal Oak has an iron upper deck, but is otherwise wood-built. The Hector and Valiant are entirely iron-built.

A first inspection of the Table might lead us to infer that the large value of D in the iron-plated ships is due to the armour-plating at the sides, but the comparison with the Royal Oak shows this not to be the case. In fact a little consideration will show that, as regards longitudinal induction, the effect of armour-plating continued from end to end is to produce a $-a$; that, as regards transverse induction, the effect of the parts which run fore and aft is to produce a small $+e$, and the effect of the transverse parts near the extremities of the ship to produce a small $-e$, so that on the whole the tendency is probably rather to diminish than to increase D. The large value of D in the iron ships is evidently attributable to the increased amount of transverse iron in decks, bulkheads, iron beams, and the iron bottom of the ship, the magnetism of which is, as it were, conducted upwards by the iron sides.

λ.

The value of λ is so closely connected with that of D that it is desirable to consider them together. In the earlier built iron vessels λ was very nearly equal to 1. In the Rainbow, at four stations distributed along nearly the whole length of the ship, λ ranged from ·972 to 1·003. In the Ironsides, the first iron-built sailing ship, it was ·917 at the steering-compass. In several iron-built ships purchased into the Royal Navy from ten to fifteen years after Mr. AIRY's observations, λ averages at present about ·930. In the iron-plated ships of the present day it ranges from ·700 to ·900.

The following are its values in the iron-plated ships before mentioned.

	Warrior.	Black Prince.	Achilles.	Defence.	Resistance.	Hector.	Valiant.	Royal Oak (wood-built).
Standard compass	·873	·783	·822	·822	·758	·814	·859	·907
Starboard steering	·833	·760	·777	·794	·703	·791	·817	·906
Main deck	......	·757	·755	·759	·782	·726	·722	·862

The large value in the Warrior is evidently owing to the rifle tower, the small value in the Resistance, as compared to the value in the Defence, to the position of the compasses with respect to the armour bulkheads as above described, and with reference to the armour-plating generally.

Familiarity with the values of $\mathfrak{D}$ and λ in vessels of different classes, is of great importance in enabling us to deduce $\mathfrak{B}$ and $\mathfrak{C}$ by observations made without swinging.

The mathematical theory from which the values of $\mathfrak{D}$ and λ are derived, supposes that the transient induced magnetism to which $\mathfrak{D}$ and $1-\lambda$ owe their values, is instantaneously developed, and as instantaneously destroyed or altered as the ship assumes a new position. This we cannot suppose to be exactly true; but whether the time required for the soft iron to receive its new magnetic state as the ship swings is appreciable has been a matter of doubt. The opinion of the authors of the Report of the Liverpool Compass Committee (an opinion entitled to the greatest weight) was, that an appreciable time was required, and that the value of D in particular might be different according as the vessel was swung slowly or quickly; we have not, however, been able to detect any difference in the values of D which can be attributed to any cause of this nature.

The most remarkable feature, however, in λ and $\mathfrak{D}$ is the change which takes place with the lapse of time, indicating apparently a change in the molecular structure of the soft iron by which it becomes less susceptible of induced magnetism. This is shown clearly in the following Table:—

			Standard.		Starboard steering.		Main deck.	
			λ	$\mathfrak{D}$	λ	$\mathfrak{D}$	λ	$\mathfrak{D}$
Achilles	October	1864	·822	+·121	·777	+·154	·755	+·214
	December	1864	·854	+·116	·819	+·137	·804	+·188
Black Prince.	November	1861	·716	+·145				
	September	1862	·783	+·134	·760	+·184		
	April	1864	·846	+·137				
	November	1864	·849	+·122	·881	+·144		
Defence	February	1862	·822	+·122	·794	+·179	·759	+·254
	December	1863	·853	+·122	·842	+·180	·810	+·230
	April	1864	·857	+·112	·853	+·159	·828	+·233
	October	1864	·852	+·112	·830		·842	+·230
Resistance ...	August	1862	·758	+·111			·782	+·244
	December	1863	·850	+·122			·880	+·219
Royal Oak ...	March	1863	·861	+·047				
	April	1863	·907	+·061	·887	+·067		
	June	1863	·907	+·055	·906	+·031		
Dromedary...	July	1862	·841	+·104				
	December	1862	·861	+·097				

These changes, and particularly that in the value of λ, seem far too great, far too regular, and far too consistent, to be attributed to any cause except some molecular change in the structure of the iron which, with the lapse of time, renders it less susceptible of induced magnetism. Whether this change is accompanied by any change which can affect the strength, the liability to oxidation, or any other qualities of the iron, is a point on which we are not able to offer any information, but we beg to suggest it as a question deserving a careful experimental investigation.

Heeling Error.

As the heeling coefficient depends partly on vertical induction in transverse iron, partly on the mean vertical force arising from permanent magnetism and vertical induction in vertical iron, and as the two conspire when the vertical force of the ship acts downwards, or when μ is greater than unity, and counteract each other when the vertical force acts upwards, or when μ is less than unity, we may expect great differences in the heeling coefficient in different ships. In those which have been built head North, we may expect a large heeling error in compasses near the stern, and a smaller one in compasses near the bow, and the converse in ships built head South. This we find to be the case.

In these cases the uniformity of the heeling coefficients from transverse iron is remarkable, and they are, as might be expected, all of the same sign; the differences, it will be seen, are nearly all in the part which arises from vertical force; this varies from 1° 6′ in the Warrior to −1° 9′ in the Enterprise.

It will be seen that in the wood-built iron-plated ships the vertical force is generally diminished. This is doubtless the effect of the iron plating, which acts as a $-k$. No doubt in iron-plated iron-built ships the effect is the same, and the heeling error is probably diminished and not increased by the effect of the iron plating. Observations of vertical force have not been made in the main-deck compasses of these ships; but probably there the heeling error would be small, and possibly be a heeling error to leeward.

We must observe that there has not been an opportunity of making an exact comparison of the values of the heeling coefficient deduced from theory with those deduced from actually heeling and swinging the ship. The great amount of labour and time required to heel a ship of the class we are discussing, and swing her, has prevented such observations being made in more than a very small number of cases. In the case of the Warrior, Black Prince, and Defence, advantage was taken of their being heeled at

Class of Ship.	Date.	Name of Ship.	Direction of Head in building.	μ	g	Heeling coefficient from		Heeling coefficient to windward.
						vertical induction in transverse iron.	vertical force and induction in vertical iron.	
Iron ships, iron-plated.	July 1862.	Warrior	N. 3° E.	1·399	+·069	+0° 43′	+1° 06′	+1° 49′
	Jan. 1863.	,, *Lisbon*	,,	—	+·106	+0 32	+0 50	+1 22
	Sept. 1862.	Black Prince	S. 20° E.	·945	+·118	+1 01	−0 11	+0 50
	Jan. 1863.	,, *Lisbon*	,,	—	+·262	+0 43	+0 09	+0 52
	April 1864.	,,	,,	·971	+·111	+0 48	−0 05	+0 43
	Oct. 1864.	Achilles (Standard aft)	S. 51° 40′ E.	·870	+·194	+0 50	−0 23	+0 27
	Dec. 1864.	,,	,,	·896	+·210	+0 43	−0 18	+0 25
	Oct. 1864.	,, (Standard forward)	,,	1·217	−·172	+0 49	+0 40	+1 29
	Dec. 1864.	,,	,,	1·240	−·165	+0 37	+0 41	+1 18
	Feb. 1862.	Defence	S. 47° W.	1·040	+·138	+0 51	+0 08	+0 59
	Jan. 1863.	,, *Lisbon*	,,	—	+·117	+0 33	−0 03	+0 30
	April 1864.	,,	,,	·968	+·157	+0 42	−0 06	+0 36
	Aug. 1862.	Resistance	S. 86½° W.	1·071	+·176	+1 04	+0 14	+1 18
	Dec. 1863.	,,	,,	1·044	+·190	+0 45	+0 08	+0 53
	Feb. 1864.	Hector	S. 20° E.	·983	...	+0 48	−0 03	+0 45
	Jan. 1865.	Valiant	S. 87° W.	1·061	+·120	+0 37	+0 11	+0 48
Wood ships, iron-plated.	April 1863.	Royal Oak	Plated S. 49° E.	·896	+·045	+0 24	−0 17	+0 07
	June 1863.	,,	,,	·882	+·127	+0 23	−0 19	+0 04
	Feb. 1864.	Prince Consort	Plated S. 39° W.	·848	+·038	+0 16	−0 24	−0 08
	Aug. 1864.	Ocean	Plated S. 79° E.	·929	+·112	+0 19	−0 34	−0 15
	June 1864.	Enterprise (Iron topsides)	Built and plated S. 56° W.	·622	+·152	+0 37	−1 09	−0 29
Iron ships.	July 1863.	Orontes	N. 66° W.	1·164	+·056	+0 36	+0 28	+1 04
	Nov. 1863.	Tamar	West	1·117	+·147	+0 31	+0 20	+0 51
		,, (Binnacle over rudder)	,,	1·248	+·294	+0 28	+0 42	+1 10
	Sept. 1863.	Wye	Probably to E.S.E.	1·195	+·252	+0 27	+0 34	+1 0
	Feb. 1863.	Caradoc	Probably to N. by W.	1·002	...	+0 14	+0 01	+0 15
	Feb. 1863.	Clyde	Probably to N.E.	1·275	...	+0 35	+0 47	+1 22
	Mar. 1863.	Industry	Probably to S. by E.	·859	...	+0 18	−0 23	−0 05
	June 1863.	City of Sydney	Probably to W.N.W.	1·246	...	+0 46	+0 45	+1 31

Lisbon for the purpose of cleaning the bottoms, to swing them at the same time, and the heeling coefficients so obtained correspond very satisfactorily with those obtained in England from observations of horizontal and vertical force. But, unfortunately, at present we have no instances in which the horizontal and vertical forces were observed at the time and place at which the ship was heeled and swung; and it seems very desirable that the theory should be put to the practical test, though there seems no reason to doubt that the results of the two methods would agree within the limits of errors of observation.

g.

g is one of those quantities which it is of importance to be able to estimate with some approach to accuracy, in order that the value of the mean vertical force, or μ, may be determined by observations of the vertical force made with the ship's head on one point only.

The Tables show that this may be done; g, as might be expected, is larger the nearer the stern the Standard Compass is placed, and is negative in compasses placed near the bow.

Achilles	$+\cdot194$
Resistance	$+\cdot176$
Defence	$+\cdot157$
Black Prince	$+\cdot118$
Warrior	$+\cdot069$
Achilles (Standard forward)	$-\cdot172$

There are indications of changes in the value of the heeling coefficient and in the value of g from the lapse of time, corresponding to the changes in the values of $\mathfrak{D}$ and λ; but more extended observations are necessary to show the amount and law of these changes.

To afford a clear view of the general structure of the armour-plated ships, and the position of the several compasses, profile sketches of these ships are given (Plate XI.), and it may be deemed of sufficient interest to add a brief description of their general arrangements as affecting their magnetic characteristics.

The Warrior, Black Prince, and Achilles, of 6100 tons, are types of the largest size iron-built and iron-plated ships of war; they are 380 feet long, 58 feet beam, 26 feet draught of water, propelled by engines of 1250 horse-power, and carry from forty to twenty heavy guns. 3750 tons of iron is used in the construction of the hull, which varies in thickness from $1\frac{1}{4}$ inch near the keel to $\frac{5}{8}$ inch behind the armour-plates. For the Achilles 1200 tons of iron $4\frac{1}{2}$ inches thick was employed for the armour-plating.

The Hector and Valiant of 4100 tons, and the Defence and Resistance of 3700 tons, are types of the medium and smaller-sized iron-built and iron-plated ships of war. In the general features of construction they are similar to the Warrior, Black Prince, and Achilles; all are frigate-built, or with a main deck for the principal battery of guns, and the only wood used in the hulls, with the exception of teak-wood backing to the armour plates, is for the surface covering of the iron decks, and for the personal arrangements and accommodation of the crews.

In the Warrior, Black Prince, Defence, and Resistance, the armour-plating of 4½-inch iron is not continued to the bow or stern, but where it terminates is continued from side to side of the ship as an armour bulkhead. In the Achilles, Hector, and Valiant, the armour plating is continued round the ship, but of smaller dimensions near the bow and stern, and with corresponding smaller transverse-armour bulkheads.

The Royal Oak, Prince Consort, Caledonia, and Ocean, of 4050 tons, 800 to 1000 horse-power engines, and carrying thirty-five heavy guns, are types of the largest-sized wood-built iron plated-ships; the hull, with the exception of the iron upper deck and its supporting iron beams and uprights, is entirely constructed of wood; the exterior of the hull to 4 feet below the water-line (in this respect similar to the iron-built ships) is plated with 4½-inch iron entirely round.

The Enterprise, of 993 tons, is the type of the smaller-sized wood-built ship; she is constructed to carry four heavy guns within a square battery of 4½-inch iron, and has a continuous armour belt of 4½-inch iron round the ship; the upper deck, deck beams, and top sides are of thin plate-iron.

The Royal Sovereign, of 3765 tons, is an experimental class of vessel; she was originally a wood-built three-decked ship of 110 guns, but now cut down to the lower-gun deck, plated continuously round with 5½-inch iron, and with an iron upper deck and bulworks. The armament of five guns of large calibre is worked within four turrets; the iron frame of these turrets varies in thickness from 5½ to 10 inches; and the largest, arranged to carry two guns, weighs 146 tons.

The internal arrangements of all these classes of ships allow little room for selection in the position of the compasses. The accurate drawings, kindly furnished by the Department of the Controller of the Navy, enables their several positions to be shown with reference to the most important masses of iron.

TABLES OF COEFFICIENTS.

I. Iron-plated, Iron-built Ships.

II. Iron-plated, Wood-built Ships.

III. Iron-built Ships, Her Majesty's Navy.

IV. Iron-built Ships, Mercantile Marine.

Table of Terrestrial Magnetic Elements employed in discussion of magnetic coefficients.

TABLE I.—Iron-plated, Iron-built Ships.

Ship.	Compass.	Place.	Date.	Approximate coefficients.					Exact coefficients.				
				A	B	C	D	E	𝔄	𝔅	ℭ	𝔇	𝔈
				° ′	° ′	° ′	° ′	° ′					
WARRIOR. (6109 tons), Iron-plated, iron hull, 40 guns, 1250 horse-power. Built at Blackwall, River Thames; head N. 3° E. magnetic. Launched Dec. 29, 1860. Plated with head generally to N.W.	Standard.	Greenhithe ...	Sept. 16, 17, 1861	+1 7	−24 15	− 7 42	+ 9 23	+0 39	+·019	−·449	−·124	+·164	+·010
		Portsmouth...	Oct. 15, 17, 1861	−1 0	−22 12	− 5 52	+ 8 56	+0 44	−·017	−·409	−·092	+·155	+·013
		Gibraltar ...	*Feb.* 1862	...	−15 51	− 6 0	+ 8 20	+0 23	...	−·293	−·095	+·145	+·006
		Portsmouth...	July 28, 29, 1862	−0 12	−17 24	− 7 9	+ 8 27	+1 8	−·003	−·321	−·114	+·148	+·020
		Gibraltar ...	*Nov.* 1862	+1 0	−14 39	− 4 50	+ 8 25	−0 43	+·017	−·272	−·077	+·146	−·012
		Lisbon	*Heeled* 7½° *to Port*	+0 50	−14 43	+ 6 45	+ 8 9	+0 59	+·015	−·272	+·108	+·143	+·017
			Upright, Jan. 1863 ...	+0 50	−14 33	− 3 34	+ 7 48	−0 24	+·015	−·269	−·057	+·136	−·007
			Heeled 7½° *to Starboard*	+0 44	−15 36	−13 37	+ 8 17	−0 45	+·013	−·287	−·216	+·145	−·013
		Devonport ...	May 1, 1863	−0 12	−17 10	− 8 18	+ 8 26	−0 32	−·003	−·317	−·132	+·146	−·009
		Madeira	*Dec.* 28, 30, 1863	−1 59	−12 56	− 2 48	+ 7 15	−0 4	−·035	−·239	−·046	+·126	−·001
		Plymouth ...	June 1864	+0 25	−16 45	− 3 24	+ 8 44	−0 19	+·007	−·311	−·054	+·152	−·005
		Portland	Oct. 28, 1864	−0 17	−16 35	− 4 33	+ 8 45	−0 41	−·005	−·307	−·072	+·152	−·012
	Starboard steering.	Greenhithe ...	Sept. 16, 17, 1861	+0 20	−20 19	− 7 35	+15 28	−0 7	+·006	−·395	−·111	+·268	−·002
		Portsmouth...	Oct. 15, 17, 1861	+0 12	−20 37	− 6 37	+15 51	−0 11	+·003	−·402	−·098	+·273	−·003
		Portsmouth...	July 28, 1862......	−1 48	−15 31	− 7 50	+11 56	+0 43	−·031	−·296	−·121	+·208	+·012
		Devonport ...	May 1, 1863	−0 7	−16 28	−10 24	+12 3	−0 31	−·002	−·312	−·160	+·210	−·009
	Main deck.	Greenhithe ...	Sept. 11, 1861......	−0 30	−25 56.	−12 6	+10 58	−1 15					
		Greenhithe ...	Sept. 16, 17, 1861	+0 55	−22 34	− 7 49	+11 43	−1 46					
BLACK PRINCE. (6109 tons), Iron-plated, iron hull, 41 guns, 1250 horse-power. Built at Glasgow; head S. 20° E. magnetic. Launched Feb. 27, 1861. Plated head South.	Standard.	Greenock[1] ...	Nov. 1861	+0 10	+23 0	+ 3 41	+ 8 19	+0 25	+·003	+·422	+·058	+·145	+·007
		Portsmouth...	Sept. 2, 1862	+0 49	+20 59	+ 4 40	+ 7 38	0 0	+·014	+·383	+·074	+·134	·000
		Lisbon	*Heeled* 6½ *to Port*	+0 57	+15 31	+ 9 11	+ 6 45	+1 50	+·016	+·282	+·148	+·117	+·032
			Upright, Jan. 1863 ...	−0 1	+15 39	+ 3 12	+ 7 24	−1 20	·000	+·288	+·052	+·129	−·023
			Heeled 6½° *to Starboard*	+0 1	+15 14	− 2 6	+ 7 14	−1 25	·000	+·280	−·034	+·126	−·025
		Portland	June and July 1863	+0 2	+21 8	+ 4 10	+ 7 6	+0 51	·000	+·384	+·067	+·124	+·015
		Madeira	*Jan.* 1864	−0 25	+13 12	+ 4 29	+ 7 19	−0 6	−·007	+·243	+·072	+·128	−·002
		Lisbon	*Jan. and Feb.* 1864	+0 2	+15 8	+ 3 59	+ 7 20	−0 38	·000	+·278	+·064	+·128	−·011
		Portland	Mar. and Apr. 1864	+1 22	+21 16	+ 5 24	+ 7 54	−0 24	+·024	+·389	+·086	+·137	−·007
		Portland	Oct. 1864	+0 30	+19 5	+ 3 5	+ 7 2	−0 3	+·009	+·349	+·050	+·122	−·001
	Starboard steering.	Portsmouth...	Sept. 2, 1862	+2 59	+20 9	+ 8 10	+10 32	+1 37	+·052	+·379	+·136	+·184	+·028
		Plymouth ..	Nov. 1864	+2 19	+19 40	+ 6 13	+ 8 18	+1 45	+·040	+·363	+·103	+·144	+·030
	Main deck.	Portsmouth...	Sept. 2, 1863	−1 9	+27 25	+ 8 4	+13 16	+0 2	−·020	+·516	+·120	+·231	·000

---

[1] λ observed at Greenock = ·804, multiplied by earth's horizontal force ·89 = ·716.

TABLE I.—Iron-plated, Iron-built Ships.

Maximum of semicircular deviation $\frac{\sqrt{B^2+C^2}}{\text{Horizontal force of ship } \sqrt{\mathfrak{B}^2+\mathfrak{C}^2}}$*			Mean force to North, λ †	$\frac{1}{\lambda}$	Coefficients of horizontal induction.		Part of D from		Mean Vertical force, μ ‡	Heeling coefficient to *windward*, χ	Heeling coefficients from		$\frac{g}{\tan\theta}$ ‡	g †
Amount.		Direction.			Fore-and-aft, a †	Transverse e †	Fore-and-aft induction.	Transverse induction.			Vertical induction in transverse iron.	Vertical force and induction in vertical iron.		
°		°					° ′	° ′		° ′	° ′	° ′		
25½	·466	195½												
23	·419	192½												
16	·308 ·409	198												
19	·341	199½	·873	1·145	+·002	−·256	+0 6	+ 8 24	1·399	+1 49	+0 43	+1 06	+·028	+·069
15½	·282 ·375	196												
......	·293	158¼												
15	·275 ·345	192	......	......	......	......	......	......	......	+1 22	+0 32	+0 50		
......	·360	217												
19	·344	203	·860	1·163	−·015	−·265	−0 28	+ 8 52						
12½	·243 ·328	191												
17	·314	190												
17¼	·316	193												
21¾	·410	195½												
21¾	·414	193½												
17½	·320	202	·833	1·201	+·006	−·340	+0 14	+11 46						
19½	·352	207	·878	1·139	+·062	−·306	+2 0	+10 04						
23¼	·426	8	·804 ·716	1·396	−·180	−·388	−7 15	+15 40						
20½	·390	11	·783	1·277	−·112	−·322	−4 4	+11 42	·945	+0 50	+1 1	−0 11	+·048	+·118
......	·318	27½												
16	·293 ·369	10½ ...	......	......	.. ...	......	......	.. ...	......	+0 52	+0 43	+ 0 9		
......	·282	353												
20½	·390	10												
14	·254 ·343	16½												
15¾	·286 ·360	13	·778	1·285	−·122	−·322	−4 28	+11 53						
22	·399	12½	·846	1·182	−·038	−·270	−1 19	+ 9 11	·971	+0 43	+0 48	−0 5	+·045	+·111
19½	·354	8	·849	1·178	−·047	−·255	−1 36	+ 8 38						
21¾	·404	20	·760	1·316	−·100	−·380	−3 47	+14 28						
20½	·377	16	·881	1·135	+·008	−·246	+0 14	+ 8 4						
28½	·530	13	·757	1·321	−·068	−·418	−2 35	+15 58						

* Mean force to North (λH) being unit. † Earth's Horizontal force (H) being unit. ‡ Earth's Vertical force (Z) being unit.

TABLE I. (continued).—Iron-plated, Iron-built Ships.

Ship.	Compass.	Place.	Date.	Approximate coefficients.					Exact coefficients.				
				A	B	C	D	E	𝔄	𝔅	ℭ	𝔇	𝔈
				° ′	° ′	° ′	° ′	° ′					
ACHILLES*. (6121 tons). Iron-cased, iron hull, 20 guns, 1250 horse-power. Built at Chatham, and fully plated in dock; head S. 51° 40′ E. magnetic. Floated out of dock Dec. 24, 1863.	Standard (aft).	Sheerness	Oct. 12, 13, 1864	−0 16	+19 54	+ 2 56	+ 6 58	−0 56	−·005	+·362	+·047	+·121	−·016
		Plymouth †	Dec. 5, 1864	−0 35	+19 54	+ 7 38	+ 6 41	−0 32	−·010	+·361	+·123	+·116	−·009
	Standard (forward).	Sheerness	Oct. 12, 13, 1864	−0 10	+21 42	+ 1 11	+ 7 19	−0 31	−·003	+·396	+·019	+·128	−·009
		Plymouth	Dec. 5, 1864	+0 39	+19 51	+ 6 15	+ 5 44	−1 01	+·011	+·357	+·102	+·100	−·018
	Starboard steering.	Sheerness	Oct. 12, 13, 1864	+0 07	+23 31	+ 4 10	+ 8 51	−1 20	+·002	+·432	+·061	+·154	−·023
		Plymouth	Dec. 5, 1864	−0 55	+23 30	+10 04	+ 7 51	−0 30	−·016	+·427	+·160	+·137	−·009
	Main deck (starboard).	Sheerness	Oct. 12, 13, 1864	−0 47	+12 42	+ 2 19	+12 13	+0 21	−·014	+·244	+·031	+·214	+·006
		Plymouth	Dec. 5, 1864	−1 11	+14 17	+ 3 49	+10 46	+1 23	−·021	+·271	+·059	+·188	+·024
DEFENCE. (3720 tons). Iron-plated, iron hull, 16 guns, 600 horse-power. Built on River Tyne; head S. 47° W. magnetic. Launched Apr. 24, 1861. Plated with head S. 19° E. magnetic.	Standard.	Sheerness	Feb. 17, 18, 1862	−0 28	+25 43	+ 0 17	+ 7 0	+0 5	−·008	+·464	+·005	+·122	+·001
		Baltic Sea	*July and Aug.* 1862	−0 17	+25 35	− 0 25	+ 6 25	−0 41	−·005	+·463	−·007	+·112	−·012
		Gibraltar	*Nov.* 15, 1862	+0 16	+15 21	− 4 15	+ 6 9	+0 25	+·005	+·280	−·069	+·107	+·007
		Lisbon	*Heeled 7½° to Port*	+1 47	+16 39	+ 2 49	+ 7 18	+1 50	+·031	+·305	+·045	+·127	+·032
		Lisbon	*Upright, Jan.* 1863	+1 41	+16 26	− 1 5	+ 7 4	+0 42	+·029	+·302	−·018	+·123	+·012
		Lisbon	*Heeled 7½° to Starboard*	+1 38	+16 27	− 4 40	+ 7 0	−0 5	+·028	+·301	−·075	+·122	+·001
		Flushing & Portsmouth	March 3, 21, 1863	+0 5	+20 50	− 2 8	+ 6 50	−0 11	+·001	+·379	−·034	+·119	−·003
		Plymouth	Dec. 1863	+1 6	+22 18	− 0 57	+ 6 59	−0 7	+·019	+·403	−·016	+·122	−·002
		Teneriffe	*Jan.* 2, 3, 1864	......	......	......	......	......	...	+·292	−·040	+·114	...
		Gibraltar	*Jan.* 9, 13, 1864	−1 0	+15 25	− 1 44	+ 6 30	−0 46	−·017	+·282	−·028	+·113	−·013
		Lisbon	*Jan. and Feb.* 1864	+0 40	+16 37	− 1 18	+ 6 22	−0 7	+·012	+·303	−·021	+·111	−·002
		Portland	Mar. and Apr. 1864	+0 21	+21 37	− 0 24	+ 6 26	−0 24	+·005	+·391	−·007	+·112	−·007
		Portland	Oct. 1864	−0 23	+20 55	− 2 6	+ 6 23	+0 10	−·007	+·379	−·034	+·112	+·003
	Starboard steering.	Sheerness	Feb. 17, 18, 1862	+0 16	+36 14	+ 0 56	+10 16	+1 7	+·005	+·653	+·014	+·179	+·019
		Plymouth	Dec. 1863	+1 4	+31 18	− 1 21	+10 19	+0 36	+·019	+·572	−·020	+·180	+·010
		Portland & Downs	Apr. and May 1864	......	......	......	......	......	+·014	+·586	−·030	+·159	+·009
		Devonport	Nov. 1864	......	......	......	......	......	...	+·546	−·056	+·159	...
	Main deck.	Sheerness	Feb. 17, 18, 1862	−0 51	+36 23	+ 0 42	+14 35	−0 55	−·015	+·669	+·010	+·254	−·016
		Plymouth	Dec. 1863	+1 16	+26 44	+ 0 34	+13 10	−0 6	+·022	+·505	+·009	+·230	−·002
		Portland & Downs	Apr. and May, 1863	......	......	......	......	......	+·019	+·450	+·004	+·233	−·013
		Devonport	Nov. 1864	......	......	......	......	......	...	+·486	−·030	+·230	...

* ACHILLES. Dec. 23, 1863 {In dock at Chatham; by observations of deviation and horizontal force on one point, and employing λ and 𝔇 of Oct. 1864 (no machinery on board, or internal fittings) .... } = +·464 +·323

Sept. 26, 1864 {Complete in equipment; by observations of deviation and horizontal force on one point, Head moored N. 62° W., same λ and 𝔇 as above .... } = +·377 +·037

Oct. 11, 1864. Same observations, λ and 𝔇 as above. Head moored S. 54° 40′ E. .... = +·355 +·062

† After having remained in dry dock 25 days. Head S. 79° E. magnetic.

TABLE I. (continued).—Iron-plated, Iron-built Ships.

Maximum of semicircular deviation $\sqrt{B^2+C^2}$ / Horizontal force of ship $\sqrt{\mathfrak{B}^2+\mathfrak{C}^2}$*. Amount.		Direction.	Mean force to North, λ †	$\frac{1}{\lambda}$	Coefficients of horizontal induction. Fore-and-aft, a †	Transverse e †	Part of D from Fore-and-aft induction.	Transverse induction.	Mean Vertical force, μ ‡	Heeling coefficient to *windward*, χ	Heeling coefficients from Vertical induction in transverse iron.	Vertical force and induction in vertical iron.	$\frac{g}{\tan\theta}$ ‡	g †
°		°					° ′	° ′		° ′	° ′	° ′		
20¼	·365	7½	·822	1·216	−·079	−·277	−2 45	+ 9 40	·870	+0 27	+0 50	−0 23	+·079	+·194
21	·381	18½	·854	1·171	−·047	−·245	−1 36	+ 8 17	·896	+0 25	+0 43	−0 18	+·084	+·210
21¾	·397	2½	·831	1·202	−·063	−·275	−2 7	+ 9 30	1·217	+1 29	+0 49	+0 40	−·070	+·172
20¾	·371	16	·872	1·147	−·041	−·215	−1 22	+ 7 7	1·240	+1 18	+0 37	+0 41	−·066	+·165
24	·437	8	·777	1·287	−·103	−·343	−3 47	+12 43						
25½	·458	20¾	·819	1·221	−·069	−·293	−2 24	+10 15						
13	·246	7¼	·755	1·325	−·083	−·407	−3 9	+15 36						
14¾	·278	12½	·804	1·244	−·045	−·347	−1 36	+12 28						
25¾	·464	360¼	·822	1·217	−·078	−·278	−2 42	+ 9 44	1·040	+0 59	+0 51	+0 8	+·056	+·138
25½	·463 ·440	359¾												
16	·288 ·383	346												
16¾	·317	15½												
16½	·303 ·381	356½	……	……	……	……	……	……	……	+0 30	+0 33	−0 3		
17	·311	346												
21	·391	355												
22½	·403	358	·853	1·172	−·043	−·251	−1 26	+ 8 28						
……	·294 ·408	352	·846	1·182	−·058	−·250	−1 57	+ 8 31						
15½	·283 ·376	354	·853	1·172	−·051	−·243	−1 40	+ 8 13						
16½	·304 ·382	356	·827	1·209	−·081	−·265	−2 49	+ 9 16						
21½	·392	359	·857	1·167	−·071	−·263	−1 33	+ 8 0	·968	+0 36	+0 42	−0 6	+·064	+·157
21	·381	355	·852	1·174	−·053	−·243	−1 46	+ 8 13						
36½	·654	361¼	·794	1·258	−·064	−·348	−2 17	+12 35						
31½	·572	358	·842	1·118	−·006	−·310	−0 14	+10 36						
……	·586	357	·853	1·172	−·036	−·308	−0 21	+ 9 33						
……	·558	348½	·830											
36½	·669	361	·759	1·318	−·048	−·434	−2 10	+16 58						
26¾	·505	361	·810	1·235	−·004	−·376	−0 8	+13 24						
……	·450	360½	·828	1·208	+·021	−·365	+0 41	+12 42						
……	·487	356½	·842	1·188	+·036	−·352	+1 12	+12 4						

* Mean force to North (λH) being unit. † Earth's Horizontal force (H) being unit. ‡ Earth's Vertical force (Z) being unit.

TABLE I. (continued).—Iron-plated, Iron-built Ships.

Ship.	Compass.	Place.	Date.	Approximate coefficients.					Exact coefficients.				
				A	B	C	D	E	𝔄	𝔅	ℭ	𝔇	𝔈
				° ′	° ′	° ′	° ′	° ′					
RESISTANCE. (3710 tons), Iron-plated, iron hull, 16 guns, 600 horse-power. Built at Millwall, River Thames; head S. 86½° W. magnetic. Launched April 11, 1861. Plated with head generally to Westward.	Standard.	Sheerness......	Aug. 25, 26, 1862...	+0 36	+ 8 9	− 9 41	+ 6 17	+0 8	+·010	+·149	−·158	+·111	+·010
		Lisbon	*Jan.* 1863	+1 54	+ 4 5	− 6 27	+ 6 54	+0 59	+·033	+·075	−·105	+·120	+·017
		Portsmouth...	June 19, 1863......	+0 44	+ 8 21	− 8 24	+ 5 48	−0 54	+·013	+·152	−·138	+·101	−·016
		Portsmouth...	Dec. 1863	+1 1	+ 5 46	− 7 22	+ 7 0	−1 59	+·018	+·106	−·120	+·122	−·034
		Malta	*Jan.* 1864	−0 19	+ 1 36	− 6 13	+ 6 45	−1 20	−·005	+·030	−·102	+·117	−·023
		Malta	*Dec.* 27, 1864	−0 4	+ 2 30	− 6 43	+ 5 58	−1 8	−·001	+·044	−·116	+·104	−·020
	Starboard steering.	Sheerness......	Aug. 25, 26, 1862...	+0 24	+ 7 55	−17 15	+ 8 28	+1 9	+·007	+·147	−·274	+·148	+·020
		Portsmouth.	.June 19, 1863......	+1 20	+10 41	−13 23	+ 8 56	−0 51	+·023	+·198	−·212	+·155	−·015
		Portsmouth...	Dec. 1863	+2 10	+ 9 42	−12 25	+ 9 43	−0 49	+·038	+·181	−·196	+·170	−·014
	Main deck.	Sheerness......	Aug. 25, 26, 1862...	−0 18	+ 9 24	−17 9	+14 0	+0 21	−·005	+·181	−·260	+·244	+·006
		Portsmouth...	June 19, 1863......	+2 9	+ 9 6	−13 6	+13 25	+1 12	+·020	+·175	−·200	+·232	+·021
		Portsmouth...	Dec. 1863	+3 5	+ 3 41	−11 11	+12 39	−3 30	+·054	+·070	−·173	+·219	−·061
HECTOR (¹). (4089 tons), Iron-cased, iron hull, 28 guns, 800 h.-p. Built at Glasgow; head S. 20° E. magnetic. Launched Sept. 26, 1862. Plated with head N. 55° W. and S. 49° W.	Standard.	Portsmouth...	Feb. 16, 1864	−0 24	+21 53	+ 4 54	+ 5 24	−0 39	−·007	+·392	+·079	+·094	−·011
	Starboard steering.	Portsmouth...	Feb. 16, 1864	+0 37	+30 36	+10 37	+ 8 24	−0 16	+·011	+·545	+·164	+·147	−·005
	Main deck.	Portsmouth...	Feb. 16, 1864	+0 16	+31 22	+13 50	+ 9 47	−0 50	+·004	+·520	+·239	+·170	−·014
VALIANT. (4144 tons), Iron-plated, iron hull, 28 guns, 800 h.-p. Built at Millwall, River Thames; head S. 87° W. Launched Oct. 14, 1863. Plated with head generally to Westward.	Standard.	Sheerness	...Jan. 12, 16, 1865...	+1 2	+ 2 30	−12 44	+ 4 54	−0 43	+·018	+·046	−·211	+·085	−·012
	Starboard steering.	Sheerness	...Jan. 12, 16, 1865...	+2 7	+ 7 35	−20 12	+ 6 52	−0 14	+·037	+·138	−·325	+·120	−·004
	Main deck (Starboard).	Sheerness	...Jan. 12, 16, 1865...	+2 35	+ 5 29	−18 39	+ 8 5	−0 12	+·045	+·101	−·297	+·142	−·003

(¹) Hector, June 9, 1863. In basin at Portsmouth, by observations of Deviation and Horizontal force on one point, and employing λ and D of February 1864, B=+·398, C=+·159.

TABLE I. (continued).—Iron-plated, Iron-built Ships.

Maximum of semicircular deviation $\sqrt{B^2+C^2}$	Horizontal force of ship $\sqrt{\mathfrak{B}^2+\mathfrak{C}^2}$*.		Mean force to North, λ †	$\frac{1}{\lambda}$	Coefficients of horizontal induction.		Part of D from		Mean Vertical force, μ ‡	Heeling coefficient to *windward*, χ	Heeling coefficients from		$\frac{g}{\tan\theta}$ ‡	g †
Amount.		Direction.			Fore-and-aft, a †	Transverse e †	Fore-and-aft induction.	Transverse induction.			Vertical induction in transverse iron.	Vertical force and induction in vertical iron.		
°		°					° ′	° ′		° ′	° ′	° ′		
12¼	·218	313	·758	1·319	−·158	−·326	−5 55	+12 21	1·071	+1 18	+1 4	+0 14	+·071	+·176
7¾	·129 ·162	305½												
11¾	·205	317¾												
9½	·160	311½	·850	1·176	−·046	−·254	−1 33	+8 34	1·044	+0 53	+0 45	+0 8	+·076	+·190
6½	·107 ·158	285½												
7¾	·124 ·183	291												
19	·312	298	·703	1·423	−·193	−·401	−7 53	+16 33						
17¼	·290	313												
15¾	·266	313												
19½	·316	305	·782	1·279	−·027	−·409	−1 2	+15 11						
16	·266	311												
11¾	·187	292	·880	1·136	+·073	−·313	+2 25	+10 15						
24½	·400	12½	·814	1·228	−·109	−·263	−3 51	+ 9 15	·983	+0 45	+0 48	−0 3	−·005	−·013
33½	·568	16½	·791	1·264	−·093	−·325	−3 23	+11 49						
34¼	·572	25	·726	1·377	−·151	−·397	−5 58	+15 54						
13	·216	282½	·859	1·164	−·068	−·214	−2 14	+ 7 11	1·061	+0 48	+0 37	+0 11	+·048	+·120
21½	·353	293	·817	1·224	−·085	−·281	−2 59	+ 9 54						
19½	·313	288½	·722	1·385	−·176	−·380	−6 56	+15 14						

* Mean force to North (λH) being unit. † Earth's Horizontal force (H) being unit. ‡ Earth's Vertical force (Z) being unit.

TABLE II.—Iron-plated, Wood-built Ships.

Ship.	Compass.	Place.	Date.	Approximate coefficients.					Exact coefficients.				
				A	B	C	D	E	𝔄	𝔅	ℭ	𝔇	𝔈
				° ′	° ′	° ′	° ′	° ′					
ROYAL OAK. Iron-cased, wood-built, 4056 tons, 35 guns, 800 horse-power. Iron-plated; head S. 49° E. Floated out of dock March 19, 1863.	Standard.	Chatham …	Mar. 19, 1863……	……	……	……	……	……	…	+·253	+·287	+·047	…
		Chatham ……	Apr. 11, 1863……	……	……	……	……	……	…	+·231	+·197	+·061	…
		Sheerness……	June 2, 1863 ……	−0 39	+13 56	+ 7 26	+ 3 9	+0 1	−·011	+·248	+·128	+·055	·000
		Plymouth …	Jan. 8, 1864 ……	−0 12	+12 20	+10 9	+ 2 19	+0 20	−·003	+·218	+·172	+·040	+·006
		Malta ………	*Mar.* 1, 1864 ……	−1 9	+ 8 8	+ 6 1	+ 2 58	−0 48	−·020	+·143	+·108	+·052	−·014
	Starboard steering.	Chatham ……	Apr. 11, 1863……	……	……	……	……	……	…	+·377	+·379	+·067	…
		Sheerness……	June 2, 1863 ……	+0 15	+24 5	+14 25	+ 1 47	+1 17	+·004	+·414	+·241	+·031	+·022
	Main deck.	Sheerness……	June 2, 1863 ……	−1 54	+32 22	+12 47	+ 1 28	−0 11	−·033	+·546	+·210	+·026	−·003
PRINCE CONSORT. Iron-cased, wood-built, 4045 tons, 35 guns, 1000 horse-power. Iron-plated; head S. 39° W.	Standard.	Milford ……	May 25, 1863……	−0 6	+33 39	−13 41	+ 2 18	−0 4	−·001	+·569	−·222	+·040	−·001
		Plymouth …	Feb. 9, 1864 ……	−0 28	+25 36	− 3 53	+ 3 6	−0 33	−·008	+·447	−·064	+·054	−·010
CALEDONIA. Iron-cased, wood-built, 4125 tons, 35 guns. 1000 horse-power. Iron-plated; head S. 26° W.	Standard.	Sheerness……	June 15, 1864……	+0 18	+25 47	− 8 21	+ 2 57	+0 20	+·005	+·448	−·138	+·051	+·006
OCEAN. Iron-cased, wood-built, 4047 tons, 35 guns. 1000 horse-power. Iron-plated; head S. 79° E.	Standard.	Devonport …	Aug. 3, 1864 ……	+0 8	+13 2	+15 23	+ 2 31	−0 4	+·002	+·229	+·259	+·044	−·001
ROYAL SOVEREIGN. Iron-cased, wood-built, turret ship of 5 guns, 3765 tons, 800 horse-power. Iron-plated; head S. 72° E.	Standard.	Portsmouth…	July 21, 22, 1864	−0 3	+12 38	+13 39	+ 7 41	+0 7	−·001	+·233	+·219	+·134	+·002
	Steering wheel (upper deck).	Portsmouth…	July 21. 22, 1864	−1 8	+23 30	−19 40	+13 3	−9 14	−·022	+·487	−·323	+·238	−·159
	Steering wheel (Cap.'s cabin).	Portsmouth…	July 21, 22, 1864	−0 25	+20 11	+ 4 56	+ 6 20	−5 10	−·007	+·364	+·086	+·110	−·090
	Starbᵈ forward (lower deck).	Portsmouth…	July 21, 22, 1864	−0 37	−13 15	+40 15	+15 43	−4 42	−·004	−·277	+·563	+·272	−·078
	Port, forward (lower deck).	Portsmouth…	July 21, 22, 1864	+6 42	−14 35	− 7 8	+13 23	+4 38	+·117	−·286	−·119	+·233	+·081
	Suspended over fore-turret.	Portsmouth…	July 21, 22, 1864	+1 0	−19 33	+ 9 23	+ 8 9	+ 0 1					
ENTERPRISE[1]. (993 tons), 4 guns, 160 h.-p. screw. Built and plated at Deptford; head S.56°W. Launched February 1864.	Standard.	Greenhithe …	June 7, 1864 ……	+1 24	+14 42	−18 45	+ 2 34	+0 35	+·025	+·257	−·312	+·045	+·010
WOLVERENE[2]. (703 tons), 21 guns, 400 h. p. screw. Built at Woolwich; head S.S.W. Launched in 1863.	Standard.	Greenhithe …	May 31, 1864……	+0 23	+14 10	− 2 11	+ 3 20	+0 46	+·007	+·253	−·036	+·058	+·013

---

[1] Wood bottom, Iron-cased, with central iron battery. Iron topsides, decks and beams.

[2] Wood hull, iron beams and stanchions.

TABLE II.—Iron-plated, Wood-built Ships.

Maximum of semicircular deviation $\sqrt{B^2+C^2}$ / Horizontal force of ship $\sqrt{\mathfrak{B}^2+\mathfrak{C}^2}$*.			Mean force to North, λ †	$\frac{1}{\lambda}$	Coefficients of horizontal induction.		Part of D from		Mean Vertical force. μ ‡	Heeling coefficient to *windward*, χ	Heeling coefficients from		$\frac{g}{\tan\theta}$ ‡	g †
Amount.		Direction.			Fore-and-aft, a †	Transverse e †	Fore-and-aft induction.	Transverse induction.			Vertical induction in transverse iron.	Vertical force and induction in vertical iron.		
°		°					° ′	° ′		° ′	° ′	° ′		
……	·382	48¾	·861	1·162	−·098	−·178	−3 16	+ 6 2						
……	·304	40⅓	·907	1·102	−·038	−·148	−1 12	+ 4 39	·896	+0 7	+0 24	−0 17	+·018	+·045
15¾	·280	27⅛	·907	1·102	−·043	−·143	−1 19	+ 4 32	·882	+0 4	+0 23	−0 19	+·052	+·127
16	·278	38												
10	·179 ·264	37												
……	·534	45	·887	1·127	−·054	−·172	−1 43	+ 5 30						
28	·480	30	·906	1·104	−·066	−·122	−2 7	+ 3 51						
34¾	·586	21	·862	1·160	−·116	−·160	−3 51	+ 5 20						
36½	·612	339	·840	1·190	−·126	−·194	−4 18	+ 6 36						
26	·452	352	·950	1·053	+·001	−·101	0 0	+ 3 6	·848	−0 8	+0 16	−0 24	+·015	+·038
27	·469	343	·895	1·117	−·059	−·151	−1 53	+ 4 46						
20⅓	·346	48½	·923	1·083	−·036	−·118	−1 9	+ 3 40	·929	−0 15	+0 19	−0 34	+·045	+·112
18½	·320	43½	·912	1·097	+·044	−·204	+1 5	+ 6 36						
30½	·584	326	·980	1·020	+·202	−·242	+5 58	+ 7 7						
20¾	·374	13½	·917	1·091	+·028	−·184	+0 34	+ 5 45						
42½	·629	116	·783	1·277	−·003	−·431	−0 2	+15 55						
16¼	·310	203	·811	1·233	·000	−·379	0 0	+13 25						
23¾	·406	309½	·817	1·224	−·146	+·220	−5 6	+ 7 44	·622	−0 29	+0 37	−1 9	+·062	+·152
14¼	·256	352	·962	1·039	+·018	−·094	+0 35	+ 2 45	·953	+0 7	+0 14	−0 7		

* Mean force to North (λH) being unit. † Earth's Horizontal force (H) being unit. ‡ Earth's Vertical force (Z) being unit.

TABLE III.—Iron-built Ships, Her Majesty's Navy.

Ship.	Compass.	Place.	Date.	Approximate coefficients.					Exact coefficients.				
				A	B	C	D	E	𝔄	𝔅	ℭ	𝔇	𝔈
				° ′	° ′	° ′	° ′	° ′					
ORONTES. (2812 tons), 4 guns, 500 h.-p., screw. Built at Birkenhead; head N. 66° W. magnetic. Launched Nov. 22, 1862.	Standard.	Plymouth	...May 26, 1863	−0 31	− 7 45	−12 20	+5 46	−0 24	−·009	−·141	−·203	+·100	−·007
		Portsmouth	...July 7, 1863	−0 2	− 6 55	−12 0	+5 30	−0 13	·000	−·125	−·198	+·096	−·004
		C. of Good Hope	*Nov.* 1864	−1 40	− 9 39	−10 41	+5 49	0 0	−·029	−·177	−·178	+·101	·000
	Starboard steering.	Portsmouth	...July 7, 1863	−0 43	−10 27	−13 7	+7 16	−0 22	−·012	−·191	−·213	+·126	−·006
TAMAR. (2812 tons), 4 guns, 500 h.-p., screw. Built at Millwall, River Thames; head West. Launched Jan. 5, 1863.	Standard.	Sheerness	......Nov. 21, 23, 1863	+0 18	+ 1 42	−10 49	+3 18	+0 33	+·005	+·031	−·184	+·058	+·010
		Portsmouth	...Oct. 1864	+0 4	+ 2 11	− 5 26	+3 11	+0 22	+·001	+·038	−·095	+·056	+·006
	Starboard steering.	Sheerness	......Nov. 21, 23, 1863	−1 50	+ 7 15	−17 14	+3 27	+0 8	−·032	+·128	−·288	+·060	+·002
ADVENTURE. (1794 tons), 400 horse-power, screw. Built at Birkenhead. Launched Feb. 17, 1855.	Standard.	Greenhithe	...April 26, 1862 ...	+0 2	− 4 5	+10 59	+2 56	+0 26	·000	−·073	+·186	+·051	+·007
		Greenhithe	...Oct. 28, 1862	+0 8	− 3 59	+10 59	+2 53	+0 10	+·002	−·071	+·186	+·050	+·003
		Yokohama, Japan	...*Nov.* 11, 1864.	......	− 3 28	+ 8 4	+2 49	−0 19	......	−·061	+·139	+·049	+·005
DROMEDARY. (647 tons), 100 horse-power, screw.	Standard.	Greenhithe	...July 8, 1862	+0 32	+ 5 0	−11 50	+6 0	+0 14	+·009	+·091	−·194	+·104	+·044
		Greenhithe	...Dec. 16, 1862	+0 21	+ 4 59	−10 55	+5 33	+0 44	+·006	+·091	−·179	+·097	+·013
WYE. (700 tons), 100 horse-power, screw.	Standard.	Greenhithe	...Sept. 1, 1863	+0 25	+ 3 24	+10 50	+1 31	+0 5	+·007	+·059	+·186	+·026	+·001
CARADOC. (676 tons), Paddle-wheel, 350 h.-p. Built at Blackwall. Launched July 1847.	Standard.	Greenhithe	...Feb. 12, 1863	−0 43	−13 28	− 2 54	+2 3	−0 7	−·012	−·238	−·049	+·036	−·002
INDUSTRY. (638 tons), Screw, 80 horse-power. Built at Blackwall. Launched 1854.	Standard.	Greenhithe	..March 14, 1863 ...	−0 13	+11 32	− 2 16	+2 58	−0 6	−·004	+·206	−·038	+·052	−·002
SUPPLY. (638 tons), Screw, 80 horse-power. Built at Blackwall. Launched June 1854.	Standard.	Greenhithe	...Oct. 17, 1863	−0 12	−13 32	− 1 40	+2 55	+0 16	−·003	−·240	−·028	+·051	+·004

TABLE III.—Iron-built Ships, Her Majesty's Navy.

Maximum of semicircular deviation $\sqrt{B^2+C^2}$ Horizontal force of ship $\sqrt{\mathfrak{B}^2+\mathfrak{C}^2}$*.			Mean force to North, λ †	$\frac{1}{\lambda}$	Coefficients of horizontal induction.		Part of D from		Mean Vertical force, μ ‡	Heeling coefficient to *windward*, χ	Heeling coefficients from		$\frac{g}{\tan\theta}$ ‡	g †
Amount.		Direction.			Fore-and-aft, a †	Transverse e †	Fore-and-aft induction.	Transverse induction.			Vertical induction in transverse iron.	Vertical force and induction in vertical iron.		
°		°					° ′	° ′		° ′	° ′	° ′		
14½ 14 14¼	·247 ·234 {·251 ·293	235 238 225	·875	1·143	−·041	−·209	−1 22	+6 40	1·164	+1 4	+0 36	+0 28	+·023	+·056
17	·286	228	·862	1·160	−·029	−·247	−0 58	+8 13						
11 6	·187 ·102	279½ 292	·870	1·150	−·080	−·180	−2 38	+5 58	1·117	+0 51	+0 31	+0 20	+·060	+·147
18¾	·315	294	·886	1·129	−·061	−·167	−2 0	+5 27	1·248	+1 10	+0 28	+0 42	+·120	+·294
11¾ 11¾ 9	·200 ·199 {·151 ·249	111 111 ·113	·922 ·918	1·085 1·090	−·031 −·035	−·125 −·129	−1 0 −1 8	+3 56 +4 1						
12¾ 12	·215 ·201	295 297	·841 ·861	1·186 1·161	−·072 −·056	−·246 −·222	−2 21 −1 50	+8 21 +7 28						
11½	·195	72	·869	1·151	−·108	−·154	−3 34	+5 3	1·195	+1 0	+0 27	+0 34	+·103	+·252
13¾	·243	191½	·945	1·058	−·021	−·089	−0 38	+2 42	1·002	+0 15	+0 14	+0 1		
11¾	·209	349½	·937	1·067	−·014	−·112	−0 41	+3 40	·859	−0 5	+0 18	−0 23		
13¾	·242	186½	·925	1·081	−·028	−·122	−0 55	+3 47						

* Mean force to North (λH) being unit. † Earth's Horizontal force (H) being unit. ‡ Earth's Vertical force (Z) being unit.

TABLE IV.—Iron-built Ships, Mercantile Marine.

Ship.	Compass.	Place.	Date	Approximate coefficients.					Exact coefficients.				
				A	B	C	D	E	𝔄	𝔅	ℭ	𝔇	𝔈
				° ′	° ′	° ′	° ′	° ′					
RAINBOW[1]	Station No. 1	Deptford......	July and Aug. 1838	+0 40	−50 36	−11 4	+1 23	+0 38	+·012	−·802	−·173	+·024	+·011
	„ No. 2	......		+0 35	−18 45	−12 57	+2 30	+0 2	+·010	−·327	−·217	+·044	+·001
	„ No. 3	......		+0 42	−15 46	−10 39	+3 7	−0 2	+·012	−·279	−·181	+·054	−·001
	„ No. 4	......		+0 5	− 8 5	− 9 33	+3 26	+0 2	+·001	−·145	−·161	+·060	+·001
IRONSIDES[2].........	Binnacle, or steering.	Liverpool ...	Oct. 27, 1838	0 0	−24 16	+20 50	+2 15	−0 1	·000	−·416	+·346	+·039	·000
GREAT EASTERN[3]	Standard position.	River Thames...	Sept. 7, 1859 ...	−0 10	+23 13	+25 38	+4 21	−0 37	−·003	+·402	+·408	+·076	−·011
		Portland	Sept. 12, 1859...	−1 3	+22 42	+16 43	+4 44	−0 45	−·018	+·400	+·272	+·082	−·013
	Compass aft on platform.	River Thames...	Sept. 7, 8, 1859	−1 40	+13 34	+22 41	+7 55	−0 12	−·029	+·247	+·359	+·138	−·003
	Compass on fore bridge.	River Thames...	Sept. 8, 1859 ...	+0 3	+31 56	+17 47	+4 31	−0 9	+·001	+·551	+·282	+·079	−·003
CLYDE	Standard position.	Greenhithe ...	Feb. 21, 1863	+0 41	− 7 56	+ 7 25	+4 43	+0 8	+·012	−·143	+·124	+·082	+·002
CITY OF SYDNEY...	Standard position.	Greenhithe ...	June 13, 1863......	+1 27	− 3 29	−18 51	+4 32	+0 23	+·025	−·063	−·311	+·079	+·007

[1] Station No. 1, (near the binnacle) 13 ft. 2 in. distant from the extreme part of stern, 4 ft. 0½ in. from deck.
„ 2, 31 9 „ „ „ „
„ 3, 48 3 „ „ „ „
„ 4, 151 6 „ „ „
47 0 from knight head of stem „ „
See Philosophical Transactions, 1839, Part I. p. 167.

[2] See Philosophical Transactions, 1839, Part I. p. 206.

[3] See Philosophical Transactions, 1860, Part II. p. 375.

Table of Terrestrial Magnetic Elements. [1864.]

Place.	In British absolute units.		Dip θ.	Tan θ.	Horizontal force at Greenwich being unit*.	
	Horizontal force.	Vertical force.			Horizontal force.	Vertical force.
			° ′			
Greenwich	3·83	+ 9·53	+68 7	+2·49	1·00	+2·49
Greenhithe.........	3·84	+ 9·50	+68 5	+2·48	1·00	+2·48
Sheerness	3·83	+ 9·50	+68 2	+2·48	1·00	+2·48
Portsmouth	3·86	+ 9·48	+67 50	+2·45	1·01	+2·47
Portland............	3·88	+ 9·50	+67 45	+2·44	1·01	+2·47
Plymouth	3·86	+ 9·54	+67 58	+2·47	1·01	+2·49
Milford	3·62	+ 9·80	+69 44	+2·71	·95	+2·56
Greenock	3·38	+10·04	+71 23	+2·97	·88	+2·63

* NOTE.— For British absolute units multiply by 3·83.
For Foreign absolute units multiply by 1·76.

TABLE IV.—Iron-built Ships, Mercantile Marine.

Maximum of semicircular deviation $\sqrt{B^2+C^2}$	Horizontal force of ship $\sqrt{\mathfrak{B}^2+\mathfrak{C}^2}$*		Mean force to North, λ †	$\frac{1}{\lambda}$	Coefficients of horizontal induction.		Part of D from		Mean vertical force, μ ‡	Heeling coefficient to *windward*, χ	Heeling coefficients from		$\frac{g}{\tan\theta}$ ‡	g †
Amount.		Direction.			Fore-and-aft a †	Transverse e †	Fore-and-aft induction.	Transverse induction.			Vertical induction in transverse iron	Vertical force and induction in vertical iron.		
°		°					° ′	° ′		° ′	° ′	° ′		
52	·822	192	·984	1·016	+·008	−·040	+0 14	+1 9						
22¾	·392	213	·972	1·029	+·015	−·071	+0 24	+2 7						
19	·332	213	1·003	·997	+·057	−·050	+1 40	+1 26						
12½	·217	228	·999	1·001	+·060	−·060	+1 43	+1 43						
32	·542	140	·914	1·094	−·050	−·122	−1 33	+3 50						
34½	·574	45½	·791	1·264	−·072	−·192	−3 50	+8 13						
28	·484	34½	·775	1·291	−·082	−·209	−4 4	+8 48						
26½	·438	55½	·897	1·115	+·066	−·182	+0 38	+7 18						
36½	·619	27	·892	1·121	−·038	−·178	−4 38	+9 16						
11	·189	139	·870	1·149	−·059	−·201	−1 57	+6 39	1·275	1 22	+0 35	+0 47		
19½	·158	258½	·816	1·225	−·120	−·248	−4 8	+8 44	1·246	1 31	+0 46	+0 45		

* Mean force to North (λH) being unit. † Earth's Horizontal force (H) being unit. ‡ Earth's Vertical force (Z) being unit.

Table of Terrestrial Magnetic Elements. [1864.]

Place.	In British absolute units.		Dip θ.	Tan θ.	Horizontal force at Greenwich being unit §.	
	Horizontal force.	Vertical force.			Horizontal force.	Vertical force.
Lisbon	4·82	+8·46	+60° 23′	+1·76	1·26	+2·21
Gibraltar	5·09	+7·89	+57 9	+1·55	1·33	+2·06
Madeira	5·17	+8·27	+57 55	+1·60	1·35	+2·16
Teneriffe	5·44	+8·10	+56 10	+1·49	1·42	+2·12
Malta	5·65	+7·29	+52 20	+1·29	1·47	+1·90
Simons Bay, Cape of Good Hope	4·48	−6·43	−55 8	−1·44	1·17	−1·68
Yokohama, Japan	6·32	+7·08	+48 10	+1·12	1·65	+1·85

§ NOTE.—For British absolute units multiply by 3·83.
For Foreign absolute units multiply by 1·76.

ON THE EFFECT ON THE COMPASS OF PARTICULAR MASSES OF SOFT IRON IN A SHIP*.

The form of the general equations for the effect of the soft iron of a ship on the compass does not, as we have seen, depend on the form, position, or inductive capacity of the iron. They involve, it is true, nine coefficients which depend on these particulars, but the data of the problem are in general not these particulars, but the effects which they cause in certain definite positions of the ship. This is fortunate, because, while the form of the general equations is obtained at once from very simple physical considerations, and while the special formulæ required are deduced from these by simple trigonometrical operations, and the coefficients are then deduced from the observations by a simple arithmetical operation, the *à priori* determination of the effect on the compass of given masses of iron is, in all but the very simplest cases, a matter of great and generally insuperable difficulty.

It is however in all cases interesting, and in some cases important, to be able to form an approximate estimate of the nature and amount of the effects on the compass of particular masses of iron, and although the precise cases of masses of iron in which the problem admits of an exact solution may not often occur, yet cases frequently occur of masses of iron sufficiently resembling them to have much light thrown on their effects by the knowledge of the effect of the simpler bodies which they most nearly resemble.

The most general case for which the problem can be solved is that of ellipsoids and ellipsoidal shells, including the forms into which these degenerate, as spheres, spheroids, plates, cylinders, &c., but the general solution is so extremely unmanageable, in its practical application, that it is more convenient to consider the simpler cases independently. The cases which we shall consider are—

1. Infinitely thin rods of finite or infinitesimal length.
2. Infinitely thin plates of finite dimensions magnetized longitudinally.
3. Infinite plates of finite thickness magnetized perpendicularly.
4. Spheres.
5. Spherical shells.
6. Infinitely long cylinders magnetized perpendicularly.
7. Infinitely long cylindrical shells magnetized perpendicularly.

A little consideration will show that there is hardly any arrangement of iron in a ship which does not bear more or less resemblance to one or other of these cases.

The physical theory of Coulomb, on which Poisson's mathematical theory is based, supposes, as is well known, that there is no separation of two kinds of magnetism except within infinitely small elements of the iron; but on this theory, if the iron be homogeneous, the result on all external bodies is precisely the same as that of a certain distribution of North and South magnetism on the surface of the iron.

---

* I beg to express my obligations to Professor W. Thomson for much of what is contained in this part of the paper, and at the same time to express my hope that he may be induced to complete the promised Treatise on the Mathematical Theory of Magnetism, part of which was published in the Phil. Trans. 1851.—A. S.

To avoid the ambiguity which arises from the use of the terms "North" and "South" magnetism, we shall speak of the magnetism of the north end of the needle and the southern hemisphere of the earth as *red* magnetism, of the south end of the needle and the northern hemisphere as *blue* magnetism.

I. *An infinitely thin rod.*

Let S be the area of a section of the rod, F the component of the earth's force in the direction of the rod, and $\varkappa$ a coefficient depending on the inductive capacity of the iron.

Each end of the rod will have a quantity of free magnetism $=\varkappa\mathrm{SF}$, the magnetism being red at the north end, blue at the south end of the rod.

If x, y, z be the coordinates, r the distance of the blue end, x', y', z' the coordinates, r' the distance of the red end, l the length of the rod, X, Y, Z the components of the earth's force, then the effect of the rod on a red particle at the origin is a force

$$\text{Towards } x=\varkappa\mathrm{S}\left(\frac{x}{r^3}-\frac{x'}{r'^3}\right)\left\{\frac{x'-x}{l}\mathrm{X}+\frac{y'-y}{l}\mathrm{Y}+\frac{z'-z}{l}\mathrm{Z}\right\},$$

$$\text{Towards } y=\varkappa\mathrm{S}\left(\frac{y}{r^3}-\frac{y'}{r'^3}\right)\left\{\frac{x'-x}{l}\mathrm{X}+\frac{y'-y}{l}\mathrm{Y}+\frac{z'-z}{l}\mathrm{Z}\right\},$$

$$\text{Towards } z=\varkappa\mathrm{S}\left(\frac{z}{r^3}-\frac{z'}{r'^3}\right)\left\{\frac{x'-x}{l}\mathrm{X}+\frac{y'-y}{l}\mathrm{Y}+\frac{z'-z}{l}\mathrm{Z}\right\}.$$

If the rod be infinitely short, and $x'-x=dx$, $y'-y=dy$, $z'-z=dz$, $l=ds$, then force

$$\text{Towards } x=\varkappa\mathrm{S}\frac{ds}{r^3}\left\{3\frac{x}{r}\left(\frac{x}{r}\frac{dx}{ds}+\frac{y}{r}\frac{dy}{ds}+\frac{z}{r}\frac{dz}{ds}\right)-\frac{dx}{ds}\right\}\left\{\frac{dx}{ds}\mathrm{X}+\frac{dy}{ds}\mathrm{Y}+\frac{dz}{ds}\mathrm{Z}\right\},$$

$$\text{Towards } y=\varkappa\mathrm{S}\frac{ds}{r^3}\left\{3\frac{y}{r}\left(\frac{x}{r}\frac{dx}{ds}+\frac{y}{r}\frac{dy}{ds}+\frac{z}{r}\frac{dz}{ds}\right)-\frac{dy}{ds}\right\}\left\{\frac{dx}{ds}\mathrm{X}+\frac{dy}{ds}\mathrm{Y}+\frac{dz}{ds}\mathrm{Z}\right\},$$

$$\text{Towards } z=\varkappa\mathrm{S}\frac{ds}{r^3}\left\{3\frac{z}{r}\left(\frac{x}{r}\frac{dx}{ds}+\frac{y}{r}\frac{dy}{ds}+\frac{z}{r}\frac{dz}{ds}\right)-\frac{dz}{ds}\right\}\left\{\frac{dx}{ds}\mathrm{X}+\frac{dy}{ds}\mathrm{Y}+\frac{dz}{ds}\mathrm{Z}\right\}.$$

If the rod be in the plane of x, y and parallel to the axis of x, then z, dy and $dz=0$, and force

$$\text{Towards } x=\frac{\varkappa\mathrm{S}l}{r^3}\left(3\frac{x^2}{r^2}-1\right)\mathrm{X},$$

$$\text{Towards } y=\frac{\varkappa\mathrm{S}l}{r^3}\,3\frac{xy}{r^2}\mathrm{X},$$

$$\text{Towards } z=0.$$

If the rod be in the axis of x, then $x=r$, and the force is

$$2\frac{\varkappa\mathrm{S}l}{r^3}\mathrm{X} \text{ in the direction of } +x.$$

If the rod be in the axis of y, then $x=0$, and the force is

$$\frac{\varkappa\mathrm{S}l}{r^3}\mathrm{X} \text{ in the direction of } -x.$$

The product $\varkappa\mathrm{S}l\mathrm{X}$ is called the *moment* of the magnetic rod.

We will now pause to state what is known of the value of $\varkappa$ for iron of different kinds.

The coefficient $\varkappa$ is the quantity so designated by NEUMANN in Crelle's Journal, vol. xxxvii. p. 21, WEBER in Götting. Trans. vol. vi. p. 20, and THALEN in Nov. Act. Soc. Reg. Upsal. 1861.

It is related to the k of POISSON'S papers in the fifth volume of the 'Mémoires de l'Institut,' and to the g of GREEN'S celebrated "Essay on the Mathematical Theories of Electricity and Magnetism" (Nottingham, 1828; reprinted in Crelle's Journal, vol. xlvii.), by the equation

$$k=g=\frac{\frac{4\pi}{3}\varkappa}{1+\frac{4\pi}{3}\varkappa}.$$

GREEN, in the essay referred to, finds, from some experiments of COULOMB on steel wire,

$$g=\cdot986636,$$

whence

$$\varkappa=17\cdot625.$$

WEBER finds the following values of $\varkappa$:

Steel tempered to glass hardness and already magnetized . . .	4·091
Steel tempered to glass hardness with no permanent magnetism .	4·934
Soft steel	5·61
Soft iron	36

THALEN finds, from six specimens of soft iron carefully annealed, the following values:

Specimen.	$\varkappa$.
1	34·58
2	27·24
3	45·26
4	32·25
5	44·23
6	36·96
Mean . . .	36·75

From observations of iron bars given by SCORESBY in his 'Magnetical Investigations,' vol. ii. p. 320, we derive

	$\varkappa$.
Iron rod, not struck	16·77
Iron rod, struck	44·07

From observations which we have made with a rod of iron $\frac{7}{16}$ths of an inch in diameter, 3 feet long, we have found

	$\varkappa$.
Iron, not struck	12·48
Iron, struck several sharp blows, about	80

Hence probably in the iron plates used in ship-building $\varkappa$ may vary from 10 to 30.

2. *An infinitely thin plate of finite dimensions magnetized longitudinally.*

If F be the component of the earth's magnetism in the plane, and perpendicular to any part of the edge, we shall have a distribution of red magnetism on the northern edge of the plate, of blue magnetism on the southern; and if m be the thickness of the plate, then the force exerted by a part of the blue edge of length ds, or a red particle at a distance r, will be

$$\varkappa \mathrm{F}\frac{m ds}{r^2},$$

and the effect of the whole edge will be given by ordinary integration. Such a plate may in fact be considered as a collection of thin iron rods laid side by side, parallel to the direction of the component of the earth's force which we are considering.

3. *An infinite plate of finite thickness magnetized perpendicularly.*

Let F be the component of the earth's force perpendicular to the plate.

The northern surface of the plate will have a distribution of red free magnetism, the southern surface of blue; the amount of each on an element of surface $=d\mathrm{S}$ being

$$\frac{\varkappa \mathrm{F}}{1+4\pi\varkappa}d\mathrm{S}.$$

Each surface will exercise a force in a direction perpendicular to the plate of $\frac{2\pi\varkappa}{1+4\pi\varkappa}\mathrm{F}$ on a red particle anywhere situate.

Hence the effect of the one surface, in the case of an *external* particle, will be to neutralize the effect of the other.

On an *internal* particle, both surfaces acting in the same direction, the force will be

$$\frac{4\pi\varkappa}{1+4\pi\varkappa}\mathrm{F} \text{ to South.}$$

4. *Sphere.*

The distribution of free magnetism on the surface of a sphere will of course be symmetrical with regard to two poles and an axis parallel to the direction of dip, the free magnetism being red in the northern half of the sphere, blue in the southern; the amount on a unit of surface at either pole will be

$$\mathrm{I}=\frac{\varkappa}{1+\frac{4}{3}\pi\varkappa}\mathrm{F},$$

and at a point at the extremity of a radius making an angle α with the axis

$$\mathrm{I}\cos\alpha=\frac{\varkappa}{1+\frac{4}{3}\pi\varkappa}\,.\,\mathrm{F}\cos\alpha.$$

The effect on a red particle at a distance r from the centre of the sphere, and in a direction making an angle α with the axis, p being the radius of the sphere, is

$$\frac{\frac{4\pi}{3}\kappa}{1+\frac{4\pi}{3}\kappa}\mathrm{F}\frac{p^3}{r^3}(3\cos^2\alpha-1)\ \textit{to North},$$

$$\frac{\frac{4\pi}{3}\kappa}{1+\frac{4\pi}{3}\kappa}\mathrm{F}\frac{p^3}{r^3}3\sin\alpha\cos\alpha\ \textit{from axis}.$$

Hence, at the pole, *but outside the sphere,* it is

$$\frac{\frac{8\pi}{3}\kappa}{1+\frac{4\pi}{3}\kappa}\mathrm{F}\ \ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ \ \text{to North.}$$

At the equator, and anywhere inside the sphere, it is

$$\frac{\frac{4}{3}\pi\kappa}{1+\frac{4}{3}\pi\kappa}\mathrm{F}\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ \ \text{to South.}$$

The sphere, therefore, acts on external particles precisely as an infinitely small magnet of moment $\frac{\frac{4}{3}\pi\kappa}{1+\frac{4}{3}\pi\kappa}p^3\mathrm{F}$ held in the direction of the lines of force.

Here we may pause to observe the very remarkable fact that while the effect of a thin rod or plate magnetized in a direction parallel to itself is nearly proportional to κ, the effect of a plate magnetized at right angles to its plane, or of a solid sphere, is almost independent of the value of κ.

Thus, taking WEBER'S observations, the values of κ for steel and soft iron are nearly 5 and 36. A soft iron rod or thin plate magnetized in the direction parallel to itself would therefore have more than seven times the effect of a steel rod or plate of the same dimensions; but in the case of spheres the proportion of the effects would be

$$\frac{5}{5\cdot24}:\frac{36}{36\cdot24}=\cdot954:\cdot993$$
$$=\ 24\quad 25\ \text{nearly;}$$

or the effect of the hardest steel sphere is within 4 per cent. of the effect of a similar sphere of soft iron, and within 5 per cent. of the effect of a similar sphere of a substance infinitely susceptible of induction, and hammering such a sphere would make no perceptible difference in its effect.

At the equator outside, and anywhere in the interior, the force of the sphere, as we have said, is

$$\frac{\frac{4\pi}{3}\kappa\mathrm{F}}{1+\frac{4\pi}{3}\kappa}\ \text{to South;}$$

this force therefore would, within 5 per cent. in the case of a steel sphere, and within 1 per cent. in the case of a soft iron sphere, neutralize the effect of the earth's magnetism.

5. *Spherical Shell.*

Let p be the radius of the outer surface, q of the inner.

There will be a distribution of free magnetism on the outside similar to that on the sphere, but in the case of the shell

$$\mathrm{I}=\kappa\mathrm{F}\frac{1+\frac{8\pi}{3}\kappa\left(1-\frac{q^3}{p^3}\right)}{1+4\pi\kappa+\frac{4\pi}{3}\cdot\frac{8\pi}{3}\kappa^2\left(1-\frac{q^3}{p^3}\right)}.$$

There will be a similar distribution of free magnetism, but of the opposite kind, in the interior surface, such that if I′ represent the amount of *blue* magnetism on a unit of surface at the north pole of the interior surface,

$$\mathrm{I}'=\kappa\mathrm{F}\frac{1}{1+4\pi\kappa+\frac{4\pi}{3}\cdot\frac{8\pi}{3}\kappa^2\left(1-\frac{q^3}{p^3}\right)}.$$

Hence for an external particle the coefficient will be

$$\mathrm{I}-\mathrm{I}'\frac{q^3}{p^3}=\kappa\mathrm{F}\frac{\left(1+\frac{8\pi}{3}\kappa\right)\left(1-\frac{q^3}{p^3}\right)}{1+4\pi\kappa+\frac{4\pi}{3}\cdot\frac{8\pi}{3}\kappa^2\left(1-\frac{q^3}{p^3}\right)}$$

$$=\frac{\kappa\mathrm{F}}{1+\frac{4\pi}{3}\kappa}\cdot\frac{1-\frac{q}{p}}{1-\frac{q}{p}+\frac{3}{8\pi\kappa}}$$

nearly, if κ be large and $1-\frac{q}{p}$ small.

If $1-\frac{q}{p}$ be infinitely small, the intensity both outside and inside at the North end is $=\frac{\kappa}{1+4\pi\kappa}\cdot\mathrm{F}$, or the same as in a plate, as might be expected.

Mr. Barlow found that in a shell of $\frac{1}{30}$th of an inch thick and 10 inches diameter the effect was $\frac{2}{3}$ that of a solid sphere, whence

$$\frac{\frac{1}{150}}{\frac{1}{150}+\frac{3}{8\pi\kappa}}=\frac{2}{3},$$

$$\text{or } \kappa=\frac{112{\cdot}5}{\pi}$$

$$=35{\cdot}8,$$

which agrees closely with the previous results.

The coefficient for the force on a point in the interior of the spherical shell is

$$
\begin{aligned}
&-\frac{4\pi}{3}.(\mathrm{I}-\mathrm{I}')\\
&=-\frac{4\pi}{3}\varkappa\mathrm{F}\frac{\frac{8\pi}{3}\varkappa\left(1-\frac{q^3}{p^3}\right)}{1+4\pi\varkappa+\frac{4\pi}{3}\cdot\frac{8\pi}{3}\cdot\varkappa^2\left(1-\frac{q^3}{p^3}\right)}\\
&=-\mathrm{F}\frac{1-\frac{q}{p}}{1-\frac{q}{p}+\frac{3}{8\pi\varkappa}}
\end{aligned}
$$

nearly, when $\varkappa$ is large and $1-\frac{q}{p}$ small; and the whole directive force in the interior will in that case be

$$=\frac{\mathrm{F}}{1+\frac{8\pi\varkappa}{3}\left(1-\frac{q}{p}\right)};$$

and therefore if the shell be thick it will be nearly zero, the residual force being inversely as the thickness; if the shell be thin, the loss of force will be nearly proportional to the thickness.

6. *Infinite cylinder, magnetized at right angles to its length.*

Radius $=p$.

The intensity of red magnetism on a point in the surface at an angle α from North is

$$\mathrm{I}=\frac{\varkappa\mathrm{F}\cos\alpha}{2\pi\varkappa+1}.$$

The effect on a red particle at a distance r, r making an angle α with the North and South axis of a perpendicular section, is

$$\frac{2\pi\varkappa}{2\pi\varkappa+1}\mathrm{F}\frac{p^2}{r^2}\cos 2\alpha \quad . \quad . \quad . \quad \text{to North,}$$

$$\frac{2\pi\varkappa}{2\pi\varkappa+1}\mathrm{F}\frac{p^2}{r^2}\sin 2\alpha \quad . \quad . \quad . \quad \text{to East.}$$

7. *Infinite cylindrical shell.*

External radius p, internal radius q.

The distribution of free magnetism will be similar to that on the solid cylinder, except that, as in the case of a spherical shell, the free magnetism on the interior surface will be of the opposite kind to that at corresponding points of the external surface.

For the external surface (red at North),

$$\mathrm{I}=\varkappa\mathrm{F}\frac{1+2\pi\varkappa\left(1-\frac{q^2}{p^2}\right)}{1+4\pi\varkappa+4\pi^2\varkappa^2\left(1-\frac{q^2}{p^2}\right)}.$$

Internal surface (blue at North),

$$I'=\kappa F\frac{1}{1+4\pi\kappa+4\pi^2\kappa^2\left(1-\frac{q^2}{p^2}\right)}.$$

Hence for an external particle the coefficient will be $2\pi\left(I-\frac{q^2}{p^2}I'\right)$,

$$2\pi\left(I-I'\frac{q^2}{p^2}\right)=2\pi\kappa F\frac{(1+2\pi\kappa)\left(1-\frac{q^2}{p^2}\right)}{1+4\pi\kappa+4\pi^2\kappa^2\left(1-\frac{q^2}{p^2}\right)}$$

$$=\frac{2\pi\kappa F}{2\pi\kappa+1}\;\frac{1-\frac{q}{p}}{1-\frac{q}{p}+\frac{1}{2\pi\kappa}}$$

nearly, when κ is large and $1-\frac{q}{p}$ small.

In the interior of the cylinder the coefficient is

$$-2\pi\{I-I'\}=-2\pi\kappa F\frac{2\pi\kappa\left(1-\frac{q^2}{p^2}\right)}{1+4\pi\kappa+4\pi^2\kappa^2\left(1-\frac{q^2}{p^2}\right)}$$

$$=-F\frac{1-\frac{q}{p}}{1-\frac{q}{p}+\frac{1}{2\pi\kappa}}$$

nearly, if κ be large and $1-\frac{p}{q}$ be small; or whole force in interior

$$=\frac{F}{1+2\pi\kappa\left(1-\frac{q}{p}\right)}.$$

Application to particular cases.

As we know from the general equations that the effect of any masses of soft iron may be represented by means of the coefficients a, b, c, d, e, f, g, h, k, and as we are in possession of formulæ which give the different parts of the deviation in terms of these coefficients, by far the most convenient mode of expressing the effect of any given mass of soft iron is to find the a, b, c, d, e, f, g, h, k to which it gives rise; and in what follows we shall suppose the formulæ involving these quantities and connecting them with the deviation-coefficients to be known.

Thus from the expressions we have given for the effect of a finite or infinite rod, we at once derive the coefficients a, b, c, they being the factors of X, Y, Z in the expressions for the force towards x, and so of the others. From these we might derive the coefficients $\mathfrak{A}$, $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, $\mathfrak{E}$, λ, χ; but there would be no interest in the general solution, as the rods we have to deal with in practice are always parallel to one of the principal axes, and these we shall therefore consider separately.

Transverse longitudinal masses of Iron extending from side to side as Iron beams.

Let m be the length of the beam, or in general the breadth of the vessel, r the distance of either end of the beam from the compass, S the area of the section of the beam. It is easily seen that such a beam will give no coefficient except

$$e=-\frac{\varkappa \mathrm{S} m}{r^3}.$$

Every such beam therefore diminishes the directive force and produces a + quadrantal deviation, the effect being directly proportional to the mass of the beam, inversely proportional to the cube of the distance of its ends.

If we have a rectangle of four beams, two fore-and-aft and two transverse, the compass being in or directly above or below the centre of the rectangle, l being the length of the two fore-and-aft beams, m of the two transverse beams, we shall have

$$a=-2\varkappa \mathrm{S}\frac{l}{r^3},$$

$$e=-2\varkappa \mathrm{S}\frac{m}{r^3},$$

whence

$$\lambda=1-\varkappa \mathrm{S}\,\frac{l+m}{r^3},$$

$$\mathfrak{D}=-\frac{\varkappa s}{\lambda}\,\frac{l-m}{r^3}.$$

Such beams may be compared to the armour-plating of a ship, and we thus see that for a compass near the centre of the ship, l being greater than m, the effect of such plating will be to diminish the quadrantal deviation.

In accordance with this result, we find that in the wood-built iron-plated ships, when the compasses are inside the rectangle of the armour-plating, the quadrantal deviation is very small.

When, as in the case of the Warrior and Black Prince, the plating does not extend from end to end, and the compasses are near or even outside one end, the case is different.

Thus if the fore-and-aft coordinates of the ends be x' and x, and the distances from the compass r' and r, we shall have

$$a=2\varkappa \mathrm{S}\left\{-\frac{x'}{r'^3}+\frac{x}{r^3}\right\},$$

$$e=2\varkappa \mathrm{S}\left\{-\frac{y}{r'^3}+\frac{y}{r^3}\right\},$$

$$\mathfrak{D}=\frac{\varkappa s}{\lambda}\left\{\frac{x+y}{r^3}-\frac{x'-y}{r'^3}\right\}.$$

When the plating extends abaft the compass x is negative, and when this is the case, x' being of course greater than y, so long as x is greater than y, or *so long as the plating* *extends half the breadth of the ship abaft the compass*, it will diminish the quadrantal deviation.

$$\text{When } x = -\left\{y-(x'-y)\frac{r^3}{r'^3}\right\},$$

or *when the armour-plating extends a little less than half the breadth abaft the compass*, its effect on the quadrantal deviation vanishes, and when the distance is less than that last mentioned, it increases the quadrantal deviation.

If the central part of a beam be cut out, and if y and y' be the transverse coordinates, r, r' the distances of its outer and inner extremities from the compass,

$$e = 2\varkappa S\left\{\frac{-y}{r^3}+\frac{y'}{r'^3}\right\}.$$

Hence if such a beam be near the compass so that $\frac{y}{r^3} < \frac{y'}{r'^3}$, it will increase the directive force and diminish the quadrantal deviation; if distant it will have the opposite effect.

A vertical rod, z being the vertical coordinate of the upper, z' of the lower end, x and y being the horizontal coordinates, will produce

$$c = \varkappa Sx\left(\frac{1}{r^3}-\frac{1}{r'^3}\right),$$

$$f = \varkappa Sy\left(\frac{1}{r^3}-\frac{1}{r'^3}\right),$$

$$k = \varkappa S\left(\frac{z}{r^3}-\frac{z'}{r'^3}\right).$$

The effect which is of most interest is that of k, as it affects the heeling error.

If z be negative, z' positive, or if the upper end of the beam be above and the lower end below the level of the compass, we see that k will be negative, and will in general diminish the heeling error.

If the rod be a short one of length n,

$$k = \frac{\varkappa Sn}{r^3}\left(3\frac{z^2}{r^2}-1\right);$$

here k will be $\pm$, as

$$\frac{z}{r} \gtrless \frac{1}{\sqrt{3}},$$

or, in other words, if the centre of the rod be within the cone traced out by a line through the compass, making an angle of 54° 45′ with the vertical, k will be positive, and the force of the rod will act downwards and increase the heeling error. On the other hand, if the centre of the rod be without the cone, k will be negative, and the force will act upwards and decrease the heeling error.

Hence we see that in all cases, except when the compass is raised very much above the upper part of the armour-plates, the effect of armour-plating will be to diminish the heeling error.

Thin Plate magnetized in its plane.

If the compass be above or below the centre of a rectangular plate, which may represent the iron deck of a ship, $2x$ being the length, $2y$ the breadth, n the thickness, z the height of the compass above it, r the distance from the compass to one corner, and v the volume of the plate,

$$a = -\frac{4\varkappa nxy}{r(x^2+z^2)} = -\frac{\varkappa v}{r} \cdot \frac{1}{x^2+y^2},$$

$$e = -\frac{4\varkappa nxy}{r.(y^2+z^2)} = -\frac{\varkappa v}{r}\,\frac{1}{y^2+z^2},$$

$$\mathfrak{D} = \frac{\varkappa v}{2r}\left\{\frac{1}{y^2+z^2} - \frac{1}{x^2+z^2}\right\},$$

or such a plate will always produce a diminution of the directive force, and if $x > y$, or if its length be in the fore-and-aft direction, a *positive* quadrantal deviation.

A vertical thin plate, such as a transverse bulkhead, may, as regards transverse induction, be considered as a series of thin horizontal beams giving a $-e$, diminishing λ and increasing $\mathfrak{D}$. As regards vertical induction, it may be considered as a series of vertical rods giving a $+c$ if before the compass, a $-c$ if abaft, and a $+k$ or $-k$ according nearly as the centres of the supposed vertical rods are within or without the cone we have described. There would be no difficulty in computing the effect of such a bulkhead of given position and thickness if $\varkappa$ were known.

Thick Plate magnetized perpendicularly.

If the length and breadth of the plate be infinite or very great compared to the distance of the compass, such a plate will produce no effect on the compass, the effect of one surface being exactly neutralized by that of the other.

When the dimensions of the plate are finite we may arrive at an approximate result, by supposing lines drawn from the compass to every point on the edge of the further surface. The parts of the two surfaces within the pyramid bounded by these lines will neutralize each other, leaving only a margin of the nearer surface to act on the compass. The effect of this may be easily computed, by computing the effect of four such red or blue lines, as the case may be, the free magnetism in a unit of length being

$$\frac{\mathrm{F}}{4\pi} \times \text{breadth of margin.}$$

From these considerations we see that the effect of even a thick armour-plating, magnetized perpendicularly, will not be great.

The effect of a thick transverse armour bulkhead, on a compass immediately above and near it, will be to produce a $-a$, which may be easily computed, as we may suppose the dimensions of the plate in every direction below its upper surface to be infinite.

If l be the thickness of the bulkhead, n the height of the compass above its centre,

$$a = -\frac{1}{2\pi}\,\frac{l}{n}.$$

Sphere.

Let the centre of the sphere be at a distance r from the centre of the compass, and let r make angles α, β, γ with the coordinate axes to head, to starboard, and to nadir, and let

$$\frac{\frac{4\pi}{3}\varkappa}{1+\frac{4\pi}{3}\varkappa}\frac{p^3}{r^3}=\mathrm{M}.$$

Then

$$\begin{aligned}a&=\mathrm{M}(3\cos^2\alpha-1),\\ b=d&=\mathrm{M}\;3\cos\alpha\cos\beta,\\ c=g&=\mathrm{M}\;3\cos\alpha\cos\gamma,\\ e&=\mathrm{M}(3\cos^2\beta-1),\\ f=h&=\mathrm{M}\;3\cos\beta\cos\gamma,\\ k&=\mathrm{M}(3\cos^2\gamma-1),\end{aligned}$$

whence

$$\begin{aligned}\lambda&=1+\frac{\mathrm{M}}{2}\{1-3\cos^2\gamma\},\\ \mathfrak{A}&=0,\\ \mathfrak{B}&=\frac{\mathrm{M}}{\lambda}3\cos\alpha\cos\gamma\tan\theta,\\ \mathfrak{C}&=\frac{\mathrm{M}}{\lambda}3\cos\beta\cos\gamma\tan\theta.\\ \mathfrak{D}&=\frac{\mathrm{M}}{\lambda}\cdot\frac{3}{2}(\cos^2\alpha-\cos^2\beta),\\ \mathfrak{E}&=\frac{\mathrm{M}}{\lambda}3\cos\alpha\cos\beta,\end{aligned}$$

From these we see that a sphere, wherever placed, will increase λ and give a $-k$ if

$$\cos\gamma<\frac{1}{\sqrt{3}}$$

or

$$\gamma>54^\circ\,45',$$

and will decrease λ and give a $+k$ if $\gamma<54^\circ\,45'$.

Hence if, as before, we suppose a double cone traced out by a line passing through the compass, making an angle 54° 45′ with the vertical, all spherical masses of iron whose centres are placed without the cone will increase the directive force and diminish the usual heeling error. All spherical masses whose centres are placed within the cone will diminish the directive force and increase the heeling error. Hence, as far as possible, no iron should be either below or above the compass within an angle of 54° 45′ of the vertical passing through the compass.

If $\cos\alpha>\cos\beta$, or if the centre of the sphere be in either fore-and-aft quadrant, the effect of the sphere is to increase the quadrantal deviation; if in the starboard or port quadrant, it will decrease the quadrantal deviation.

If we have two spheres, one on each side and at the level of the compass, $\alpha=90^\circ$, $\gamma=90^\circ$, $\beta=0^\circ$ and 180°, whence

$$\lambda=1+\mathrm{M},$$

$$\mathfrak{D}=-\frac{3\mathrm{M}}{1+\mathrm{M}}=\frac{3}{1+\left(\frac{r}{p}\right)^3} \text{ nearly.}$$

Hence we get the following for the effect of two such spheres according to the number of semidiameters which their centres are distant from the centre of the compass.

r.	$\mathfrak{E}$.	D.
$2p$	·333	19° 30′
$3p$	·107	6 10
$4p$	·046	2 40
$5p$	·023	1 20

Hence also we find the distance of the spheres required to correct any given quadrantal deviation $\mathfrak{D}$,

$$r=p\sqrt[3]{\frac{3}{\mathfrak{D}}-1}.$$

As we have supposed $\dfrac{\frac{4\pi}{3}\kappa}{1+\frac{4\pi}{3}\kappa}=1$, the deviation which two balls of iron of the usual kind will correct will be one or two per cent. less than the above.

When the sphere is in either of the diagonal planes, $\alpha=45^\circ$, $\beta=45^\circ$, or $\alpha=-45^\circ$, $\beta=135^\circ$,

$$\mathfrak{D}=0, \text{ and } \mathfrak{E}=\pm\frac{3}{2}\frac{\mathrm{M}}{\lambda},$$

or $\mathfrak{E}$ is the same as the $\mathfrak{D}$ when the sphere is in a principal plane. This we should of course anticipate.

From the expression $\mathfrak{B}=\frac{\mathrm{M}}{\lambda}\,3\cos\alpha\cos\gamma\tan\theta$, we see that in the northern hemisphere, if the sphere be below and before, or above and abaft the compass, we have a + semicircular deviation; if above and before, or below and abaft, a − semicircular deviation.

Spherical Shell.

The effect, if the compass be exterior to the shell, will be precisely the same as that of a sphere if for M we substitute

$$\mathrm{M}\,\frac{\left(1+\frac{4}{3}\pi\kappa\right)\left(1+\frac{8}{3}\pi\kappa\right)\left(1-\frac{q^3}{p^3}\right)}{1+4\pi\kappa+\frac{4}{3}\pi\kappa\,\frac{8}{3}\pi\kappa\left(1-\frac{q^3}{p^3}\right)},$$

or nearly, when κ is large and $1-\frac{q}{p}$ small,

$$M\frac{1-\frac{q}{p}}{1-\frac{q}{p}+\frac{3}{8\pi\kappa}}.$$

Hence we see that the force of the shell will be half that of a sphere of equal external radius if κ be 12 and the thickness of the shell be $\frac{1}{100}$ of the semidiameter, or if $\kappa=24$ and the thickness be $\frac{1}{200}$ of the semidiameter, or if $\kappa=36$ and the thickness of the shell be $\frac{1}{300}$ of the semidiameter.

Hence the effect of a tank $\frac{1}{10}$th of an inch thick and 4 feet diameter would probably be about one-third that of a solid mass of the same dimensions.

The effect of such a mass as a rifle-tower $4\frac{1}{2}$ inches thick and 10 feet in diameter will be nearly the same as if it were of solid iron. Such a tower placed in front of a compass, as in the Warrior, will give a considerable $+a$, a $-e$ of half the amount, and therefore increase λ and $\mathfrak{D}$; and if the compass be neither much above nor below it, decrease the heeling error.

Infinite cylinder magnetized perpendicularly to its length.

A compass placed at a considerable height above the deck, near an iron mast or funnel, may be considered as acted on by a vertical cylinder or cylindrical shell of infinite length. If r be the distance of its centre from the centre of the compass, p and q the radii of the outer and inner surfaces of the cylinder, then when the cylinder is solid,

$$M=\frac{2\pi\kappa}{1+2\pi\kappa}\frac{p^2}{r^2}$$

and when the cylinder is hollow

$$M=\frac{2\pi\kappa}{1+2\pi\kappa}\frac{p^2}{r^2}\frac{(1+2\pi\kappa)^2\left(1-\frac{q^2}{p^2}\right)}{1+4\pi\kappa+4\pi^2\kappa^2\left(1-\frac{q^2}{p^2}\right)}$$

$$=\frac{2\pi\kappa}{1+2\pi\kappa}\frac{p^2}{r^2}\frac{1-\frac{q}{p}}{1-\frac{q}{p}+\frac{1}{2\pi\kappa}}$$

nearly, if κ is large and $1-\frac{q}{p}$ small.

Also

$$a = M,$$
$$e = -M;$$

hence

$$\lambda = 1,$$
$$\mathfrak{D} = M;$$

whence we get the remarkable result, that a long vertical cylinder or a cylindrical shell does not alter the mean directive force on a compass placed near its centre as regards elevation.

It may be interesting to compare the effect of two solid stanchions placed one on each side of the compass with that of two solid spheres, in correcting the quadrantal deviation. The effect of the stanchions would be nearly

$$2\frac{p^2}{r^2},$$

whence

r.	$\mathfrak{D}$.	D.
		° ′
$2p$	·500	30 0
$3p$	·222	12 50
$4p$	·125	7 10
$5p$	·080	4 36

A mast or stanchion placed as we have supposed would generally diminish the heeling error.

We may compare the effect on the directive force of a compass on the main deck of an iron ship with the effect on a compass in the interior of a spherical shell.

In some ships the value of λ at the main-deck compass is about ·75.

Comparing this value with the expression for the force in the interior of a spherical shell, viz.,

$$\frac{F}{1+\frac{8\pi\kappa}{3}\left(1-\frac{q}{p}\right)},$$

we have

$$\frac{1}{\cdot 75}=\frac{4}{3}=1+\frac{8\pi}{3}\kappa\left(1-\frac{q}{p}\right),$$

or

$$1-\frac{q}{p}=\frac{1}{8\pi\kappa};$$

taking κ as 24,

$$1-\frac{q}{p}=\frac{1}{600}$$

nearly, or the effect is the same as if the compass were inclosed in a spherical shell of an inch thick and 50 feet radius, or half an inch thick and 25 feet radius.

We may observe that at present one of the great difficulties in deducing numerical results as to the effect of rods or plates of iron, arises from our ignorance of the value of κ for iron used for building or plating ships. We hope to be able on some future occasion to be able to communicate to the Royal Society the result of observations made for the purpose of determining this value in plates of iron of different kinds.

GENERAL CONCLUSIONS.

The following appear to be the principal conclusions to be drawn from the application of observation and theory to the magnetic phenomena in iron ships.

1. The original semicircular deviation depends principally on the direction of the ship's head in building, and consists principally in an attraction of the north point of the needle to the part of the ship which was (nearly) south in building.

2. This attraction is caused by the subpermanent magnetism induced in the ship when building, by the horizontal force of the earth.

3. If we consider separately, first, the effect of the subpermanent magnetism induced by the fore-and-aft component of the horizontal force, and secondly, the effect of the subpermanent magnetism induced by the transverse component of the horizontal force, the first is relatively less than the second. This, if the direction of the ship in building does not coincide with a cardinal point, modifies the direction of the semicircular deviation produced.

4. A third part, being the remainder of the semicircular deviation, is independent of the direction of the ship in building. It is the effect of the subpermanent and transient magnetism induced in the ship by the vertical force of the earth, and it consists in an attraction of the north point of the needle to the bow or stern.

In the usual place of the Standard Compass this part is, in the northern hemisphere, an attraction of the north point of the needle towards the bow; but if the compass is placed nearly in front of a large vertical mass of iron, as the stern-post, it may be towards the stern.

5. The first and second parts of the semicircular deviation diminish rapidly after the ship has been launched, the second generally most rapidly; but after a time, which may be taken roughly as a year, if the ship has been allowed to swing on all azimuths, they attain a very fixed and permanent amount, from which they do not afterwards vary to any great extent.

The third part changes little, if at all, so long as the ship remains in the same latitude.

6. The changes which take place in the semicircular deviation of a ship built East and West are generally relatively greater than in one built North and South.

7. The transient magnetism induced by the earth's horizontal force adds to the effect of the subpermanent magnetism induced by the same force, when she is on the stocks, and afterwards when her head is in the same direction in which it was while building.

8. The effect of the subpermanent and transient magnetism induced by the horizontal force when the ship is on the stocks is principally, and if the ship is built on a cardinal point entirely, to produce a diminution of the directive force on the needle. and very little, and if built on a cardinal point not at all, to produce deviation.

9. The same effect (nearly) is produced at a subsequent time if the ship's head is placed on the direction in which it was while building.

10. This diminution of the directive force is greater if the ship has been built East and West than if built North and South.

11. The deviations in an iron ship which has been built East or West are more prejudicial than in a ship built North or South in the following respects:—

1. They are less symmetrical and regular, and therefore more perplexing to the seaman.
2. They change more relatively after launching.
3. They diminish the directive force more when the ship is on particular points.

12. When a ship has been built head North, the upper part of the stern and the lower part of the bow are strongly magnetized; the upper part of the bow and the lower part of the stern are weakly magnetized. When a ship has been built head South, the upper part of the bow and the lower part of the stern are strongly, the upper part of the stern and the lower part of the bow are weakly magnetized.

Consequently in ships built head North, a compass placed near the stern will have a large semicircular deviation.

13. In the last case there will be a large downward force on the north point of the needle, which will produce a large heeling error. In ships built head South, both the last errors will probably be small.

14. On the whole, for compasses to be placed in the after part of the ship, the best direction for building is head South. For compasses near the centre of the ship, the directions head North and head South are nearly equally good.

15. The diminution of the mean directive force is the mean of the diminution caused by the transient magnetism induced by the horizontal force when the ship's head is North or South, and that induced when her head is East or West, *i. e.* it is the mean of the thrust from the north end and from the north side.

16. The quadrantal deviation is caused by the excess of the latter over the former, *i. e.*, by the excess of the thrust from the north side over the thrust from the north end.

17. The diminution of the directive force and the amount of the quadrantal deviation are nearly the same at the same level in different parts of the ship. They increase in descending from the position of the Standard Compass to the compasses on the upper and main decks. They diminish with the lapse of time.

18. By substituting wood for iron in the part of the deck below and above the compass, and within an angle of 35° 15′ of the vertical line passing through the compass, and having no masses of iron with their centres within 54° 15′ of the same vertical line, the directive force is increased and the quadrantal and heeling error generally diminished.

19. In selecting a place for the Standard Compass, care should be taken to avoid as much as possible the proximity of the ends of elongated masses of iron, particularly if placed vertically; or, if they cannot be avoided, then a place should be selected where they diminish instead of increasing the semicircular deviation.

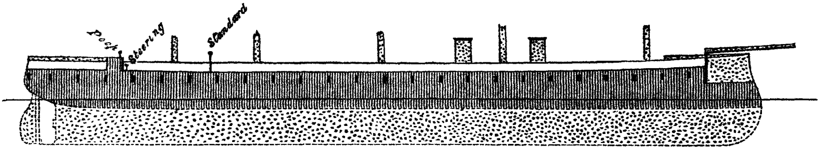
The neighbourhood of rifle and gun turrets in ships carrying them should be as much as possible avoided.

20. In the construction of iron-built and iron-plated ships, regard should be had to the providing a suitable place for the Standard Compass. It is not difficult for any one who has studied the question, to suggest arrangements which would greatly mitigate the injurious effects of the iron of the ship; the difficulty is to reconcile them with the requirements of construction and of working.

POSTSCRIPT.

Since the foregoing paper was read, additional observations of deviations have been made in the Achilles and Defence, and observations in two new iron-built armour-plated ships, the Minotaur and Scorpion, the results of which are contained in the annexed Table. The observations in the Achilles show a continued diminution in the value of 𝔅 and a continued tendency in ℭ to return to its original value. The Defence continues to show great permanence both in 𝔅 and ℭ.

The Minotaur, of which it has been thought desirable to give a woodcut drawn to the same scale as the ships represented in Plate XI., illustrates in a very remarkable manner some of the principles deduced from other ships. The Minotaur is the first iron-built ship completely plated from end to end; her quadrantal deviation is consequently small. Having been built and plated head north, the original deviations in all the compasses were very large. In the steering and poop compasses the maximum deviation was above 60°. With deviations of this amount the compass becomes useless unless corrected by magnets, and magnets were consequently applied, which removed almost entirely the semicircular deviation. Probably in a very short time we shall find the original —𝔅 of these compasses to have so far diminished that the compasses will be found to be greatly over corrected and to have a considerable +𝔅. Magnets were also applied to the Standard Compass. The heeling error at the poop compass is very large, 2° 46′. This arises from the compass being so near the stern of the ship, built and plated head north, and also from its being elevated above the armour-plating. It is interesting to contrast it with the heeling error of the steering compass, where from the peculiar configuration of the armour-plating being such as to give a—k, the heeling error is diminished and of a moderate amount.



The Scorpion is a remarkable instance of the change which takes place in the semicircular deviation from a change of position in a new iron-built vessel. Having been built head N. 76° W., or S. 254° E., the original value of 𝔅 was —·246, and the original starboard angle was 233½°. After lying four months head S. 47° W. or S. 313° E., the value of 𝔅 changed its sign and became +·225, and the starboard angle increased to 303½°, thus following very nearly the direction of the south line in the ship. The Scorpion is an instance of the successful correction of the heeling error by means of a vertical magnet. This reduced the heeling error from 1° 38′ to 2′ for each degree of heel.

Ship.	Compass.	Place.	Date.	Approximate coefficients.					Exact coefficients.				
				A	B	C	D	E	𝔄	𝔅	ℭ	𝔇	𝔈
				° ′	° ′	° ′	° ′	° ′					
MINOTAUR. (6621 tons), 1350 horse-power. 26 guns, Iron-cased, iron hull. Built on same slip as Warrior; head N. 3° E. magnetic. Launched Dec. 12, 1863. Plated head N. 22° E. in Victoria Docks.	Standard.	Victoria Docks,	March 28, 1865	*By deviation and horizontal force on one point: λ and D of March 30 adopted*						−·487	+·174	...	...
		River Thames,	March 30, 1865 ...	−0 47	−23 26	+ 6 4	+ 5 41	−0 54	−·014	−·420	+·099	+·100	−·016
		Sheerness	April 10, 1865 ..	−0 5	−20 30	+ 4 25	+ 5 43	−0 26	−·001	−·379	+·069	+·100	−·007
				−0 5	+ 0 44	− 0 16	+ 5 43	−0 26	*after correction by magnets.*				
	Starboard steering.	Sheerness	April 10, 1865 ..	. ....	−61 0	+ 0 45	......	......	+·009	−·965	+·015	+·103	−·002
				+0 32	− 0 28	+ 2 8	+ 5 56	−0 7	*after correction by magnets.*				
	Poop (on fore part),	Sheerness	April 10, 1865 ..	......	−60 0	+ 2 0	......	......	+·022	−·948	+·038	+·086	−·001
				+1 16	− 1 55	+ 5 8	+ 4 55	−0 5	*after correction by magnets.*				
SCORPION. (1857 tons), 350 horse-power, 4 guns. Iron-cased turret ship, iron hull. Built at Birkenhead; head N. 76° W.	Standard.	Birkenhead ...	October 31, 1864	*By deviation and horizontal force on one point: λ and D of March 1865 adopted, with small allowance for lapse of time*						−·246	−·355	+·190	*assumed.*
		Birkenhead	March 14, 1865	*From observations made in one quadrant after ship had been lying four months S. 47° W.*						+·225	−·341	+·180	..
			March 15, 1865	−0 53	+ 0 32	+ 1 43	+10 47	−0 52	−·015	+·009	+·030	+·187	−·015
				after correction by magnets.									
ACHILLES (continued).	Standard.	Portland	April 1865	......	+16 50	+12 30	+ 6 40	... ..	...	+·322	+·191	+·115	...
		Lisbon	*May* 4, 1865	......	......	......	......	......	...	+·274	+·132	...	...
DEFENCE (continued).	Standard.	Portland	April 3, 1865	+0 13	+20 19	− 0 14	+ 6 09	−0 36	+·004	+·367	−·004	+·107	−·010
		Lisbon	*May* 1, 1865	+0 23	+16 51	− 1 15	+ 6 16	+0 04	+·007	+·307	−·021	+·109	+·001

Maximum of semicircular deviation $\sqrt{B^2+C^2}$ / Horizontal force of ship $\sqrt{\mathfrak{B}^2+\mathfrak{C}^2}$*.			Mean force to North, λ †	$\frac{1}{\lambda}$	Coefficients of horizontal induction.		Part of D from		Mean Vertical force, μ ‡	Heeling coefficient to *windward*, χ	Heeling coefficients from		$\frac{g}{\tan\theta}$ ‡	g †
Amount.		Direction.			Fore-and-aft, a †	Transverse e †	Fore-and-aft induction.	Transverse induction.			Vertical induction in transverse iron.	Vertical force and induction in vertical iron.		
°		°					° ′	° ′		° ′	° ′	° ′		
..	·516	160½												
24¼	·432	166½	·876	1·142	–·036	–·212	–1 12	+ 6 57						
21¼	·385	169½	·892	1·121	–·019	–·197	–0 38	+ 6 51	1·442	+1 21	+0 35	+0 46		
61	·965	179	·811	1·233	–·106	–·272	–3 43	+ 9 42	1·091	+1 7	+0 50	+0 17		
60¼	·950	177¾	·826	1·211	–·103	–·245	–3 33	+ 8 30	1·660	+2 46	+0 46	+2 0		
……	·434	233½	·810 *assumed.*		……	……	… .	…..	1·472	+1 39				
. ..	·406	303½	……	. ..	…..	……	……	……	1·636	+1 38	+1 02	+0 36	–·050	
.. ..	.. …	. ….	·838	1·193	–·037	–·350	–0 7	+10 57	·826 *after correction by vertical magnet.*	+0 2				
21	·374	30½	·844	1·185	–·059	–·253	–1 57	+ 8 38						
……	·306 ·384	26	·820	1·219	–·086	–·274	–3 2	+ 9 37						
20⅓	·367	359½	·875	1·143	–·031	–·219	–1 02	+ 7 13						
16¾	·308 ·387	356	·855	1·169	–·052	–·238	–1 46	+ 8 0						

* Mean force to North (λH) being unit. † Earth's Horizontal force (H) being unit. ‡ Earth's Vertical force (Z) being unit.

VI. *On some Foraminifera from the North Atlantic and Arctic Oceans, including Davis Straits and Baffin's Bay. By* W. KITCHEN PARKER, *F.Z.S., and Professor* T. RUPERT JONES, *F.G.S. Communicated by Professor* HUXLEY, *F.R.S.*

Received April 26,—Read May 12, 1864.

TABLE OF CONTENTS.

§ I. Introduction :—

Page
1. Soundings from Baffin's Bay. (Table I.) 325
2. Dredgings from the Hunde Islands. (Table II.) 326
3. Dredgings from Norway. (Tables III. & IV.) 329
4. Soundings from the North Atlantic. (Tables V. & VI.) 331
5. General Remarks 334
§ II. Descriptions. Genera, Species, and Varieties. (Table VII.) 336
Descriptions of the Plates 412
Appendix I.—Additional North Atlantic Foraminifera 422
Appendix II.—Professor J. W. BAILEY'S Researches on the "Virginian" Foraminifera of the North Atlantic. (Table VIII.) 423
Appendix III.—Further Researches by Professor J. W. BAILEY 428
Appendix IV.—Mr. POURTALES' Researches on North Atlantic Foraminifera 429
Appendix V.—The Foraminifera of the "Celtic" and "Virginian" Provinces of the North Atlantic, as a Fauna. (Table IX.) 430
Appendix VI.—Distribution of Foraminifera. (Tables X. & XI.) 434
Appendix VII.—The North-Atlantic Soundings. (Table XII.) 439
(Map [Plate XII.] and Plates XIII. to XIX.)

Introduction.

THE specimens here described are comprised in four collections; namely—

1. From Baffin's Bay, between 76° 30′ and 74° 45′ North Latitude. These specimens are derived from seven deep-sea soundings made during one of the Arctic Expeditions under Sir EDWARD PARRY. These soundings were confided to us by Professor HUXLEY, of the Museum of Practical Geology, Jermyn Street, to which Institution they had been given in April 1853 by Mr. J. W. LOWRY, who received them of Mr. FISHER, Assistant-Surgeon in the Expedition alluded to. The Foraminifera obtained by us from these soundings are tabulated in Tables I., IV., and VII.

This material from the "Arctic Province" of Naturalists is but scanty. None of the Foraminifera here obtained are numerous, except *Polystomella striatopunctata*, *Nonionina Scapha*, *Truncatulina lobatula*, and *Cassidulina lævigata*; the first two of which are at home in Arctic waters: and none have attained here a large size except *Lituolæ*. The material from 150 fathoms yielded these relatively large and numerous specimens.

Table I.—Table of the Soundings from Baffin's Bay.

No.		Depth. fathoms.	Condition of bottom, &c.	Genera and subgenera of Foraminifera.
1.	Lat. 75° 10′, Long. 60° 12′	?	Fine grey syenitic sand, with syenitic fragments ¼ inch and less in length.	Nodosarina (Dentalina), Lagena, Planorbulina (Truncatulina), Polystomella (and Nonionina), Cassidulina, Miliola (Quinqueloculina), Lituola.
2.	Lat. 76° 30′, Long. 77° 52′	150	Greyish muddy micaceous sand, with angular syenitic fragments ½ inch and less in length.	Globigerina, Planorbulina (Truncatulina), Pulvinulina, Polystomella (and Nonionina), Cassidulina, Lituola.
3.	Lat. 74° 45′, Long. 59° 17′	250	Greysandymud; sand, quartzose, angular and rounded.	No Foraminifera.
4.	Lat. 75° 25′, Long. 60°	314	Syenitic sand, with fragments of syenite ¼ inch and less in length.	Miliola (Triloculina), Lituola.
5.	Lat. 76° 20′, Long. 76° 27′	?	....	No Foraminifera.
6.	Lat. 75°, Long. 59° 40′	230	Grey mud, with quartzose sand, partly rounded, and with several partly rounded fragments of lava-rock.	Planorbulina (Truncatulina), Polystomella (and Nonionina), Miliola (Quinqueloculina), Lituola.
7.	Lat. 76° 10′, Long. 76°	..	Sand from an iceberg. Grey, heavy, fine, micaceous, syenitic sand, with fragments (¾ in. largest); some grains slightly worn.	No Foraminifera.

2. From the Hunde Islands, in South-east or Disco Bay, on the west coast of Greenland (lat. 68° 50′ W., long. 53° N.). Five soundings taken by Dr. P. C. Sutherland (now Surveyor-General of Natal) in 1850, and confided to us by Professor Huxley of the Museum of Practical Geology, to which Museum they were given by Dr. Sutherland in 1853.

Dr. P. C. Sutherland's observations on the Arctic Regions visited by him were published in his 'Journal of a Voyage in Baffin's Bay and Barrow Straits in the years 1850-51,' 2 vols. 8vo, 1852; and in the Quart. Journ. Geol. Soc. vol. ix. p. 296, &c.

See Tables II., IV., VII. for the Foraminifera from the Hunde Islands.

TABLE II.—Table of the Dredgings and Foraminifera from the Hunde Islands, Disco Bay.

No.		Depth. fathoms.	Character of bottom.	Genera and subgenera of Foraminifera.
1.	Hunde Islands . .	25 to 30	Pale-grey micaceous clay; more than half small mica-flakes. With vegetable matter (fucal); Hydrozoa (*Sertularia*); Polyzoa (*Berenicea*, &c.); Entomostraca (*Cythere*, &c.); Bivalve and univalve Mollusks. (About an ounce.)	Polymorphina, Planorbulina (Truncatulina), Pulvinulina, Polystomella (and Nonionina), Nummulina, Cassidulina, Bulimina, Textularia (and Verneuilina), Cornuspira, Miliola (Quinqueloculina, Triloculina), Lituola.
2.	"	28 to 30	Gravel of hornblende-schist and syenite (largest fragments 1½ inch long). Seaweed (*Fucus*); Nullipores; fragments of *Balanus* (predominant); Crustacea (*Talitrus*, *Cythere*, &c.); spines and plates of *Echinus*; Polyzoa; Univalves and Bivalves. (About 4 ounces.)	Globigerina, Planorbulina (Truncatulina), Pulvinulina, Discorbina, Polystomella (and Nonionina), Cassidulina, Miliola (Quinqueloculina), Lituola.
3.	"	30 to 40	Shelly sandy mud. Syenitic fragments (¼ inch and less), some rather rounded; fragments of *Balani*; *Serpulæ*; spines of *Echinus*; Bivalves and Univalves. (About 2 ounces.)	Nodosarina (Nodosaria, Cristellaria), Lagena, Polymorphina, Uvigerina, Globigerina, Planorbulina (Truncatulina), Pulvinulina, Discorbina, Polystomella (and Nonionina), Cassidulina, Bulimina (and Virgulina and Bolivina), Textularia (and Verneuilina), Patellina, Trochammina, Miliola (Quinqueloculina), Lituola.
4.	"	50 to 70	Shelly fine sand (syenitic). *Serpula*; Bivalves and Univalves. (About 1 ounce.)	Lagena, Polymorphina, Uvigerina, Planorbulina (Truncatulina), Pulvinulina, Discorbina, Polystomella (and Nonionina), Cassidulina, Patellina, Miliola (Quinqueloculina), Lituola.
5.	"	60 to 70	Shelly sandy mud (syenitic). *Serpula*; *Balanus* (predominant); Bivalves and Univalves. (About 1 ounce.)	Nodosarina (Dentalina, Cristellaria), Lagena, Polymorphina, Uvigerina, Globigerina, Planorbulina (Truncatulina), Pulvinulina, Discorbina, Polystomella (and Nonionina), Cassidulina, Bulimina (and Virgulina and Bolivina), Textularia (and Bigenerina and Verneuilina), Spirillina, Patellina, Trochammina, Cornuspira, Miliola (Quinqueloculina, Triloculina), Lituola.

Mr. G. S. BRADY, of Sunderland, has examined the Bivalved Entomostraca from these dredgings, and has determined the following:—

Cytheridea Bradii, *Norman*.
—— setosa, *Baird*.
Cythere costata, *Brady*.
—— protuberans, *Brady*.
—— plicata, *Reuss*.
Cythere clathrata, *Reuss*.
—— septentrionalis, *Brady*.
Jonesia simplex, *Norman*.
Cytherideis pulchra, *Brady*[*].

---

[*] The new species of Entomostraca from the Hunde Islands, from Norway (p. 329), and from the Atlantic (p. 334) are described and figured by Mr. BRADY in the Zool. Soc. Trans. vol. v. part 5.

Shells, &c. from the Hunde Islands, Davis Straits.

(Dredged by Dr. SUTHERLAND, October 1852: named by Dr. S. P. WOODWARD.)

Box I. 28–30 fathoms.

Balanus porcatus, *DC.* } probably: fragments much
—— crenatus, *Brug.* } water-worn.
Mya truncata. Fragment.
Saxicava arctica. Small valve.
Tellina calcaria (=proxima =lata). Fragment.
Echinus, sp. Fragments of plates and spines.

Box II. 30–40 fathoms.

Leda minuta. Odd valve (large) and fry.
Crenella decussata. Small.
Limatula sulcata.
Astarte striata. Young.
—— semisulcata. Young.
Saxicava. Fry.
Rissoa castanea.
—— scrobiculata.
Scissurella crispata.
Turritella lactea. Young.
Margarita undulata.
—— cinerea. Young.
Echinus. Small spine.
Spirorbis. Whorls furrowed.

Box III. 25–50 fathoms.

Saxicava arctica. Adult.
Lyonsia striata. Fry.
Astarte striata. Adult and fry.
Leda truncata. Fragments.
—— pygmæa. Fry.
Crenella decussata.
—— faba
Nucula tenuis. Fry.
Cardium elegantulum.
Natica pusilla (Grœnlandica). Fry.
Cylichna Gouldii. Young.
Rissoa scrobiculata.
Spirorbis.
Echinus. Spine.

Box IV. 50–70 fathoms.

Pilidium fulvum.
Acmæa. Fragment.
Chiton albus? Two valves.
Astarte striata. Fry.
Spirorbis nautilus?
——. Sulcated.

Box V. 60–70 fathoms.

Pecten Islandicus. Fragments.
Mya truncata.
Astarte borealis, var. semisulcata. Young.
—— striata.
Saxicava. Fry.
Crenella decussata.
Limatula sulcata.
Turritella lactea. Fragment.
Rissoa castanea.
—— scrobiculata.
Margarita helicina.
—— undulata. Fragment and fry.
—— cinerea. Fry.
Scissurella crispata.
Litorina obtusata. Fry.
Cemoria Noachina. Fry.
Pilidium fulvum.
Serpula.
Spirorbis.
Balanus porcatus. Tergum, and fragments of parietes.
Echinus. Fragments of spines.

The five specimens of sea-bottom above-mentioned, taken at depths of from 25 to 70 fathoms, and consisting mainly of shelly muddy sands, afford a good local example of the Foraminiferal fauna of the "Arctic Province" of Naturalists, at the "Coralline-zone" (15–50 fathoms) and the "Coral-zone" (50–100 fathoms) of Davis Straits.

Lagenæ abound in these dredgings at from 30 to 70 fathoms; *Polymorphina* is small here and rather common: *Uvigerina* common at from 30 to 70 fathoms, but small. *Globigerinæ* are not rare at the same depths, but are very small. *Truncatulina* flourishes at all the depths (25 to 70 fathoms). *Pulvinulina* is freely represented by the small *P. Karsteni*. *Discorbina* gets more abundant with the greater depth. The simpler forms of *Polystomella*, including the feeble *Nonioninæ*, have their home evidently in this region. *Cassidulina* abounds, but is not large. A small *Nummulina*, the feeble representative of a once highly potent species, still abounding in some warm seas, is not wanting in the "Coralline-zone." The essentially Arctic form of *Bulimina* (*B. elegantissima*) flourishes at from 30 to 70 fathoms at the Hunde Islands, and other varieties are not wanting, though not abundant. The *Textulariæ* are represented by some small specimens of the type, and by three of its modifications in small but numerous individuals. *Spirillina* is very rare and small. *Patellina* is small and common from 30 to 70 fathoms. *Trochammina* is common, though small, in the deepest sounding. *Cornuspira* is common at the least and the greatest depths. *Quinqueloculina* is common, but not large, throughout. *Triloculina* occurs freely at 25 to 30 fathoms. *Lituola* abounds from 25 to 70 fathoms.

3. From the coast of Norway, between North Cape and Drontheim, from 69° to 63° N. lat. Dredgings made by Messrs. MACANDREW and BARRETT in the summer of 1855.

One portion of these materials[*] was received from the late Mr. LUCAS BARRETT, in small boxes, numbered, and labelled with the depths and localities of the dredgings; another portion, received from Dr. WOODWARD, was the sandy refuse from a jar in which specimens of Mollusks, &c. had been preserved in spirits; and, thirdly, Dr. BOWERBANK favoured us with a packet of shelly sand obtained when preparing sponges taken in the same dredgings. The latter lots of sand were manipulated and examined together[†], no particular depths and localities being noted for these mixed results of dredgings in from 30 to 200 fathoms.

The series of which the exact localities and depths are known comprises seven lots; these with their characters and contents are arranged in the following Table (No. III.).

The Bivalved Entomostraca from these dredgings have been determined by Mr. G. S. BRADY, as follow:—

Cythere Minna, *Baird.*
—— spinosissima, *Brady.*
—— clathrata (varieties), *Reuss.*
Cythere catenata, *Brady.*
Cytheridea Bradii, *Norman.*
Cytherella Beyrichi, *Reuss.*

---

[*] These Norwegian Foraminifera have already been noticed and illustrated by us in the Annals of Nat. Hist. 2 ser. vol. xix. pp. 273, &c., pls. 10 & 11 (1857); we are, however, desirous of emending some of the descriptions there given, as well as the nomenclature and classification in several points; and these Foraminifers are here brought into association with their allies of the neighbouring ocean.

[†] The specimens from this mixed material are grouped together in pl. 10 of the Ann. Nat. Hist. 2 ser. vol. xix.

TABLE III.—Table of the Norwegian Dredgings and Foraminifera.

No.	Locality.	Depth in fathoms.	Character of sea-bottom, &c.	Genera, &c.
1.	East of Rolfs Oe, or Bred Sound, Finmark. Lat. 71°, long. 24°.	30	Gravel	Miliola (Biloculina, Quinqueloculina), Lituola, Polymorphina, and Planorbulina (Truncatulina and Anomalina).
2.	Omnoes Oe, Nordland (half-a-mile from the shore; Woodward's 'Manual,' p. 434). Lat. 66° 45′, long. 13° 25′.	40	Gravel	Miliola (Quinqueloculina), Planorbulina (Truncatulina and Anomalina).
3.	West Fjord, Nordland. About lat. 68° 15′, long. 14° 30′.	60	Sand......	Miliola (Quinqueloculina), Nodosarina (Dentalina), Pulvinulina, Planorbulina (Truncatulina).
4.	Bodoe, Nordland. Lat. 67° 15′, long. 14° 18′.	70–100	Sand......	Miliola (Biloculina, Quinqueloculina), Planorbulina (Truncatulina and Anomalina).
5.	Vigten Islands (Inner Passage), Drontheim. Lat. 65° 47′, long. 11° 5′.	100	On sponge .	Pulvinulina.
6.	Finmark (half-a-mile from shore: see Woodward's 'Manual,' p. 435).	150	Sand......	Miliola (Quinqueloculina), Planorbulina (Truncatulina).
7.	Arctic Circle, Nordland. Lat. 66° 30′, long. 12° 45′.	160	Mud	Miliola (Biloculina), Nodosarina (Glandulina, Nodosaria, Dentalina, Marginulina, Cristellaria), Planorbulina (Truncatulina and Anomalina).
8.	Various localities between the North Cape and Drontheim.	20–300	Various ..	Miliola (Quinqueloculina), Lituola, Lagena (and Entosolenia), Nodosarina (Dentalina), Nummulina (Operculina), Polystomella (and Nonionina), Discorbina, Spirillina, Planorbulina (Truncatulina and Anomalina), Globigerina, Polymorphina, Uvigerina, Cassidulina, Bulimina, Textularia, Valvulina.

The Norwegian Foraminifera are tabulated with those from Baffin's Bay and Davis Straits in Table IV., and with those from the North Atlantic in Table VII.

Mr. MACANDREW, who has kindly supplied us with latitude and longitude of the localities in the foregoing list, informs us that "these dredgings were all taken in sheltered situations among the islands and near shore; occasionally a mile or two from land, and frequently nearer. That at Omnoes Oe was made from the boat, and *commenced* very near shore. The others in the list were made from the yacht, when we required more room."

Compared with the group of Foraminifera obtained at the Hunde Islands at similar depths, those from the Norway coast present considerable differences; and this is mainly owing to the fact that the specimens given us from the seven Norwegian dredgings were only the larger and more conspicuous of a probably rich fauna; but also, partly, because the coast of Norway (excepting the neighbourhood of North Cape) lies in the "Boreal Province," and is far less under the chilling influence of floating ice than the American coasts to the westward. The dredging from Rolfs Oe was taken within the "Arctic Province." The mixed sands obtained from the shells and sponges of Messrs. MACANDREW and BARRETT's dredgings, and examined by ourselves, yielded many representatives of the forms native to the Coralline- and the Coral-zone, though chiefly of small size.

The most interesting fact to be pointed out is the relatively great abundance of large

TABLE IV.—DISTRIBUTION OF FORAMINIFERA IN THE ARCTIC OCEAN, OFF THE COASTS OF GREENLAND AND NORWAY.

rl. Rather large. *l.* Large. *vl.* Very large. *m.* Middle-sized. *s.* Small. *vs.* Very small. VC. Very common. C. Common. RC. Rather common.
RR. Rather rare. R. Rare. VR. Very rare.

GENERA, SPECIES, AND VARIETIES.	1 Hunde Islands, Davis Straits, 25 to 30 fms.	2 Hunde Islands, Davis Straits, 28 to 30 fms.	3 Hunde Islands, Davis Straits, 30 to 40 fms.	4 Hunde Islands, Davis Straits, 50 to 70 fms.	5 Hunde Islands, Davis Straits, 60 to 70 fms.	6 Baffin's Bay, No. 1. Lat. 75° 10′ N., Long. 60° 12′ W.	7 Baffin's Bay, No. 2, 150 fms. Lat. 76° 30′ N., Long. 77° 52′ W.	8 Baffin's Bay, No. 3, 250 fms. Lat. 74° 45′ N., Long. 59° 17′ W.	9 Baffin's Bay, No. 4, 314 fms. Lat. 75° 25′ N., Long. 60° W.	10 Baffin's Bay, No. 5. Lat. 76° 20′ N., Long. 76° 27′ W.	11 Baffin's Bay, No. 6, 220 fms. Lat. 75° N., Long. 59° 40′ W.	12 Baffin's Bay, No. 7. Sand from an Iceberg. Lat. 76° 10′ N., Long. 76° W.	13 Norway, No. 1, 30 fms. East of Rolfs Oe, or Bred Sound, Finmark.	14 Norway, No. 2, 40 fms. Omnoes Oe, Nordland.	15 Norway, No. 3, 60 fms. West Fjord, Nordland.	16 Norway, No. 4, 70 to 100 fms. Bodoe, Nordland.	17 Norway, No. 5, 100 fms. Vigten Island (Inner Passage), Drontheim.	18 Norway, No. 6, 150 fms. Finmark.	19 Norway, No. 7, 160 fms. Arctic Circle, Nordland.	20 Norway, No. 8, 30 to 200 fms. Var. loc. between Drontheim and N. Cape.
Glandulina lævigata, D'O.	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	*l* C	
Nodosaria Radicula, Linn.	..	..	*rl* R	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	*l* C	
Dentalina pauperata, D'O.	..	..	..	..	*m* R	*m* R														
—— communis, D'O.	..	..	..	..	*l* R	*l* R	..	..	..	..	..	..	..	..	..	..	..	..	..	*vs* VR
—— guttifera, D'O.	..	..	..	..	..	..	..	..	..	..	..	..	..	..	*l* R	..	..	..	*l* R	
Vaginulina linearis, Montag.	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	*s* RC
Marginulina Lituus, D'O.	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	*vl* C	
Cristellaria Crepidula, F. & M.	..	..	*s* C	..	*s* C															
—— cultrata, Montf.	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	*l* C	
—— rotulata, Lam.	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	*l* C	
Lagena distoma, P. & J.	..	..		..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	*l* RC
—— polita, P. & J.	..	..		..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	*s* RC
—— lævis, Montag.	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	*s* R
—— semistriata, Will.	..	..	..	*m* RC																
—— striatopunctata, P. & J.	..	..	*l* RC																	
—— sulcata, W. & J.	..	..	*l* C	*m* C	*vl* VC	..	..	..	..	..	..	..	..	..	..	..	..	..	..	*m* RC
—— Melo, D'O.	..	..	*m* C	*m* C	*m* C	..	..	..	..	..	..	..	..	..	..	..	..	..	..	*m* RC
—— globosa, Montag.	..	..	*m* C	..	*m* C	*l* R														
—— caudata, D'O.	..	..	*s* R																	
—— squamosa, Montag.	..	..	..	*s* RC	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	*s* RC
—— marginata, Montag.	..	..	*m* C	*m* C	*m* C	..	..	..	..	..	..	..	..	..	..	..	..	..	..	*s* RC
Polymorphina lactea, W. & J.	*s* C	..	..	*s* RC	*s* C	..	..	..	..	..	..	..	..	..	..	..	..	..	..	*s* RC
—— compressa, D'O.	..	..	*s* R	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	*s* RC
—— tubulosa, D'O.	..	..	..	..	..	..	..	..	..	..	..	..	*m* R	..	..	..	..	..	..	*m* R
Uvigerina pygmæa, D'O.	..	..	*s* C	*s* RC	*s* C	..	..	..	..	..	..	..	..	..	..	..	..	..	..	*m* C
—— angulosa, Will.	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	*m* C
Globigerina bulloides, D'O.	..	*s* RR	*vs* C	..	*s* RC	..	*vs* R	..	..	..	..	..	..	..	..	..	..	..	..	*s* RR
Truncatulina lobatula, W. & J.	*s* C	*l* VC	*l* C	*l* C	*l* C	*m* C	*m* C	..	..	..	*s* RC	..	*l* C	*l* C	*l* C	*l* C	..	*l* C	*l* C	*m* C
Anomalina coronata, P. & J.	..	..	..	..	..	..	..	..	..	..	..	..	*l* C	*l* C	..	*l* C	..	..	*l* C	*m* C
Pulvinulina punctulata, D'O.	..	..	..	..	..	..	..	..	..	..	..	..	..	..	*vl* R	..	*vl* C			
—— Karsteni, Reuss	*s* R	*s* RC	*m* VC	*s* C	*m* C	..	*s* RC													
—— Micheliniana, D'O.	*s* R																			
Discorbina obtusa, D'O.	..	*l* R	*l* RC	..	*l* C															
—— globularis, D'O.	..	..	*s* RC	*s* C	*m* C	..	..	..	..	..	..	..	..	..	..	..	..	..	..	*m* C
Polystomella crispa, Linn.	*s* R																			
—— striatopunctata, F. & M.	*m* C	*m* C	*m* VC	*m* C	*m* VC	*vs* R	*m* C	..	..	..	*vs* VR	..	..	..	..	..	..	..	..	*s* RR
—— Arctica, P. & J.	*m* C	*l* C	*l* C	*l* C	*l* C															
Nonionina asterizans, F. & M.	*s* C																			
—— Faba, F. & M.	*m* RR	..	*l* RC	..	*l* C															
—— Scapha, F. & M.	*m* C	..	*l* C	*m* RC	*l* C	*m* R	*m* VC													
—— depressula, W. & J.	*s* C	*s* C	..	*s* C	*m* C															
—— stelligera, D'O.	*s* C	*s* C	*s* C	*s* C	*s* C	..	..	..	..	..	..	..	..	..	..	..	..	..	..	*s* C
—— umbilicatula, Montag.	..	..	..	..	..	..	..	..	..	..	*m* R	..	..	..	..	..	..	..	..	*m* VC
Pullenia sphæroides, D'O.	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	*vs* RC
Nummulina planulata, Lam.	*s* RC																			
Operculina ammonoides, Gronov.	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	*s* VC
Cassidulina lævigata, D'O.	*m* VC	*m* VC	*m* VC	*m* C	*m* VC	*m* R	*m* C	..	..	..	..	..	..	..	..	..	..	..	..	*s* C
—— crassa, D'O.	..	*s* C	*m* C	*s* C	*s* C	..	*s* C													
Bulimina Pyrula, D'O.	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	*l* C
—— aculeata, D'O.	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	*m* C
—— elegantissima, D'O.	*m* R	..	*l* C	..	*l* C															
Virgulina Schreibersii, Czjzek	..	..	*l* R																	
—— squamosa, D'O.	..	..	*s* R	..	*s* R	..	..	..	..	..	..	..	..	..	..	..	..	..	..	*l* R
Textularia agglutinans, D'O.	*s* RC	..	*s* R	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	*m* RC
—— pygmæa, D'O.	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	*m* C
—— biformis, P. & J.	..	..	..	..	*s* C															
Bigenerina Nodosaria, D'O.	..	..	..	..	*s* C															
Verneuilina polystropha, Reuss	*s* C	..	*s* C	..	*s* VC															
Valvulina conica, P. & J.	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	*s* R
Spirillina vivipara, Ehrenb.	..	..	..	..	*s* R	..	..	..	..	..	..	..	..	..	..	..	..	..	..	*s* R
Patellina corrugata, Will.	..	..	*s* C	*s* RC	*s* C															
Trochammina squamata, P. & J.	..	..	*s* R	..	*s* C															
—— gordialis, P. & J.	..	..	..	..	*s* C															
Cornuspira foliacea, Phil.	*m* C	..	..	..	*s* C															
Quinqueloculina Seminulum, Linn.	*s* C	*m* C	*s* C	*s* C	*m* C	*vs* R	..	..	..	..	*s* R	..	*l* C	*l* C	*l* C	*l* C	..	*l* C	*l* C	*m* C
—— Ferussacii, D'O.	..	..	*m* C	..	*m* C															
—— agglutinans, D'O.	..	..	*m* R	..	*m* C	*m* R														
—— subrotunda, Montag.	..	..	..	..	*s* C															
Triloculina cryptella, D'O.	..	..	..	..	..	..	..	..	*m* C											
—— tricarinata, D'O.	*s* C	..	..	..	*s* R															
—— oblonga, Montag.	*s* C																			
Biloculina ringens, Lam.	..	..	..	..	..	..	..	..	..	..	..	..	*l* C	..	..	*l* C	..	..	*l* C	
Lituola Canariensis, D'O.	*l* VC	*l* R	*l* C	*s* C	*l* VC	*m* C	*vl* C	..	*m* C	..	*s* R	..	*l* R	..	..	..	..	..	..	*s* RC
—— globigeriniformis, P. & J.	..	..	*s* C	*s* RC	*s* RC	*s* C	..	..	*s* R	..	*s* R									
—— Scorpiurus, Montf.	*l* C	..	*l* C	..	..	*m* C	*vl* RC													

Nodosarinæ, at 160 fathoms, just within the Arctic Circle,—such forms as are known under the subgeneric names of *Glandulina*, *Nodosaria*, *Dentalina*, *Marginulina*, and *Cristellaria*, and are abundant in some warm seas at less depths, and in the fossil state in the Chalk and other deposits of Secondary and Tertiary age. Where the "Celtic Province" (under the name "Virginian") impinges on the American coast of the Atlantic, between lat. 30° and lat. 50° N., some soundings made by the Coast-survey of the United States, at from 20 to 105 fathoms, yielded to Professor BAILEY's search several *Dentalinæ*, *Marginulinæ*, and *Cristellariæ* of good size. (See Appendix II.)

The Mollusca obtained by Messrs. MACANDREW and BARRETT at Omnoes Oe, Nordland, at from 30 to 50 fathoms, half-a-mile from shore (the dredging No. 2 in our list above), are enumerated in Dr. WOODWARD's 'Manual of Mollusca, Recent and Fossil,' p. 434; and a list of the shells from an equivalent dredging to our No. 6 (if not the same) is given at p. 435.

4. From the North Atlantic Ocean, between 52° 25′ and 48° north latitude. Deep-sea soundings in the North Atlantic between Ireland and Newfoundland, made in Her Majesty's Ship 'Cyclops,' by Lieut.-Commander JOSEPH DAYMAN, in June and July 1857. See the Admiralty Report, with map and plates, and an Appendix by Professor HUXLEY, 8vo, 1858. Thirty-nine of these soundings, from 43 to 2350 fathoms, were examined. See Table V. and Map, Plate XII.

The materials confided to us were small portions (about thimblefuls) of thirty-nine selected soundings, from out of a hundred and two.

This collection affords as fair an exposition of the Foraminiferal fauna of the particular tract of sea-bottom examined as the limited amount of material brought up by the sounding-machine can be expected to give. The other materials (organic and inorganic) besides Foraminifera are shown in Tables VI. & XII.

Three soundings, at from 43 to 90 fathoms off the coast of Ireland, at about 30 miles, 60 miles, and 75 miles off shore respectively (Nos. 39 [102], 38 [100], 37 [99]), indicate the Foraminifera there inhabiting the "Coral-zone"; here the *Nodosarinæ* are rare and small; *Lagenæ* rather more common; *Orbulina* still more common; *Globigerina* rare; the *Rotalinæ* (*Planorbulina*, *Discorbina*, *Rotalia*, and *Pulvinulina*) are represented, though not at all abundantly. *Polystomella* has its northern form (*P. striatopunctata*) here and little else; *Cassidulina*, *Uvigerina*, *Bulimina*, and *Textularia* are plentiful; *Miliola* and *Lituola* are comparatively poor both in number and size.

At different depths, ranging from 223 to 415 fathoms further westward along the line of soundings, and nearly to the brink of the marginal plateau, this same fauna, with some exceptions and a few additions, continues; but *Globigerina* increases in size and numbers; and so do *Planorbulina Ungeriana* and *Pulvinulina Menardii*, with its subvariety *Micheliniana*.

Beyond and at the foot of the marginal plateau, the first sounding (15° 6′ W. long.) is at 1750 fathoms, and here we find very few Foraminifera, only *Orbulina*, *Globigerina*, *Pulvinulina Canariensis*, and *Cassidulina*, the two latter being small and rare. Further westward, however, along the wide abyssal depths (to about 45° 30′ W. long.), even at more than 2000 fathoms, we find a larger fauna, of but few species, among which *Orbulina* and *Globigerina* are characteristically abundant (especially the latter), and are accompanied by *Lagena* (rare), *Discorbina*, *Uvigerina*, *Rotalia Soldanii*, *Pulvinulina Menardii*, *P. Micheliniana*, and *P. Canariensis*, occasional specimens of *Pullenia*, a few *Nonioninæ* and *Polystomellæ* (*P. striatopunctata*), a few *Buliminæ*, very few *Textulariæ*, and scattered small *Miliolæ* and *Lituolæ*. In the western portion of this territory the fauna is somewhat poorer, where naturalists have drawn the southern portion of their "Boreal Province."

Rising the western slope from the abyss (40° 45′ to 49° 23′ W. long., parallel to the northern end of the Bank of Newfoundland), we enter the great southern angle of the "Arctic Province," and the Foraminiferal fauna continues to have much the same elements; but *Globigerina* and *Orbulina* have become rarer; *Miliolæ* are very rare; *Planorbulina* comes in, *Pulvinulinæ* disappearing after the first upslant of the bottom at 45° 45′ W. long.

From 50° 14′ 30″ to 52° 44′ W. long., we are still off the northern edge of the Newfoundland Bank; and, though the depth decreases from 405 fathoms to 161 and then to 112 fathoms, Foraminifera are extremely rare, owing, without doubt, chiefly to the coldness of ice-laden water. *Truncatulina*, *Pulvinulina*, *Polystomella*, and *Uvigerina* seem to struggle for existence here, where "Arctic" conditions are extended southwards.

At 52° 56′ and thence to 53° 57′ 35″ W. long. the line of soundings is in Trinity Bay, with depths varying from 124 to 195 fathoms. Only very scarce *Globigerinæ*, a few *Pulvinulinæ*, some *Nonioninæ*, rather more of the very persistent *Cassidulinæ*, and a very few *Uvigerinæ*, *Buliminæ*, and *Lituolæ* appear to inhabit this unfavourable locality at the depths examined. In fact this region belongs to the "Arctic Province," which is here prolonged southwards towards the Bank of Newfoundland by the influence of cold currents and icebergs.

With the exception of the westerly soundings, these deep-sea gatherings from the North Atlantic illustrate the Foraminifera of the "Celtic Province"; but necessarily lack, as a fauna, the complementary shallow-water forms,—namely, those living in the Coralline, Laminarian, and Littoral Zones, at depths less than 40 fathoms.

The materials from Davis Straits (Hunde Islands) above-mentioned serve to illustrate only for the "Arctic Province" the Foraminiferal inhabitants of the Coralline-Zone; and therefore do not fulfil the requirements of this case. We may take, however, as a term of comparison the list of the Recent Foraminifera of the British Isles, described by Professor WILLIAMSON, but classified (and partly renamed) after the plan here adopted, and augmented by later researches (including those by Mr. H. B. BRADY, F.L.S.); and we thus have before us, in these combined lists, a synopsis of the Foraminiferal fauna of the "Celtic Province." (See Table IX. in Appendix V.)

The deep-sea Foraminiferal fauna of the North Atlantic differs from the fauna of the Coralline, Laminarian, and Littoral Zones of the "Celtic Province" chiefly in having fewer varieties and (generally) smaller individuals of *Nodosarina*, *Lagena*, *Polystomella*

TABLE V.—Table showing the presence of Foraminifera in 39 specimens of sea-bottom from the North Atlantic (Dayman's Soundings); also their abundance or rarity, and their relative size. [See Appendix VIII.]

vl. Very large. *l.* Large. *m.* Middle-sized. *rs.* Rather small. *s.* Small. *vs.* Very small. VC. Very common. C. Common. RC. Rather common. RR. Rather rare. R. Rare. VR. Very rare.

Typical Species.	Genera, Species, and Varieties.	Arctic Province (Trinity Bay).									Arctic (North of Newfoundland Bank).										
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
		Lat. 48° 0' 30" N. Long. 53° 27' 35" W.	Lat. 48° 0' 10" N. Long. 53° 26' 36" W.	Lat. 48° 9' N. Long. 53° 15' W.	Lat. 48° 15' 30" N. Long. 53° 13' W.	Lat. 48° 9' 45" N. Long. 53° 10' 50" W.	Lat. 48° 11' N. Long. 53° 7' 50" W.	Lat. 48° 14' 22" N. Long. 53° 14' W.	Lat. 48° 18' N. Long. 52° 56' W.	Lat. 48° 21' N. Long. 52° 44' W.	Lat. 48° 28' 30" N. Long. 52° 19' 30" W.	Lat. 48° 40' N. Long. 51° 45' W.	Lat. 47° 57' 20" N. Long. 51° 31' 30" W.	Lat. 49° N. Long. 50° 48' 30" W.	Lat. 49° 2' N. Long. 50° 14' 30" W.	Lat. 49° 23' 30" N. Long. 49° 55' W.	Lat. 49° 26' N. Long. 49° 48' W.	Lat. 49° 16' 30" N. Long. 49° 17' W.	Lat. 49° 18' N. Long. 49° 12' W.	Lat. 49° 23' N. Long. 48° 48' W.	Lat. 49° 38' N. Long. 48° 5' W.
		No. 53, 195 fms.	No. 49, 129 fms.	No. 47, 190 fms.	No. 39, 124 fms.	No. 45, 150 fms.	No. 41, 129 fms.	No. 61, 167 fms.	No. 59, 133 fms.	No. 55, 112 fms.	No. 65, 102 fms.	No. 69, 146 fms.	No. 63, 145 fms.	No. 73, 161 fms.	No. 33, 405 fms.	No. 77, 221 fms.	No. 78, 329 fms. [330 fms.]	No. 32, 740 fms. [742 fms.]	No. 79, 725 fms.	No. 31, 954 fms.	No. 30, 1203 fms.
1. Nodosarina Raphanus, Linn.	Nodosaria Raphanus, Linn.																				
	—— scalaris, Batsch.																				
	Dentalina communis, D'O.																				
	Cristellaria Crepidula, F. & M.																				
	—— cultrata, Montf.																				
2. Lagena sulcata, W. & J.	Lagena sulcata, W. & J.																				
	—— lævis, Montag.																*m* VR				
	—— caudata, D'O. striated																				
	smooth																				
	—— globosa, Montag.																				
	—— squamosa, Montag.																				
	—— marginata, Montag.																	*s* R			
3. Orbulina universa, D'O.	Orbulina universa, D'O.																*m* R				
4. Globigerina bulloides, D'O.	Globigerina bulloides, D'O.					*vs* VR			*vs* VR								*vs* VR	*rs* C	*s* RR	*s* C	*s* C
5. Planorbulina farcta, F. & M.	Truncatulina lobatula, W. & J.												*s* R					*m* R			
	Planorbulina Mediterranensis, D'O.																				
	—— Haidingerii, D'O.																				
	—— Ungeriana, D'O.																				
6. Discorbina Turbo, D'O.	Discorbina Berthelotiana, D'O.																				
	—— rosacea, D'O.																				
7. Rotalia Beccarii, Linn.	Rotalia Beccarii, Linn.																				
	—— Soldanii, D'O.																				
	—— orbicularis, D'O.																				
8. Pulvinulina repanda, F. & M.	Pulvinulina Menardii, D'O.																*m* VR				
	—— Karsteni, Rss.								*l* R												
	—— Micheliana, D'O.																				
	—— elegans, D'O.								*m* RR												
	—— Canariensis, D'O.				*s* VR				*s* R					*s* VR							
	—— pauperata, P. & J.																				
9. Sphæroidina bulloides, D'O.	Sphæroidina bulloides, D'O.																				
10. Pullenia sphæroides, D'O.	Pullenia sphæroides, D'O.																				*vs* VR
11. Polystomella crispa, Linn.	Nonionina asterizans, F. & M.																	*s* R			
	—— Scapha, F. & M.				*m* RC	*m* R			*m* RR				*m* R	*m* VR			*m* VR		*m* VR	*vs* VR	
	—— turgida, Will.																				
	—— umbilicatula, Montag.					*vs* VR											*vs* VR				
	Polystomella striatopunctata, F. & M.												*s* R	*vs* VR				*m* RC	*s* RR	*s* R	
	—— crispa, Linn.																		*s* VR		
12. Nummulina perforata, Montf.	Operculina ammonoides, Gm.																				
13. Cassidulina lævigata, D'O.	Cassidulina lævigata, D'O.		*m* RC		*m* RC	*s* R			*m* RR	*m* RR	*m* VR		*m* RR					*m* RC			
	—— crassa, D'O.																				
14. Uvigerina pygmæa, D'O.	Uvigerina pygmæa, D'O.					*vs* R				*m* RR				*s* VR							
	—— angulosa, Will.																				
15. Bulimina Presli, Rss.	Bulimina ovata, D'O.					*vs* VR												*m* R			
	—— aculeata, D'O.																				
	—— marginata, D'O.																				
	—— Buchiana, D'O.																				
	Virgulina Schreibersii, Czjzek																		*s* VR	*s* R	
	Bolivina punctata, D'O.																				
	—— costata, D'O.																				
16. Textularia agglutinans, D'O.	Textularia abbreviata, D'O.																				
	—— Sagittula, Defr.																				
	—— pygmæa, D'O.																				
	—— carinata, D'O.																				
	Bigenerina Nodosaria, D'O.																				
	—— digitata, D'O.																				
17. Miliola Seminulum, Linnè	Spiroloculina planulata, Lam.																				
	—— limbata, D'O.																				
	Quinqueloculina tenuis, Czjzek																				
	—— Seminulum, Linn.																			*m* R	
	—— triangularis, D'O.																				
	—— agglutinans, D'O.																				
	Triloculina oblonga, Montag.																				
	Biloculina depressa, D'O.																				
	—— elongata, D'O.																				
18. Lituola nautiloidea, Lam.	Lituola Canariensis, D'O.																				*m* R
	—— globigeriniformis, P. & J.								*m* R				*vs* VR							*s* VR	
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

Typical Species.	Genera, Species, and Varieties.	Boreal (Abyssal).						Celtic (Abyssal).						Celtic (Marginal).						
		21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39
		Lat. 50° 6' N. Long. 45° 45' W.	Lat. 50° 25' N. Long. 44° 19' W.	Lat. 50° 46' N. Long. 42° 20' W.	Lat. 52° 11' N. Long. 31° 29' W.	Lat. 51° 29' N. Long. 38° 1' W.	Lat. 51° 30' N. Long. 38° W.	Lat. 52° 16' 30" N. Long. 29° 28' 30" W.	Lat. 52° 25' N. Long. 28° 10' W.	Lat. 52° 33' 30" N. Long. 21° 16' W.	Lat. 52° 16' N. Long. 16° 46' W.	Lat. 52° 16' N. Long. 16° 42' W.	Lat. 52° 21' 30" N. Long. 15° 6' W.	Lat. 52° 16' N. Long. 14° 30' W.	Lat. 52° 11' N. Long. 13° 45' W.	Lat. 52° 8' 30" N. Long. 12° 31' W.	Lat. 52° 0' 30" N. Long. 12° 7' 30" W.	Lat. 52° 1' N. Long. 11° 14' 40" W.	Lat. 51° 59' N. Long. 11° W.	Lat. 51° 57' N. Long. 10° 30' 30" W.
		No. 80, 1450 fms. [1405 fms.]	No. 26, 2330 fms.	No. 25, 2250 fms. [2050 fms.]	No. 19, 2035 fms. [2030 fms.]	No. 81, 2350 fms. [Deepest.]	No. 22, 1660 fms. [2250 fms.]	No. 85, 2176 fms.	No. 86, 1950 fms. [2050 fms.]	No. 15, 1776 fms.	No. 90, 2050 fms.	No. 13, 2050 fms.	No. 12, 1750 fms.	No. 93, 200 fms.	No. 95, 223 fms.	No. 98, 415 fms.	No. 7, 338 fms.	No. 99, 90 fms.	No. 100, 78 fms.	No. 102, 43 fms.
1. Nodosarina Raphanus, Linn.	Nodosaria Raphanus, Linn.																		*s* R	
	—— scalaris, Batsch.													*rs* R	*s* RR	*s* R		*s* R	*s* R	
	Dentalina communis, D'O.									*vs* R										
	Cristellaria Crepidula, F. & M.																			*s* R
	—— cultrata, Montf.																		*m* R	
2. Lagena sulcata, W. & J.	Lagena sulcata, W. & J.		*m* R			*s* R									*s* R					*s* RC
	—— lævis, Montag.														*l* R					
	—— caudata, D'O. striated	*m* R																		
	smooth					*s* R		*s* R												
	—— globosa, Montag.															*l* R				
	—— squamosa, Montag.	*m* R																		
	—— marginata, Montag.	*m* R				*l* RC										*l* R		*s* RC	*s* C	*s* C
3. Orbulina universa, D'O.	Orbulina universa, D'O.					*m* RC	*m* R	*l* R	*m* R	*m* C	*m* C	*m* R	*l* R		*m* R	*l* C		*m* R	*m* R	
4. Globigerina bulloides, D'O.	Globigerina bulloides, D'O.	*m* VC	*m* VC	*m* VC	*m* VC	*m* VC	*m* VC	*m* RC	*m* VC	*m* VC	*m* VC	*m* VC	*m* VC	*s* RR	*m* C	*m* C	*l* C	*s* R	*s* C	*s* RC
5. Planorbulina farcta, F. & M.	Truncatulina lobatula, W. & J.							*s* R									*s* R		*m* RC	*m* C
	Planorbulina Mediterranensis, D'O.																			*s* R
	—— Haidingerii, D'O.									*m* R										
	—— Ungeriana, D'O.	*s* R	*s* R	*s* R		*s* C	*m* RR	*s* C	*s* R	*s* R	*s* RC			*m* RC	*l* C	*m* C		*m* R	*s* RR	*s* RC
6. Discorbina Turbo, D'O.	Discorbina Berthelotiana, D'O.																		*s* RC	*s* R
	—— rosacea, D'O.																			*m* RC
7. Rotalia Beccarii, Linn.	Rotalia Beccarii, Linn.																		*s* R	
	—— Soldanii, D'O.				*m* R	*s* R				*vs* R		*s* R			*m* RR	*m* C				*s* R
	—— orbicularis, D'O.								*vs* VR											
8. Pulvinulina repanda, F. & M.	Pulvinulina Menardii, D'O.	*m* RC	*m* RC	*m* R		*m* RC	*m* RR	*m* C	*m* RC	*m* C	*m* C	*m* R			*m* RC	*m* RC	*m* R		*s* R	
	—— Karsteni, Rss.																			
	—— Micheliana, D'O.	*s* R	*m* RC		*m* R	*m* RC	*m* RR	*m* R	*l* RC	*m* C	*m* C	*m* C		*m* RC	*m* RC	*m* C			*s* R	
	—— elegans, D'O.	*m* RC					*s* R												*s* C	
	—— Canariensis, D'O.	*s* RC	*vs* C	*s* R	*m* RC	*s* RC	*s* RR	*m* C	*m* RC	*s* RC	*m* C		*s* R			*m* R	*s* R		*s* C	
	—— pauperata, P. & J.	*m* R						*s* R												
9. Sphæroidina bulloides, D'O.	Sphæroidina bulloides, D'O.		*vs* VR												*s* R					
10. Pullenia sphæroides, D'O.	Pullenia sphæroides, D'O.		*vs* VR		*s* R			*vs* RR		*vs* R				*s* R						
11. Polystomella crispa, Linn.	Nonionina asterizans, F. & M.																			
	—— Scapha, F. & M.														*s* R					
	—— turgida, Will.														*s* R					*s* R
	—— umbilicatula, Montag.					*s* R		*s* RC	*m* RC	*s* R	*s* RC				*m* C	*m* C			*m* C	
	Polystomella striatopunctata, F. & M.														*s* R				*vs* VR	*s* C
	—— crispa, Linn.																			
12. Nummulina perforata, Montf.	Operculina ammonoides, Gm.													*s* R	*s* C	*s* C		*s* C	*s* VC	*s* C
13. Cassidulina lævigata, D'O.	Cassidulina lævigata, D'O.												*s* R	*s* R	*s* RC	*s* C		*m* C	*m* C	*m* C
	—— crassa, D'O.														*vl* VR					
14. Uvigerina pygmæa, D'O.	Uvigerina pygmæa, D'O.													*l* C	*l* VC	*l* VC	*l* C	*l* VC	*l* VC	*l* VC
	—— angulosa, Will.														*m* C				*m* RC	*s* R
15. Bulimina Presli, Rss.	Bulimina ovata, D'O.								*vs* R	*vs* RR									*s* R	
	—— aculeata, D'O.														*m* C					
	—— marginata, D'O.														*s* C	*s* C		*m* C	*m* C	*l* C
	—— Buchiana, D'O.																		*s* R	
	Virgulina Schreibersii, Czjzek		*l* R						*m* R											
	Bolivina punctata, D'O.														*s* RC	*s* R				*s* R
	—— costata, D'O.														*l* R					
16. Textularia agglutinans, D'O.	Textularia abbreviata, D'O.														*s* R	*s* RC		*m* RC	*m* C	*m* C
	—— Sagittula, Defr.																		*m* C	
	—— pygmæa, D'O.	*vs* VR			*s* R									*s* R	*m* R	*s* R	*m* R	*s* RC	*s* RC	
	—— carinata, D'O.																	*s* C	*s* RC	
	Bigenerina Nodosaria, D'O.																	*m* C	*m* C	
	—— digitata, D'O.																		*s* R	
17. Miliola Seminulum, Linnè	Spiroloculina planulata, Lam.		*s* R								*m* R									*m* R
	—— limbata, D'O.																		*s* R	
	Quinqueloculina tenuis, Czjzek										*s* R					*s* RC				
	—— Seminulum, Linn.				*s* R	*s* R					*s* R							*s* R	*s* C	*s* C
	—— triangularis, D'O.		*vs* VR				*vs* VR	*s* R		*vs* R										
	—— agglutinans, D'O.	*s* VR								*vs* R						*m* C				
	Triloculina oblonga, Montag.		*vs* VR																	
	Biloculina depressa, D'O.	*m* R				*m* R	*m* R	*s* R											*s* R	*s* R
	—— elongata, D'O.		*s* R						*vs* VR			*s* R								
18. Lituola nautiloidea, Lam.	Lituola Canariensis, D'O.														*m* R					*s* R
	—— globigeriniformis, P. & J.						*m* R													
		21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39

TABLE VI.—Table showing the presence and proportion of Organic and Inorganic substances in 100 parts of dry sea-bottom from the North-Atlantic. See also Table XII.

	Arctic Province (Trinity Bay).									Arctic (North of Newfoundland Bank).										
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
	No. 53, 195 fms.	No. 49, 129 fms.	No. 47, 190 fms.	No. 39, 124 fms.	No. 45, 150 fms.	No. 41, 129 fms.	No. 61, 167 fms.	No. 59, 133 fms.	No. 55, 112 fms.	No. 65, 102 fms.	No. 69, 146 fms.	No. 63, 145 fms.	No. 73, 161 fms.	No. 33, 405 fms.	No. 77, 221 fms.	No. 78, 329 fms.	No. 32, 740 fms.	No. 79, 725 fms.	No. 31, 954 fms.	No. 30, 1203 fms.
Molluscs—small, with fragments																				
Annelids—sand-tubes and Serpulæ													sand-tube							
Polyzoa—fragments																				
Entomostraca—valves																				
Echinoderms—spines and plates							a trace.													a trace.
Sponges—spicules									a trace,				2 spicules.		a trace.					
Foraminifera a. Globigerina bulloides					a trace												8·		0·7	1·5
Foraminifera b. Other species		a few.			a few.			a few.	a few.	1 spec.		a trace.	a trace.	a few.		a trace.	a few.	a few.	0·3	0·5
Polycystineæ*																		a few.		
Diatomaceæ †			some		some.	some.	some.	some.	some.		some.	some.	some.					some.	some.	some.
Whitish mud																				49·
Grey mud	99·	50·5	99·	2·	49·5	60·	1·			33·	25·	33·	33·	2·5	1·	25·	88·	98·	75·	
Sand, mostly quartz a. Fine	1·	30 5	1·	96·	49·5		49·			10·	35·	33·	30·	97·5	49·	72·	3·	1·5	12·	49·
Sand, mostly quartz b. Coarse		18·				20·	50·		49·	23·	40·	33·	36·		50·	2·			12·	
Small gravel stones						20·			50·	33·										

	Boreal (Abyssal).						Celtic (Abyssal).						Celtic (Marginal).						
	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39
	No. 80, 1450 fms.	No. 26, 2330 fms.	No. 25, 2250 fms.	No. 19, 2035 fms.	No. 81, 2350 fms.	No. 22, 1660 fms.	No. 85, 2176 fms.	No. 86, 1950 fms.	No. 15, 1776 fms.	No. 90, 2050 fms.	No. 13, 2050 fms.	No. 12, 1750 fms.	No. 93, 200 fms.	No. 95, 223 fms.	No. 98, 415 fms.	No. 7, 338 fms.	No. 99, 90 fms.	No. 100, 78 fms.	No. 102, 43 fms.
Molluscs—small, with fragments																	fragments of shells	fragts.	shells and fragments
Annelids—sand-tubes and Serpulæ																			
Polyzoa—fragments																			fragts.
Entomostraca—valves								a valve.	a valve.	a valve.				a few.					a few.
Echinoderms—spines and plates							a trace.	a trace.						a trace.		a spine.	fragts.	5·	15·
Sponges—spicules																			a few.
Foraminifera a. Globigerina bulloides	97·	52·	95·	97·	95·5	94·	73·5	97·5	83·	53·	49·	97·5	a few.	16·	12·	23·	a few.	4·	7·5
Foraminifera b. Other species	1·	1·	a few.	a few.	0·5	a few.	0·5	0·5	1·	1·5	a few.	a few.	5·	32·	8·	10·	5·	10·	20·
Polycystineæ*							a few.	a few.											
Diatomaceæ †							some.	some.										some.	
Whitish mud	a trace.	45·	a trace.	2·	1·5	a trace.	25·	1·5	15·	45·	50·	a trace				a trace.			
Grey mud														1·5	40·		a trace.	1·	
Sand, mostly quartz a. Fine	1·5	2·	2·	a trace.	1·5	2·	1·	0·5	a trace.	a trace.	a trace.	a trace.	5·	50·	40·	66·	92·5	70·	25·
Sand, mostly quartz b. Coarse	1·		2·		1·	2·							89·					10·	25·
Small gravel stones																			

---

* In some portions of this Atlantic ooze that Mr. Roper received from Professor Huxley, he met with many varieties of *Polycystineæ*; and in his Appendix to Commander Dayman's Report on the North-Atlantic Sea-bed, Professor Huxley states (p. 64) that, in the grey calcareous mud (ooze) of from 1700 to 2400 fathoms, he noticed among the siliceous organisms a number of *Polycystineæ*, e.g., *Flustrella*, *Eucyrtidium*, *Stylosphæra*, *Haliomma*, &c.; also the spicular bodies known as *Spongolitharia*, and the doubtful organisms *Codium* and *Rhizosalenia*.

† See Mr. Roper's letter, p. 334.

and *Nonionina*, *Rotalinæ*, *Bulimina*, *Textularia*, *Cornuspira*, *Miliola*, and *Lituola*, and no *Polymorphinæ*; and in having more *Cassidulinæ* and *Uvigerinæ*, and far more *Globigerinæ* and *Orbulinæ*, with the addition of *Pullenia*.

The Telegraph-line, in passing the northern end of the Bank of Newfoundland, enters (at about 47° W. long.) the Southern extension of the "Arctic Province"*, where the prevalence of floating ice keeps "Arctic" conditions as far south as 45° N. lat. This western extremity of the line does not belong, therefore, to the "Celtic," but to the "Arctic Province"; and the few Foraminifera occurring there may be regarded as equivalent to those mentioned above as being found at similar depths in Baffin's Bay.

The south-western extremity of the "Boreal Province," bordering the "Arctic," also invades the western part of the line of soundings†; and is coincident with a somewhat impoverished condition of the abyssal fauna common to these soundings (Nos. 21–26) and others (Nos. 27–32) to the East ("Celtic").

The accompanying Map (after Commander DAYMAN's Chart), Plate XII., illustrates the foregoing remarks. See Appendix VII.

We are fortunately able to compare the deep-sea Foraminifera of the North Atlantic with those inhabiting the shallower water of its western margin at a lower latitude than Newfoundland, where the Telegraph-soundings terminate. The late Professor BAILEY's examination‡ of some soundings made by the United States Coast-survey on the shores of New Jersey and Delaware, between lat. 50° and lat. 38° N., in 1848, affords us the means of doing this, at least to some extent.

Where the "Celtic Province," crossing the Atlantic from the British Isles, approaches North America, it takes on a modified character, and is known as the "Virginian Province"; and its north and south limits are just those of the series of soundings made by the United States Coast-survey referred to above, and thus yielding us (as far as Professor BAILEY's figures and descriptions serve) the western equivalents of the eastern margin of the "Celtic Province." See Appendix II., and Table VIII.

As far as Professor BAILEY's material shows, we find the "Virginian" fauna to be related to the "Celtic" of the Irish coast by *Orbulina universa*, *Cristellaria cultrata*, *Planorbulina Ungeriana* (abundant in the Irish and rare in these American soundings; whilst its congener, *Pl. Haidingerii*, abounds here and is wanting in the soundings off Ireland), *Pulvinulina Menardii*, *Globigerina bulloides*, and *Quinqueloculina Seminulum*. All the recorded "Virginian" forms occur in the British seas, except *Marginulina regularis*, *Verneuilina triquetra*, *Bulimina Pyrula* (represented to the eastward by *B. marginata*) and *Virgulina squamosa*.

---

* See the Map of the Molluscan Provinces, by E. FORBES, in KEITH JOHNSTON's 'Physical Atlas'; the Map in S. P. WOODWARD's 'Manual of Mollusca'; and that in FORBES and GODWIN-AUSTEN's 'Nat. Hist. of the European Seas.'

† Soundings Nos. 26, 25, 24, 23, 22, & 21. The last is close upon the southern limb of the "Arctic Province."

‡ See the 'Smithsonian Contributions to Knowledge,' vol. ii. 1861.

To the "Arctic" and "Boreal" faunæ the "Virginian" is allied by *Dentalina pauperata*, *Cristellaria cultrata*, *Globigerina bulloides*, *Bulimina Pyrula*, *Virgulina Schreibersii*, *V. squamosa*, and *Quinqueloculina Seminulum*.

Besides *Foraminifera*, the North-Atlantic soundings obtained by Commander DAYMAN have yielded us the organic and inorganic materials indicated in Tables VI. & XII.

Mr. F. C. S. ROPER, F.L.S., F.G.S., has obliged us with the following Note on the Diatomaceæ *.

"3 Carlton Villas, January 7, 1864.

"MY DEAR SIR,—I regret that I have not before this replied to yours of the 24th ult., relating to the Soundings I received from Mr. PARKER. I mounted slides from each packet, but found that they contained so few Diatoms, that I only made cursory notes upon them; and, on referring to these, find they were almost confined to specimens of Coscinodiscus, as you will see by the list enclosed. These Atlantic soundings are so transparent, and the siliceous matter apparently so wasted, that it is very trying to the eyes to hunt over a succession of slides with high powers, to seek the few Diatoms contained in them; and I was compelled from the fear of injury to my sight to abstain from an exhaustive examination of them.

"Believe me very truly yours,
"F. C. S. ROPER."

Clays.

No. 30. A few fragments of Coscinodisci.
No. 31. „ „
No. 61. A large Cocconeis.
A few Coscinodisci, apparently C. radiatus.
A Rhabdonema.
No. 63. A few fragments of Coscinodisci.
No. 85. Coscinodiscus. C. perforatus?
No. 100. Coscinodiscus eccentricus.
Coscinodiscus radiatus.
Orthosira marina.
Actinocyclus undulatus.
Pleurosigma transversale?
No. 41. Coscinodiscus radiatus.
A Nitzschia.
A Rhabdonema.
No. 45. Fragments of Coscinodisci.
No. 69. „ „
No. 73. Large Coscinodiscus.
No. 79. „
No. 86. Coscinodisci, a few.

The remainder little else than Foraminifera and sand.

Sands.

No. 47. Coscinodiscus? sp.
Rhabdonema.
Grammatophora marina.
No. 59. A few Coscinodisci.
No. 64. „

The remainder nearly all sand with Foraminifera.

The following Entomostraca from these soundings have been determined by Mr. G. S. BRADY.

Cythere scabra, *Münster*; 2050 fathoms. Lat. 52° 16′ N., long. 16° 46′ W.
—— rhomboidea *Brady*; 43 fathoms. Lat. 51° 57′ N., long. 10° 30′ W.
—— mamillata, *Brady*; 110 fathoms. Lat. 52° 59′ N., long. 14° 10′ W.
Bairdia Bosquetiana, *Brady*; 470 fathoms; off Ireland.

5. Besides the description and illustration of the Foraminifera obtained from the four sets of soundings and dredgings above mentioned, and the tabulation of the species and varieties, showing their depth of water and relative size and abundance, we also point

---

* The Diatoms found in the "Virginian Province" are noticed by Professor BAILEY in the memoir above referred to.

out, to some extent, their distribution in other seas (see Table VII.), and their occurrence in the fossil state; thus providing some materials towards a correct knowledge of their distribution in Time and Space.

With this in view, we have endeavoured to simplify the nomenclature of the Foraminifera by adhering as strictly as possible to the plan of study laid down by WILLIAMSON* and CARPENTER†, and followed by ourselves in former memoirs‡.

Using the classification and nomenclature § proposed in the 'Introduction to the Study of the Foraminifera,' we have, under generic and specific heads, a limited number of Foraminiferal groups, possessing among themselves very different features, whilst the members of each group are formed on one simple plan, almost infinitely modified in its details, and often producing imitations of members of the other groups, just as mimetic resemblances occur in Mollusca, and in other Classes of the Animal and Vegetable Kingdoms.

By recognizing these mimetic resemblances among distinct varieties and species, and laying but little stress on non-essential features, we seem to be able to grasp the multitudinous varieties and subvarieties, modified, disguised, and transitional, with something like satisfactory results; and they fall into natural recognizable groups, having more or less fixed habits and places of growth, instead of escaping from us as an illimitable cloud of differing though related individuals, almost unknown in reality, though nearly each has been endowed by writers with a separate binomial title.

In determining the species and varieties of the Foraminifera under notice, we have, as far as possible, used already published materials; and in comparing our specimens with figured forms, we have been satisfied when a *near* approach to identity is shown; minute differences are ignored, such differences not being of essential value.

There have been many naturalists who have helped on our knowledge of these Microzoa. D'ORBIGNY first classified them sufficiently well to enable himself and others to group their acquired material in an orderly, though artificial manner; and by his care an enormous number of forms, specific and varietal, from different parts of the earth, recent and fossil, have been arranged in good lithograph plates, serving as a museum for reference. Since D'ORBIGNY, few have collected such great stores of Foraminifera, and illustrated them so abundantly, as Professor Dr. A. E. REUSS; providing naturalists with, as it were, available collections of hundreds of forms. Professor REUSS's latest observations have led him in a great degree to concur with (and in some cases to anticipate, we believe) the classification propounded in the 'Introduction to the Study of the

---

* On the Recent Foraminifera of Great Britain; by Professor W. C. WILLIAMSON, F.R.S. (Ray Society) 4to. 1858.

† Introduction to the Study of the Foraminifera; by W. B. CARPENTER, M.D., F.R.S., assisted by W. K. PARKER, Esq., and T. RUPERT JONES, F.G.S. (Ray Society) 4to. 1862.

‡ Papers on the Nomenclature of the Foraminifera, in the Annals of Natural History, from 1859 to 1863.

§ The concise and well-digested remarks on classification and nomenclature in Dr. WOODWARD's 'Manual of Mollusca' are in great part applicable to Rhizopodal studies.

Foraminifera.' To D'ORBIGNY and REUSS, then, references will be continually made in this memoir for illustrations of the species and varieties; and the titles and dates of their works, and of those of other authors treating of Foraminifera, are given in the books and memoirs above mentioned, in which all the species adopted by the older authors (LINNÈ, GMELIN, WALKER, JACOB, MONTAGU, FICHTEL, MOLL, LAMARCK, DE MONTFORT, DE BLAINVILLE, and DEFRANCE) have been critically determined.

If ever the Foraminifera of all seas come to be collected and examined with care, there is little doubt that they will afford to the Naturalist as satisfactory results as the bathymetrical study of mollusks affords; they will be perhaps even more useful to the Geologist, in aiding him to form correct notions as to the depth and other conditions of water in which strata have been formed; whilst the accurate comparison of the long-enduring Foraminiferal species of past and of present time, with their ever-varying modifications, according to climate, depth, and food, cannot fail to be a source of instruction to the Biologist.

II. *Description of Species and Varieties.*

In the following list, the species and varieties described in this memoir are enumerated in their natural order as nearly as their nature permits; the more important of the typical forms not represented in the Arctic and North-Atlantic fauna, but required to complete the series as a natural group, being added in brackets.

List of Genera, Species, and Varieties of Foraminifera from the Arctic and North Atlantic Oceans.

Genus NODOSARINA.

	Arctic.	North Atlantic.
[SPECIES. Nodosarina (Marginulina) Raphanus.]		
Subspecies. N. (Nodosaria) Raphanus		Plate XVI. fig. 1.
Variety. N. (Nodosaria) scalaris		Plate XVI. fig. 2.
N. (Glandulina) lævigata	Plate XIII. fig. 1.	
N. (Nodosaria) Radicula	Plate XIII. figs. 2–7.	
N. (Dentalina) communis	Plate XIII. fig. 10.	
Subvariety. N. (D.) consobrina		Plate XVI. fig. 3.
N. (D.) pauperata	Plate XIII. figs. 8, 9.	
N. (D.) guttifera	Plate XIII. fig. 11.	
[Subspecies. N. (Vaginulina) Legumen.]		
Variety. N. (V.) linearis	Plate XIII. figs. 12, 13.	
[SPECIES. Nodosarina (Marginulina) Raphanus.]		
Variety. N. (M.) Lituus	Plate XIII. fig. 14.	
[Subspecies. N. (Cristellaria) Calcar.]		
Variety. N. (C.) Crepidula	Plate XIII. figs. 15, 16.	Plate XVI. fig. 4.
N. (C.) cultrata	Plate XIII. figs. 17, 18.	Plate XVI. fig. 5.
N. (C.) rotulata	Plate XIII. fig. 19.	

Genus LAGENA.

		Arctic.	North Atlantic.
SPECIES.	Lagena sulcata	Plate XIII. figs. 24, 28–32.	Plate XVI. figs. 6, 7 *a*.
Variety.	L. globosa	Plate XIII. fig. 37.	Plate XVI. fig. 10.
	L. lævis	Plate XIII. fig. 22.	Plate XVI. fig. 9 *a*.
	L. semistriata	Plate XIII. fig. 23.	
	L. striatopunctata	Plate XIII. figs. 25–27.	
	L. Melo	Plate XIII. figs. 33–36.	
	L. squamosa	Plate XIII. figs. 40, 41.	Plate XVI. fig. 11.
	L. marginata	Plate XIII. figs. 42–44.	Plate XVI. fig. 12.
	L. distoma	Plate XIII. fig. 20.	
Subvariety.	L. polita	Plate XIII. fig. 21.	
Variety.	L. caudata	Plate XIII. figs. 38, 39.	Plate XVI. figs. 7, 8, 9.

Genus POLYMORPHINA.

SPECIES.	Polymorphina lactea	Plate XIII. figs. 45, 46.	
Variety.	P. compressa	Plate XIII. figs. 47–51.	
	P. tubulosa	Plate XIII. fig. 52.	

Genus UVIGERINA.

SPECIES.	Uvigerina pygmæa	Plate XIII. figs. 53–57.	Plate XVII. fig. 65.
Variety.	U. angulosa	Plate XIII. fig. 58.	Plate XVII. fig. 66.

Genus ORBULINA.

SPECIES.	Orbulina universa		Plate XVI. figs. 13, 14.

Genus GLOBIGERINA.

SPECIES.	Globigerina bulloides	Plate XIV. figs. 1, 2.	Plate XVI. fig. 15.
Variety.	Gl. inflata		Plate XVI. figs. 16, 17.

Genus PULLENIA.

SPECIES.	Pullenia sphæroides	Plate XIV. fig. 43.	Plate XVII. fig. 53.

Genus SPHÆROIDINA.

SPECIES.	Sphæroidina bulloides		Plate XVI. fig. 52.

Genus TEXTULARIA.

SPECIES.	Textularia agglutinans	Plate XV. fig. 21.	
Variety.	T. abbreviata		Plate XVII. fig. 76.
	T. Sagittula		Plate XVII. fig. 77.
	T. pygmæa	Plate XV. fig. 22.	Plate XVII. fig. 78.
	T. carinata		Plate XVII. fig. 79.
	T. biformis	Plate XV. figs. 23, 24.	
	T. (Bigenerina) Nodosaria	Plate XV. fig. 25.	Plate XVII. fig. 80.
Subvariety.	T. (B.) digitata		Plate XVII. fig. 81.
Variety.	T. (Verneuilina) polystropha	Plate XV. fig. 26.	

Genus BULIMINA.

		Arctic.	North Atlantic.
[SPECIES.	Bulimina Presli.]		
Variety.	B. Pyrula	Plate XV. figs. 8, 9.	
	B. marginata	Plate XV. fig. 10.	Plate XVII. fig. 70.
Subvariety.	B. aculeata	Plate XV. fig. 11.	Plate XVII. figs. 68, 69.
Variety.	B. ovata		Plate XVII. fig. 67.
	B. Buchiana		Plate XVII. fig. 71.
	B. elegantissima	Plate XV. figs. 12–17.	
	B. (Virgulina) Schreibersii	Plate XV. fig. 18.	Plate XVII. figs. 72, 73.
Subvariety.	B. (Virgulina) squamosa	Plate XV. figs. 19, 20.	
Variety.	B. (Bolivina) costata		Plate XVII. fig. 75.
	B. (B.) punctata		Plate XVII. fig. 74.

Genus CASSIDULINA.

SPECIES.	Cassidulina lævigata	Plate XV. figs. 1–4.	Plate XVII. fig. 64 *a*, *b*, *c*.
Variety.	C. crassa	Plate XV. figs. 5–7.	Plate XVII. fig. 64 *d*.

Genus PLANORBULINA.

[SPECIES.	Planorbulina farcta.]		
Variety.	Pl. (Truncatulina) lobatula	Plate XIV. figs. 3–6.	Plate XVI. figs. 18–20.
	Pl. Haidingerii		Plate XVI. fig. 22.
	Pl. Ungeriana		Plate XVI. figs. 23–25.
	Pl. Mediterranensis		Plate XVI. fig. 21.
	Pl. (Anomalina) coronata	Plate XIV. figs. 7–11.	

Genus DISCORBINA.

[SPECIES.	Discorbina Turbo.]		
Variety.	D. rosacea		Plate XVI. fig. 28.
[*Variety.*	D. vesicularis.]		
Subvariety.	D. globularis	Plate XIV. figs. 20–23.	
	D. obtusa	Plate XIV. figs. 18, 19.	
[*Variety.*	D. Parisiensis.]		
Subvariety.	D. Berthelotiana		Plate XVI. figs. 26, 27.

Genus ROTALIA.

SPECIES.	Rotalia Beccarii		Plate XVI. figs. 29, 30.
Variety.	R. Soldanii		Plate XVI. figs. 31–33.
Variety.	R. orbicularis		Plate XVI. fig. 34.

Genus PULVINULINA.

[SPECIES.	Pulvinulina repanda.]		
Subvariety.	P. punctulata	Plate XIV. figs. 12, 13.	
Variety.	P. auricula.		
Variety.	P. Menardii		Plate XVI. figs. 35–37.
Subvariety.	P. Canariensis		Plate XVI. figs. 47–49.
	P. pauperata		Plate XVI. figs. 50, 51.
	P. Micheliniana	Plate XIV. fig. 16.	Plate XVI. figs. 41–43.
[*Variety.*	P. Schreibersii.]		
Subvariety.	P. Karsteni	Plate XIV. figs. 14, 15, 17.	Plate XVI. figs. 38–40.
Variety.	P. elegans.		Plate XVI. figs. 44–46.

Genus SPIRILLINA.

		Arctic.	North Atlantic.
SPECIES.	Spirillina vivipara	Plate XV. fig. 28.	

Genus PATELLINA.

		Arctic.	North Atlantic.
[SPECIES.	Patellina concava.]		
Variety.	Patellina corrugata	Plate XV. fig. 29.	

Genus NUMMULINA.

		Arctic.	North Atlantic.
[SPECIES.	Nummulina perforata.]		
Subspecies.	N. planulata	Plate XIV. fig. 45.	
[*Variety.*	N. (Operculina) complanata.]		
Subvariety.	N. (O.) ammonoides	Plate XIV. fig. 44.	Plate XVII. figs. 62, 63.

Genus POLYSTOMELLA.

		Arctic.	North Atlantic.
SPECIES.	Polystomella crispa	Plate XIV. fig. 24.	Plate XVII. fig. 61.
Variety.	P. arctica	Plate XIV. fig. 25–30.	
	P. striatopunctata	Plate XIV. figs. 31–34.	Plate XVII. fig. 60.
	P. (Nonionina) Faba	Plate XIV. fig. 36.	
	P. (N.) asterizans	Plate XIV. fig. 35.	Plate XVII. fig. 54.
Subvariety.	P. (N.) depressula	Plate XIV. fig. 39.	
	P. (N.) stelligera	Plate XIV. figs. 40, 41.	
	P. (N.) Scapha	Plate XIV. figs. 37, 38.	Plate XVII. figs. 55, 56.
	P. (N.) umbilicatula	Plate XIV. fig. 42.	Plate XVII. figs. 58, 59.
	P. (N.) turgida	....	Plate XVII. fig. 57.

Genus VALVULINA.

		Arctic.	North Atlantic.
[SPECIES.	Valvulina triangularis.]		
Variety.	V. conica	Plate XV. fig. 27.	

Genus LITUOLA.

		Arctic.	North Atlantic.
[SPECIES.	Lituola nautiloidea.]		
Variety.	L. Canariensis	Plate XV. fig. 45.	Plate XVII. figs. 92–95.
	L. globigeriniformis	Plate XV. figs. 46, 47.	Plate XVII. figs. 96–98.
	L. Scorpiurus	Plate XV. fig. 48.	

Genus TROCHAMMINA.

		Arctic.	North Atlantic.
SPECIES.	Trochammina squamata	Plate XV. figs. 30, 31.	
Variety.	T. gordialis	Plate XV. fig. 32.	

Genus CORNUSPIRA.

		Arctic.	North Atlantic.
SPECIES.	Cornuspira foliacea	Plate XV. fig. 33.	

Genus MILIOLA.

		Arctic.	North Atlantic.
[SPECIES.	Miliola (Quinqueloculina) Seminulum]	Plate XV. fig. 35.	Plate XVII. fig. 87.
Variety.	M. (Q.) agglutinans	Plate XV. fig. 37.	
	Q. Ferussacii	Plate XV. fig. 36.	
	Q. oblonga	Plate XV. figs. 34, 41.	Plate XVII. figs. 85, 86.
	Q. subrotunda	Plate XV. fig. 38.	
	Q. tenuis	....	Plate XVII. fig. 84.

	Arctic.	North Atlantic.
Variety. M. (Spiroloculina) planulata		Plate XVII. fig. 82.
Subvariety. Sp. limbata		Plate XVII. fig. 83.
Variety. M. (Biloculina) ringens	Plate XV. figs. 42–44.	
Subvariety. B. depressa		Plate XVII. fig. 89.
B. elongata		Plate XVII. figs. 88, 90, 91.
[*Variety.* M. (Triloculina) trigonula.]		
Subvariety. T. tricarinata	Plate XV. fig. 40.	
T. cryptella	Plate XV. fig. 39.	

Genus Nodosarina.

Several of the Nodosarine forms are well represented in the northern seas; but the completion of this group of hyaline, straight, or more or less bent and coiled, uniserial shells, flat, bulbous, cylindrical, or tapering, with simple septal apertures surrounded by radiating fissures, such as are comprised in our great genus *Nodosarina* (with but one true species), must be sought for in other seas. The larger *Nodosariæ* and *Cristellariæ* are wanting here, as well as the *Flabellinæ* and *Frondiculariæ*, the *Lingulinæ* also, and a host of variable *Dentalinæ*, *Vaginulinæ*, and *Marginulinæ*.

Nodosaria[*] *Raphanus*, Linnè, sp. Plate XVI. fig. 1 (North Atlantic).

A dwarf sulcate specimen with the septal lines hidden; ridges strong, oblique, and inosculating to some extent. These are not unusual features in similar but larger specimens from the Mediterranean and elsewhere, occurring at from the shore-line to 100 fathoms.

Our specimen is from 78 fathoms, lat. 51° 59′, long. 11°, North Atlantic, to the north of Newfoundland Bank.

Nodosaria scalaris, Batsch. Plate XVI. figs. 2 *a*, 2 *b*, 2 *c* (North Atlantic).

A pretty, common form, neatly striated, subcylindrical, with more or less elongate neck or stolon-tube. This is one of the varieties found by Soldani near Sienna (Testaceogr. vol. i. part 2, pl. 95, figs. *b–m*), and named *N. longicauda* by D'Orbigny (Ann. des Sciences Nat. vol. vii. p. 254, no. 28[†]. (See also page 353.)

Our figured specimens are from the North Atlantic; rare and small at 78, 90, 200, 222, and 415 fathoms (see Table V.). We have otherwise collected it principally from muds from about 100 fathoms in the northern seas.

Nodosaria (*Glandulina*) *lævigata*, D'Orbigny. Plate XIII. fig. 1 (Arctic).

This is a smooth form, and rather slender compared with that figured by D'Orbigny in the 'Annales des Sciences Nat.,' vol. vii. pl. 10, fig. 1–3; the ribbed form (*N. Glans*, D'Orb.) is represented by D'ORBIGNY'S Modèle, No. 51; both of these were from the Adriatic, and were grouped in his subgenus *Glandulina*, characterized by the short and acute-ovate shell, formed of few, close-fitting chambers, rapidly enlarging from the primordial. Similar characters, but with less regularity, are found in many specimens of *Nodosaria Radicula*, and therefore the term *Glandulina* is useful merely for convenience in distinguishing the neatest of a great number of similarly modified forms, and is nothing in a zoological sense.

---

[*] For the relationship of species and varieties in the genus *Nodosarina*, of which *Nodosaria* represents a subgroup, see the list at page 336.

[†] The priority of the name given by Batsch has been determined since this paper was read: see Ann. Nat. Hist. March 1865, p. 225.

Our specimens are from Nordland, in the Arctic Circle, at a depth of 160 fathoms (Messrs. MACANDREW and BARRETT); and they appear to be not uncommon, on a muddy bottom.

This *Glandulina* occurs also, though never abundantly, in other seas; for instance, on the muddy bed of the Gulf of Suez at 30 to 40 fathoms; in the Mediterranean, at from 30 to 100 fathoms (particularly in the Adriatic); and it has been found by Mr. H. B. BRADY in sea-sand from Shetland.

In the fossil state this form is not rare, though of extremely small size, as in several of the fossiliferous clays of the Secondary Period (where it is apt to run, on the one hand, into *Lingulina*, and, on the other, into *Nodosaria Radicula*), as in the Upper Triassic Clay of Chellaston, the Oxford Clay of Leighton, the Kimmeridge Clay of Aylesbury, and in the Chalk-marl; as also in the Tertiary strata of the Mediterranean area.

Nodosaria Radicula, Linn. sp. Plate XIII. figs. 2–7 (Arctic).

This is a Nodosarian variety closely related to the last, passing gradually from the shape of a top to that of a pupa, or from a glandiform to a cylindrical shape, thus comprising *Nodosaria humilis*, Roemer, and many other named subvarieties. These allied forms also lead out from *Nodosaria* proper to *Dentalina*; the aperture being often excentric and the axis curved. The several intermediate modifications of form have received numerous binomial appellations from authors.

Fig. 4 presents, instead of the round aperture, a transverse slit. This is a character supposed to be of generic value by D'ORBIGNY and special to *Lingulina*, this form of aperture being connected generally with a flattened or tongue-shaped form of shell. Here we have a specimen which dissolves the distinction between *Nodosaria* and *Lingulina*.

Of the specimens here illustrated, figs. 2–6 are from Nordland (MACANDREW and BARRETT), 160 fathoms, muddy bottom. They are common (about a dozen specimens), and of relatively large size. Fig. 7 is from Hunde Islands, Davis Straits, from a bottom of shelly sand, at 30–40 fathoms (Dr. P. C. SUTHERLAND).

These and numerous other closely allied forms occur in abundance in the Upper Triassic and Liassic clays, and in the clays of the Oolitic formation, but usually they are of very small size. In the Gault, Chalk-marl, and Chalk of the Cretaceous group, *Nod. Radicula* and *Nod. humilis*, connecting it with *Glandulina lævigata*, are not uncommon, and often of as large a size as those of the North Sea. In the Maestricht Chalk, also, *N. Radicula* is present and of moderate size.

Nodosaria (*Dentalina*) *communis*, D'Orb. Plate XIII. fig. 10 (Arctic).

This specimen is a dwarf *Dentalina communis** of D'ORBIGNY. The obliquity of the chambers in this shell begins early, and so does the greater excentricity of the aperture. This style of growth is well represented also by *D. inornata*, D'Orb. For. Foss. Vien. pl. 1. fig. 51, and still better by *D. Badenensis*, D'Orb. Ibid. pl. 1. figs. 48, 49; both of which are well-grown specimens of *D. communis*.

Our figured specimen is from mixed shelly sands dredged up at various spots between Drontheim and the North Cape by Messrs. MACANDREW and BARRETT. It is very small, and resembles what is usually found in nearly any muddy sand containing Foraminifera.

Dentalina communis is an extremely common variety wherever Nodosarian forms occur in the clays of the Secondary Formations, but usually it is of small size. It is larger in the Gault than in the Jurassic clays; still larger in the Chalk-marl and Chalk, and in the Maestricht Chalk, as well as in the Tertiary beds that yield *Nodosarinæ*. It is very large in the Crag of Suffolk, and in the Subapennine Tertiaries. Older than the Secondary deposits, however, it is found in the Permian limestones of England and Germany.

It is common in the recent state from the Arctic Circle to the Line; in fact, geographically and geologically, it has a very large range. It occurs in many sandy shore-deposits; but its favourite habitat is mud at 50–100 fathoms; and is continually met with in the deepest soundings, although never abundant there, and generally small.

Nodosaria (*Dentalina*) *consobrina*, D'Orbigny. Plate XVI. fig. 3 (North Atlantic).

Two joints of *Dentalina communis*, subvar. *consobrina*, D'Orb. (For. Foss. Vien. pl. 2, figs. 1–3); the chambers are longish and set on more squarely than in *D. communis* proper; representing a passage into *D. ovicula*, D'Orb. (*D. globifera*, Batsch).

This is small and rare at 1776 fathoms in the North Atlantic, lat. 52° 33′, long. 21° 16′.

Nodosaria (*Dentalina*) *pauperata*, D'Orbigny. Plate XIII. figs. 8, 9 (Arctic).

We have here a very common subvariety of *Dentalina communis*, in which the primordial chamber is relatively large, the septa but slightly oblique, and the aperture almost central; the shell is smooth, nearly cylindrical, and not constricted at the septa in the earlier portion of the shell (as shown in our figures 8 and 9); as the animal advances in growth, the chambers take on a more vesicular shape. *D. pauperata*, D'Orb. For. Foss. Vien. pl. 1. figs. 57, 58, is the same as our figured specimens; and *D. brevis*, D'Orb. Ibid. pl. 2. figs. 9 and 10, and many other named forms, are scarcely distinguishable.

---

* Annales des Sc. Nat. vol. vii. p. 254, No. 35; Mém. Soc. Géol. France, iv. p. 13, pl. 1. fig. 4.

Somewhat rare; from shelly sand, Hunde Islands, Disco Bay (Dr. P. C. Sutherland), at 60–70 fathoms; also from Baffin's Bay, lat. 75° 10′ N., long. 60° 10′ W. (Parry's soundings).

Nodosaria (Dentalina) guttifera, D'Orbigny. Plate XIII. fig. 11 (Arctic).

Passing out of *Dentalina communis* towards the perfectly moniliform subvarieties of *Nodosaria*, we have this loosely grown Dentaline form (*D. guttifera*, D'Orb. For. Foss. Vien. pl. 2. fig. 13), near *D. Pyrula*, D'Orb. It varies much in the gibbosity of the chambers.

Though curved, this *Dentalina* has an almost central aperture, as shown by a broken terminal chamber not here figured. (See Ann. Nat. Hist. 2 ser. vol. xix. pl. 19. figs. 4, 5).

We have *Dentalina guttifera* from Norway at West Fjord (Nordland), from a sandy bottom at 60 fathoms (MacAndrew and Barrett); and from a muddy bottom (Arctic Circle) at 160 fathoms. These are two fragments of two large specimens. There is no doubt that in this, as in other instances, the small quantity of materials obtained necessarily limited the number of individuals.

Forms similar or allied to this occur both in existing sea-bottoms and in fossil deposits with much the same range as that of *D. communis*; but they are not so common.

Vaginulina linearis, Montagu, sp. Plate XIII. figs. 12 *a*, 12 *b*, 13 *a*, 13 *b* (Arctic).

The straight varieties of *Marginulina Raphanus* (or the flattened forms of *Nodosaria Raphanus*, with excentric septal apertures) are known as *Vaginulinæ*; a large group, widely extending in time and space; especially abundant in the Gault and Chalk-marl. Of these *Vaginulinæ*, *V. Legumen*, Linn., is the most common among the recent; and the Adriatic Sea may be said to be its home. The subvarieties with linear costation are very variable as to their amount of ornament; but they may be all comprised under Montagu's name *V. linearis*. (See Williamson's 'Monograph Recent Foram. Great Britain,' p. 23, pl. 2. figs. 46–28.)

We have two small specimens from the mixed sands dredged up between Drontheim and North Cape (MacAndrew and Barrett).

This is not an uncommon form, occurring at moderate depths. It does not appear to be so common in the fossil as in the recent state, though it is not without close allies in the clays and other deposits of the Secondary and Tertiary formations.

Marginulina Lituus, D'Orbigny. Plate XIII. figs. 14 *a*, 14 *b* (Arctic).

One of Soldani's figured Foraminifera from the Adriatic, named *Marginulina Lituus* by D'Orbigny (Annales des Sciences Nat. vol. vii. p. 259. No. 11), well represents our specimen from the Arctic Ocean. This may be looked at as a passage-form from a simple *Vaginulina*, oval in section and but little altered from *Dentalina*, into *Cristellaria*, through innumerable gentle gradations; or it may be regarded as a medium between *Cristellaria* and *Marginulina*; and so leading to *Nodosaria*, through the flattened forms. Having the chief Nodosarine characters, the *Marginulinæ* form the central group of the *Nodosarinæ*, and *Nodosarina* (*Marginulina*) *Raphanus* is the type of all.

Very large specimens of *M. Lituus* occur at Nordland, Arctic Circle (MACANDREW and BARRETT), on a muddy bottom, at 160 fathoms. These are the largest individuals we have ever seen of this common variety of *Marginulina* or uncoiled *Cristellaria*, which is to be met with wherever the Cristellarians occur, recent or fossil, from the Lower Secondary deposits upwards.

In this case *Cristellaria cultrata* is also present; and an analogous companionship of the Cristellarian and the Marginuline *Nodosarinæ* is to be found in Professor BAILEY'S "Microscopical Examination of Soundings made by the United States' Coast-survey off the Atlantic Coast of the United States" (Smithsonian Contributions to Knowledge, vol. ii. 1851), where two forms (*Robulina D'Orbignii* and *Marginulina Bacheii*, Bailey), equivalent to the above, accompany each other in soundings of from 51 to 90 fathoms. (See above, page 331, and Appendix II.)

Cristellaria Crepidula, Fichtel and Moll, sp. Plate XIII. figs. 15, 16 *a*, 16 *b* (Arctic); Plate XVI. fig. 4 (North Atlantic).

We have here a very insensible gradation from *Marginulina Lituus* (fig. 14). In fact fig. 15 differs but little from the latter except in size; and fig. 16 is merely somewhat more closely coiled, flatter, and shorter; thus putting on the true Cristellarian form.

These specimens are from dredgings made at the Hunde Islands by Dr. P. C. SUTHERLAND; they are rather common in the sandy mud, rich with shells, at from 30 to 40 and 60 to 70 fathoms.

In recent occurrence *C. Crepidula* is as world-wide as the ordinary *Dentalinæ*. It is a feeble form of *Cristellaria* creeping up from the favourite depth of *Cristellariæ* (50 to 100 fathoms) to shallow water, and downwards to abyssal deeps.

In the fossil state also it has an equally wide range; but, like its congeners, it is met with of a larger size in the Upper than in the Lower Secondary deposits. The largest are to be found in the Subapennine and Viennese Tertiaries; some of these large fossil varieties are extremely thin.

Plate XVI. fig. 4 (North Atlantic).

A pretty little *C. Crepidula*, differing only as an individual from fig. 16 in Plate XI. Small and rare at 43 fathoms, lat. 51° 57′, long. 10° 30′, North Atlantic.

Cristellaria cultrata, Montfort, sp. Plate XIII. figs. 17 *a*, 17 *b*, 18 *a*, 18 *b* (Arctic); Plate XVI. fig. 5 (North Atlantic).

This is *Cristellaria* proper, the most nautiloid form attained by any *Nodosarina*. Here the rod-like chain of chambers seen in the simple forms (*Nodosaria*) has passed into a spiral, discoidal, symmetrical, lens-shaped shell (*Cristellaria*). In this variety, *C. cultrata*, the shell is more or less keeled; this keel becomes more developed and rowelled in *C. Calcar*, Linn., sp. When the keel is wanting, we have *Cristellaria rotulata*, Lamarck. There are no specific differences in their features.

Fig. 18 shows an irregularity of growth, and a disposition to depart from the nautiloid form towards the simpler varieties in which the greater distinction of the chambers is preserved. Several angles around the periphery of the shells are sometimes formed, rendering their outline polygonal. Other variations of growth are not uncommon; the polymorphism of these simple organisms being very great.

Plate XVI. fig. 5 (North Atlantic).

A smallish nautiloid *Cristellaria* with moderately developed keel, such as fig. 17 of Plate XIII., but differing in the non-essential features of greater obliquity of chambers and more distinct umbilical knob.

Rare at 78 fathoms, lat. 51° 59′, long. 11°, North Atlantic.

Cristellaria rotulata, Lamarck, sp. Plate XIII. fig. 19 (Arctic).

Here the keel is nearly obsolete. This carina is generally all that is left to us in these nautiloid forms of the longitudinal striæ or costæ that so frequently ornament the subspecies of the large *Nodosarina* group. Occasionally, however, the lateral faces of the shell bear raised costæ crossing the chambers, nearly at right angles, as in the ribbed *Nodosariæ* and *Marginulinæ* (typical), and in many *Vaginulinæ*, *Flabellinæ*, and *Frondiculariæ*.

The *Cristellariæ* represented by figs. 17–19 occur, common and large, in the Arctic Circle, Nordland, on a muddy bottom at 160 fathoms.

These recent northern specimens are, as regards size, equal to such as we find in those rich Cristellarian deposits, the Chalk and Chalk-marl. Like the rest of this group, however, the largest of this form are found in the Subapennine Tertiaries, the Vienna Basin, and in the Tertiary beds of Jamaica and San Domingo. Exactly similar specimens of *Cristellariæ* abound in the rich shelly bottom, at 50 fathoms, in the Port of Orotava, in the Canaries (*Robulina Canariensis*, D'Orb. For. Canar. p. 127, pl. 3. figs. 3, 4); and forms nearly as large are not at all uncommon in the Mediterranean, especially in mud at from 50 to 100 fathoms. In the Adriatic, however, this, with other *Cristellariæ*, is found of similar size in shallow water.

Of small size, these are found on our own coasts and throughout all seas. They are fossil in very many Secondary and Tertiary deposits, but of rather small size in the older strata; nevertheless in these latter beds they are exceedingly abundant and characteristic, not being mixed so much with species of other families of Foraminifera that have come in at a later epoch.

Genus LAGENA.

For full descriptions, general and special, of this genus we refer to Professor WILLIAMSON'S Memoir on *Lagenæ*, Annals Nat. Hist. 2 ser. vol. i. 1848; and his 'Monograph of British Recent Foraminifera,' 1857; to Dr. CARPENTER'S 'Introduction to the Study of Foraminifera,' 1862; and to Professor REUSS'S 'Monographie der Lagenideen,' Sitzungsber. Akad. Wiss. Wien, vol. xlvi. 1 Abth. 1863 (read June 1862); and for the strict determination of the species noticed by WALKER, JACOB, and MONTAGU, and for some special remarks on *Lagenæ*, we refer to our own Papers in the Annals Nat. Hist. 1857, &c.

On account of their extreme variability (within certain limits) as to shape and ornament, without any definite break in the range of the modifications being recognizable, it is impossible to fix on any distinctive character, or set of characters, sufficiently limited in development to be of real importance in dividing the *Lagenæ* into even two *species*. For convenience, however, we must take the best marked shapes and ornaments as indicating subordinate or varietal types, around which the diverging modifications may be grouped in an artificial classification.

This has been nearly completely accomplished, in his "Monographie" above referred to, by Professor REUSS; who, however, regards these subordinate divisions as "species." The addition of some striking varieties chiefly found in the warm seas, including the two-mouthed elongate forms, and the correction of some errors in the synonymy, arising mainly from mistakes as to WALKER'S and MONTAGU'S *Lagenæ*, would still further improve Professor REUSS'S classified and illustrated conspectus of the chief members of this group of elegant little single-chambered Foraminifera; and, without doubt, his so-called "genus" *Fissurina* is open to criticism, as we shall see further on.

Lagena, including both those that have external apertural tubes (Ectosolenian) and those with internal neck-tubes (Entosolenian), have their chief features of shape and ornament shown by globose, ovate, and fusiform shells, either smooth, partly or wholly ribbed, reticulate, or granulate and spinose; also by more or less compressed shells, of round or oval outline, with and without linear and reticulate sculpture; further, the base of the shell, opposite to the aperture, becomes apiculate, produced, and perforate, in any of the above-mentioned kinds of shell, resulting in a more or less fusiform and perforate, or distomatous, condition.

Taking the *smooth* forms, varying from egg-shaped to flask- and amphora-shaped, with or without long necks, we have the "lævigatæ" of REUSS, among which *L. globosa*, Walker and Jacob, *L. lævis*, Montagu, and *L. clavata*, D'Orbigny, represent the three best-marked stages. REUSS includes also the apiculate smooth forms in this group; but we prefer to bring them into relation with the perforate forms, to which we believe they strongly tend.

Those with furrows, riblets, and ribs are the "striatæ aut costatæ" of REUSS. They are led by *L. semistriata*, Williamson, from out of the smooth forms up to *L. sulcata*, Walker and Jacob, and even more coarsely ribbed shells, with modifications of form exactly corresponding to those of the smooth varieties; but no particular stage of shape and of ornament can be said to be permanently associated.

In the "reticulatæ" (Reuss) the longitudinal riblets become united by cross-bars, of varying strength; either regularly, so as to form rectangular meshes (*L. squamosa*, var. *catenulata*, Williamson, and *L. Melo*, D'Orbigny; or less regularly, and forming—1st, either tetragonal or hexagonal network, with the meshes one above the other from the base to the top of the shell, and divided by nearly straight longitudinal ridges or walls; 2ndly, hexagonal network, with the meshes alternately placed (honeycomb-pattern), the walls being zigzag, and equally developed along and across (*L. squamosa*, var. *hexagona*, Williamson). Lastly, hexagonal and quadrangular meshes are combined on one shell, as in *L. squamosa*, Montagu, sp., which herein well serves as the subtype.

The "asperæ" of REUSS are such as are ornamented with granules and spines. These exogenous shell-growths are, without doubt, equivalent to linear and reticulate ridges, variously modified; just as hispid *Nodosarinæ* show gradual modifications of riblets and spines. As with the "costatæ" and the "reticulatæ," no particular shape of shell specially affects this style of ornament. REUSS'S "compressæ" comprise the more or less flattened *Lagenæ*, and must include those which he separates under the name *Fissurina* on the supposition that they are distinguishable by their slit-like aperture. All *Lagenæ* that are more or less compressed have the aperture correspondingly narrowed and outdrawn, just as all *Nodosariæ* becoming flattened and "Linguline" have a more and more chink-like aperture. The transitions are extremely gradual both into "Fissurina" and "Lingulina" respectively, and are associated indiscriminately with all the other modifications of outline and ornament that belong to the species. The compressed *Lagenæ* usually take on one or more keel-like riblets at or near the margin, representing the local accumulation of the linear exogenous shell-growth so common in *Lagena*. A similar feature occurs in the *Nodosarinæ*, where a similar ornamentation obtains.

Lastly, we propose to complete this artificial classification of the *Lagenæ*, by dividing off those that, passing from a pointed or apiculate shape at the base, ultimately present a perforate or distomatous, continuously tubular shell, more or less fusiform. REUSS'S *L. apiculata* represents the smooth *apiculate* forms; D'ORBIGNY'S *L. caudata* the ribbed ones; our *L. polita* the smooth, and our *L. distoma* the costulate, *perforate* forms. (See Scheme of the *Lagenæ*, p. 348.)

Of *Lagena* it may be said, that, though apparently one of the simplest of Foraminifera, it is not one of the oldest, as far as our knowledge serves us at present. Nor can it be regarded as an arrested *Nodosaria*; rather, it may be looked on as a higher specialization of the simple repetitive Nodosarian form. It has its isomorphisms with *Nodosaria*, both in ornamentation and in its flattening.

All the large *Lagenæ* are found at about 50 fathoms (25–70) in shelly sands; the more delicate forms occur both in shallow water (which may even be brackish), in the dark muds of harbours and bays, and, on the other hand, at great depths, being not uncommon in the deposits almost wholly composed either of Foraminifera alone, or of these with Pteropods.

Scheme of the LAGENÆ.

Single-mouthed	round in section	smooth	egg-shaped	*globosa*, Montagu; Williamson, Monogr. p. 8, pl. 1. figs. 15, 16.
			flask-shaped	*lævis*, Montagu; Williamson (*L. vulgaris*), Monogr. p. 3, pl. 1. fig. 5.
			amphora-shaped	*clavata*, D'Orb. For. Foss. Vien. p. 21, pl. 1. figs. 2, 3.
		ribbed	partly	*semistriata*, Williamson, Monogr. p. 6, pl. 1. figs. 7, 9.
			delicately	*striata*, D'Orb. For. Amér. Mér. p. 21, pl. 5. fig. 12.
			strongly	SULCATA, Walker and Jacob; Williamson (*L. vulgaris*, var. *striata*), Monogr. p. 6, pl. 1. fig. 10. [The typical *Lagena*.]
			coarsely	*acuticosta*, Reuss, Sitz. Ak. Wiss. Wien, vol. xliv. p. 303, pl. 1. fig. 4.
		reticulate	square meshes	*Melo*, D'Orb. For. Amér. Mér. p. 20, pl. 5. fig. 9.
			6-sided meshes	*hexagona*, Williamson, Monogr. p. 13, pl. 1. fig. 32.
			both 4- and 6-sided meshes	*squamosa*, Montagu; Williamson, Monogr. p. 29, pl. 1. fig. 29.
		roughened	spines	*hispida*, Reuss, Sitz. Ak. Wiss. Wien, vol. xlv. p. 335, pl. 6. figs. 77–79.
			granules	*aspera*, Reuss, Sitz. Ak. Wiss. Wien, vol. xliv. p. 305, pl. 1. fig. 5.
	compressed		smooth	*marginata*, Montagu; Williamson, Monogr. p. 10, pl. 1. figs. 19–21.
			striate	*radiato-marginata*, Parker and Jones (var. nov.), Plate XVIII. fig. 3.
			reticulate	*squamoso-marginata*, Parker and Jones (var. nov.), Plate XVIII. fig. 2.
			3-keeled	*trigono-marginata*, Parker and Jones (var. nov.), Plate XVIII. fig. 1.
Passing from appendiculate or caudate to distomatous		smooth	short	*apiculata*, Reuss, Haid. ges. nat. Abhandl. vol. iv. p. 22, pl. 1. fig. 1; and Sitz. Ak. Wiss. Wien, vol. xlvi. p. 318, pl. 1. figs. 4–8, 10, 11.
			long	*distoma-polita*, Parker and Jones (var. nov.), Ann. Nat. Hist. 2 ser. vol. xix. p. 279, pl. 11. fig. 23, Plate XVIII. fig. 8.
		ribbed	short	*caudata*, D'Orb. For. Amér. Méd. p. 19, pl. 5. fig. 6.
			long	*distoma*, Parker and Jones (var. nov.), Ann. Nat. Hist. *ib.* fig. 24.
		granulate	short	*distoma-aculeata*, Parker and Jones (var. nov.), Plate XVIII. fig. 5.
			long	*distoma-margaritifera*, Parker and Jones (var. nov.), Plate XVIII. fig. 6.

The family *Lagenida* (comprising *Lagena*, *Nodosarina*, *Orthocerina*, *Polymorphina*, and *Uvigerina*) may be said to have its central home (bathymetrically speaking) at about from 50 to 100 fathoms. Of these, *Polymorphina* is almost exceptional, however; for it is, of this group, the most inclined to seek and flourish in shallow water, always avoiding abyssal depths. *Uvigerina* and *Lagena* are more capable even than *Nodosarina* of living in deeper water than 100 fathoms, and of existing even at very great depths (2000 fathoms). *Uvigerina* has its feeblest representatives in shallow water; but *Lagena* attains as fair a size in shallow water as it does at 100 fathoms; and at 1000 fathoms it is often in good condition. *Nodosarinæ* are, as to their habitat, intermediate between *Polymorphinæ* and the others. They are of large size at 100 fathoms; and are found occasionally, but small and rare, at 1000 fathoms; and in shallow water they are more abundant than in the abyssal depths, and attain a larger size.

Lagena sulcata, Walker and Jacob, Var. (*Entosolenia*) *globosa*, Montagu. Plate XIII. figs. 37 *a*, 37 *b* (Arctic); Plate XVI. figs. 10 *a*, 10 *b* (North Atlantic).

This is the simplest of the *Lagenæ*, subspherical and Entosolenian, that is, having an intus-suscepted mouth-tube. It is entirely devoid of ornament, and generally thin-walled. It may be said to be a feeble form connecting *L. lævis* with swollen varieties of *L. marginata*.

L. globosa comes from 30 to 40 fathoms, and from 60 to 70 fathoms at the Hunde Islands (Dr. SUTHERLAND); and in both dredgings it is rather common and of middling size. Also from Baffin's Bay, lat. 75° 10′ N., long. 60° 12′ W. (PARRY); here it seems to be rare, but is of large size,—a curious fact, in contrast with the occurrence of equally large individuals of this variety at very great depths (1080 fathoms) in the tropical Atlantic (lat. 2° 20′ N., long. 28° 44′ W.).

This also is a world-wide and very common *Lagena*, as we may see by Table VII. Professor REUSS has it fossil from the Chalk of Maestricht and of Lemberg, from the Septarian Clay of Pietzpuhl, the Salt-clay of Wieliczka, and the Crag of Antwerp (Monogr. Lagen. p. 318). It is of good size and rather common in the English Crag also.

L. globosa was figured and described by WALKER and BOYS, but not named by WALKER and JACOB in KANMACHER'S edition of ADAM'S 'Essays on the Microscope,' where the specific names given by WALKER and JACOB are recorded. It was named by MONTAGU, 'Test. Brit.' p. 523.

Plate XVI. figs. 10 *a*, 10 *b* (North Atlantic).

Equivalent to fig. 37 of Plate XIII., but having more neck, and like figs. 30 & 31 (*L. sulcata*) in outline and in thickness of neck.

Rare and large at 415 fathoms, lat. 52° 8′, long. 12° 31′, North Atlantic.

Lagena sulcata, Walker and Jacob, Var. *lævis* *, Montagu. Plate XIII. fig. 22 (Arctic); Plate XVI. fig. 9 *a* (North Atlantic).

Fig. 22 is the common, smooth, flask-shaped *Lagena* of authors. In this specimen pseudopodial passages are crowded about the lower third of the shell, the upper two-thirds being destitute of such foramina. We have observed that in *Lagenæ* such perforations occur only when the shell is of a certain thickness, considerable tracts of the shell-wall being often extremely thin and imperforate. In the very small-ribbed varieties (such as figs. 25–27) perforations are arranged in a row on each side of the costa, where its base is thick (*L. striatopunctata*). In the closely allied Entosolenian *L. marginata* also (as in fig. 44), perforations occur principally along the thickened margins, occasionally as a broad band; though sometimes (as in fig. 42) they are also scattered sparsely over the whole shell.

This is from the mixed sands from Norway above alluded to. It is world-wide, often found at considerable depths, but shallow water appears to be its favourite habitat. In the fossil state this smooth variety is very abundant in the Post-pliocene clays of Lincolnshire and Cambridgeshire, and in the Grignon sands (Eocene); it occurs also in the Vienna Tertiaries, and (according to REUSS, Monogr. Lagen. p. 322) in the Crag of Antwerp, the Septarium-clay of Pietzpuhl, and the Tertiary beds of Taranto (COSTA). It is rare in the English Crag.

---

* Taking this as the type of *Lagena*, WILLIAMSON thought that "lævis" was not an appropriate name for a shell that is often ornamented, and substituted the term "vulgaris"; this unnecessary change has been unfortunately adopted by REUSS (Sitzungsb. Ak. Wien, vol. xlvi. p. 321).

Plate XVI. fig. 9 *a* (North Atlantic).

This figure represents a specimen of *L. lævis* from the North Atlantic, where this variety is very rare and of middling size at 329 fathoms, lat. 49° 26′, long. 49° 48′, and rare and large at 223 fathoms, lat. 52° 11′, long. 13° 45′.

Lagena sulcata, Walker and Jacob, Var. *semistriata*, Williamson. Plate XIII. fig. 23 (Arctic).

This beautiful little *Lagena* connects the smooth with the striated varieties. Like the others, it varies much in shape and in the strength of its riblets; the specimen figured by Professor WILLIAMSON (pl. 1. fig. 9) is much more decanter-shaped than ours, and has a very long neck, with a neatly turned rim or lip; our specimen is deficient as to this latter character. We quite agree with Professor REUSS in grouping WILLIAMSON'S *L. vulgaris*, var. *perlucida* (Monogr. p. 5, pl. 1. figs. 7, 8), with this variety. MONTAGU'S *L. perlucida* is a six-ribbed *L. sulcata.* We found this specimen (fig. 23) in the shelly sand from the Hunde Islands, Davis Straits, 50 to 70 fathoms. Dr. WALLICH figures *L. semistriata* in 'The North-Atlantic Sea-bed,' pl. 5. fig. 17.

It is very common to meet with *Lagenæ*, both recent and fossil, taking on striæ and riblets to greater or less extent, as in this instance. REUSS figures finely striated specimens from the Crag of Antwerp in his paper on the *Lagenidæ*, Sitzungsb. Wien Akad. vol. xlvi. pl. 2. figs. 18–21.

Lagena sulcata, Walker and Jacob, Var. *striatopunctata*, nov. Plate XIII. figs. 25–27 (Arctic).

We have long known this variety from the Indian Ocean on Clam shell, and at 2200 fathoms, the Red Sea (372 fathoms), South Atlantic (2700 fathoms), and from the Eocene deposits of Grignon, but it has not been hitherto figured nor described.

It is a relatively small *Lagena*, and is one of the most delicate. It varies in shape, from forms more delicately elongate than the tear-shaped specimen represented by fig. 25, to those having the usual flask-shape, with longer neck than in fig. 27. The ribs are comparatively strong; they range in number from four to twelve, and in one recent specimen we have seen them spiral. The thickened base of the ribs is neatly perforated on each side by pseudopodian foramina, which also occasionally pass through the rib itself, from within outwards.

L. striatopunctata occurs rather common at the Hunde Islands, 30 to 40 fathoms, in shelly sandy mud, and here attains a size greater than those in the Indian Ocean, or those from the inside of a Grignon shell (p. 419, note); the specimens from the Red Sea, however, are as large as those from Davis Straits.

Lagena sulcata, Walker and Jacob. Plate XIII. figs. 24, 28–32 (Arctic); Plate XVI. figs. 6, 7, 7 *a* (North Atlantic).

This is the typical form of *Lagena*; its variations lead, in one direction, into the feebler forms (*L. semistriata*, *lævis*, *globosa*); on the other hand, we have varieties with reticulated, hispid, and granular ornament; and there are also compressed forms, and elongate varieties, departing more or less widely from the middle type presented by the ovate and characteristically costate *Lagenæ*.

Figs. 30 & 31 represent the best characterized forms (though not absolutely the largest) that we know of in the group of *Lagenæ*. This is shown in their boldness of growth, the strength of their ribs (twelve to fourteen in number), and particularly in the radiated structure of the aperture. This last seems to be a rare condition; we have as yet seen it only in these specimens*, but it is one among many features showing the intimate relationship between *Lagena*, *Nodosarina*, and *Polymorphina*.

L. sulcata of WALKER and JACOB, in KANMACHER'S edition of ADAM'S 'Essays,' well-figured previously by WALKER and BOYS, is a good-conditioned, strongly ribbed, and flask-shaped shell; our figs. 28–31 present less neck; but WILLIAMSON'S figure of *L. vulgaris*, var. *striata* (Monogr. p. 6, pl. 1. fig. 10), and REUSS'S figure of his *L. filicosta* (Monogr. pl. 4. figs. 50, 51), show as much or more neck and a better lip than WALKER'S figure does; but they are rather less globose, passing off into *L. Amphora*, Reuss, and *L. gracilis*, Williamson. See REUSS'S Monogr. Lagen. pl. 4, where by extreme care the ovate, flask-like, and fusiform shapes of the well-ribbed *L. sulcata* are divided into seven "species," according to their gradations of shape and modifications of ornament. It is, however, next to impossible, and of very little use, to institute minor distinctions with these *Lagenæ*.

As explained in the Annals Nat. Hist. 1859, 3 ser. vol. iv. p. 336, MONTAGU termed this form "striata," overlooking the prior name, which alone is necessary.

Figs. 28 & 29 are from the Hunde Islands, 30–70 fathoms; and from the Arctic Ocean (found in the mixed sands).

Figs. 30 & 31 represent specimens from the Hunde Islands also, three gatherings by Dr. P. C. SUTHERLAND, in shelly sandy muds, from 30–70 fathoms; within this limit *L. sulcata* is most common; and is largest at the greater depth. Perhaps the figured specimens nearest to these are *L. Isabella* and *L. raricosta*, D'Orb., from the Falkland Islands (Foram. Amér. Mérid. p. 20, pl. 5. figs. 7, 8, 10, 11). The almost exact counterpart of these fine large specimens we have found in the Upper Chalk of Maestricht, in the Crag of Suffolk, and in recent shelly sands from the Isle of Man. REUSS figures (under other names) long- and short-necked specimens, strongly ribbed, of *L. sulcata* from the Black Crag of Antwerp, and the Septarian Clay of Pietzpuhl and Hermsdorf.

Among the localities given by WILLIAMSON for the common *L. sulcata* (Monogr. p. 6) we find the Hunde and Beechey Islands; Petersburg, U.S. (fossil; Miocene); and English Crag.

---

* Professor REUSS figures this feature in some of the illustrations of his paper on the *Lagenidæ*, Sitzungsb. Ak. Wiss. Wien, Math.-Nat. Cl. vol. xlvi. 1862, Erste Abth. p. 308, &c. pl. 1–7.

This *Lagena* does not usually occur of so large a size as some of those from Hunde Islands. The most common condition is represented by figs. 28 & 29. These are smaller forms wanting the radiate structure of the aperture, but not separable from the type. Fig. 32 is a similar, but still smaller, form, and rather distorted. These feebler varieties of *L. sulcata* are common in all seas wherever *Lagenæ* are found.

Plate XIII. fig. 24 is a rather small flask-shaped *Lagena* with costulæ, having a spiral twist, which are intermediate in strength between the delicate riblets of fig. 23 and the strong ribs of the type-form, *L. sulcata.* The spiral arrangement of the costulæ is very variable in different individuals collected from various places: the obliquity and curvature of these ornaments being greater or less; and, as usual, the riblets vary in length, even in the same individual, being sometimes short, and sometimes lengthened so as to creep upwards, twining round the neck as far as the mouth. The intervals or flutings (sulci) may have a width equal to, or be far greater than, the ridges or riblets. When very small the riblets have been mistaken for minute sulci or "striæ." With regard to the rib-ornament of *Lagena*, we may observe that the costation of the flatter varieties is sometimes reduced to a mere keel (as in the Cristellarian forms of *Nodosarina*); either as a single keel; or a larger marginal, and a secondary, pair; thus making six costæ crowded at the edge (as in *Lagenæ* common in the Tertiary beds of Grignon). A somewhat similar condensing of the ordinary riblets into a few (six and even three) large ribs takes place in the cylindrical *Nodosariæ.* In one form of *Lagena* from the Grignon beds, we have three, meridional, three-edged, equal ribs (*L. trigono-marginata*, Parker and Jones, Plate XVIII. fig. 1); and in another four, strong, equal, spiral ribs (marked by pseudopodia, as in *L. striatopunctata*), this is our *L. tetragona*, Plate XVIII. fig. 14.

Fig. 24 is one of the feeble forms of *L. sulcata* (type), world-wide, and acclimatized to nearly all latitudes and depths; it is common at Hunde Islands (Dr. SUTHERLAND), at 60-70 fathoms in shelly sandy mud.

Plate XVI. figs. 6, 7, 7 *a* [including Var. *caudata*, D'Orb.] (North Atlantic).

Various modifications of the typical *Lagena*, from the North Atlantic, are shown by figs. 6, 7 *a*, 7 *b*. Fig. 6 is like fig. 29 of Plate XIII., but it is rather more globose, has rather shorter ribs, and is apiculate (non-essential differences, though the last feature makes it Var. *caudata*, D'Orb.). Fig. 7 *a* is smaller and less globular than figs. 30 & 31 of Plate XIII.

These are rare and of middling size at 2330 fathoms, lat. 50° 25′, long. 44° 19′, North Atlantic; rare and small at 223 fathoms, lat. 52° 11′, long. 13° 45′; and rather common but small at 43 fathoms, lat. 51° 57′, long. 10° 30′.

Fig. 7 *b* (Var. *caudata*, D'Orb.) has an elongate olive-like shape, and thinner costæ than the others. It was rare and of middle size at 1450 fathoms, lat. 50° 6′, long. 45° 45′; and rare and small at 2350 fathoms, lat. 51° 29′, long. 38° 1′, North Atlantic.

***Lagena sulcata*, Walker and Jacob, Var. (*Entosolenia*) *Melo*, D'Orb.** Plate XIII. figs. 33–36 (Arctic).

This is *L. sulcata* with a modified ornamentation. It has small transverse ridges between the ribs, connecting them, and forming subquadrate reticulations, which vary in different specimens.

Professor Reuss would retain Williamson's term *catenulata* for those specimens that have the cross-bars weaker than the ridges; probably a convenient, though hardly necessary, arrangement; the modifications of the relative thicknesses of the longitudinal and transverse ridges are endless, varying from a network of thin lines, equal or unequal in strength, to that with broad, flat, equal ridges, and shallow squarish pits between.

Further, our figs. 33–36, Plate XIII., show sufficiently clearly that no characteristic can be found in the disposition of the secondary or transverse riblets, whether end to end, or alternately between contiguous ribs; for in the same specimen they vary as regards this arrangement.

Fig. 34 has but few of the cross-bars, and these are oblique. In this it not only connects *L. sulcata* with *L. Melo* by the presence of secondary riblets, but the obliquity of these connecting bars shows a tendency towards the formation of the variety *L. squamosa*, next to be described, in which the ornament has a honeycomb- rather than a ladder-pattern. Dr. Wallich figures another pretty passage-form, 'North-Atlantic Sea-bed,' pl. 5. fig. 23.

Figs. 33 & 35 differ in the relative size of the areolæ; a condition dependent upon the number of the primary ribs, and very variable. From the Hunde Islands, 30–70 fathoms; and from the Arctic Ocean (mixed sands).

Fig. 36 is an extremely rare monstrosity, being a *Lagena* with a superadded chamber. It is from the Hunde Islands, from between 30 and 40 fathoms, shelly muddy sand (Dr. P. C. Sutherland). This specimen is unique in our collection. Soldani has figured a specimen extremely like this one, in his 'Testaceograph.' vol. i. part 2, pl. 95. fig. A; named *Nodosaria cancellata* by D'Orbigny (Ann. Sc. Nat. vol. vii. p. 254, No. 29).

As a rule, monstrosities of the *Lagena* are formed by the budding, as it were, of a new chamber obliquely on the side of the original chamber (Plate XVIII. figs. 10–12); these are very rare. If, however, a smooth or ribbed *Lagena* were to take on an additional chamber in the axis of the primary chamber, it would be scarcely distinguishable from a *Nodosaria*. We possess such a form (from the shallow water at Eastbourne), Plate XVIII. fig. 9, which we believe to be a monster of *Lagena lævis*. In the Tertiary Sands of Bordeaux also, rich with *Lagenæ* and small *Nodosariæ*, very puzzling forms occur, which may either be two-celled individuals of *Nodosaria scalaris*, Batsch[*], or possibly monstrous varieties of *Lagena sulcata*. In the specimen before us (Plate XIII. fig. 36) we have a mode of ornamentation never found in any Nodosarian Foraminifer;

---

[*] Well figured by Wallich in 'The North-Atlantic Sea-bed,' pl. 5. fig. 18, and in Journ. Sci. No. 1, Jan. 1864, fig. 6, in the plate illustrating his paper on the bed of the Atlantic Ocean. Figured also, for comparison, in our Plate XVIII. fig. 13.

and therefore, whilst we have some doubt as to the two-celled forms that have either no surface-ornament, or a sculpturing common to *Nodosaria* and *Lagena*, here we have satisfactory means of diagnosis.

Everywhere in the Foraminiferal group, we have most curious instances of *Isomorphism*, not merely between nearly related species, but between even the diverse forms of separate families (as between those of the Vitreous and Porcellanous Series). In the case under notice isomorphism may be said to occur between three closely cognate specific groups: thus, the specimen of *Lagena* before us has imitated a *Nodosaria*; whilst those already spoken of as taking on a second chamber obliquely have the habit of a young *Polymorphina* (see fig. 46).

Lagena Melo is not uncommon in company with other *Lagenæ*, though not so common as the smooth, sulcate, honeycombed, and marginate varieties. For its occurrence (recent and fossil) in the Mediterranean Area, see Quart. Journ. Geol. Soc. vol. xvi. Table, p. 302.

Lagena sulcata, Walker and Jacob, Var. (*Entosolenia*) *squamosa*, Montagu, sp. Plate XIII. figs. 40, 41 (Arctic); Plate XVI. figs. 11 *a*, 11 *b* (North Atlantic).

This represents a state of ornamentation peculiar to the *Lagenæ* amongst the "hyaline," and to certain varieties of *Miliola Seminulum* among the "porcellanous" Foraminifera. In *L. Melo* the cross-bars are often weaker than the longitudinal ribs, and pass straight across from rib to rib, like the secondary veins in a monocotyledonous leaf, such as *Alisma*, *Myrsiphyllum*, &c. In *L. squamosa*, however, not only have the secondary riblets become equal to the primary, but, by the zigzag inflection of the latter, a nearly regular hexagonally areolated ornament is produced, reminding one strongly of the polygonal meshes produced by the more perfect reticulation of the woody tubes in a dicotyledonous leaf. Early observers, using but imperfect microscopes, compared this retose ornament with a scaly skin of a fish (see WILLIAMSON, Monograph, p. 12).

In fig. 34 we have noticed a variety of *L. sulcata* in which a few secondary bands had united with the main ribs, commencing, as it were, the honeycomb-pattern.

Fig. 40, the largest of our specimens, is from the Hunde Islands* (Dr. P. C. SUTHERLAND), 50 to 70 fathoms; and the smaller one from the Arctic Ocean (MACANDREW and BARRETT).

L. squamosa is of world-wide occurrence; but, like *L. Melo*, is not so abundant as the long flask-shaped and the marginated forms. REUSS has it from the Black Crag of Antwerp, and we have it fossil from Castel Arquato. By far the bulkiest specimens of *L. squamosa* that we have seen are from a Tertiary sand, which, rich in many varieties of *Lagenæ*, in *Ovulites*, *Polymorphina*, and *Vertebralina*, was taken from the inside of a *Cerithium giganteum* from Grignon (page 419, note).

In this reticulate *Lagena* the neck is usually intussuscepted (Entosolenian); in the large fossil form (*L. tubifero-squamosa*, Parker and Jones, Plate XVIII. fig. 7), however, the neck is protruded in some cases to a considerable extent, and has about three secondary tubular apertures arising from it laterally, and almost at right angles to the main tube. This is an isomorphism with *Polymorphina tubulosa*, and with certain feeble bifurcating forms of *Nodosaria* from Cretaceous beds.

---

* Professor WILLIAMSON has also noted its occurrence here (Monogr. p. 12).

Plate XVI. figs. 11 *a*, 11 *b* (North Atlantic).

The specimen here figured is a little less globular than figs. 40, 41 of Plate XIII., and has its reticulation rather more regular. Rare and middle-sized at 1450 fathoms, lat. 50° 6′, long. 45° 45′, North Atlantic. In Dr. WALLICH'S 'North-Atlantic Sea-bed," pl. 5. fig. 21 seems to be *L. squamosa*.

Fig. 11 *a*, Plate XVI. has the six-sided meshes one above the other, touching by the parallel sides of the hexagon, and in so much corresponding with WILLIAMSON'S *L. scalariformis* (Monogr. p. 13. pl. 1. fig. 30), and REUSS'S *L. geometrica* (Monogr. Lag. p. 334, pl. 5. fig. 74); but this straight meridional arrangement of the meshes is lost in the less regular reticulation of such specimens as figs. 40 & 41 in Plate XIII., where square, six-sided, and irregular meshes occur, in straight, oblique, and irregular lines. Professor REUSS'S unnecessary disuse of MONTAGU'S term *squamosa* for this varietal group leads to increased confusion in any attempt to subdivide these reticulate *Lagenæ*, which have no natural divisions among themselves.

Lagena sulcata, Walker and Jacob, Var. (*Entosolenia*) *marginata*, Montagu. Plate XIII. figs. 42–44 (Arctic); Plate XVI. figs. 12 *a*, 12 *b* (North Atlantic).

These are flattened forms variable in shape; generally Entosolenian, but sometimes Ectosolenian with a long delicate neck. This compressed shape is usually associated with a trenchant margin, sometimes slightly apiculated (as in fig. 42), and sometimes dentate or rowelled (as in WILLIAMSON'S Monograph, pl. 1, figs. 21 *a*, 25, 26), reminding one of the keel of certain *Cristellariæ*. Occasionally in large well-developed specimens of *L. marginata* (recent and fossil) the margin is composed of a large predominant rib, strengthened by a pair of smaller costæ; showing that, as in other Foraminifera, especially the Nodosarine group, the exogenous costæ gather themselves to the margins, the rest of the surface becoming less and less ornamented. The pseudopodial pores also usually affect the neighbourhood of the thickened margin in these flattened forms, just as they follow the ridges of *L. striatopunctata* (figs. 25–27). Occasionally the pseudopodia have perforated the whole surface, either sparsely, as in fig. 42 *a*, or freely, as we have seen in specimens from the Indian Sea.

In some rare specimens from the Coral-reefs of Australia, and fossil at Bordeaux, we see the pseudopodia begin to enter the shell-wall near the centre, and then burrow radially to escape near the margin; the shell-surface being perfectly smooth and as polished as glass. This is our subvariety *Lagena radiato-marginata*, Plate XVIII. fig. 3. In the Crag of Suffolk there is another subvariety of *L. marginata*, in which the radiating canals are visible only at the margin.

The intussuscepted neck-tube in *L. marginata* is generally more or less oblique, somewhat trumpet-shaped, and of varying length (as may be seen in figs. 42 & 43). Fig. 44 has the tube partly protruded, and partly internal. The apparent difference in the setting on of the mouth, which we formerly thought we could detect, between *Entosolenia* and *Lagena* proper (Annals Nat. Hist. 2 ser. vol. xix. p. 279), does not really exist, for we find that in any of the subspecific groups forms may occur having either a gently tapering neck (Ectosolenian), or a tube abruptly set in (Ento-ecto-solenian), or a mouth-tube entirely intussuscepted (Entosolenian). *L. marginata* is sometimes distomatous, being open at the base, and then coming under another (artificial) subdivision.

Between such globose forms as figs. 38 & 39, and the flattened ones (figs. 42–44), there is an almost infinite number of gentle gradations shown in specimens from all parts of the world.

The specimens figs. 42–44 occur at the Hunde Islands (Dr. SUTHERLAND), in three dredgings at from 30 to 70 fathoms, and at Drontheim, North Cape (MACANDREW and BARRETT), from 30 to 200 fathoms. Rather common. Professor WILLIAMSON has already recorded the occurrence of *L. marginata* at 100 fathoms at the Hunde Islands (Monogr. pp. 10 & 11). Like other *Lagenæ*, it is world-wide; and is abundant in the Tertiaries: it is rather common, for instance, in the Crag of Suffolk. For some of its Mediterranean habitats (recent and fossil) see Quart. Journ. Geol. Soc. vol. xvi. p. 302, Table. Under the name of *Oolina compressa*, D'ORBIGNY described it as occurring with other *Lagenæ* at the Falkland Isles. It is figured by J. D. MACDONALD, Assist.-Surgeon H. M. S. Herald, in the Annals Nat. Hist. 2 ser. vol. xx. pl. 5. figs. 7–10, but not described. He found it, together with a dimorphous *Uvigerina* (with loosely set, straggling chambers), *Spiroloculina planulata*, *Quinqueloculina Seminulum*, and *Triloculina oblonga* in 440 fathoms water between Ngau and Viti-Laru, in the Feejee group of islands.

L. marginata is sometimes hexagonally areolated, like *L. squamosa*, but more feebly (*L. squamoso-marginata*, Parker and Jones, Plate XVIII. fig. 2); as we have seen in specimens from the Tertiary beds of San Domingo, and from the white mud of the Australian Coral-reefs.

Plate XVI. figs. 12 *a*, 12 *b* (North Atlantic).

Here we have a slight modification in the development of the keel, as compared with the equivalent specimens represented by figs. 42, 43, Plate XIII. In the North Atlantic *L. marginata* is rare and small at 740 fathoms; rare and middle-sized at 1450 fathoms; rather common and large at 2350 fathoms; rare and large at 415 fathoms; rather common and small at 90 fathoms; and common and small at 78 and 43 fathoms. Dr. WALLICH figures three forms of *L. marginata*, 'North-Atlantic Sea-bed,' pl. 5. figs. 19, 20, 22.

Lagena sulcata, Walker and Jacob, Var. *distoma*, nov. Plate XIII. fig. 20 (Arctic).

Fig. 20 represents a long, costulated, fusiform *Lagena*, open at both ends, with one extremity rather more tapering than the other. This variety of *Lagena* has not been previously named. It was figured and described by us in the 'Annals Nat. Hist.' ser. 2. xix. p. 279, pl. 11. f. 24. See also Trans. Linn. Soc. xxiv. p. 467, pl. 48, f. 6, BRADY.

It can only be received as a varietal form of the typical *Lagena sulcata*, Walker and Jacob; but, like other noticeable varieties of Foraminifera, it requires a distinctive binomial appellation. It is from Norway (MACANDREW and BARRETT); found in mixed sands and muds dredged at various places between Drontheim and North Cape, and at depths varying from 30 to 200 fathoms; of rare occurrence. It is very rare in deep water off Shetland, and not uncommon off the Northumberland coast (H. B. BRADY).

The exact counterpart in form, but somewhat of less size, occurs in the clay beneath the fen near Peterborough, but not in any abundance. A somewhat similar, large, two-mouthed *Lagena* is found in the Sponge-sand from Melbourne, Australia, and is rather common: it is even larger than our Arctic specimens; is never quite straight; and, instead of being covered with delicate costulæ, is richly ornamented with pearl-like grains, profusely spread over the surface, hence we call it *Lagena distoma-margaritifera*, Plate XVIII. fig. 6.

A smooth distomatous *Lagena*, of twice the size of the last mentioned, is not uncommon in the rich fossil Rhizopodal fauna so well worked out of the Crag of Sutton, Suffolk, by Mr. S. V. WOOD, F.G.S. This *Lagena* of the Crag of Suffolk is the largest of the elongate *Lagenæ* that we know.

Dr. CARPENTER supposes that the elongate distomatous *Lagenæ* may be double *Lagenæ* joined by their bases (Introd. p. 157); and Professor A. E. REUSS suggests that they are separated chambers of *Nodosariæ* or *Dentalinæ* (Sitzung. Ak. Wien, vol. xlvi. p. 315); but in these opinions we can by no means agree. Our *L. distoma* is grouped by REUSS (*loc. cit.* p. 331) with *L. gracilis*, Williamson; but our description and figure show the distinctive features.

Lagena sulcata, Walker and Jacob, Var. *distoma-polita*, nov. Plate XIII. fig. 21 (Arctic).

Another elongate, fusiform, distomatous variety of *Lagena* (fig. 21), but smooth instead of costulate, occurs in the same Norway dredgings, and in the Red Sea (PULLEN'S soundings), on the beach near Melbourne, at Swan River, on the Australian Coral-reefs, and on the Durham Coast (BRADY), and of a large size (relatively) in the Crag of Suffolk.

As fig. 20 represents a distomatous, striated, subcylindrical variety of *L. sulcata*, so fig. 21 is a smaller and smooth distomatous, but amphora-shaped, variety; the former may be said to be, in one sense, a subvariety of *L. striata*, and the latter a subvariety of *L. lævis*. In the Norway dredgings it is smaller and rarer than *L. distoma* (fig. 20). Its two extremities are not nearly so equal as those of fig. 20, and the shell is not so cylindrical; but in the hotter seas it is long and slender (Plate XVIII. fig. 8). We term it *L. distoma-polita*. In some respects it has less departed, than *L. distoma* has, from the ordinary smooth flask-like forms, especially those which are somewhat pointed at the bulbous end, as *Lagena apiculata*, Reuss (Sitzungs. Akad. Wien, vol. xlvi. p. 1, figs. 4–8, 10, 11). In fact the subdivision of these varieties is artificial, and made only for the sake of convenience.

Lagena sulcata, Walker and Jacob, Var. (*Entosolenia*) *apiculata*, Reuss, et *caudata*, D'Orbigny. Plate XIII. figs. 38, 39 (Arctic); Plate XVI. figs. 6, 7, 8, 9 (North Atlantic).

The distomatous condition of *Lagena* also obtains in the globular forms (included in the *Oolinæ* of D'ORBIGNY), which frequently have the neck-tube lengthened inwards and free (the characteristic of *Entosolenia*, Ehrenberg), see figs. 38 & 39. Among these the base of the shell is frequently drawn out or apiculate (as in fig. 39, and in the figures of *L. apiculata*, Reuss, above mentioned), and sometimes perforate, as it is in fig. 38. This also holds good in the compressed varieties (*L. marginata*). Also among the flask-like *Lagenæ* we have apiculate forms, as in *Oolina* (*Amphorina*) *caudata*, D'Orb., whether striated, as that is, or smooth; such also are *L. apiculata*, Reuss, *L. globosa*, var. *lineata*, Williamson (Monogr. pl. 1. fig. 17), *L. strumosa*, Reuss, *L. mucronata*, Reuss, &c. Any of these may be perforate. See also Plate XVI. figs. 6, 7, 8, 9.

Excepting, then, that the globular and lenticular *Lagenæ* are frequently Entosolenian, none of these characters, whether of elongation, apiculation, and perforation, or of being smooth, striated, sulcated, honeycombed, or reticulate (as we shall see with the ornamented forms), are confined to one or another set of *Lagenæ*. No specific distinctions can be based on any of these features; but, for convenience sake (as among other species of Foraminifera), several subspecies and varieties take binomial appellations. To avoid, however, too great an accumulation of such names we must adopt the published names whenever it is possible; and in this case D'ORBIGNY's *Oolina caudata* will serve as a point around which the apiculate and distomatous *Lagenæ*, of the flask-shaped and more or less globular varieties, may be conveniently grouped. The large subcylindrical and fusiform specimens, like a little rolling-pin in shape, well represented by fig. 20, will stand as a distinct variety.

Fig. 38 (Plate XIII.) differs from *L. globosa* (fig. 37) in being more elongate or olive-shaped, and in having a subsidiary tubular aperture at its base. Fig. 39 has also the fundus drawn out or apiculate, but not pervious. A large number of these apiculated forms, varying much in outline and in ornament, sometimes distomatous (as fig. 38), are not at all uncommon, and may be grouped under the name "*caudata*" given by D'ORBIGNY to one of his *Oolinæ*. Sometimes they are Entosolenian (as is seen in fig. 39 *a*), and often they are Ectosolenian, as in D'ORBIGNY's *O. caudata*, Foram. de l'Amér. Mérid. pl. 5. fig. 6, a striated form. Compare also the smooth, amphora-shaped, distomatous *Lagena*, fig. 21, above described.

From 30 to 40 fathoms at the Hunde Islands (Dr. P. C. SUTHERLAND); not common, small. World-wide. Fossil in the Tertiary formations.

Plate XVI. figs. 6, 7, 8, 9 (North Atlantic).

Allied closely to fig. 21 of Plate XIII., but more swollen; fig. 8 being more lanceolate in outline, and fig. 9 more flask-like, than fig. 21; whilst figs. 6 & 7 are striated also.

These are rare and small at abyssal depths in the North Atlantic.

A very interesting group of ten *Lagenæ* from the Falkland Isles was figured and described (as *Oolinæ*) by D'ORBIGNY in his work on the Foraminifera of South America (Voyage dans l'Amér. Mérid. partie 5me, 1839, p. 20). These represent most of the modifications shown among the Arctic and North Atlantic forms. Thus

Oolina inornata,	*op. cit.* pl. 5.	fig. 13	= Lagena globosa, *Montagu.*
lævigata,	,,	fig. 3	= L. lævis, *Montagu.*
striatocollis,	,,	fig. 14	= L. semistriata, *Williamson.*
striata,	,,	fig. 12	
Vilardeboana,	,,	figs. 4, 5	= L. sulcata, *Walker and Jacob.*
Isabella,	,,	figs. 7, 8	
raricosta,	,,	figs. 10, 11	
Melo,	,,	fig. 9	
compressa,	,,	figs. 1, 2	= L. marginata, *Montagu.*
caudata,	,,	fig. 6	

Genus POLYMORPHINA.

Polymorphina lactea, Walker and Jacob, sp. Plate XIII. figs. 45, 46 (Arctic).

Of the hyaline Foraminifera, *Polymorphina* alone forms itself persistently of a double row of alternating opposite chambers; except very rarely, when its latest chambers are uniserial (*Dimorphina*). *Uvigerina* (a closely related form) has normally three chambers in one turn of the spire, forming a triple series of alternating chambers. *Textularia* has normally a double series of chambers alternating with each other, much as in *Polymorphina*, but more regular in arrangement, and having a far more gradual increase of size. *Textularia*, however, often begins with a triserial (Verneuiline) arrangement, such as is normal in *Uvigerina*; and, like the latter, it often finishes its shell with a single row of chambers (*Bigenerina*).

In *Polymorphina*, although the arrangement of the chambers is essentially biserial, yet they are very apt to grow so loosely that a cross section through the shell will often expose three or more chambers.

This shell is normally drop-shaped, tear-shaped, and pyriform; it may, however, become flattened out into the proportions of the thick leaf of a succulent plant, or be elongated into an irregular oblong, somewhat like a wheat-ear or grass-spike. These longer forms (such as fig. 48) are isomorphic with *Textularia* proper. Of its Dimorphine condition there are Nodosarian, Textularian, and Uvigerine isomorphs.

The aperture of *Polymorphina* agrees with that of the *Nodosarinæ*, and of the well-grown *Lagenæ* (such as figs. 30 & 31), being radiated or plicated, the sarcode passing through a circular series of slits. The actual centre of the aperture is sometimes filled up with a bead of calcareous matter (fig. 52 *b*), and this occurs in *Nodosarinæ* also.

We have seen above that the varieties of *Lagenæ* are almost equally divided among those which have a gently graduating external neck, those having an entirely internal neck-tube, and those in which the tube is partly extruded and partly internal. In *Polymorphina* also this may be said to hold good to some extent; for in small and in young specimens (fig. 46) we see the Entosolenian tube, just as in the globular and flat *Lagenæ* (figs. 39, 42, 43). Indeed in specimens having five chambers we have distinguished a tube in each chamber. In large individuals the apertural plicæ grow quite as far into the chamber as they project outwards. Thus the Entosolenian character of aperture is generally present; and though the mouth does not pout so much as in many of the *Nodosariæ* and *Lagenæ*, yet the last chamber not unfrequently sends out a dendritic growth of exserted apertural tubes—a character noticed by us in a large *Lagena* also common in the Tertiary beds of Grignon (see p. 354). Nor is this feature unrepresented among the *Nodosariæ*, as shown by the dichotomous *Dentalina aculeata*, D'Orb., of the Chalk and Gault.

The shell of *Polymorphina* has usually a glassy smoothness; it rarely shows any tendency to striation; when this occurs, it is longitudinal, but feeble, and then arises from, apparently, the fusion of granules arranged in rows; whereas in the three most cognate species (*Nodosarina*, *Lagena*, and *Uvigerina*) striation and strong costation of the chamber-walls are extremely common. It has, however, at times another mode of ornament, such as is not unfrequently met with in the Nodosarine and Uvigerine groups, and especially in the *Globigerinæ* of the deep seas in low latitudes, and in *Calcarina*,—namely, prickles or long needles of shell-substance bristling over the surface. Another surface-ornament is common in large well-grown *Polymorphinæ*, especially those of the Crag of Suffolk (Mr. S. V. WOOD's Collection), which have a rich granulation of clear, polished, calcareous beads and lobules scattered over the whole surface, but most strongly on the older chamber-walls. A like granular ornament is common in the very large distomatous *Lagenæ* from the Australian shores (as already mentioned). The best example of the development of this particular ornament is seen in the great explanate *Cristellariæ* of the Tertiary beds of Malaga, Sienna, and Turin.

In the form before us (figs. 45 & 46) we have a subglobular condition of *P. lactea*, Walker and Jacob. Fig. 46 is the young, showing, by transparency, the long Entosolenian neck, as well as the radiated aperture. It has but two chambers, the second of which is relatively small; in after-growth the chambers increase in size rapidly but irregularly, and overlap each other in proportion to the gibbosity of the shell. We possess complanate or leaf-shaped forms, such as are figured by D'ORBIGNY in his For. Foss. Vienne, pl. 13. figs. 25–30, in which there is scarcely the least overlapping of the chambers.

The two chambers of fig. 46 are the "primordial" and "circumambient" chambers of other polythalamous Foraminifera. We have seen a similar double-celled condition of shell belonging to young forms within the chambers of the mother-shell, in *Truncatulina lobatula* (from south coast of England), *Peneroplis pertusus* (from India), and in large *Orbitolites complanatus* (from Fiji). In the last (some specimens of which were full an inch in diameter) we found the mother-chambers, towards the periphery of the shell, crowded with young ones*.

---

* These specimens, both old and young, may be seen in the Hunterian Museum, Royal College of Surgeons (See Catal. Mus. Plants and Invertebr. 1860, p. 96, No. A 54); and have been described by Dr. CARPENTER, Introd. Foram., Ray Soc. p. 38.

To us it appears that the Polythalamous Foraminifera are often, if not generally, viviparous, and that the young shell, when hatched, consists of two chambers. We think that the subsequent chambers of these Polythalamians are not always formed slowly, one by one, but sometimes, at least, laid down, and marked off by the growth of two or more septa, at the same time; calcification beginning at points nearest to the earlier chambers, the thickness of the chamber-wall being in direct ratio with its age. This mode of growth of more than one chamber at a time seems to be proved by the curious manner in which the sarcode passes, by many bundles, from the older chambers into the newest in those individuals of *Polymorphina lactea* which throw out tubular staghorn processes from their last chamber (*P. tubulosa*, D'Orb.); for, as may be seen in fig. 52, the newest chamber, namely, that which bears the cervicorn appendage, communicates, not merely with the ante-penultimate chamber, but, by a double row of lateral apertures, with all the chambers occurring on its own side, namely those which it overlaps. The communication of the last, outer, wild-growing chamber with the penultimate is not only by means of this double row of apertures, but (as is seen in fig. 52 *b*) by the ordinary radiated mouth. Another view, however, may be taken of the growth of such an individual as fig. 52: thus, we may suppose that absorption has taken place, opening foraminal communications between the last and the former chambers. Either hypothesis would explain the fact,—that, as we find on breaking open very large specimens of the normal *P. lactea*, the stolon-passages between the older chambers are very free and patulous; whereas the terminal mouth of the last chamber presents the radiate condition; the only passage here for the sarcode being the thin slits around the strong growth of hyaline shelly matter in the mouth.

Fig. 45 represents the group of individuals to which D'ORBIGNY applied the subgeneric term *Globulina*; but neither this term nor that of *Guttulina* (another so-called subgenus) can be separated from the ordinary, more or less oval, more or less pyriform, or more or less elongate varieties of *Polymorphina lactea*.

Figs. 45 & 46 are from the Hunde Islands (Dr. SUTHERLAND), in three dredgings from 25–70 fathoms. Rather common and of small size. Also from the Norway coast (MACANDREW and BARRETT) in the mixed sands.

Polymorphina lactea, Walker and Jacob, Var. *compressa*, D'Orbig. Plate XIII. figs. 47–51 (Arctic).

These are more or less flattened forms, ranging themselves around *P. compressa*, D'Orb. (For. Foss. Vien. pl. 12. figs. 32–34), though not exactly identical with that variety of *P. lactea*. In the relative length of the chambers, their setting on, and in the degree of exposure of the plaiting by the alternation of the double series of chambers, these *Polymorphinæ* are so very variable, that we have taken the flattened condition as a characteristic, and out of the very many names they have received, we have chosen "*P. compressa*" as a secondary centre around which to collect a certain series of more or less elongate and compressed forms, more elongate than *P. lactea* proper, and less compressed than *P. complanata*, D'Orb. (For. Foss. Vien. pl. 13. figs. 25–30); the latter being the centre of the group of leaf-like forms.

Fig. 47, though not so flat as D'ORBIGNY's figure of *P. compressa*, comes nearest to it, of these before us. Fig. 48, somewhat Textularian in its make, connects *P. compressa* with D'ORBIGNY's *P. Thouini* (Modèle, 23): the latter, however, is still more elongate and less compressed. In the Crag of Suffolk this elongation advances to such an extent that the shell at first sight looks like a *Dentalina*: it has become the isomorph of the elongate Virguline *Bulimina* of the English Gault and the German Pläner-Mergel. Figs. 49 & 51 connect *P. compressa* with D'ORBIGNY's *P. Problema* (Modèle, 61). Fig. 50, composed of about three chambers, is a young or an arrested individual of the compressed type.

At the Hunde Islands, 30–40 fathoms, these forms of *P. compressa* occur rare and small. From the Norwegian coast we have them, rather common and small, in the mixed sands.

These are amongst the commonest forms of *Polymorphina* from the Lower Secondary period up to the Recent.

Polymorphina lactea, Walker and Jacob, Var. *tubulosa*, D'Orbigny. Plate XIII. fig. 52 *a–d* (Arctic).

This condition of *P. lactea* we have already spoken of. We may add that the tubular appendages are found on *Polymorphinæ* of various shapes, but generally on the more or less spheroidal, or at least ovoidal, forms; and it is only for the sake of convenience that it can be regarded as a subcentral group and distinguished by a binomial appellation. D'ORBIGNY's figured and named specimen (For. Foss. Vien. pl. 13. figs. 15, 16) has priority among several.

Fig. 52 *a–c* is from Bred Sound, Finmark (MACANDREW and BARRETT), 30 fathoms. The fragment fig. 54 *d* is from some other part of the Norwegian coast.

Tubulose individuals of *P. lactea* occur common in the Cretaceous deposits; are very common in some of the Grignon and other Tertiary beds; and are very large in the Crag of Suffolk (Mr. S. V. WOOD's Collection). In the Australian coast-sand (Melbourne) living individuals of large size are abundant; and fine specimens live in the Bay of Biscay (50–60 fathoms) and in the English Channel. One large and interesting specimen that we have obtained in the shelly sand off Plymouth is adherent to a fragment of a bivalve shell; its tubular arms spreading radially on the shell, like the wild-growing cells of a *Planorbulina* or of a *Carpenteria*. Professor WILLIAMSON figures a fine tubulose British *Polymorphina* (*P. lactea*, var. *fistulosa*, Monogr. fig. 150), and also a small plano-convex, crenately winged form (*P. lactea*, var. *concava*, fig. 151), which he regards (with much probability) as having been parasitic. We have met with similar forms in sands of shallow waters.

Genus Uvigerina.

Uvigerina pygmæa, D'Orbigny. Plate XIII. figs. 53–57 (Arctic); Plate XVII. figs. 65 *a*, 65 *b* (North Atlantic).

Uvigerina makes up its shell normally of three series of inflated chambers, alternating somewhat irregularly on an elongated spire. Its aperture is a very distinct and round passage, generally tubular (Ectosolenian) and lipped. The lip is sometimes faintly toothed, showing a relationship to the radiated mouth of the *Polymorphina*, *Lagena*, and *Nodosarina*. To the last genus it is mostly related by its style of ornament, which, as a rule, consists of strong well-marked costæ, parallel to the axis of the shell. In all large well-developed individuals, whether of typical or dimorphous growth, these costæ are distinct and strong, just as obtains in the large *Lagenæ* and *Nodosarinæ* (Plate XVIII. figs. 16, 17). In weaker individuals the ribbing is less prominent and often becomes obsolete in the newer chambers (Plate XIII. figs. 56 & 57). Certain dimorphous forms are quite smooth (Plate XVIII. fig. 18). As in *Nodosariæ*, some *Uvigerinæ* take on the aculeate or hispid ornamentation; the ribs of each chamber either sending back one or more spines, or breaking up into prickles; or the whole surface of the shell may become spinose and bristly. The hispid forms of *Uvigerina* are generally found at great depths (common at 1000 fathoms in the Tropical Atlantic, Indian Ocean, &c.), and are frequently angular in section, belonging to the variety *U. angulosa*, Williamson. In deep water also the large *Uvigerinæ* are frequently elegantly racemose, with a prickly surface; the chambers are globular and distinct, and the tubular mouth much elongated: this botryoidal form is, as far as shape is concerned, the most deserving of the generic term "*Uvigerina*" given originally to the really typical costate *U. pygmæa*, such as we have before us. Large *Uvigerinæ* of the typical form are especially abundant and well-grown in the southern and eastern parts of the Mediterranean, at from 100–300 fathoms: the home of *Uvigerina* seems to be in warm seas at this depth, but it is found also in shallower water (Coralline-zone), but is then of the small size. Feeble forms creep upwards, as it were, into shallow water, and downwards to great depths; still the abyssal forms predominate over the littoral, the latter retaining the greatest resemblance to the typical *U. pygmæa*; whilst the deep-water forms, whether angular or inflated, are prickly, the angular forms in shallow water are ribbed.

In the elongated form, of feeble growth and faint striation (fig. 57), we may see a tendency to a biserial and even a uniserial growth; the chambers ceasing to retain a definite triserial alternation; and, becoming loose in their setting on, they present such a condition as leads ultimately to a uniserial row of chambers in the newer part of the shell. Such a dimorphous condition is clearly seen in certain figures, given by Soldani, of Italian *Uvigerinæ*, named *U. nodosa* by D'Orbigny (Ann. Sc. Nat. vol. vii. p. 269); and we also possess similar forms both from the recent and the fossil deposit of the Mediterranean area, Plate XVIII. fig. 15. These dimorphous specimens present a growth of either one, two, or three chambers in a straight line in the younger part of the shell (still retaining the same kind of aperture), and with or without the intervention of a biserial arrangement of chambers. This dimorphism of the Uvigerine type is seen best, however, in specimens from shell-beds in the tropical and subtropical parts of the Indian and Atlantic Ocean; but in these the triserial mode of growth is obsolete, and even the biserial is but feebly developed; the result being a shell which, at first sight, might easily be mistaken for a *Nodosaria Raphanus*. Close examination, however, shows the short, wide, strongly labiate aperture of *Uvigerina*, markedly developed, and a plaiting of the early chambers*. D'ORBIGNY has figured, under the name of *Sagrina pulchella*, Foram. Cuba, pl. 1, figs. 23, 24, a specimen which was either the young, or an arrested individual of such a biformed *Uvigerina*. *Bigenerina* amongst the *Textulariæ* is the isomorph of the above described dimorphous *Uvigerina* (*Sagrina*).

Not only is our Nodosariform *Uvigerina* connected with the typical *U. pygmæa* (figs. 53–56) through *Sagrina pulchella*, D'Orb., but an intermediate condition between it and the feebler dimorphs of the Mediterranean area occurs in the mud brought up by the sounding-lead from the Abrohlos Bank (*U. dimorpha*).

Altogether, this latter group of forms shows how great the affinity is between the always hyaline *Uvigerina* and the porous sandy *Textularia*.

The specimens figured in Plate XIII. figs. 53–57 are very common forms. The finest individuals (figs. 53, 54) are from the mixed sands of the Norwegian coast. The feebler specimens (figs. 55–57) are common in shell-sands from 30–70 fathoms at the Hunde Islands, Davis Straits.

In the North Atlantic *Uvigerina pygmæa* (Plate XVII. fig. 65) is large and common throughout the eastern marginal plateau; wanting at great depths; rare and middle-sized north of the Bank; and rather common and of middle size in Trinity Bay.

Uvigerina pygmæa is world-wide in its distribution, and goes back at least to the Middle Tertiary period.

Uvigerina pygmæa, D'Orb., Var. *angulosa*, Williamson. Plate XIII. fig. 58 (Arctic); Plate XVII. figs. 66 *a*, 66 *b* (North Atlantic).

Of this we have spoken above, page 363. This compressed condition turns up wherever *Uvigerinæ* are at all common; the ribbed or striated forms belonging to moderate depths.

In the mixed sands from Norway specimens were rather common.

In the North Atlantic *U. angulosa* is rare and small; it occurs on the eastern marginal plateau to the north of the Bank, and in Trinity Bay; but was not found in the Abyssal area.

Genus ORBULINA.

Orbulina universa, D'Orbigny. Plate XVI. figs. 13, 14 (North Atlantic).

This is a monothalamous hyaline Foraminifer, globular and porous, of world-wide distribution, found in shallow water in the Adriatic and other warm seas, but usually frequent on sandy and muddy bottoms at not less than 30 fathoms and down to at least 2350 fathoms. It is richest in numbers where there is nothing but the calcareous matter of Foraminifera. In the shallow water of our coasts *Orbulina* is poorly developed.

---

* A ribbed form from the East Indian Seas is our *Uvigerina* (*Sagrina*) *Raphanus*, Plate XVIII. figs. 16, 17; and a smooth one from the Abrohlos Bank is our *U.* (*S.*) *dimorpha*, Plate XVIII. fig. 18.

We have not recognized it fossil in strata older than the Middle Tertiary period.

In the North Atlantic the deep-sea soundings indicate that at 78, 90, 223, 329, 1660, 1950, and 2050 fathoms *O. universa* is rare and of middling size; at 2350 fathoms it is middle-sized and rather common; at 1776 and 2050 it is middle-sized and common; at 415 fathoms it is large and common; and at 1750 and 2176 fathoms it is large but rare.

Genus GLOBIGERINA.

Globigerina bulloides, D'Orbigny. Plate XIV. figs. 1 & 2 (Arctic); Plate XVI. fig. 15, and Var. *inflata*, figs. 16, 17 (North Atlantic). [See also Professor HUXLEY's Appendix to Commander DAYMAN's Admiralty Report, p. 65, pl. 4.]

Globigerina bulloides is the type of a distinct species, which is related to the monothalamous *Orbulina* on one hand, and to the polythalamous *Rotalinæ* on the other. Its shell is composed of a series of hyaline and perforated chambers, of a spheroidal form, arranged in a spiral manner, and each opening by a large aperture around the umbilicus, in such a manner that the apertures of all the chambers are apparent on that aspect of the shell, and form a large "umbilical vestibule." This opening of the chambers into one common vestibule is also characteristic of *Carpenteria balaniformis*. The extraordinarily wild manner of growth of the latter is, to a certain degree, represented in many of the larger specimens of *Globigerina*, which, losing the vesicular or botryoidal form, become flat, outspread, and loosely lobulated or palmate. Although in these respects, and also in the close resemblance of the young shells, these two species show a near alliance, yet *Globigerina* seems, on the whole, from its general neat habit of growth, and from its peculiar varietal groups, to be most nearly related to the *Rotalinæ* (*Planorbulina* and *Discorbina*). In fact, *Globigerina* and its varieties form an interesting group, which may be regarded as central to the Planorbuline and Discorbine species and their varieties, as well as certain species (*Pullenia* and *Sphæroidina*) which were not until lately recognized as related to the *Rotalinæ*.

The chief varieties of *Globigerina* are peculiarly isomorphic of these other forms. The large, extremely thick-walled, compact *Globigerinæ*, of the deepest waters, may stand as the isomorphs of the equally abyssal solid specimens of *Sphæroidina*; nor are the two forms dissimilar at first sight. The smooth-walled compact *Globigerinæ*, such as have been named *Gl. inflata*, D'Orb. (Foram. Canaries, pl. 2. figs. 7–9), come near in structure to the highly polished, flush-celled, somewhat gigantic specimens of *Pullenia obliquiloculata*, Parker and Jones* (the type of which is the so-called *Nonionina sphæroides*, D'Orb.) from great depths. We have already mentioned the wild-growing *Globigerina*

---

* CARPENTER's 'Introd. Foram.,' p. 183. See also Plate XIX. fig. 4.

(*Gl. helicina*, D'Orb. Ann. Sc. Nat. vol. vii. p. 277, after SOLDANI) as representing in its own group a type of structure which has its completeness in *Carpenteria*. Like certain varieties of *Planorbulina farcta*, hereafter to be described (Plate XIV. figs. 7–11), and of *Discorbina Turbo*, *Globigerina* has nearly symmetrical (nautiloid) varieties (*Gl. hirsuta*, D'Orb. For. Canar. pl. 2. figs. 4–6, and *Gl. pelagica*, D'Orb., sp., For. Am. Mér. pl. 3. figs. 13, 14): by the possession of these forms *Globigerina* touches isomorphically several other specific types, amongst which is *Pullenia*, its near relation, above referred to, typically symmetrical. Such an assumption of symmetry in these simple, vesicular, discoidal Foraminifers is interesting, as suggestive of a tendency to attain the more regular nautiloid form, normal amongst the higher forms, such as *Nummulina*, *Cycloclypeus*, *Heterostegina*, *Polystomella*, and others, which, on their part, when feebly developed, are apt to be asymmetrical. Indeed in this respect we have a connecting link between the higher and the lower group in *Amphistegina*, a congener of the true *Nummulinæ*, but simpler in structure and essentially asymmetrical.

The foregoing observations on the relationships of *Globigerina* will assist us in elucidating the alliances of many of the species and varieties about to be described, lying between the simple monothalamous *Orbulina* and those highest in the scale (*Polystomella* and *Nummulina*), which give the fullest expression of the type of structure possessed by this Rhizopodous order.

The affinities and isomorphisms of *Globigerina*, however, are not exhausted by the consideration of the groups above referred to; for the small and feebly developed individuals of the typical *Globigerina bulloides*, which are so extremely abundant in the deep seas, mixed with large specimens, are imitated by the small, vesicular, weakly grown *Textulariæ*, *Uvigerinæ*, *Buliminæ*, and *Cassidulinæ*; and we might even include the dwarf vesicular *Lituolæ* of deep waters (see Plate XV. figs. 46, 47, and Plate XVII. figs. 96–98).

Figs. 1 & 2 are relatively small specimens of *Globigerina bulloides*, such as are found in shallow seas all the world over, and also (as above mentioned) in abyssal depths, where they are in company with large individuals; the latter live in deep water only.

There is but little exogenous growth on the primary perforated chamber-wall of such *Globigerinæ* as those before us; but in deeper water, as a rule, a large proportion of the individuals have a thick deposit of exogenous shell-matter, which generally rises into reticulate ridges, surrounding the pseudopodian passages, and giving the surface a roughly honeycombed appearance. Sometimes these ridges are developed into asperities, prickles, needles, and even large tubules. The latter are sparsely scattered; are formed of the divergent growth of the whole areola around the pseudopodian passage; and occur on the symmetrical, nautiloid forms, such as occur at 1600–1700 fathoms between Malta and Crete. The acicular appendages arise at the junctions, or on the edges of the areolæ, and are found on some symmetrical varieties. Such are very abundant in the Red Sea at from 300–700 fathoms; and here the needles are often so long on the peripheral parts of the older chambers that they subdivide the large arched aperture of the last chamber into narrow oblong openings.

The chamber-walls attain their greatest thickness in those close-set and rough-shelled varieties which occur in great abundance at from 1600 to 2400 fathoms in the North Atlantic, between Ireland and Newfoundland (Plate XVI. fig. 15; and Professor Huxley's plate in the Admiralty Report on the Telegraph-soundings in the North Atlantic), and at lat. 5° 37′ S., long. 61° 33′ E. in the Indian Ocean (2200 fathoms). These are the nearest isomorphs of *Sphæroidina dehiscens*, Parker and Jones, Plate XIX. fig. 5, which is found with them in the tropical parts of the Atlantic and in the Indian Ocean, and not in the North Atlantic. Those smooth forms (*Gl. inflata*, D'Orb., from the Canaries) having moderately thick chamber-walls, and which are nearest to *Pullenia* in style of growth, abound in the North Atlantic (Plate XVI. figs. 16, 17), and are very plentiful in the Southern Mediterranean, at about 700 fathoms, and in the Indian Ocean, lat. 36° 58′ S., long. 51° 49′ E. (900 to 1120 fathoms). *Gl. bulloides* is small and very abundant at 2700 fathoms in the South Atlantic; the greatest depth for its habitat that we know of.

The complanate form of *Globigerina*, with more or less limbate septal lines, is figured by D'Orbigny, as living on the coast of Cuba, with the name of *Rosalina Linnæi* (Foram. Cuba, pl. 5. figs. 10–12). It is common in the Chalk, and is known as *Rosalina marginata*, Reuss (Charakt. Kreid. Ostalpen, Denksch. Akad. Wien, vii. pl. 26. fig. 1), and *Rosalina canaliculata*, Reuss (Ibid. fig. 4).

Plate XIV. figs. 1 & 2 represent specimens obtained at three places among the Hunde Islands by Dr. P. C. Sutherland (28–30, 30–40, and 60–70 fathoms), rather common and small; and others found (rare and very small) in the most northern soundings we have examined, namely, Baffin's Bay, lat. 76° 30′ N., long. 77° 52′ W. (Parry) at 150 fathoms; and others from the coast of Norway, few and small in the mixed sands (MacAndrew and Barrett).

In the North Atlantic *Globigerina bulloides*, including its variety *Gl. inflata*, D'Orb. (Plate XVI. figs. 16, 17), is spread broad-cast; but is abundant and of good size only at the greater depths ("Virginian Province," and the "Celtic" and "Boreal" abyssal areas, at upwards of 2000 fathoms in some places), and at 223, 338, and 415 fathoms on the eastern marginal plateau: elsewhere on this plateau it is small and varying in numbers. On the western plateau (north of the Bank of Newfoundland) it is small, though sometimes common; whilst in Trinity Bay it is very small and very rare.

The oldest known *Globigerinæ* are those in the Gault.

Globigerina bulloides, Var. *inflata*, D'Orbigny. Plate XVI. figs. 16, 17 (North Atlantic).

In this *Globigerina* (For. Canar. p. 134, pl. 2. figs. 7–9), peculiar for its large gaping aperture, the newer chambers are relatively larger than usual, and cover the former ones to a great extent (see figs. 16, 17). It is variable in its details, and does not differ specifically from *Gl. bulloides*. It has already been referred to above (page 365).

This variety abounds and is large on the North Atlantic, and on deep muddy bottoms in the Mediterranean (Dayman's soundings). Professor Bailey noticed it in soundings from off the Coast of New Jersey (see Appendix). D'Orbigny had it from the Canaries; it is plentiful in the Indian Ocean (see above).

From some mounted specimens lent to us by Mr. F. GALTON, F.R.S., we may add the following notes as to the *Globigerinæ* of the North Atlantic. See also Appendix I.

At 1650 fathoms the deep-sea ooze consists chiefly of *Globigerinæ*, many of them of large growth (as if well-nourished), thick-shelled and rough, the sarcode remaining (brown) in most of the larger shells; and at the same time there are very many small and delicate individuals (just as is the case with other Foraminifera,—minute dwarfs accompanying full-grown specimens of one and the same type). With *Globigerina* at this depth occur a rather small *Rotalia Beccarii*, a very small *Bulimina* (?), and siliceous Sponge-spicules. At 1600 fathoms *Globigerinæ* as above, with a small *Spirillina*. At 1500 fathoms *Globigerinæ* appear as at 1650 fathoms. The thickness of the chamber-wall is relatively great. A sponge-gemmule was also found here.

Dr. G. C. WALLICH has well illustrated *Globigerina* and *Orbulina* in plate 6 (undescribed) of the First Part of 'The North-Atlantic Sea-bed,' 1862.

Globigerinæ are (as is well known) among the most characteristic of *deep-sea* Foraminifera (*Abyssina*); and these form a group that love to live at from 1000 to 2500 fathoms. They are *Pullenia*, *Sphæroidina*, *Globigerina*, and its monothalamous congener *Orbulina*

The first three are always rare and small in shallow water; and *Orbulina* usually has similar conditions.

Cassidulina is also an abyssal form; but lives well up to 30 fathoms, though in flatter and more delicate forms than it has lower down.

Genus PULLENIA.

Pullenia sphæroides, D'Orbigny, sp. Plate XIV. figs. 43 *a*, 43 *b* (Arctic); Plate XVII. fig. 53 (North Atlantic).

For an account of *Pullenia*, one of the deep-sea forms, probably allied to *Globerigina*, though resembling *Nonionina*, see CARPENTER's Introd. Foram. p. 184; it is the *Nonionina sphæroides*, D'Orb. Modèles, No. 43, Ann. Sc. Nat. vol. vii. p. 293, No. 1; and *N. bulloides* of the same author, For. Foss. Vienn. p. 107, pl. 5. figs. 9, 10, and Ann. Sc. Nat. vol. vii. p. 293, No. 2.

Our figure 43 is of normal shape, but small size, as are all those which we find in the Arctic and North Atlantic seas. Another form of *Pullenia* has the chambers set on obliquely (*P. obliquiloculata*, Parker and Jones, Plate XIX. fig. 4). In the mixed sands from Norway *Pullenia sphæroides* is rather common and small: it is rare and small, often very small, at 1776, 2035, 2176, and 2330 fathoms in the North Atlantic; also at 1203 fathoms north of Newfoundland Bank, and at 200 fathoms on the plateau off Ireland.

Fig. 53 is the *Nonionina quinqueloba*, Reuss, Zeitsch. Deutsch. Geol. Ges. vol. iii. pl. 5. fig. 31, an enfeebled, somewhat flattened form, of looser growth than usual. It occurs also in the Eocene Clays of Hants and the Isle of Wight (H. B. BRADY), in the 'Septarian Clay' (Eocene) near Berlin (REUSS), and recent in the Red Sea.

Pullenia sphæroides lives in the Mediterranean, the Red Sea, and South Atlantic at from 30–320 fathoms.

Genus SPHÆROIDINA.

Sphæroidina bulloides, D'Orbigny, sp. Plate XVI. fig. 52 (North Atlantic).

This peculiar species (of which *Sph. dehiscens* is another variety) is related to *Globigerina*; and, together with *Pullenia*, *Orbulina*, and *Globigerina*, essentially of deep-water habits, is small and rare in the North Atlantic, but large in the Tropics.

Sphæroidina has a small spire, somewhat irregularly wound, the vesicular chambers (of which only three or four are visible) hiding the spiral arrangement. REUSS has figured many specimens (*Sph. Austriaca*) in pl. 51, Denkschr. K. Akad. Wissen. Wien, vol. i. 1850.

Sphæroidina dehiscens, Parker and Jones, is largish, thick-shelled; the chambers not closely applied, and their edges roughly everted and jagged (Plate XIX. fig. 5).

Sph. bulloides is rare and small at 223 fathoms on the marginal plateau off Ireland; very rare and very small at 2330 fathoms in mid-ocean.

In the Mediterranean it occurs at 320 fathoms, in the Red Sea at 372, in the Tropical Atlantic at 1080, in the Southern Atlantic at 260 and 940, and in the Indian Ocean at 2200 fathoms.

Genus TEXTULARIA.

Textularia agglutinans, D'Orbigny. Plate XV. fig. 21 (Arctic).

Textularia agglutinans, D'Orbig. (Foram. Cuba, p. 144, pl. 1. fig. 17, 18, 32-34), in its ordinary and moderately developed condition, gives a fuller idea of the species than any other variety.

We have it in the mixed sands from Norway rather common and of middle size; and at the Hunde Islands it is small, rare at 30–40 fathoms, rather common at 25–30 fathoms.

Textularia agglutinans is world-wide; and has its representatives in many Tertiary and Secondary strata.

Textularia agglutinans, Var. *abbreviata*, D'Orbigny. Plate XVII. figs. 76 *a*, 76 *b* (North Atlantic).

T. abbreviata, D'Orb. (For. Foss. Vien. p. 249, pl. 15. figs. 7–12), is a short form, intermediate to *T. gibbosa*, D'Orb. Modèles, No. 28, and *T. agglutinans*, D'Orb., and smaller than either; but, like them, it is sandy.

We have it from the marginal plateau of the Atlantic off Ireland, where it is common and middle-sized at 43 and 78 fathoms; rather common and middle-sized at 90 fathoms; rare and small at 223 fathoms; rather common and small at 415 fathoms.

T. abbreviata has much the same range as its type *T. agglutinans*.

Textularia agglutinans, D'Orbigny, Var. *Sagittula*, Defrance. Plate XVII. figs. 77 *a*, 77 *b* (North Atlantic).

T. Sagittula, Defrance (see 'Annals Nat. Hist.' 3rd ser. vol. xi. p. 91, &c.), is the common, often small, sandy, triangular variety of *T. agglutinans*, D'Orb.

Our figures indicate a normal specimen of this form from the marginal plateau off Ireland, where it is common and of middle size at 78 fathoms.

T. Sagittula is world-wide, and common in many Tertiary deposits.

Textularia agglutinans, Var. *pygmæa*, D'Orbigny. Plate XV. fig. 22 (Arctic); Plate XVII. figs. 78 *a*, 78 *b* (North Atlantic).

This is the common, small, hyaline or clear-shelled, perforate *Textularia*; its sandy analogue is *T. Sagittula*. Normal specimens are figured here.

We have it in the mixed sands from Norway, common and middle-sized.

In the North Atlantic it is rather common and small at 78 and 90 fathoms on the marginal plateau; and it is rare and small at 200 and 415 fathoms, rare and middling at 223 and 338 fathoms on the same ground: in the abyssal depth (Boreal) it is rare and small at 2033 fathoms; and nearer to the Bank it is very rare and very small at 1450 fathoms.

T. pygmæa, D'Orb. Modèles, No. 7 (the same as *T. aciculata*, D'Orb., Ann. Sc. Nat. vol. vii. p. 263, pl. 11. figs. 1–4), has a distribution similar to that of the other chief varieties.

Textularia agglutinans, Var. *carinata*, D'Orb. Plate XVII. figs. 79 *a*, 79 *b* (N. Atlantic).

The shell of *T. carinata*, D'Orb. (For. Foss. Vienn. p. 247, pl. 15. figs. 32–34), is flatter than that of either *T. pygmæa* or *T. Sagittula*; the edges becoming very thin and more or less produced into a sharp keel; and the chambers extend backwards irregularly. The specimen figured is a small and feeble individual of this variety. Still more flattened is our new variety *T. Folium*, from the Australian coast, Plate XIX. fig. 19.

T. carinata in the London Clay frequently has a spiral arrangement of its earliest chambers, such as is seen also in many other varieties of *Textularia*. In fig. 79 *a* a faint tendency to a coil is seen at the apex of the specimen.

On the marginal plateau off Ireland *T. carinata* occurs rather common and small at 78 and 90 fathoms. It is found in the Adriatic and other seas, extremely large between Socotra and Kurachee; also fossil in the Tertiary deposits.

Textularia agglutinans, D'Orb., Var. *biformis*, nov. Plate XV. figs. 23, 24 (Arctic).

These very small *Textulariæ* have a sandy shell, often of a rusty colour, with scarce any shell-substance proper. They have a spiral commencement (a not uncommon feature in *Textularia*), and the later chambers are subquadrate, arranged alternately. This may be regarded as an arrested form of *T. annectens*, Parker and Jones (Annals Nat. Hist. 3rd ser. vol. xi. p. 92, fig. 1); for, if better developed and carried on with uniserial chambers, it would be equivalent to that variety. It is common in the Gault and Chalk with *T. annectens*.

Textularia biformis is common and small at the Hunde Islands in 60 to 70 fathoms.

Textularia agglutinans, Var. (*Bigenerina*) *Nodosaria*, D'Orb. Plate XV. fig. 25 (Arctic); Plate XVII. figs. 80 *a*, 80 *b* (North Atlantic).

Bigenerinæ are *Textulariæ* that commence with alternate biserial chambers and complete themselves with a uniserial set, the aperture becoming terminal, central, round, and sometimes pouting.

Bigenerina Nodosaria, D'Orb. (Ann. Sc. Nat. vol. vii. p. 261, pl. 11. figs. 9–12; and Modèle, No. 57), is usually sandy, and commences with flat interlacing of chambers, as in *T. agglutinans*, D'Orb.; whilst *B. digitata*, D'Orb. (Modèle, No. 58), begins with a conical set of chambers, as in *T. gibbosa*, D'Orb.

At the Hunde Islands (Dr. SUTHERLAND) *B. Nodosaria* is extremely small, but common, at 60 to 70 fathoms.

On the marginal plateau off Ireland it is common at 78 and 90 fathoms, coarsely arenaceous and of fair size.

B. Nodosaria lives in the Mediterranean and other seas, being widely distributed; it keeps a good size, and prefers muddy bottoms, flourishing down to 200 or 300 fathoms.

Textularia agglutinans, Var. (*Bigenerina*) *digitata*, D'Orbigny. Plate XVII. fig. 81 (North Atlantic).

B. digitata, D'Orb. (Modèle, No. 58), may be said to be a smooth, rusty subvariety of *B. Nodosaria*, with a conical instead of flattened apex.

On the marginal plateau of the North Atlantic *B. digitata* is rare and small at 78 fathoms; the figured specimen is obscure, and may be regarded as feebly developed.

B. digitata occurs in company with *B. Nodosaria* in the Mediterranean and elsewhere.

Textularia agglutinans, D'Orb., Var. (*Verneuilina*) *polystropha*, Reuss, sp. Plate XV. fig. 26 (Arctic).

When *Textulariæ* have a triple row of alternating chambers, as is not unusual with them, they are termed *Verneuilinæ*; having commenced triserially, they may afterwards take on a biserial or uniserial arrangement of chambers, and are known as *Gaudryinæ*, *Clavulinæ*, &c. Some that have a triple series of chambers are so much twisted on the axis as to have a Buliminoid aspect; a slight approach to this condition is shown in *Verneuilina polystropha* (*Bulimina polystropha*, Reuss, Böhm. Kreid. vol. ii. p. 109, pl. 24. fig. 53; *Polymorphina silicea*, Schultze; *Bulimina arenacea*, Williamson). In *Verneuilinæ* the aperture ceases to be transverse, becoming drawn upwards, as it were, across the septal plane more and more in the later chambers, until it ceases to be even a notch, and becomes terminal and round, as it is in *Bigenerinæ*.

V. polystropha may be said to be a small, vesicular, arrested Verneuiline *Textularia*; sandy, twisted on its axis, and very red in colour. It is of wide distribution, living in all latitudes; and is found fossil in the Tertiary and Cretaceous beds.

It is often of much larger size than our figured specimen, which is from the Hunde Islands (Dr. SUTHERLAND); where *V. polystropha* is common and small at 25–40 fathoms, and very common and small at 60–70 fathoms.

Genus BULIMINA.

Bulimina Presli, Reuss, Var. *Pyrula*, D'Orbigny. Plate XV. figs. 8, 9 (Arctic).

In describing the *Buliminæ* that form part of the Rhizopodal Fauna of the Arctic and North Atlantic Oceans, we have not occasion to treat so largely of the special characters of the genus, nor the relationships of the subspecific groups, as is necessary in the case of the *Nodosarinæ*, *Lagenæ*, *Polymorphinæ*, *Uvigerinæ*, *Globigerinæ*, *Rotalinæ*, and *Polystomellæ*; chiefly because these relationships and characters are not difficult to be understood, with the help of the figures before us, and because they have been clearly stated in CARPENTER's 'Introd. Foram.,' p. 195, &c.

As the best medium-form of the very variable *Buliminæ* we take REUSS's *B. Presli* (Verst. Böhm. Kreid. pl. 13. fig. 72; Haiding. Abhandl. iv. pl. 10. fig. 10; and CARPENTER's 'Introd.' pl. 12. fig. 18). *B. Pyrula*, D'Orb. (For. Foss. Vien. pl. 11. figs. 9, 10), of which we have some Norwegian specimens before us, is one of the varieties (for we cannot see evidence of the existence of more than one species of *Bulimina*) that have the greatest tendency to overlap their chambers, and so hide the primary segments by the later ones closing over them. It is usually prickled at the apex.

We have it common and large in the mixed sands from the coast of Norway (MACANDREW and BARRETT). It lives in the Mediterranean, and is large between Socotra and Kurachee. It is found fossil in the Vienna Tertiaries (where it is large) and the London Clay. A *Bulimina* of very similar shape occurs also in the Upper Triassic Clay of Chellaston, Quart. Journ. Geol. Soc. xvi. p. 457, pl. 20. fig. 45.

Bulimina Presli, Reuss, Var. *marginata*, D'Orbigny. Plate XV. fig. 10 (Arctic); Plate XVII. fig. 70 (North Atlantic).

The neat, little, acute-ovate *Buliminæ* that next come under notice are characterized by the exogenous growth of shell-matter, in the form of prickles, on the primordial chamber (as in *B. Pyrula* also) and at the posterior edges of the other chambers to a greater or less degree.

The edges of the chambers may be pinched up, crenulated, serrated, toothed, or spined; the spines may be few or numerous along the sharpened border or on the surface of the chambers, and they may be present on all of them or limited to the earlier ones; intermediate conditions in every respect being observable. No real division can be made amongst these modifications; but for convenience-sake those edged with prickles are grouped under *B. marginata*, D'Orb. Ann. Sc. Nat. vol. vii. p. 269, No. 4, pl. 12. figs. 10–12; whilst *B. aculeata*, D'Orb. (after SOLDANI), Ann. Sc. Nat. vol. vii. p. 269, No. 7, takes those with fewer spines. WILLIAMSON's *B. pupoides*, var. *spinulosa*, Monogr. p. 62, pl. 5. fig. 128, has many fine long spines along the margins. The crenate and prickly margins are found associated with more contracted forms of *Buliminæ** than those above-mentioned; but the exogenous growths belong to thick-shelled specimens, and probably indicate favourable habitats; on the thin-shelled and the attenuate forms there is little or no fringing or other ornament.

---

* Such as *B. pulchella*, D'Orb. (For. Amér. Mér. p. 50, pl. 1. figs. 6, 7), a very small subcylindrical form, with pinched and fringed chambers; living in the Pacific, from the equator to 34° S. lat.; and *B. Patagonica*, D'Orb. (Ibid. p. 50, pl. 1. figs. 8, 9), a very rare form (contracted and fringed at first, irregularly globuliform afterwards), found at the Bay of San Blas, Patagonia.

Fig. 10. Plate XV. has the chambers somewhat extended by their produced spiny edge or prickly fringe, and has a long apical spine; such forms, with others (as fig. 11) with less of the marginal spines, occurred common and of middle size in the mixed sands from Norway (MacAndrew and Barrett).

Plate XVII. figs. 70 *a*, 70 *b*, 70 *c* (North Atlantic).

Figs. 70 *a* & 70 *b*, Plate XVII., differ somewhat one from the other and from fig. 10, Plate XV., in the marginal and caudal spines; but no two specimens, even among many, are exactly alike.

They are common and large at 43 fathoms; common and middling at 78 and 90 fathoms; and common and small at 223 and 415 fathoms, on the plateau off Ireland in the North Atlantic.

B. marginata lives in all seas, at no great depths.

Bulimina Presli, Reuss, Var. *aculeata*, D'Orbigny. Plate XV. fig. 11 (Arctic); Plate XVII. figs. 68 & 69 (North Atlantic).

In these specimens the chambers have a well-marked globosity, and favourable conditions of growth have given them a rapid rate of increase, as in the foregoing subvariety; the exogenous prickles, however, are less largely developed, being confined for the most part to the earliest chambers.

Fig. 11, Plate XV. is an intermediate form, from Norway (mixed sands), with fewer marginal spines than some of its congeners; and though more spinous than figs. 68 & 69, yet, as these are essentially *marginatæ* also, and as there is a difference of degree and not of kind, not only among these, but between these and others presently to be described, it is placed under *B. aculeata* as its fittest place in the grouping. Its chambers have sharp posterior edges, drawn out into comparatively few spines, short and strong; and it has a strong double caudal spine.

B. aculeata, D'Orb., is sufficiently well figured by Soldani, Testac. vol. i. part 2, pl. 127. fig. 1, pl. 130. fig. *v v*, and pl. 131. fig. *x x* (the last has been unnecessarily separated by D'Orbigny as *B. trilobata*).

Plate XVII. figs. 68 & 69 (North Atlantic).

In figs. 68 & 69, Plate XVI. the chambers are globose, and the earliest alone are armed with spiny excrescences. A less developed form appears in our next variety (fig. 67).

Figs. 68 & 69 are from the eastern marginal plateau at 223 fathoms, where *B. aculeata* is common and of middle size.

B. aculeata is found everywhere with *B. marginata* and *B. ovata*.

Bulimina ovata, D'Orbigny. Plate XVII. figs, 67 *a*, 67 *b* (North Atlantic).

Among the *Buliminæ* that fall short of the fair growth of the type (*B. Presli*, Reuss) are *B. ovata*, D'Orb., *B. pupoides*, D'Orb., and others which have a more or less subcylindrical form owing to the somewhat slow rate of increase in the successive chambers. Professor WILLIAMSON took *B. pupoides* as the type when describing the British *Buliminæ*, 'Monograph,' p. 61, &c.

B. ovata, D'Orb., For. Foss. Vien. p. 185, pl. 11. figs. 13 & 14, is just such a varietal form as occurs in the North Atlantic; on the Irish plateau, rare and small at 78 fathoms; rare and very small in the abyssal area at 1776 and 1950 fathoms; rare and middle-sized at 740 fathoms, north of the Bank; very rare and very small at 150 fathoms in Trinity Bay.

It is a British form (*B. pupoides*, var. *fusiformis*, Williamson, Monogr. p. 63, pl. 5. figs. 129, 130), together with the almost identical *B. pupoides*, D'Orb.; both of which are found fossil (and large) in the Vienna Tertiaries. It is large also in fossil beds at Jamaica (BARRETT). In Captain PULLEN'S Soundings from between Socotra and Kurachee it is very large (sometimes thin-shelled). *B. ovata* accompanies the other *Buliminæ*. They prefer muddy bottoms; flourishing in depths as great as 100 or 150 fathoms; and in the fossil state they are found in clays, corresponding to mud-beds.

Fig. 67 *a* shows a slight amount of exogenous growth on the early chambers, sufficient to indicate the close relationship of habit between this and its better grown allies (figs. 68 & 69).

Bulimina Presli, Reuss, Var. *Buchiana*, D'Orb. Plate XVII. fig. 71 (North Atlantic).

In this elegant little form we find the largest relative proportion of shell-matter among *Buliminæ*, which, on the other hand, are often very thin-shelled, but often thicker in deep seas. The chambers are here laid closely one on another, fitting well, nearly hiding their septa, and bearing vertical superficial ridges, sparse and strong, in which the marginal spines, seen in other varieties, are lost; just as spinose *Lagenæ*, *Nodosariæ*, &c. pass into ribbed varieties by modifications of the ornament. *B. Buchiana* is the most Uvigerine, both in shell-structure and shape, of all the *Buliminæ*.

B. Buchiana, D'Orb., For. Foss. Vien. pl. 11. figs. 15–18, is widely distributed; though never common. It is found in the Mediterranean; but, in comparison with *B. ovata* and *B. marginata*, it is rare: it is fossil near Vienna.

On the marginal plateau off Ireland it is rare and small at 78 fathoms.

Bulimina Presli, Reuss, Var. *elegantissima*, D'Orbigny. Plate XV. figs. 12–17 (Arctic).

Some *Buliminæ* have their segments or chambers lengthened sideways and set on very obliquely to the axis of the spine, the greater part of the shell being made up of the last whorl of from seven to ten chambers. More especially in short and gibbose individuals some of these many chambers are smaller than others in the whorl, and appear to interdigitate or to be intercalated. *Bulimina elegantissima*, D'Orb., For. Amér. Mérid. p. 51, pl. 7. figs. 13, 14, and *Robertina arctica*, D'Orb., For. Foss. Vien. p. 203, pl. 21. figs. 37, 38, both belong to this group of *Buliminæ* (see Carpenter's Introd. Foram. p. 195, &c.), and the differences of modification are so slight that we include the latter in the former.

Our Arctic specimens of *B. elegantissima* are relatively large in size and thin-walled. In the Indian seas *B. elegantissima* occurs smaller, and with thicker walls; but from the Australian seas we have it more elongate and stronger than the Arctic form. The elongate form is found also on the British coasts (see Williamson's 'Monograph,' p. 64, pl. 5. figs. 134, 135). *B. elegantissima* occurred to D'Orbigny in the sea-sands from the Pacific coast of South America; and he had *Robertina arctica* from the North Cape.

B. elegantissima is rare and of middling size at 25–30 fathoms, and common and large at from 30–70 fathoms, at the Hunde Islands (Dr. P. C. Sutherland's dredgings).

It is fossil at Grignon; also in the Eocene sandy clays of Hants and Isle of Wight (H. B. Brady), and in the Pliocene clay under the fens near Peterborough. In the recent state it is world-wide,—the British coasts, the Mediterranean, Red Sea, Tropical Atlantic, Australia, and Fiji.

Bulimina Presli, Reuss, Var. (*Virgulina*) *Schreibersii*, Czjzek. Plate XV. fig. 18 (Arctic); Plate XVII. figs. 72, 73 (North Atlantic).

Virgulinæ are such *Buliminæ* as are very much outdrawn, with thin shells, and having long loop-like apertures, with inverted lips, as in *Bulimina* proper. The chambers are arranged less compactly than in the other *Buliminæ*, in consequence of the elongation of the shell, and are scarcely more than biserial, or even only irregularly so. *V. Schreibersii*, Czjzek, Haid. Abhandl. vol. ii. pl. 12. figs. 18–21, is of irregular growth, intermediate between the long and loose-growing varieties of *B. ovata*, D'Orb., and the Textulariform *Virgulina squamosa*, D'Orb., next described. It is an isomorph of *Polymorphina*, as *V. squamosa* is isomorphic with *Textularia*.

We have it rare and large from the Hunde Islands, where Dr. Sutherland dredged it in 30–40 fathoms; and in the North Atlantic it is rare and middle-sized at 1950 fathoms; rare and large at 2330 fathoms (Boreal portion of the Abyss); and rare and small at 954 and 725 fathoms north of Newfoundland Bank.

V. Schreibersii and its subvarieties are not rare in existing seas, both of warm and cold climates; and it occurs fossil in the Tertiary beds of Sienna, Vienna, and Turin.

Some allied forms occur in the Chalk and in the Clays of the Oolite, which are isomorphs of the Dentaline or Virguline *Polymorphinæ* of the Sutton Crag.

Bulimina Presli, Reuss, Var. (*Virgulina*) *squamosa*, D'Orbigny. Plate XV. fig. 19 *a*, 19 *b*, 20 (Arctic).

Although the arrangement of the chambers has become almost regularly biserial, and alternate, as in *Textularia*, yet this variety retains the true Bulimine aperture; and gradual modifications in form lead us from *Virgulina squamosa*, D'Orb. (Modèle, No. 64), before us, through *V. Schreibersii* (fig. 18), to the more regular *Buliminæ.*

This variety has the same world-wide distribution as *V. Schreibersii*; but is never common: at the Hunde Islands it is rare and small at 30–40 and 60–70 fathoms; and it was rare and large in the mixed sands from Norway.

As an enfeeblement of *Bulimina*, it points in one direction to *V. Schreibersii*, and in another to the *Bolivinæ.* Fig. 20 is a specimen that can scarcely be separated from *Bolivina punctata.*

Bulimina Presli, Reuss, Var. (*Bolivina*) *costata*, D'Orbigny. Plate XVII. fig. 75 (North Atlantic).

"A more decided modification of the Bulimine type is presented by those forms which have been ranked by D'ORBIGNY in his genus *Bolivina*; the arrangement of the segments being here regularly biserial and alternating, as in *Textularia*; but the aperture never loses the elongation and the inversion of its lips, characteristic of the Bulimine type, and its direction is usually somewhat oblique. In the *B. costata* of D'ORBIGNY (For. Amér. Mérid. p. 62, pl. 8. figs. 8, 9) there is a set of right parallel costæ, running continuously from one segment to another along the entire length of the shell, giving to it a very peculiar aspect" (CARPENTER, 'Introd.' p. 196).

The inversion of the lip of the aperture, characteristic of *Bulimina*, and homologous with the intussusception of the neck-tube in *Lagena*, is well seen in some young transparent *Bolivinæ.*

B. costata is rare and large at 223 fathoms on the marginal plateau off the coast of Ireland. D'ORBIGNY found it common at 20 mètres at Cobija, South America; an allied and small variety, *B. plicata* (*op. cit.* pl. 8. figs. 4–7), he found in deeper water at Valparaiso.

B. costata lives on muds and is found fossil in clays, like other *Buliminæ*; flourishing down to about 100 fathoms; it is never common, but is found on the west coast of Scotland, and from the south coast of England (Eastbourne) to the tropics.

Bulimina Presli, Reuss, Var. (*Bolivina*) *punctata*, D'Orbigny. Plate XVII. fig. 74 (North Atlantic).

The figured specimen is a short and vesicular subvariety of *B. punctata*, D'Orbigny. (For. Amér. Mérid. p. 63, pl. 8. figs. 10–12), which is the centre of a group of many forms. The one before us is perfectly Textulariform, and can be diagnosed as a *Bulimina* only by the shape and subobliquity of its aperture.

We find it rare and small at 43 and 415 fathoms, and rather common and small at 223 fathoms, on the marginal plateau off Ireland.

D'ORBIGNY got it rather common at from 40 to 50 mètres at Valparaiso.

B. punctata is world-wide, reaching as low as 100 fathoms. In the Mediterranean area it is both recent and fossil. It is present in the Oxford and Kimmeridge Clays.

Genus CASSIDULINA.

Cassidulina lævigata, D'Orbigny. Plate XV. figs. 1–4 (Arctic); Plate XVII. figs. 64 *a*, 64 *b*, 64 *c* (North Atlantic).

Cassidulina, related to *Bulimina* and *Textularia*, is described in CARPENTER's Introd. Foram. p. 197. It is of world-wide distribution, on muddy bottoms in both shallow and deep waters. In the Indian Ocean (between Socotra and Kurachee) *Cassidulina* takes on the uncoiled condition (*Cassidulina Pupa*, D'Orb., *Ehrenbergina serrata*, Reuss); and in the tropical deep seas it passes into thick-walled, flush-shelled, and uncoiled forms, isomorphic of *Bolivinæ*. It occurs in Tertiary deposits. Deep-sea forms are usually thick-walled.

C. lævigata, D'Orb. (Modèles, No. 41, Ann. Sc. Nat. vii. p. 282, pl. 15. figs. 4, 5 *bis*) is common and small in the mixed sands from Norway (MACANDREW and BARRETT); common and middle-sized at the Hunde Islands, from 30 to 70 fathoms*; common and middle-sized in 150 fathoms, 76° 30′ lat., 77° 52′ long., Baffin's Bay, and rare and middling at 75° 10′ lat., 60° 12′ long.

In the North Atlantic it is rare and small at 1750 fathoms in the central area; north of the Bank it is rare and of middle size at 102, 112, and 145 fathoms, and rather common at 740 fathoms; in Trinity Bay it is rare and small at 150 fathoms, middle-sized and not very common at 124, 133, and 192 fathoms.

On the Newfoundland Bank *Cassidulinæ* are few and probably dead, just as *Nonionina Scapha* occurs. *Cassidulina* is also a Middle Tertiary form.

Cassidulina lævigata, D'Orb., Var. *crassa*, D'Orb. Plate XV. figs. 5, 6, 7 (Arctic); Plate XVII. fig. 64 *d* (North Atlantic).

This thicker form accompanies the typical *C. lævigata* in its wide-spread occurrences. D'ORBIGNY first described and figured *C. crassa* from off Cape Horn (160 mètres), and, in company with *C. Pupa*, from the Falkland Isles ("at a considerable depth"). Professor WILLIAMSON's *C. obtusa* (Monogr. p. 69, pl. 6. figs. 143, 144), from the British coasts, and from the Hunde Islands, is the same as *C. crassa*, excepting a slight difference in the variable aperture.

C. crassa, D'Orb. (For. Amér. Mér. p. 56, pl. 7. figs. 18–20) is small at 28–30 and 50–70 fathoms, and of middle size at 30–40 fathoms at the Hunde Islands, and common throughout; it is common and small at 150 fathoms in Baffin's Bay, 76° 30′ lat., 77° 52′ long.

On the eastern plateau of the North Atlantic it is very rare and very small at 223 fathoms.

C. crassa has its finest development (as far as we know) at 1100 fathoms in the Tropical Atlantic; like *C. lævigata* it is often among the deep-sea forms; it is found also in the Mediterranean and in Bombay Harbour.

---

* Professor WILLIAMSON (Monogr. p. 68) notices the umbonate and transparent condition of the *Cassidulinæ* from the Hunde and Beechey Islands.

Genus PLANORBULINA.

Planorbulina farcta, Fichtel and Moll, sp. (Varieties). Plate XIV. figs. 3–11 (Arctic); Plate XVI. figs. 18–25 (North Atlantic).

This is a very common variety of a species belonging to the Rotaline group of Foraminifera. In endeavouring to elucidate the relationships of the *Rotalinæ*, we have been impressed with the distinctiveness of nine specific groups, six of which have more or less of the well-known Rotalian shape, and are extremely rich in varietal forms (see Dr. CARPENTER'S 'Introd. Study Foram.' Ray Soc. 1862, pp. 198, &c.). A great proportion of these varieties have been described by authors under the generic term "Rotalia"; others have been grouped under the leading names of *Rosalina*, *Planorbulina*, *Gyroidina*, *Anomalina*, *Truncatulina*, and several others, supposed to be of subgeneric, or even of generic, value. An artificial classification and extreme confusion have been the consequence. After a long examination of the subject in its bibliographic aspect, and having carefully studied large numbers of the actual organisms, recent and fossil, we find that they range themselves around six specific centres, which may also be regarded as types of so many genera; and with these are allied three other specific forms, not so Rotalian in aspect (*Tinoporus*, *Patellina*, and *Polytrema*).

The protean variability of all the six Rotalian types being great, and isomorphism, or similarity of form among the varieties and subvarieties of these several specific groups, being of very frequent occurrence, we still use binomial terms, in a subgeneric sense, for members of this great group; and often, in ordinary descriptions, we retain, for the sake of convenience, binomial appellations (without direct reference to their exact zoological value) for striking specimens even of varieties and subvarieties. Thus *Truncatulina lobatula* is a distinct binomial term for the common variety of *Planorbulina farcta* first to be noticed (page 381).

The old name *Rotalia* is retained for one of these six genera; and we arrange the whole as a subfamily with the appellation of ROTALINÆ*.

Discorbina Turbo, *D'Orb.*, sp.
Planorbulina farcta, *Fichtel and Moll*, sp.
Pulvinulina repanda, *Fichtel and Moll*, sp.
Rotalia Beccarii, *Linn.*, sp.
Cymbalopora Poyei, *D'Orb.*, sp.
Calcarina Spengleri, *Gmelin*, sp.
Tinoporus lævis, *Parker and Jones*, sp.
Patellina concava, *Lam.*, sp.
Polytrema miniaceum, *Esper*, sp.

Each of the six Rotaliform genera is represented by one typical species, which carries with it a large number (from 50 to 200 or 300) of divergent forms, most of them having special names, which we must in many instances retain for convenience, though we refer them to one or the other of the six species above mentioned.

---

* See CARPENTER'S 'Introd. Foram.' Ray Soc. 1862, pp. 198, &c.

In nature these Foraminifera are never absolutely strict in their adherence to any one of the chief varietal forms; but the latter are serviceable as subspecific centres, around which may be arranged a large number of modifications, more and more gentle and mutually confluent; so that when we speak of *Truncatulina lobatula* or of *Discorbina vesicularis* (and the same may be said of the varietal groups of any true Foraminiferal species), we do not mean to say that the specimen which we have before us necessarily answers exactly to any figure or description in the literature of the subject, but that it is nearer to some one of the accepted illustrations than to any other. To attempt greater exactness would be useless; indeed the classification of these little creatures is very similar to what that of vegetables would be if we had only the separate leaves for our guides.

From 100 fathoms to shallow water (seaweed-belt, 10 fathoms and less) is the best home for the Rotaliform *Rotalinæ*. Certain varieties of *Pulvinulina repanda* attain a good size at 2400 fathoms. The varieties of *Planorbulina farcta*, also, are not uncommon at very great depths. *Discorbina Turbo*, *Rotalia Beccarii*, *Calcarina Spengleri*, and *Cymbalopora Poyei* avoid great depths (with few exceptions), the best developed specimens keeping themselves above the Coralline-zone or 25 fathoms.

Planorbulina has a coarsely porous shell (more so than any of its congeners), often of a relatively large size, consisting of from 15 to 200 or more chambers, with single septa, and very slight rudiments of the canal-system: it is usually complanate (*Pl. Mediterranensis*) and parasitic on sea-weeds and shells; but many of its varieties are plano-convex (*Truncatulina*), and some become almost subnautiloid (*Anomalina*). The shell is mostly smooth; rarely limbate (*Planulina*); and frequently granulate (*Pl. vulgaris* and *Pl. larvata*): the aperture varies from an open to a contracted slit, and is often produced and lipped.

Scheme of the chief members of the Rotaline genus PLANORBULINA.

Fully developed forms; becoming concentric, with alternating chambers built over the apertures of the penultimate ring.	*vulgaris*, D'Orb. For. Foss. Canar. pl. 2. fig. 30; Carpenter, Introd. For. pl. 13. figs. 13–15.
	Mediterranensis, D'Orb. Modèles, No. 79.
	retinaculata, Parker and Jones (sp. nov.); Carp. Int. For. p. 209. Plate XIX. fig. 2.
	larvata, P. and J. (sp. n.), Ann. Nat. Hist. 3 ser. vol. v. p. 68. Plate XIX. fig. 3.
Intermediate forms.	FARCTA, Fichtel and Moll, sp. (the type of *Planorbulina*), Test. Micr. pl. 9. figs. *g–i*.
	lobatula (*Truncatulina*), Walker and Jacob, sp., D'Orbigny's Modèles, No. 37.
	refulgens, Montfort, sp., D'Orbigny's Modèles, No. 77.
Quasi-rotalian and subnautiloid forms.	*Haidingerii*, D'Orb., sp., For. Foss. Vien. pl. 8. figs. 7–9.
	Ungeriana, D'Orb., sp., For. Foss. Vien. pl. 8. figs. 16–18.
	ammonoides, Reuss, sp., Böhm. Kreid. pl. 8. fig. 53.
	reticulata, Czjzek, sp., Haiding. Abhandl. ii. pl. 13. figs. 7–9.
	coronata (*Anomalina*), Parker and Jones, Ann. Nat. Hist. 2 ser. vol. xix. p. 294, pl. 10. figs. 15, 16.
	Ariminensis (*Planulina*), D'Orb., sp., Modèles, No. 49.

Figs. 3 to 11 include two striking varieties of *Planorbulina farcta*,—a type perhaps the richest of all the Rotalines in modification; and which not only developes the largest chambers, but produces the largest shells (some with a diameter of a quarter of an inch, *P. vulgaris*, D'Orb.). The disk and the chambers are so large in some specimens from tropical seas, that individuals have been mistaken for Polyzoa, and this mistake has been strengthened by the pouting of the marginal apertures.

Both of the varieties here under consideration, though attaining considerable size, are arrested and few-chambered varieties. They have attained the simple Rotaline form without as yet taking on the more characteristic features of the more outspread *Planorbulinæ*, although their somewhat free mode of growth, the coarseness of their shell-walls, and their relatively large aperture afford the connecting links to the observer; more especially when we find the same shells having the aperture firstly lipped, then protrusive, and gradually (among numbers of individuals) acquiring a neck and distinct rim. The typical development of this *Planorbulina*, with a subtubular chief aperture and supernumerary necked and lipped apertures on the periphery of the shell, is rarely found in the northern seas; by far the most common variety is the well-known form, figs. 3–6, long ago described as *Truncatulina lobatula*. This, as a rule, grows on a shell or other substance having a smooth surface, and during the growth of the shell the little parasitical Foraminifer occasionally becomes more or less imbedded in its substance. This plano-convex variety represents in the temperate climes the many-chambered plano-convex *Planorbulina Mediterranensis*. The latter swarms on seaweeds and shells in the shallow water of the Mediterranean; and it is in company with it (especially when growing on the larger bivalves, such as *Pinna flabellum*) that *Pl. lobatula* is seen to take on a wild-growing condition, with subsidiary marginal necks, becoming *Pl. farcta* and *Pl. variabilis*, without developing nearly so many chambers as are seen in its associate, although exceeding the latter in size. In tropical and subtropical seas *Pl. farcta* grows on to be the great *Pl. vulgaris*, D'Orb., the arrested Truncatuline forms being comparatively rare.

In the seas of hot climates a large amount of exogenous granular matter is formed on the surface of the shell (as in *Pl. larvata* *, Parker and Jones); far different to the smooth, polished shells in the Mediterranean and northern seas. There is one parasitical form (*Pl. retinaculata* †, Parker and Jones) which, besides being scabrous with granulation, developes so large a number of peripheral, subsidiary, tubular apertures, connecting together, and still keeping apart, the sarcode-chambers, and forming a kind of irregular network over the surface of the shells on which it grows, like certain Polyzoa, that the features of this *Planorbulina* are extremely different from that of its type; and it can scarcely be connected with the simple varieties of the species without a knowledge of the real relationships of the great and widely extended Rotaline group. The same structure really exists in the great *Pl. vulgaris*, D'Orb. For. Canar. pl. 2. fig. 30, and Carpenter, Introd. For. pl. 13. figs. 13–15; but here the connexion of the chambers is masked in some degree by the obesity of the chambers themselves; the retinaculate variety developing smaller and more depressed lobes of sarcode. On *Chama gigas* there is often a wild-growing parasitic *Tinoporus* isomorphous with *Pl. retinaculata*, but still larger.

---

* Plate XIX. fig. 3. Ann. Nat. Hist. 3 ser. vol. v. p. 68.

† Plate XIX. fig. 2. CARPENTER'S Introd. Foram. p. 209.

The oldest known *Planorbulinæ* are found in the Lias.

Planorbulina farcta, Fichtel and Moll, sp., Var. (*Truncatulina*) *lobatula*, Walker and Jacob, sp. Plate XIV. figs. 3–6 (Arctic); Plate XVI. figs. 18–20 (North Atlantic).

Planorbulina lobatula has been described above to some extent; we may add that it is an exceedingly unstable form, even whilst keeping its simple character; for frequently it has only half the thickness seen in fig. 5 *b*, which is an average specimen for such as live at from 30 to 160 fathoms in the Northern Seas; but at about 60 to 70 fathoms it frequently assumes a modified condition, taking a high conical shape (*Pl. refulgens*, Montfort, sp.), its smoothness and polish being much greater than in these flatter forms; and the apex of the shell is on the umbilical aspect (as in *Pl. lobatula*); the whole coil of chambers being seen on the base of the shell. *Pl. lobatula* also passes insensibly into an extremely thin scale-like variety, nearly symmetrical, with limbate septal lines and square edges, which has been described as *Planulina Ariminensis*, D'Orb. (Modèles, No. 49). Other forms gradually lose the plano-convex, or Truncatuline, character; the edges become rounded, the primary and succeeding chambers become elevated above the margin of the shell, which thus grows biconvex or lenticular; for instance, *Planorbulina Haidingerii*, D'Orb., sp. (For. Foss. Vien. pl. 8. figs. 7–9), and *Pl. Ungeriana*, D'Orb., sp. (Ibid. figs. 16–18), common forms at from 60 to 300 fathoms. We here omit any notice of the intermediate varieties, which have been extensively named as species.

Like *Pulvinulina repanda*, as seen in its variety *P. Micheliniana* (Plate XIV. fig. 16), the Truncatuline forms of *Pl. farcta* have the spiral arrangement of the chambers marked on the flat face of the shell; on the other hand, the plano-convex varieties of *Discorbina Turbo* have the umbilical surface flat; the apex of the cone being formed of the primordial chamber: an approach to this condition is seen in Plate XIV. figs. 18, 19, *Discorbina obtusa*, D'Orb., sp. (For. Foss. Vien. pl. 11. figs. 4–6), a variety of *D. Turbo*, D'Orb., sp. (Modèles, No. 73).

Plate XIV. figs. 3–6 represent specimens of *Pl. lobatula* from the Hunde Islands, in five dredgings by Dr. P. C. SUTHERLAND (25 to 70 fathoms), where they are very common and generally of good size; from Baffin's Bay, at three places; lat. 75° 10′ N., long. 60° 12′ W., and lat. 76° 30′ N., long. 77° 52′ W., of middling size and common, and at lat. 75° N., long. 59° 40′ W. (220 fathoms), where they are small and rather common; and from seven out of the eight dredgings by MACANDREW and BARRETT on the Norwegian coast we have them large and common. We have already indicated that this variety is world-wide; fossil, it occurs in the Chalk-marl, Chalk, and many later deposits.

Fig. 6 shows a condition of the parasitic forms of *Planorbulina farcta* very common, both in this arrested Truncatuline variety and in the outspread *Planorbulinæ*. Two young individuals, establishing themselves close to each other, grow on until their shells become blended and confused; this is still better seen in the many-chambered *Planorbulinæ*, two or more of which, growing into each other, form lichen-like patches on shells.

Plate XVI. figs. 18–20 (North Atlantic).

Truncatulina lobatula belongs essentially to shallow waters, and it becomes smaller when in deeper water than usual (as is the case with the specimens before us), and is then more compact and neat, takes on a limbation (exogenous edging to the chamber-walls, fig. 19), and soon approaches the conical and shapely *Tr. refulgens*, Montfort, sp.

On the eastern marginal plateau of the North Atlantic *Truncatulina lobatula* is common and of middle size at from 43 to 78 fathoms, rare and small at 338 fathoms. It is absent from the abyssal depths. To the north of Newfoundland Bank ("Arctic" tract) it is rare and small at 145 fathoms, and rare and middle-sized at 740 fathoms.

Planorbulina farcta, Fichtel and Moll, sp., Var. *Haidingerii*, D'Orbigny, sp. Plate XVI. figs. 22 *a*, 22 *b* (North Atlantic).

This is a variety of *Planorbulina farcta* near to *Pl. lobatula*, but biconvex and having more chambers and a more solid and symmetrical make. It is usually larger and more ventricose than these Atlantic specimens.

This and *Pl. Ungeriana* are two closely allied, compact, and flush-chambered varieties of *Pl. farcta*, more Rotaliform than *Pl. lobatula*, and inhabiting moderately deep seas. In the North Atlantic *Pl. Haidingerii* is rare and of middle size at 1776 fathoms in the Abyssal area. It is more abundant in the "Virginian Province" on the coast of New Jersey (see page 333 and Appendix II.). The two are fossil together in Tertiary beds.

Pl. Haidingerii is world-wide, like the type, and bears the same relation to it that *Rotalia Soldanii* does to *R. Beccarii*,—a rather large and moderately deep-sea variety.

Planorbulina farcta, Fichtel and Moll, sp., Var. *Ungeriana*, D'Orbigny, sp. Plate XVI. figs. 23–25 (North Atlantic).

This variety has relatively narrower chambers and more limbation than its congener *Pl. farcta*, var. *Haidingerii*, D'Orb., sp., above-mentioned.

It is widely distributed in the Atlantic. On the marginal plateau off the Irish coast it is rare and small in the shallow, common and largest in the deeper parts. In the Abyssal tract ("Celtic") it is common but small; and throughout the "Boreal" portion of that tract (1400–2300 fathoms) it is rare and small. It is figured in Dr. WALLICH'S 'North-Atlantic Sea-bed,' pl. 6. figs. 20, 21.

Pl. Ungeriana is world-wide, like the last; but, as a weaker and smaller shell, it takes the place of the type in deepest waters, where also *Rotalia orbicularis* represents *R. Beccarii*. *Pl. Culter*, nov., Plate XIX. fig. 1, is a rare, keeled subvariety, living at great depths.

Planorbulina farcta, Fichtel and Moll, sp., Var. *Mediterranensis*, D'Orbigny. Plate XVI. fig. 21 (North Atlantic).

This explanate *Planorbulina* is of small size in the North Atlantic, as usual in North Temperate seas; it is rare off the Irish coast at 43 fathoms.

It is spiral at first, then excentric, and ultimately concentric; always orderly in its growth, with bipolar chambers; not having exogenous matter, nor a free growth of marginal apertures. It flourishes in the warmer temperate seas; is starved in the British area; abounds in the Mediterranean and Australian seas; but in the latter is less plentiful than *Pl. vulgaris*, with which it is associated. It forms a tiny scale on flat-fronded sea-weeds, and has a livid pinkish colour, both from its contained sarcode and from the shell-substance being actually coloured.

Planorbulina farcta, Fichtel and Moll, sp., Var. (*Anomalina*) *coronata*, Parker and Jones. Plate XIV. figs. 7–11 (Arctic).

This has been termed *Anomalina coronata*, Ann. Nat. Hist. 2 ser. vol. xix. p. 294; but it belongs to *Planorbulina*, and the term *Anomalina* is not really wanted, however convenient it may be as a term for the subsymmetrical or somewhat biconvex arrested *Planorbulinæ*, as *Truncatulina* indicates the plano-convex few-chambered forms. To make the so-called genus *Anomalina*, D'Orbigny took several of the minor forms of *Planorbulina farcta*, namely those which are somewhat symmetrical and subnautiloid, with one variety of *Discorbina Turbo* (*A. elegans*, Modèle, 42).

On taking into consideration the evident passage of form from the plano-convex (Truncatuline) to the biconcave (Anomaline) condition of the shell, shown by figs. 8, 10, 9, 7, & 11, the observer may at once see the force of the above remarks.

This variety, *Pl. coronata*, has the same kind of shell-substance, thick, subtransparent, and coarsely perforated, as *Pl. lobatula*; it has a greater tendency to develope clear, non-perforate, exogenous shell-matter on both faces of the shell, sometimes hiding the septal lines; the pseudopodia chiefly passing from the periphery of the chambers and through the lacunæ in the superadded coating, both on the umbilical (fig. 10) and the flatter spiral surface (fig. 8). The presence of these lacunæ is highly interesting, as being the first rough outline of the great vascular or interseptal canal-system which attains such perfection in the highly developed *Rotalinæ*, *Polystomellæ*, and *Nummulinæ*.

Pl. coronata is not so common as *Pl. lobatula*; it abounds, however, in MacAndrew and Barrett's Norway dredgings (at five places); and it is abundant at certain places in the Mediterranean, especially at about 100 fathoms. At such depths it is that *Pl. coronata* takes the place of *Pl. lobatula*, by living independently and developing its surfaces more or less freely, whilst but few of the parasitical variety are left on the rare shells of deep water. *Pl. coronata* has been found abundantly in the North British seas by Mr. H. B. Brady.

Pl. vulgaris also has a subnautiloid form in its young state; and throughout its growth the chambers are more or less convex both on the attached and the free face.

Planorbulina vulgaris grows on rough shells (such as *Tridacna* and *Hippopus*); and its under surface touches but at points, not lying flat (as in *Pl. Mediterranensis* on sea-weed-fronds, and *Pl. lobatula* on smooth shells and algæ).

In the fossil state we have *Pl. coronata* from the Grignon sands (Eocene), of large size, rivalling in size the *Discorbina trochidiformis* of that deposit.

Under the names of *Rotalia*, *Rosalina*, *Anomalina*, and *Truncatulina*, have been described a great number of subnautiloid forms which are evidently some of them enfeeblements of *Pl. coronata* (the nearest being *Truncatulina vermiculata*, D'Orb. Foram. Amér. Mérid. pl. 6. figs. 1–3), whilst others are either young or arrested modifications of *Pl. vulgaris*. In deep water *Planorbulina* scarcely ever takes its true Planorbuline character; this many-chambered condition seeming to require sea-weeds or shell-surfaces for support. Mixed with these, and at still greater depths, we get numbers of small subsymmetrical nautiloid forms of this species, such as have passed under the names of *Rotalia Clementiana*, D'Orb., and *Rotalia ammonoides*, Reuss; as well as many other forms ranging between the latter and *Planulina Ariminensis*. *Planorbulina ammonoides* of the Lias, Gault, and Chalk takes on the symmetrical (subnautiloid) character so distinctly as to be mistaken for small *Nonioninæ*. These small, more or less symmetrical *Planorbulinæ*, so common in some deposits of the Secondary period, are abundant enough in the existing seas at from 100 to 1000 fathoms, or even more. We may suppose that the sea-weeds and bivalves of the shallow water of the Secondary period were abundantly encrusted with *Planorbulinæ* as littoral representatives of the deep-sea forms now fossil in the clays of that period.

Genus DISCORBINA.

Discorbina Turbo, D'Orbigny (Varieties). Plate XIV. figs. 18–23 (Arctic); Plate XVI. figs. 26–28 (North Atlantic).

Discorbina presents a simple Rotaline form of shell, having from 7 to 30 more or less vesicular chambers, with double septa when the chambers are discrete, and with rudiments of the canal-system. The shell is coarsely porous (coarser than that of *Cymbalopora*, and less so than *Planorbulina*); somewhat conical in shape; the upper side the thickest; the margin rather sharp; but some varieties are complanate with square edges. The aperture is a large arched slit, usually occluded by an umbilical process or flap, which is sometimes developed into a subsidiary umbilical chamber; and the flaps or chamberlets of the successive chambers give a star-like or Asterigerine aspect in the umbilicus. Exogenous shell-growth sometimes thickens the septal lines of the spire; but it frequently ornaments, and even masks, the umbilical lobes.

The many varieties of this porous and flap-bearing Rotaline species are so intimately connected one with the other, that the following classification is little more than suggestive and provisional.

Scheme of the arrangement of the chief subspecific forms of DISCORBINA.

1. Conical.	*trochidiformis*, Lam., sp., Ann. Mus. viii. pl. 62. fig. 8. Fossil from Grignon. Coarsely perforate, valvular or flapped, valvules marked by a mass of granules. It is an isomorph of *Polystomella craticulata*.
	TURBO, D'Orb., sp., Modèles, No. 73. Fossil from Grignon. Coarsely perforate. Valvules distinct. This is the typical species.
	rosacea, D'Orb., sp., Modèles, No. 39. Fossil from Bordeaux (=*Asterigerina Planorbis*, D'Orb.). It is delicately perforate; valvules distinct.
	Pileolus, D'Orb., sp., For. Amér. Mér. pl. 1. figs. 15–17. From India, Australia, &c., and fossil from Grignon. Small; conical or hemispherical: chambers vertical: granulate ornament in radiating lines. Connecting *D. rosacea* with *D. Parisiensis*. It has its extreme flatness in (*Ros.*) *semistriata*, D'Orb., For. Cuba, pl. 3. figs. 15–17.
2. Vesicular: valves feeble in the feeble vesicular forms, especially in *globularis* and its poor relations.	*vesicularis*, Lamk. Ann. Mus. viii. pl. 62. fig. 7 (=(*Rot.*) *Gervillii*, D'Orb. Modèles, No. 72). From Australia, and fossil at Grignon. Carpenter, Introd. For. pl. 13. figs. 2, 3.
	rimosa, Parker and Jones (Carpenter, Introd. For. p. 205). Fossil at Grignon: recent from India to Australia, including Fiji. Plate XIX. fig. 6.
	dimidiata, Parker and Jones (Carpenter, Introd. For. p. 201. fig. 32, B.). Plano-convex. Plate XIX. fig. 9.
	elegans, D'Orb. Modèles, No. 42. Fossil from Bordeaux (=(*Rot.*) *complanata*, For. Foss. Vien. pl. 10. figs. 10–15). Passing insensibly into *D. vesicularis*.
	globularis, D'Orb. Modèles, No. 69 =(*Rot.*) *semiporata*, Egger, sp. Miocene, Germany.
	obtusa, D'Orb. For. Foss. Vien. pl. 11. figs. 4–6.
	globigerinoides, Parker and Jones. Extreme of *D. vesicularis*, running into *D. elegans*. It is an isomorph of *Cymbalopora bulloides*, D'Orb. (*Rosalina*, Cuba, pl. 3. figs. 2–5). Plate XIX. fig. 7.
	Binkhorsti, Reuss, Sitz. Akad. Wien. xliv. pl. 2. fig. 3. This is an isomorph of *Pulvinulina caracolla*, Roem., sp. Limbate.
3. Outspread (more or less complanate): valves feeble in the small outspread forms.	*Parisiensis*, D'Orb., sp., Modèles, No. 38. Fossil at Grignon. Ornamented with granular lines.
	Cora, D'Orb., sp., For. Amér. Mér. pl. 6. figs. 19–21. Complanate, and round-edged; probably representing a somewhat starved condition.
	Berthelotiana, D'Orb., sp., For. Canaries, pl. 1. figs. 28–30.
	biconcava, Parker and Jones (Carpenter, Introd. Foram. p. 201. fig. 32, G). Complanate, with raised square edges. Plate XIX. fig. 10.

The oldest known are *Discorbina Turbo* and *D. Binkhorsti*, both in the Maestricht Chalk.

Discorbina Turbo, Var. *rosacea*, D'Orbigny, sp. Plate XVI. figs. 28 *a*, 28 *b* (North Atlantic).

Discorbina rosacea, D'Orb., sp. (Modèles, No. 39), has an exquisitely sculptured, and more delicately porous shell than usual (the margin only may be porous); its astral flaps form sometimes a secondary system of chambers. These characters are developed largely in *D. Turbo*, D'Orb., sp., the type of the whole group, from which this flat variety has no essential distinction. *D. rosacea* is rather common and of the middle size on the Irish plateau at 43 fathoms.

D'ORBIGNY'S *Asterigerina Planorbis* (For. Foss. Vien. pl. 11. figs. 1–3) supplies a very good representation of this elegant form: see also WILLIAMSON'S Monogr. pl. 4. figs. 109–111 (*Rotalina Mamilla*), and his pl. 4. fig. 112, and pl. 5. fig. 113 (*R. ochracea*). The most exquisite specimens of this variety are from San Domingo (fossil), where it abounds in the Miocene beds. It is always small; but is larger and coarse on the Australian shores, passing insensibly into *D. Turbo*. It is common in the Grignon Tertiary deposits, rare in our Crag, and world-wide in the present seas.

Discorbina Turbo, D'Orbigny, sp., Var. *vesicularis*, Lam., sp., Subvar. *globularis*, D'Orbigny, sp. Plate XIV. figs. 20–23 (Arctic).

This small vesicular form of *D. Turbo*, D'Orb., sp., is *D. globularis*, D'Orb., sp. (Modèle, No. 69), from the Atlantic; and the same as EGGER'S *Rosalina semipunctata*, Neues Jahrb. 1857, pl. 4. figs. 1–3. It is smaller than *D. vesicularis*, Lamarck, sp. (=*D. Gervillii*, D'Orb., sp., Modèles, No. 72), and has fewer chambers.

It is a world-wide form in shallow water and down to 70 fathoms, at which depth, west of the Bay of Biscay, it abounds; it is, however, flatter here than when nearer the shore. In deeper water it becomes *D. Berthelotiana* and *D. rosacea*, D'Orb., spp.

At the Hunde Islands (SUTHERLAND'S Soundings) it is large and rather common at from 30–40 fathoms; and middle-sized and common at from 50–70.

Discorbina Turbo, D'Orbigny, sp., Var. *vesicularis*, Lamarck, sp., Subvar. *obtusa*, D'Orbigny, sp. Plate XIV. figs. 18, 19 (Arctic).

Discorbina presents a simple Rotaline form of shell, usually having more or less vesicular chambers, with porous walls, and with the septal apertures in many cases guarded by flaps or plates, which sometimes form small secondary umbilical chambers.

The specimen here figured is near to *D. globularis*, D'Orb., sp. (Modèle, No. 69), but may either be regarded as a swollen condition of the beautiful *D. Parisiensis*, D'Orb., sp. (Modèle, No. 38), or, rather, as *D. vesicularis* with the style of ornament characteristic of *D. Parisiensis*. The nearest approach to it among published figures is made by *D. obtusa*, D'Orb., sp., For. Foss. Vien. pl. 11. figs. 4–6. The coarseness of its pores, its few and subvesicular chambers, its large central chamber, and its peculiar ornamentation, are the chief characters of the variety before us. In the Arctic specimens the ornament appears as obscure, irregularly radiating, minutely granular lines on the lower face [not well shown in the figure]; in *D. Parisiensis* the under surface has an exquisite sculpturing of minutely granulate lines or ridges; D'ORBIGNY'S *D. obtusa* has a granular ornament in radiating lines [not well shown in the figure]. *D. globigerinoides*, Plate XIX. fig. 7, a new variety of *D. Turbo*, also has this kind of ornament, being thickly covered on the septal plane with sinuous exogenous rugæ, having large pores opening out of them, thus presenting a rudiment of the canal-system.

At the Hunde Islands, *D. obtusa* is large and rare at 28 to 30 fathoms; large and rather common at 30 to 40; and large and common at 60 to 70 fathoms.

Discorbina Turbo, D'Orbigny, sp., Var. *Parisiensis*, D'Orbigny, sp., Subvar. *Berthelotiana*, D'Orbigny, sp. Plate XVI. figs. 26, 27 (North Atlantic).

Discorbina Berthelotiana, D'Orb., sp. (For. Canar. pl. 1. figs. 28–30), may be regarded either as a compressed and more or less limbate form of *D. globularis*, D'Orb., sp., or, rather, as intermediate to *D. globularis*, D'Orb., sp., and *D. Parisiensis*, D'Orb., sp. (Modèles, No. 38), but without the ornamentation below. It is generally small; usually showing an umbilical flap or angle; but in fig. 27 a granule represents it. This variety makes a near approach to the strongly limbate *Discorbina Binkhorsti*, Reuss, sp. (Sitz. Akad. Wien. 1861, vol. xliv. pl. 2. fig. 3), of the Maestricht Chalk; and, though it resembles some of the margined *Globigerinæ* of the Chalk, it has no relationship with them.

Our fig. 26 is much more limbate than the specimen figured by D'ORBIGNY; but they are essentially the same.

D. Berthelotiana occurs on the marginal plateau off Ireland, small and rather common at 78 fathoms, and small and rare at 43 fathoms.

Genus ROTALIA.

Rotalia Beccarii, Linn., sp. (Varieties). Plate XVI. figs. 29–34 (North Atlantic).

Rotalia has a finely porous shell (coarser than that of *Pulvinulina* and finer than *Calcarina*); biconvex (lowest side thickest), with round margin; made up of from thirteen to forty chambers, with double septa; canal-system present. Septal lines and umbilicus often beaded with exogenous granules, sometimes to a great extent. Aperture a slit (occasionally subdivided), sometimes notched at the umbilical margin of the septal plane, as in *Pulvinulina*, *Discorbina*, and arrested *Planorbulinæ*. Shell rarely prickly; occasionally Asterigerine; generally small, compared with most other Rotalines; or, rather, it does not attain to quite as great a size.

Scheme of the chief subspecies of ROTALIA.

ROTALIA Schrœteriana, *Parker and Jones*. See Carpenter, Introd. For. pl. 13. figs. 7–9.
—— ornata, *D'Orb.*, sp., For. Amér. Mér. pl. 1. figs. 18–20.
—— craticulata, *Parker and Jones*. Plate XIX. fig. 12. (Fiji.)
—— annectens, *Parker and Jones*. Plate XIX. fig. 11. (Hong Kong.)
—— pulchella, *D'Orb.*, sp., For. Cuba, pl. 5. figs. 16–18. See Carpenter, Introd. For. p. 213.
—— dentata, *Parker and Jones*. Plate XIX. fig. 13. (Bombay Harbour.)
—— BECCARII, *Linn.*, sp., D'Orbigny's Modèles, No. 74. This is the Type species.
—— ammoniformis, *D'Orb.* Ann. Sci. Nat. vol. vii. p. 276. No. 53. (After SOLDANI.)
—— lobata, *D'Orb.*, sp., For. Cuba, pl. 5. figs. 19–21. See Carpenter, Introd. For. p. 213.
—— carinata, *D'Orb.*, sp., For. Cuba, pl. 5. fig. 25, pl. 6. figs. 1, 2.
—— Soldanii, *D'Orb.* Modèles, No. 36.
—— umbilicata, *D'Orb.* Ann. Nat. Sci. vol. vii. p. 278. No. 4, and Mém. Soc. Géol. Fr. vol. iv. pl. 3. figs. 4–6.
—— orbicularis, *D'Orb.* Modèles, No. 13.

Rotalia affords us a good example of the parallelism that may be traced between the members of one and another Foraminiferal species (just as occurs in other natural groups). Thus, contrasted with *Polystomella*, we have an interesting series of representative forms.

Parallelism of ROTALIA BECCARII *and* POLYSTOMELLA CRISPA.

Varieties of ROTALIA BECCARII.	*Varieties of* POLYSTOMELLA CRISPA.
Rotalia Schrœteriana, *Parker and Jones.*	Polystomella craticulata, *Fichtel and Moll.*
—— Beccarii, *Linn.* (large typical form).	—— crispa, *Linn.*
—— ammoniformis, *D'Orb.* (flat var. Rimini).	—— macella, *Fichtel and Moll.*
—— Beccarii, *Linn.* (small smooth var.).	—— striato-punctata, *Fichtel and Moll.*
—— dentata, *Parker and Jones.*	—— strigilata (var. β), *Fichtel and Moll.*
—— Soldanii, *D'Orb.*	—— (Nonionina) asterizans, *Fichtel and Moll.*
—— orbicularis, *D'Orb.*	—— (Nonionina) pompilioides, *Fichtel and Moll.*
—— (Calcarina) pulchella, *D'Orb.*	—— unguiculata, *Gmel.*
—— (Asterigerina) lobata, *D'Orb.*	—— (Nonionina) stelligera, *D'Orb.*

The nearness of the two specific groups is also seen in our new *Rotalia craticulata* (Plate XIX. fig. 12) being separable from *Polystomella crispa* chiefly by its want of symmetry; and, further, *R. Schrœteriana* passes into *R. craticulata* by a greater differentiation of the canal-system, which approaches its most perfect condition in the higher *Polystomellæ*.

Rotalia Beccarii, Linn., sp. Plate XVI. figs. 29, 30 (North Atlantic).

Figs. 29 & 30 present a strongly granular condition on the lower surface, and may be said to be passing into the smaller varieties that belong to deep water; indeed, they are intermediate between the common *R. Beccarii* of shallow water and the variety known as *R. Soldanii*, D'Orb. (Modèles, No. 36), that inhabits deep water. With flattened and adpressed chambers on the upper side, and without granules on the lower, figs. 29 & 30 would be *R. Soldanii*; such modifications are common. *R. Beccarii* passes into *R. Soldanii* in deep seas everywhere; but in hot seas it also passes into the large, conical, craticulate form (*R. Schrœteriana*, Parker and Jones) with pseudopodial passages, as in *Polystomella*.

Both in its estuarine and its abyssal varieties *R. Beccarii* is feeble, being delicate in shell and small in size. Its smallest and most abyssal variety is *R. orbicularis*, D'Orb. (fig. 34), which is not abundant. In about 100 fathoms *R. Soldanii*, with a diameter three times as great as that of *R. orbicularis*, is abundant enough, and is of stronger make. The shell becomes larger, more vesicular and more granular in the best habitat of *R. Beccarii* (20 to 40 fathoms in warm seas); and in shallow waters it is smaller (of the size of *R. Soldanii*), less strong in its structure, even more vesicular, and extremely abundant (even in some brackish waters).

Rotalia Beccarii from the Lido (Venice) and Rimini, both on the Adriatic, is very smooth and complanate (although large and well-developed), compared with specimens in the same latitude on the *western* shores of Italy and in fossil deposits (formed in shallow water) near Sienna; whilst the same species in the south-eastern parts of the Mediterranean has much thicker and more granular varieties than those in the *west* of Italy, and becomes very like the great *Rotalia Schrœteriana*, Parker and Jones (Ann. Nat. Hist. 3 ser. vol. v. p. 68, and Carpenter's 'Introd. Foram.' p. 213, pl. 13. figs. 7–9).

As we approach our own shores from the Mediterranean area, *Rotalia Beccarii* becomes gradually smaller but is still numerous: to the north it deteriorates more and more.

Rotalia Beccarii is rare and small at 78 fathoms on the Irish marginal plateau.

Rotalia Beccarii, Linn., sp., Var. *Soldanii*, D'Orbigny, sp. Plate XVI. figs. 31–33 (North Atlantic).

This may be described as *Rotalia Beccarii* becoming flush-chambered, conical (flat above), with a strong shell: in this form it inhabits deep water, about 100 fathoms (from 50 to 300 fathoms). D'Orbigny illustrated *R. Soldanii* by his Model, No. 36.

It is the *isomorph* of *Pulvinulina Micheliniana* and of *Planorbulina* (*Truncatulina*) *refulgens*, which are the deep-sea forms of their respective species.

R. Soldanii is rare and small at 43 fathoms, rather rare and middle-sized at 223 fathoms, and common and middle-sized at 415 fathoms, on the western plateau. It is rare and small at 1776, 2035, 2050, and 2350 fathoms in the abyssal area.

It is very common in the Mediterranean (at 100 fathoms), and fossil in the Subapennine clays. Generally it is not so flat at the top as our figured specimens are; but the upper faces of the cells are convex and separated by sulci (see D'Orbigny's Model).

Rotalia Beccarii, Linn., sp., Var. *orbicularis*, D'Orbigny, sp. Plate XVI. fig. 34 (North Atlantic).

This extremely delicate and minute abyssal variety of *R. Beccarii* is but little removed from *R. Soldanii*; but it is smaller, and has its upper face still flatter and smoother than in *R. Soldanii*. It is in shape half an oblate spheroid, having the upper side flat, the lower forming a low rounded cone. It may be said to be the starved abyssal variety of its species. It occurs, but sparsely, in deep-sea soundings in all latitudes—tropical to north-temperate; and it has been brought up from even 1000 fathoms and more, retaining its exquisite salmon-coloured sarcode.

D'Orbigny got his specimen, illustrated by Model No. 13, from the Adriatic.

The best localities for it are the Red Sea, where it has degenerated from *R. ornata* and *R. Schrœteriana*, and in the Mediterranean area, where it is ancestrally related to *R. Beccarii*. It becomes extremely small, one of the smallest even among starved Foraminifera; and, as such, is very rare at Shetland and in the Irish Sea (Brady).

In the abyssal area of the Atlantic it occurs very rare and very small at 1950 fathoms.

Genus PULVINULINA.

Pulvinulina repanda, Fichtel and Moll, sp. (Varieties). Plate XIV. figs. 12–17 (Arctic); Plate XVI. figs. 35–51 (North Atlantic).

Pulvinulina repanda is the type of a group of *Rotalinæ*, as above mentioned (page 378), of which we have here five varieties. Each of these belongs to a separate subspecific group; and, though they are few among many, yet they are of considerable importance in their several sub-groups, and may well serve as a basis for a general account of *Pulvinulina repanda* specifically considered.

P. repanda, when well developed, has its shell-structure dense and minutely perforated, compared with that of other *Rotalinæ*; more so than *Rotalia Beccarii* and *Calcarina Spengleri*, and much more so than *Discorbina Turbo* and *Planorbulina farcta.* In the delicacy of its tubuli (almost as fine as those of dentine) it rivals *Nummulina* and *Heterostegina*; whilst the loose coarse structure of some of the larger specimens of *Discorbina* and *Planorbulina* remind us of that of the Echinoderm and Madrepore.

Pulvinulina is most apt to take on an extra growth of shell-matter on the septal lines and the margins of the shell (limbation), and among its very numerous varieties there are many that are strongly limbate, and are more or less compact in growth; whilst other varieties are delicate, and become thin, outspread, Spirilline, and vermiculate. The shell has from seven to nearly thirty cells, with single septa and but little trace of the canal-system: it is rarely prickly; the umbilicus is often ornamented by granules, or by a boss, or a star of shelly matter; the aperture is a large fissure, often arched, and notched; and the septal face often bears numerous coarse subsidiary perforations. The shell is usually biconvex; the upper side the thickest; the margin more or less angular and subcarinate; some varieties are complanate, with square edges, as in RŒMER's figures of *P. caracolla* and its allies from the Hils Clay and the Gault; similar forms to these occur also in the Kimmeridge Clay of Kimmeridge.

We may divide the *Pulvinulinæ* into five groups, as follows:—

First Group, or that including *P. repanda* proper.—In its typical form *P. repanda* is a spiral coil of chambers, forming a low conical shell, showing the spire, with a more or less open umbilicus; some of the older chambers usually having limbate septa. The shell has generally an irregularly oblong form; the chambers rarely forming a symmetrical disk, never flush at the edges, but set on loosely, and usually increasing in size in a somewhat rapid ratio; they present often a curved or sickle-shaped outline both above and below, or are curved and narrow above, broad and irregularly triangular below. The umbilical portions of the chambers are generally very attenuate, fitting neatly as they converge to the centre. Occasionally these lobes are separated by narrow chinks; sometimes they are deficient, leaving a large umbilical gap. The septal face is either gently convex, or flat; in the latter case it is perforated with proportionally large holes. The aperture is a large arched slit, occasionally notched at its upper margin. Granulate ornament is not uncommon on the upper surface of the shell; below, exogenous matter may either fill the umbilical cavity, or affect the borders of the umbilical lobes, even to their union by a bridge-like growth. Limbation is seldom absent from the border of the shell; frequent on the older part of the spire; and not uncommon with the later chambers. Figs. 101–103 in Professor WILLIAMSON'S 'Monograph of British Recent Foraminifera' represent a common condition of this typical form.

The members of Group No. 1 inhabit depths of about 10 to 100 fathoms. The varieties affecting the shallow water are less neat in their make than those of greater depths.

Second Group, characterized by *P. Auricula* and *P. oblonga*.—In this group the shells are far more oblong in shape, from the very rapid increase of size of the chambers; and, as a rule, they are much more delicate and frail than the foregoing, although some small deep-sea varieties of this subtype are unusually dense. The septa and borders are rarely limbate. The septal face of the last chamber is usually drawn out and inflated, but narrow, and, by an umbilical process, overlaps the alar terminations of the older chambers. This feature has caused D'ORBIGNY to class several varieties of this subtype as species of his genus *Valvulina*. In some cases a portion of the septal face near the umbilicus is flattened and pertused; and this feature is usually associated with some degree of limbation of the upper septal lines. The whole of the septal face is flattened and coarsely perforate in certain forms lying between *P. Auricula* and *P. repanda*. The aperture is similar to that in Group No. 1; but occasionally there is a large subsidiary notch. The umbilical lobes terminate in a similar manner to what obtains in the typical group; and the umbilicus, as in the former, may either be closed, by the meeting of the lobes, or remain slightly open, or be largely excavate. The varieties in which the last-named feature occurs are small deep-sea forms, having dense shell-tissue, a flattened hispid upper surface, with flush chambers; the under surface being gently convex and highly polished.

As a rule, in each of the subgroups of *P. repanda*, here described, the thick-set varieties are those that inhabit deep water.

The members of the Group No. 2 have their best home at 50 to 70 fathoms; but they range from shallow water (algal zone) to 500 fathoms or more.

The Third Group, including *P. Menardii*.—This is an assemblage of closely related varieties, differing however considerably in feature. Some are very flat and scale-like, some conical, some biconvex. The flat forms have usually a somewhat oblong outline; but the members of this group are mostly circular, with indented septal lines; the chambers are sometimes triangular on both surfaces; though sometimes narrow and curved, or oblong, or even square above and more or less triangular below. *P. Menardii* and its nearest allies are margined and limbate on the upper surface, and often granular, scabrous, or hispid. These features are less striking in other varieties which pass gradually into feebly marked, smooth, thick, small, untypical forms. The septal face is still large in this group, gently convex or flat; sometimes sinking in at a spot near the aperture, which is often boldly notched. The chambers of these shells are fewer than in the "repanda-" or "type-group"; but in the better developed specimens they have the same rapid increase of size, with the same neat convergence of the umbilical lobes; the lines between them, however, being usually straighter. The conditions of the umbilicus resemble those of the typical group; but the contracted form of the shell, in certain varieties, raises up the umbilical portions of the chambers into the apex of a cone, the base of which is the neat and almost flat spiral surface.

The members of this group, all of which are mutual companions, are obtained from abyssal depths, 100 to 2700 fathoms.

Fourth Group, characterized by *P. Schreibersii.*—These shells have more numerous chambers than we find in the foregoing groups, nor do they enlarge with age so rapidly. The lower surface shows but few chambers (5–11), in contrast with those seen above (15–30); whilst in groups Nos. 1–3, all except the four or five earliest chambers are seen on the umbilical as well as on the spiral surface, on account of the spire being subdiscoidal, whilst in *P. Schreibersii* and its allies the spire is helical or subturreted. There is also a greater tendency to limbation (exogenous shell-growth on the septal lines and the margin), especially about the umbilicus, where a knob, a group of granules, or a star-like ornament is not unusual; hence this may be termed the "stellar" group. These, moderately deep-sea forms for the most part, have often the thickest shells of any among the subtypes, especially *P. Schreibersii* itself, as found in the muds of the Gulf of Suez at about 40 fathoms. This group has a very extensive bathymetrical range.

Fifth Group, with *P. elegans* as the leading form.—This is closely allied to the last group in its general features, and may be said to represent a further development of its peculiarities. We have here a series of neat, compact, more or less biconvex, and for the most part limbate *Pulvinulinæ.* The limbation is less constant on the upper (spiral) than on the lower surface, on which latter a symmetrical wheel-like ornament is often found, imitating such as occurs on some nautiloid *Cristellariæ.* On the upper surface the limbation is sometimes strongly developed, both on the septal lines and the margin, and in some cases (*P. D'Orbignii,* Rœmer, and *P. ornata,* Roemer) masks the spire altogether. On the other hand the limbation may be but slight; and in *P. Cordieriana,* excepting as regards the umbilical boss, it is nearly obsolete. Some subvarieties of *P. elegans* itself appear with little exogenous or limbate ornament.

In this group the shell is polished to the utmost; and in the same gatherings from very deep water *P. Menardii* will be in its roughest condition and *P. elegans* will be highly enamelled and glistening. It is always neat and nautiloid. The group ranges from 70 to 1000 fathoms.

Scheme of the chief Members of the Genus PULVINULINA.

1st Group. The type or repanda group. 10–100 fathoms.	*vermiculata*, D'Orb. (after Soldani). Carpenter, Introd. pl. 13. figs. 4–6.
	sinuata, Fichtel and Moll, sp., Test. Micros. pl. 10. figs. *a–c.*
	REPANDA, Fichtel and Moll, sp., Test. Micr. pl. 3. figs. *a–d.* (The type of *Pulvinulina.*)
	pulchella, D'Orb., sp., Modèles, No. 71.
	punctulata, D'Orb., sp., Modèles, No. 12.
	Caribœa, D'Orb., sp., For. Cuba, pl. 5. figs. 1–3.
	Boueana, D'Orb., sp., For. Foss. Vien. pl. 7. figs. 25–27.
	concentrica, Parker and Jones; Soldani, Test. i. pl. 37. fig. B.
2nd Group. Auricula or oblonga group. 10–500 fathoms (70 fathoms best).	*Auricula*, Fichtel and Moll, sp., pl. 20. figs. *a–f.*
	Sagra, D'Orb., sp., For. Cuba, pl. 5. figs 13–15.
	oblonga, Williamson, sp., Monogr. pl. 4. figs. 98–100.
	Brongniartii, D'Orb., sp., For. Foss. Vien. pl. 8. figs. 22–24.
	Hauerii, D'Orb., sp., For. Foss. Vien. pl. 7. figs. 22–24.
	contraria, Reuss, sp., Zeitsch. Deutsch. Geol. Ges. iii. pl. 5. fig. 37, *a, b, c.*
	deformis, D'Orb., sp., For. Cuba, pl. 4. figs. 9–11.
	inœqualis, D'Orb., sp. (*Valvulina*), For. Amér. Mér. pl. 7. figs. 10–12.
	oblonga, D'Orb., sp. (*Valvulina*), For. Canar. pl. 1. figs. 40–42.
	excavata, D'Orb., sp. (*Valvulina*), For. Canar. pl. 1. figs. 43–45.
	scaphoidea, Reuss, sp., Neue For. Oester. Tert. pl. 47. fig. 3, *a, b, b'.*
	Auris, D'Orb., sp. (*Valvulina*), For. Canar. pl. 2. figs. 15–17.
3rd Group. Menardii group, Abyssal group. 100–2700 fathoms.	*Menardii*, D'Orb., sp., Modèles, No. 10.
	cultrata, D'Orb., sp., For. Foss. Cuba, pl. 5. figs. 7–9.
	umbonata, Reuss, sp., Zeitschr. d. g. G. iii. pl. 5. fig. 35, *a–c.*
	crassa, D'Orb., sp., For. Craie bl. Fr. pl. 3. figs. 7, 8.
	dubia, D'Orb., sp., For. Cuba, pl. 2. figs. 29, 30, pl. 3. fig. 1.
	Canariensis, D'Orb., sp., For. Canar. pl. 1. figs. 34–36.
	pauperata, Parker and Jones, nov. sp. Plate XVI. figs. 50, 51.
	Micheliniana, D'Orb., sp., For. Craie bl. Fr. pl. 3. figs. 1–3.
	nitida, Reuss, sp., Böhm. Kreid. pl. 12. fig. 20, *a, b.*
	truncatulinoides, D'Orb., sp., For. Canar. pl. 2. figs. 25–27.
4th Group. Schreibersii group, Stellar group. 30–2700 fathoms.	*Schreibersii*, D'Orb., sp., For. Foss. Vien. pl. 8. figs. 4–6.
	Antillarum, D'Orb., sp., For. Cuba, pl. 5. figs. 4–6.
	concava, Reuss, sp., For. Ostalp. Kreid. pl. 26. fig. 3, *a–c.*
	Badensis, Czk., sp., Fos. For. Wien, pl. 13. fig. 1–3.
	Peruviana, D'Orb., sp., For. Am. Mér. pl. 2. figs. 3–5.
	Karsteni, Reuss, sp., Zeit. d. g. G. vii. pl. 9. fig. 6, *a–c.*
	squamiformis, Reuss, sp., For. Kreid. Ostalp. pl. 26. fig. 2, *a–c.*
	Alvarezii, D'Orb., sp., For. Am. Mér. pl. 1. fig. 21, pl. 2. figs. 1, 2.
	spinimargo, Reuss, sp., Neue For. Oester. Tert. pl. 47. fig. 1, *a, b.*
	Patagonica, D'Orb., sp., For. Amér. Mér. pl. 2. figs. 6–8.
5th Group. Elegans group, strongly limbate. 70–1000 fathoms.	*elegans*, D'Orb., sp., Ann. Sc. Nat. p. 276, No. 54.
	caracolla, Nils., sp., Rœmer's Nord-Deuts. Kreid. pl. 15. fig. 22.
	Partschiana, D'Orb., sp., For. Foss. Vienn. pl. 8. figs. 1–3.
	Berthelotiana, D'Orb., sp., For. Canar. pl. 1. figs. 31–33.
	Cordieriana, D'Orb., sp., For. Craie bl. Paris, pl. 3. figs. 9–11.
	ornata, Nils., sp., Rœmer's Nordd. Kr. pl. 15. fig. 25.
	D'Orbignii, Nils., sp., Rœmer's Nordd. Kr. pl. 15. fig. 24.
	stelligera, Reuss, sp., For. Kreid. Ostalp. pl. 25. fig. 15, *a–c.*
	Partschiana, D'Orb. sp., var., Borneman, Fauna Septar.-Thones Hermsd. pl. 16. fig. 6, *a–c.*

Pulvinulina repanda, Fichtel and Moll, sp., Var. *punctulata*, D'Orbigny, sp. Plate XIV. figs. 12, 13 (Arctic).

Though flatter, this is essentially the same as *Pulvinulina punctulata*, D'Orb., sp., Modèle, No. 12. When smaller, more limbate, and less compact in growth, it passes into more ordinary varieties, such as *P. repanda*, Fichtel and Moll, sp. (*Rotalina concamerata*, Williamson, Monogr. pl. 4. figs. 101–103).

In our former description of the Norwegian Foraminifera, we mistook this large variety for a large growth of *Discorbina vesicularis*, Lam., sp. It is represented, in Messrs. MacAndrew and Barrett's dredgings, by one specimen from sand at West Fiord (Nordland) from 60 fathoms depth, and eight specimens that occurred on sponge from 100 fathoms at Vigten Island, Inner Passage (Drontheim).

It lives also in the Adriatic (D'Orbigny) and at Orotava (Canaries); and is abundant and large off Sicily, and in the Levant, and in many other parts of the world at moderate depths. The huge specimens from the Crag, larger than our Norwegian specimens, lean more to the looser and few-celled type figured by Williamson.

Pulvinulina repanda, Fichtel and Moll., Var. *Menardii*, D'Orbigny, sp. Plate XVI. figs. 35–37 (North Atlantic).

Pulvinulina Menardii, D'Orb., Modèles, No. 10, is a deep-sea form of *P. repanda*; it is in best condition at from 100–500 fathoms, but lives well at even three miles depth; in shallow water (algal belt) it becomes either conus-shaped, or much depressed with a large keel (*P. pauperata*, Parker and Jones, Plate XVI. figs. 50, 51); whilst *P. repanda* (the type) becomes vermiculate, abounding in the Mediterranean as *Pulvinulina vermiculata*, D'Orb., sp. (*Planorbulina vermiculata*, D'Orb., Ann. Sc. vii. p. 280, No. 3; after Soldani). At from about 30–100 fathoms in the Mediterranean the typical *P. repanda* abounds; and in the same sea the obtusely conical *P. Micheliniana* represents the species abundantly at from 500–1500 fathoms on muddy tracts, whilst the flatter form (*P. Menardii*) common in the depths of the great oceans seems to be wanting there. *P. Micheliniana* is also potent in the Arctic seas and North Atlantic; and is fossil in great numbers in the Chalk.

P. Menardii is generally limbate and granulo-aciculate; the specimens before us are more or less limbate and have roughish shells. They are not numerous, nor have they attained the fulness of size and beauty that belong to the species in lower latitudes; the further north, the poorer they are; for those in the Mid-Atlantic (Dayman) are generally somewhat larger than those in the North Atlantic (Wallich's Collection); and this is the case with other species and varieties. In the Atlantic the proportion of *Pulvinulinæ* to the Foraminiferal fauna is perhaps not $\frac{1}{10}$th of what will be found in the deep water of tropical and subtropical seas.

In the North Atlantic *Pulvinulina Menardii* is widely distributed. On the marginal plateau off Ireland it is rare and small in the shallow, less rare and larger in the deeper part. It is of middle size and common in the "Celtic" portion, and rather rare throughout the "Boreal" portion of the abyssal tract (1400–2300 fathoms); and neither large nor common at 329 fathoms north of Newfoundland Bank. Mr. Brady has some fine specimens from the Irish Sea.

Pulvinulina repanda, Fichtel and Moll, sp., Var. *Menardii*, D'Orbigny, sp., Subvar. *Canariensis*, D'Orbigny, sp. Plate XVI. fig. 47–49 (North Atlantic).

Pulvinulina Canariensis, D'Orb., For. Canar. pl. 1. figs. 34–36, is a dwarf form of *P. Menardii*, common but distinct among the larger specimens in deep water, and widely distributed from the north to the Tropics. It is more attenuate than well-grown specimens of the subtype (*P. Menardii*), and usually is very imperfectly limbate. D'Orbigny's figure has a limbate upper surface, and the mouth more patent on the lower plane than in our specimen: but these modifications are of continual occurrence. *P. Canariensis* may be said to be a starved form among well-fed ones (as happens with *Globigerinæ* and many other Foraminifera); yet it is well to keep it apart with a name, as, should it occur without *P. Menardii*, it would bespeak an unfavourable habitat.

In the North Atlantic *Pulvinulina Canariensis* is wide-spread. On the eastern marginal plateau it is common and small at 78 fathoms, rare and small at 338 fathoms, and rare and middle-sized at 415 fathoms. In the "Celtic" abyssal tract it is rather common; throughout the "Boreal" portion also (1400–2300 fathoms) it is rather common, but smaller. North of the Bank, at 161 fathoms, and in Trinity Bay, it is rare and small.

Pulvinulina repanda, Fichtel and Moll, sp., Var. *Menardii*, D'Orb. sp., Subvar. *pauperata*, nov. Plate XVI. figs. 50, 51 *a*, 51 *b* (North Atlantic).

Pulvinulina pauperata is rare, usually small, and nearly symmetrical; found at great depths (2000 fathoms) in both high and low latitudes, and is often much larger in the latter than in the former. It presents a feeble, and, as it were, accidental condition, in which the thin film of sarcode surrounding the few feebly marked chambers has been calcified beyond their verge. Though it is very small here, we have seen this variety (from subtropical seas) as large as the largest *P. Menardii*. In tropical seas (Tropical Atlantic and Indian Ocean) it is large but rare.

This variety occurs in company with *P. Menardii* and *P. Canariensis*, which are found taking on a margined condition, with feebly developed chambers, thus connecting the depauperated variety under notice with themselves. Comparing this deep-sea attenuated form with those of shallow water, we see that the latter become vermiculate, losing the power of forming separate chambers.

P. pauperata is rare in the North Atlantic (the figured specimens are all we met with); in the "Boreal" tract, towards Newfoundland Bank it is middle-sized at 1450 fathoms; and in the Abyssal "Celtic" tract it is small.

Pulvinulina repanda, Fichtel and Moll, sp., Var. *Menardii*, D'Orbigny, sp., Subvar. *Micheliniana*, D'Orbigny, sp. Plate XIV. fig. 16 (Arctic); Plate XVI. figs. 41–43 (North Atlantic).

This small compact conical *Pulvinulina* occurs in deep water. Its deepest known habitat is at 2700 fathoms (South Atlantic). It is very common in the North Atlantic. In the Mediterranean it flourishes at 400–500 fathoms on muddy bottoms, being larger there than our figured specimens; it then takes the place of *P. Menardii*. In shallow water it degenerates into bizarre varieties.

P. Micheliniana abounds fossil in the Chalk and Gault, and was first described by D'Orbigny in his Memoir on the Foraminifera of the White Chalk of Paris, Mém. Soc. Géol. de France, vol. iv. pl. 3. figs. 1–3, together with another closely allied variety of *P. Menardii* (*P. crassa*, D'Orb., sp., *loc. cit.* figs. 7–8); as well as a third variety (*P. Cordieriana*, D'Orb., *loc. cit.* figs. 9–11), a member of the *P. elegans* group of *P. repanda*.

At the Hunde Islands this usually deep-sea form, *P. Micheliniana*, is represented by rare and small individuals at 25–30 fathoms.

Plate XVI. figs. 41–43 (North Atlantic).

From the Arctic Ocean we had but very few specimens of *P. Micheliniana*, owing to the paucity of deep-sea soundings. In the North Atlantic it is very common; and generally very rough or scabrous in its shell-tissue; in fact it may be said to be here *P. truncatulinoides*, D'Orb., sp. (For. Canar. pl. 2. figs. 25–27), and the two forms are scarcely worth separating by distinct names.

On the Irish marginal plateau it is rare and small in the shallow, rather common and large in the deep parts. In the "Celtic" abyssal depths it is common and rather large; but in the "Boreal" tract (at upwards of 2000 fathoms) it is smaller and rarer; and nearer to the Bank it is rare and small at 1450 fathoms.

Pulvinulina repanda, Fichtel and Moll, sp., Var. *Karsteni*, Reuss, sp. Plate XIV. figs. 14, 15, & 17 (Arctic); Plate XV. figs. 38–40 (North Atlantic).

This is a neat, many-chambered, moderately conical variety of *P. repanda*, with some degree of limbation bordering the chambers, especially beneath, where a wheel-like system of exogenous shell-matter characterizes the shell.

This occurs in each of the soundings at the Hunde Islands (Sutherland), and is common and of middling size in most of them. It is found also at 150 fathoms in Baffin's Bay, lat. 76° 30′, long. 77° 52′ (Parry). It is small at Shetland (Brady).

Plate XVI. figs. 38–40 (North Atlantic).

Pulvinulina Karsteni, Reuss, sp. (Zeitsch. deutsch. geol. Ges. 1855, vol. vii. pl. 9. fig. 6), is usually smaller and more conical than *P. Menardii*, also rounder, quite smooth, and free from the limbation on its upper face, which is present in *P. Menardii*; on its lower face, however, the margin and sometimes the septal furrows are limbate (a feature usually wanting in *P. Menardii*); an umbilical knob is sometimes present also; and with this as a nave, and the septa for spokes, the shell has a wheel-like aspect.

A closely allied and still more conical form (*R. Schreibersii*, D'Orb., For. Foss. Vien., pl. 8. figs. 4–6), having a stellate umbilicus and neatly radiating septa, is the leading member of the group of varieties of *P. repanda*, among which *P. Karsteni* is arranged; it is found recent in the muds of the Gulf of Suez and the Red Sea (at 40 fathoms and thereabouts), and is fossil in the Tertiary beds of Tuscany and the Vienna Basin.

Though differing from it a little in details, the North Atlantic specimens here figured are still more like REUSS's figure than is the Arctic specimen, Plate XIV. fig. 15, which in some respects is nearer to D'ORBIGNY's figure of *Pulvinulina Antillarum* (Foram. Cuba, pl. 5. figs. 4–6), an allied form. REUSS's figure is intermediate to the Arctic and North Atlantic specimens.

In Trinity Bay *P. Karsteni* is rare but large at 133 fathoms, lat. 48° 18′, long. 52° 56′. It occurs at 2700 fathoms in the South Atlantic.

Pulvinulina repanda, Fichtel and Moll, sp., Var. *elegans*, D'Orbigny, sp. Plate XVI. figs. 44–46 (North Atlantic).

Our specimens show an unusually non-limbate condition of *Pulvinulina elegans*, which is a subtype of the *P. repanda* group, and was chosen as a species by D'ORBIGNY from amongst SOLDANI's figures (Sagg. Oritt. pl. 2. fig. 2, R; Ann. Sc. Nat. vii. p. 276, No. 54). *P. elegans* has a neat, smooth, and highly polished shell, varying always in limbation and conicity. The excess of characters in this subtype is found in *P. caracolla*, Rœmer, sp., *P. ornata*, Rœm., sp., and *P. D'Orbignii*, Rœm., sp. (Norddeutsch. Kreid. pl. 15. figs. 22, 24, 25), of the Cretaceous deposits. In our specimens we have nearly an equality with *P. Partschiana*, D'Orb., sp. (For. Fos. Vien. pl. 8. figs. 1–3), excepting as to limbation: and, further, we may regard our specimens as feeble forms of *P. elegans* with a tendency towards *P. umbonata*, Reuss (Zeitsch. d. g. Ges. vol. iii. pl. 5. fig. 35).

P. elegans abounds at from 100 to 200, and even to 300 fathoms. Forms intermediate to *P. elegans* and *P. Karsteni* are common in clays of the Secondary Formations (Oxford and Kimmeridge Clays, and Upper Trias of Chellaston).

In the North Atlantic *P. elegans* is common, but small, at 78 fathoms on the eastern plateau; rare and small at 1660 fathoms in the abyssal area ("Boreal"); but rather common and larger at 1450 fathoms. It is small at 15 fathoms in the Irish Sea (BRADY).

Genus SPIRILLINA.

Spirillina vivipara, Ehrenberg. Plate XV. fig. 28 (Arctic).

For an account of *Spirillina*, see Ann. Nat. Hist. 2 ser. vol. xix. p. 284, and CARPENTER's Introduct. Foram. p. 180. There is often a difficulty in distinguishing this form from its isomorph, the vermiculate *Pulvinulina*; the numerous and non-segmented whorls decide the doubt in this instance.

Sp. vivipara is rare anywhere, and always small. We have it in the mixed sands from Norway (MACANDREW and BARRETT), and from 60 to 70 fathoms, Hunde Islands (Dr. SUTHERLAND); in deep water it is represented by the better developed *Sp. margaritifera*, Williamson.

Genus PATELLINA.

Patellina corrugata, Williamson. Plate XV. fig. 29 *a*, 29 *b*, 29 *c* (Arctic).

This species has been well figured and described by Professor WILLIAMSON (Monogr. p. 46, pl. 3. figs. 86–89); see also CARPENTER's Introd. Foram. p. 230.

We have *P. corrugata* from the Hunde Islands (Dr. SUTHERLAND's dredgings), at from 30 to 70 fathoms; where it is common and small throughout. Professor WILLIAMSON had it from the same source, and found it in several sands on the British coasts. It is present in most sea-beds that are rich with Foraminifera, from the littoral zone down to 500 fathoms; but is rarely in great abundance.

Genus NUMMULINA.

Nummulina perforata, Montfort, sp., Var. *planulata*, Lamarck. Plate XIV. figs. 45 *a*, 45 *b* (Arctic).

From the Red Sea FICHTEL and MOLL got two little *Nummulinæ* very similar to the specimens before us; Professor WILLIAMSON also has similar specimens from the British coast; and in Mr. JUKES's Australian dredgings *Nummulinæ* of like character abound, but larger, and passing into *Operculinæ*. These are degenerate forms of *Nummulina planulata*, once so abundant in the Eocene (or Nummulitic) Tertiary period, and existing still later in, at least, the Vienna area (Middle Tertiaries). *N. planulata* itself is a simple form of the better-developed *N. perforata*, Montfort, which in its extreme growth became *N. nummularia*, Brug. (*N. complanata*, Lam.).

This small form of *N. planulata* (subvar. *radiata*, Fichtel & Moll.) is rather common at the Hunde Islands in 25 to 30 fathoms. See also Ann. Nat. Hist. ser. 3. vol. v. pp. 105–107.

Besides the above-mentioned localities, the Abrolhos Bank in the South Atlantic and Bombay Harbour are places where *N. planulata* has been found.

Nummulina perforata, Montf., sp., Var. (*Operculina*) *ammonoides*, Gronovius, sp. Plate XIV. figs. 44 *a*, 44 *b* (Arctic); Plate XVII. figs. 62, 63 (North Atlantic).

This is the diminutive and northern representative of the much larger *Operculina complanata*, Defrance, sp., which is a varietal form of *Nummulina*. The last (*Nummulina*) is but poorly represented now-a-days (as far as our knowledge goes); but *Operculina* is sometimes almost, if not quite, as large in the Australian, New Zealand, and Philippine seas as ever it was in the Cretaceous, Eocene, and Miocene times. See Ann. Nat. Hist. 3 ser. vol. viii. p. 220, &c. Dr. CARPENTER has specially studied the structure of *Operculina*, Phil. Trans. 1859; and Introd. Foram. p. 247, &c. pl. 11.

Operculina ammonoides is very common in the mixed sands from Norway (MACANDREW and BARRETT). On the Irish plateau of the North Atlantic it is common at 43, 78, 90, 223, and 415 fathoms; and rare at 200 fathoms. It abounds in the North British seas; in Professor WILLIAMSON'S Monogr. it appears under the name of *Nonionina elegans*. It is found also in the Mediterranean and Red seas, and at Australia and Fiji.

Genus POLYSTOMELLA.

Polystomella crispa, Linn., sp. Plate XIV. fig. 24 (Arctic); Plate XVII. fig. 61 *a*, 61 *b* (North Atlantic).

Polystomella comprises many closely allied forms, which, on account of their apparent dissimilarity, have been usually grouped under *Nonionina* and *Polystomella*. Their differences, however, are not sufficient to destroy the value of their correspondences in structure. The shells are symmetrically discoidal, either lenticular or subglobular, more or less Nautiloid, having from about fifteen to thirty, or many more, neatly fitting, more or less sickle-shaped chambers, with the aperture at the base of the septa; and this may be either a simple low arch-like opening, or it may be crossed by bars so as to be a grating, or a row of pores: this multiplicity of stolon-passages is the condition which gave the name to this genus in particular, and to the "Foraminifera" altogether*. The gradations from the simply notched septum of some *Nonioninæ*, to the barred apertures of others (*N. Faba*, Fichtel and Moll, sp.), and thence to the curved row of pores in *Polystomella* proper, are very well marked in numerous modified varieties. Another feature of the genus is the masking of the septal furrows of the shell, by "retral processes," or lobes on the posterior edges of the chambers, connected by bridges of exogenous shell-matter to the fronts of the preceding chambers, and thus forming pits or "fossettes" along the septal lines. The mouths of the canal-system open into the "fossettes;" but the latter are not a part of that system. The processes and the bridges or bands vary much in thickness, in proportion to the higher standing of the more strongly grown varieties of this species; and this increase of shell-matter on the surface of the shell, until it has a sculptured or basket-work appearance, accompanied more or less with keel, spines, and umbones, is also traceable through very gentle gradations.

The "bridges" occur freely, in *P. Arctica* and other forms, when the retral lobes are nearly obsolete, and thus they form crenulations on the edges of the chambers.

As the soft parts of the animal afford us no distinctive specific characters, all these modifications of shell-structure fall into a series of varietal differences among the individuals of one species, subject to different conditions of existence and consequent modes of growth.

In its symmetry of shell *Polystomella* resembles *Nummulina*, but it has a canal-system different from that of the latter; and, though the aperture in *Nummulina* is in the same position (at the base of the septum) as in Nonionine *Polystomellæ*, yet the very slight attempt to modify it by subsidiary pores in *Nummulina* is sufficient to indicate an inability to depart from a special plan. The feebler *Polystomellæ* (*Nonioninæ*) are, with their neat shell and simple aperture, isomorphic with some *Nummulinæ*, especially if we compare some of the more strongly limbate of the former with the small "Operculine" or "Assiline" varieties of the latter (*Nonionina limba* compared with *Operculina ammonoides*); but the shell-tissue is more dense and tubuliferous in the latter (as in *Nummulina* proper), and the perfect marginal rim and the canal-system are wanting in the former.

---

* As being distinct in so much from the *single-tubed* Cephalopods, with which they were classed.

Again, both in some of its higher (*Polystomella macella*) and lower forms (*Nonionina turgida*) *Polystomella* loses its horizontal symmetry, which *Nummulina* (except in some Operculine individuals) never does; the asymmetrical ally of *Nummulina* (*Amphistegina*) being sufficiently differentiated as to canal-system and other points to be regarded as specifically distinct.

The close linking of *Nonioninæ* with *Polystomella*, especially by means of the graduated subdivision of aperture, and modification of lateral fossettes, retral processes, and septal bridges, is too strong to be in any way antagonized by the merely isomorphic resemblances of the former with *Nummulina*; and "Nonionina" is rightly suppressed as a generic term, being merged in "Polystomella," which well represents the peculiar features of the fairly developed, but not exaggerated, natural type. See Ann. Nat. Hist. 3rd ser. vol. v. p. 103, &c.; CARPENTER'S 'Introduct.' p. 286, &c.

Scheme of the POLYSTOMELLÆ.

A. Canal-system, retral processes of the chambers, and the septal bridges and apertural bars, all highly developed.

Polystomella craticulata, Fichtel and Moll, sp.

B. Canal-system feebly developed; but the retral processes, septal bridges, and apertural bars perfect.

P. CRISPA, Linn., sp.
P. strigillata, Fichtel and Moll, sp.
P. unguiculata, Gmel., sp.
P. macella, Fichtel and Moll, sp., &c.

C. Canal-system, the septal bridges, and apertural bars well-developed, but the retral processes abortive.

P. Arctica, Parker and Jones.

D. Canal-system and retral processes feebly developed, but the bridges over the septal lines and the bars across the aperture perfect.

P. striatopunctata, Fichtel and Moll, sp., and *P. Faba*, Fichtel and Moll, sp.

E. Canal-system, retral processes, septal bridges, and apertural bars all abortive more or less.

Nonionina limba, D'Orb.
N. stelligera, D'Orb.
N. asterizans, Fichtel and Moll, sp.
N. Scapha, Fichtel and Moll, sp.
N. depressula, Walker and Jacob, sp.

F. Canal-system, retral processes, septal bridges, and apertural bars all obsolete: there may, however, be granular shell-growth on the umbilici.

N. granosa, D'Orb. *N. umbilicatula*, Montagu, sp. *N. turgida*, Williamson, sp.

Both the feeble (Nonionine) and the well-grown varieties of *Polystomella* are distributed very widely, but avoid great depths. The thick-shelled *P. craticulata* is found in tropical seas; the medium-conditioned *P. crispa* is extensively spread about in temperate seas; *P. Arctica* and *P. striatopunctata* are the best of the species found in cold seas. The *Nonioninæ* accompany their better-grown congeners; *N. asterizans* and *N. depressula* affecting temperate climates; *N. Scapha* and *N. umbilicatula* being found more often in the warmer seas.

Polystomella crispa stands midway between those *Nonioninæ* that begin to take on a barred aperture and perforated septal furrows, and those that have cribriform septa and a surface masked with septal bridges and other exogenous shell-matter; it is therefore a good type, showing the generic and specific characters without exaggeration. It has been well illustrated and described by WILLIAMSON, CARPENTER, and SCHULTZE; and its many modifications, in the recent and fossil state, have received as many names. In some Tertiary beds *P. crispa* is plentiful; and it abounds at the present day in temperate and warm seas.

We find *P. crispa* in the dredgings from the Hunde Islands (at 25 to 30 fathoms) rare and small; and very rare and small in the North Atlantic at 725 fathoms, north of the Newfoundland Bank.

Polystomella crispa, Linn., sp., Var. *Arctica*, nov. Plate XIV. figs. 25–30 (Arctic).

One of the varietal stages presented by the simpler *Polystomellæ* is characterized by double pores for the canals in lines along the septal furrows of the shell, an advance upon the simple single pores of *P. striatopunctata*, and an approach to the higher *Polystomellæ*. These double-pored furrows belong to a rounded, bun-like, Nonionine shell, with barred aperture, sparsely perforated septa, and a tendency to irregularity of growth; the neat, definite, lenticular, sharp-edged, discoidal shell of *Polystomella* proper being but poorly represented as yet. The essential characters, however, of pores in the furrows and septal apertures are not to be mistaken, although the retral processes of the chambers and the intervening fossettes are very rudimentary. The spiral lamina is finely perforate.

This form differing from the smaller *P. striatopunctata*, Fichtel and Moll, sp., in having double pores for its lateral canals, shows thus much a differentiation of the shell-structure in relation to the forking tubes, which are single in *P. striatopunctata* (figs. 31–34). With this exception, and with some additional apertures, *P. Arctica* keeps to the *simple* type; but it attains a semigigantic size, having a similar relation to *P. striatopunctata* that *P. craticulata* has to *P. crispa*.

One individual (fig. 27) shows a tendency to produce rough exogenous accumulations of shell-substance, as is the habit of *P. craticulata*.

P. Arctica is peculiar to the most northern seas, and occurs plentifully at the Hunde Islands at from 30 to 40 and 60 to 70 fathoms (SUTHERLAND) in company with *P. striatopunctata*. Mr. H. B. BRADY has found it in Mr. JEFFREYS's dredgings made at Shetland, in some abundance, and of a brown colour.

Polystomella crispa, Linn., sp., Var. *striatopunctata*, Fichtel and Moll, sp. Plate XIV. figs. 31–34 (Arctic); Plate XVII. fig. 60 *a*, 60 *b* (North Atlantic).

This is a smooth, round-edged, Nonionine shell, variable in its thickness and in the number of bridges over the septal furrows. The aperture is more or less divided by bars, and may have supplemental pores.

Individuals presenting two stages in this variety are described and figured by WILLIAMSON under the name of *Polystomella umbilicatula* and *P. umbilicatula*, var. *incerta*, Monograph, p. 42, &c., pl. 3. figs. 81, 82, 82 *a*. Some of our figures (Plate XIV. figs. 32–34) show but little of the septal markings; but in fig. 31, and Plate XVII. fig. 60, these are much more apparent, for the furrows are more distinctly bridged over by the posterior crenulation and retral processes of the chambers, and conspicuous fossettes are formed. SCHULTZE has also illustrated this form (EHRENBERG's *Geoponus Stella-borealis*, well figured by him in the Berlin Acad. Trans. 1841) and some near allies in his 'Org. Polyth.' pl. 6. figs. 1–9 (*Polystomella gibba*, *P. Stella-borealis*, and *P. venusta*).

P. striatopunctata is widely distributed in both warm and cold seas, but not in deep water. It occurs in Tertiary and Post-tertiary deposits, sometimes abundantly, and is a characteristic fossil of the Post-pliocene clays of Canada (DAWSON) and of the coast of Scotland (Quart. Journ. Geol. Soc. vol. xiv. p. 521, note).

We have *P. striatopunctata*, rather rare and small in the mixed Norwegian sands (MACANDREW and BARRETT's dredgings); in all Dr. SUTHERLAND's dredgings from the Hunde Islands (25–70 fathoms), where it is usually common and large. Also from Baffin's Bay (PARRY), lat. 75° 10′, long. 60° 12′, rare and very small; lat. 76° 30′, long. 77° 52′, 150 fathoms, common and middle-sized; lat. 75°, long. 59° 40′, 220 fathoms, very rare and very small. In the North Atlantic it is found on the eastern marginal plateau at 43 fathoms common and small; at 78 fathoms very rare and very small; at 223 fathoms rare and small; and north of the Newfoundland Bank it occurs rare and small at 145 fathoms, very rare and very small at 161, rather common and middle-sized at 740; rather rare and small at 725; rare and small at 954 fathoms.

Polystomella crispa, Linn., sp., Var. (*Nonionina*) *Faba*, Fichtel and Moll, sp. Plate XIV. fig. 36 (Arctic).

Nonionina Faba is a small, delicate, ovate-oblong shell, with the later chambers much larger than those first formed. The septal furrows are bridged by little processes from the advancing chambers, and the septal aperture is barred or subdivided. In these latter features *N. Faba* shows an advance of structure beyond *N. Scapha* towards *Polystomella* proper, in which the septa are cribriform and the surface of the shell fenestrated.

It occurs both fossil and recent in the Mediterranean area. We have it from the Hunde Islands, where it is rather rare and of middle size at from 25 to 30 fathoms; rather common and large at 30 to 40; and common and large at 60 to 70 fathoms (SUTHERLAND's dredgings).

N. Faba among these delicate oblong *Nonioninæ*, and *P. striatopunctata* among the less feeble Nautiloid forms make advances towards the true Polystomellan characteristics; thus showing that they certainly are within one and the same specific limits; moreover, the next variety, *N. Scapha*, is seldom quite free from bridges across the divisions of its chambers on each spiral lamina, as may be seen in figs. 37 and 38, Plate XIV.

Polystomella crispa, Linn., sp., Var. (*Nonionina*) *asterizans*, Fichtel and Moll, sp. Plate XIV. fig. 35 (Arctic); Plate XVII. figs. 54 *a*, 54 *b* (North Atlantic).

This is a small, many-chambered. Nautiloid *Nonionina*, somewhat variable in its features, but having a slight umbilical growth of exogenous shell-matter often radiating along the septal furrows for some distance. This star-like limbation is much exaggerated in *N. Limba*, D'Orb. (Modèles, No. 11), and curiously modified with flaps in *N. stelligera*, D'Orb. (For. Canar. pl. 3. figs. 1, 2). *N. asterizans* varies as to its granulations and stellate umbo, readily passing into *N. granosa* and into *N. stelligera*. Fig. 35 is of a stronger make than the latter, and is such as frequents deeper water than that does. It is from the Hunde Islands (SUTHERLAND's dredgings) at from 25 to 30 fathoms, where it is common but small. *N. asterizans* is common in the British seas in shallow water.

Plate XVII. fig. 54 differs from the Arctic specimen as to the umbo, but is not separable. It is from 740 fathoms north of Newfoundland Bank.

The tribe of small *Nonioninæ* converging round *Nonionina asterizans*, although conveniently considered as a subspecific group, yet in reality are essentially of the same specific type as that to which *Polystomella crispa* belongs. They may be said to present arrested or feebly developed conditions of the form in which, under other circumstances, a luxuriant growth of exogenous shell-matter symmetrically bridges over the septal lines, and otherwise thickens and ornaments the shell. *Nonionina Limba*, D'Orb., belongs to this group, and is very apt to take on the characters of the type in connexion with its own, and thus to pass insensibly into it. It is a Tertiary form, at Grignon, Bordeaux, &c.

Polystomella crispa, Linn., sp., Var. (*Nonionina*) *depressula*, Walker and Jacob, sp. Plate XIV. figs. 39 *a*, 39 *b* (Arctic).

This is a delicate feeble form of *Nonionina asterizans*, Fichtel and Moll, sp., with the stellation of the umbilici imperfect.

It is common in the shallow sea-zone and in the brackish water of river-mouths and salt-marshes of the British area; and is the commonest shell in the clay of our Eastern Counties fen-district, excepting at the margin of that sub-recent deposit, for there *Trochammina inflata* attains its highest development and abounds most. This form is very apt to turn up, all the world over, in such shallow water as is rendered somewhat unfit for rhizopodal life by the presence of large quantities of earthy or vegetable matter,—for instance, in bays, harbours, estuaries, &c.

We have it from the Hunde Islands (SUTHERLAND's dredgings) common and small at from 25 to 30 and 50 to 70 fathoms; common and middle-sized at from 60 to 70 fathoms.

Polystomella crispa, Linn., sp., Var. (*Nonionina*) *stelligera*, D'Orb., sp. Plate XIV. figs. 40, 41 (Arctic).

This delicate and variable *Nonionina* was first described by D'ORBIGNY as occurring at the Canaries (For. Canar., p. 123*, pl. 3. figs. 1, 2). It differs from *N. asterizans* in being altogether more delicate and feeble, and in the exogenous matter having the form of a radiating series of thin flaps, which cover over the inner half of the septal sulci on each face of the shell.

It inhabits shallow waters of the Atlantic and the Australian coast. We find it in the dredgings from the Hunde Islands, throughout, from 25 to 70 fathoms, and in the mixed sands from Norway.

Polystomella crispa, Linn., sp., Var. (*Nonionina*) *Scapha*, Fichtel and Moll., sp. Plate XIV. figs. 37-38 (Arctic); Plate XVII. figs. 55, 56 (North Atlantic).

In this, almost the lowest form of *Nonionina* (the small and more or less oblique *N. turgida* being still feebler), the successive chambers enlarge at a greater ratio than they do in *N. asterizans* and its allies; hence the shell is ovato-oblong instead of discoidal; it has the shape of the *Argonauta*, instead of that of the *Nautilus*. It is *N. communis*, D'Orb. The shell varies from the complanate condition (fig. 37) to the gibbose (fig. 38), and to the subglobose (figs. 55, 56); occasionally faint traces of the septal fossettes characteristic of *Polystomella* can be recognized (fig. 38 *a*); but the aperture is still a simple arch-like slit (fig. 38 *b*); whilst in the next stage (*N. Faba*, fig. 36) the fossettes and the barred aperture occur together.

N. Scapha occurs in warm seas rarely at great depths; it is found in the British seas; and the Arctic dredgings show that it also lives at high latitudes. It occurs in Baffin's Bay at lat. 75° 10′, long. 60° 12′, rare and of middling size; lat. 76° 30′, long. 77° 52′, at 150 fathoms, very common and of middling size. At the Hunde Islands it is abundant at from 25 to 70 fathoms, sometimes of large size, usually middling.

It abounds in many Tertiary deposits, Grignon, Bordeaux, Subappennines, San Domingo, English Crag, &c.

Plate XVII. figs. 55, 56 (North Atlantic).

Nonionina Scapha is rare and small at 225 fathoms on the Irish plateau of the North Atlantic; absent apparently in the central area; rare and of middle-size at 145 fathoms north of the Bank; very rare and middling at 161, 329, and 725 fathoms, and very rare and very small at 954 fathoms along the same tract; in Trinity Bay it is rare and middle-size at 124, 133, and 150 fathoms.

The very gibbose specimen, figs. 55, 56, is the same as *N. Labradorica*, Dawson (Canad. Geol. Nat. vol. v. 1860, p. 192, fig. 4), found by him both recent in the Gulf of St. Lawrence and fossil in the Post-pliocene clays of Labrador and Maine.

---

* In the text the name given is "stelligera," in the Plate it is "stellifera"; of course the former should be received.

The specimens from Newfoundland Bank are rare and have a deadish look, as if drifted from their more favourable northern habitats.

Polystomella crispa, Linn., sp., Var. (*Nonionina*) *umbilicatula*, Montagu, sp. Plate XIV. figs. 42 *a*, 42 *b* (Arctic); Plate XVII. figs. 58, 59 (North Atlantic).

This is a small, neat, many-chambered, Nautiloid *Nonionina*, with hollow umbilici. See Ann. Nat. Hist. 3rd ser. vol. iv. pp. 346 & 347, and vol. v. p. 101, &c., for a comparison of this and other *Nonioninæ*. It is common at greater depths than most other *Nonioninæ*, except *N. Scapha*, affect; it is found in warm seas, and occurs in many Tertiary deposits.

We have it in the mixed sands from the Norway coast (MacAndrew and Barrett). In the North Atlantic *N. umbilicatula* is common and of middle-size on the marginal plateau off Ireland, at 78, 90, 223, and 415 fathoms: in the abyssal depths it is rare and small at 1776, rather common and middle-sized at 1950, rather common and small at 2050 and 2176 fathoms; and at 2350 fathoms in the "Boreal" part of the abyss it is rare and small: north of Newfoundland Bank, at 329 fathoms, and in Trinity Bay at 150 fathoms, it is very rare and small; cold water having as bad an influence on it as abyssal depth.

This form, being flush-celled, is more thoroughly changed in character from the type than the feeble varieties found in shallow water, such as *P. stelligera* and *P. depressula*. In these the vesicularity of the chambers allows of the formation of some rudiments of the retral processes, the overlying bridges, and the intervening fossettes; but in this deeper-sea variety the septal walls of contiguous chambers become perfectly adapted, and their edges grow close together at the surface of the shell. This is well shown in the recent and fossil specimens of this kind from the Mediterranean area; further north, however, it scarcely holds its own, and intermediate forms are always turning up, which connect this with the vesicular varieties.

Polystomella crispa, Linn., sp., Var. (*Nonionina*) *turgida*, Williamson, sp. Plate XVII. figs. 57 *a*, 57 *b*, 57 *c* (North Alantic).

A delicate ovate *Nonionina*; the chambers increasing so rapidly in size that the discoidal form is lost, and we have the shape of the *Argonauta* instead of the *Nautilus*. The latter chambers, too, in adult specimens are apt to be swollen at the umbilical margin, concealing the spiral parts of the shell, and hanging over a little more on one side than the other.

Our figured specimen is much thicker and more symmetrical than Professor Williamson's *Rotalina turgida* (Monogr. p. 50, pl. 4. figs. 95–97), but they both belong to the same variety.

N. turgida is found in shallow and brackish water in the British area; and occurs especially in the sub-recent clay of Peterborough Fen, rather common, but extremely small, starved, and one-sided.

We have it from the Irish plateau of the North Atlantic at 43 and 223 fathoms, rare and small.

Genus VALVULINA.

Valvulina triangularis, D'Orbigny, Var. *conica*, nov. Plate XV. fig. 27 (Arctic).

This is a very simple condition of *Valvulina*. The triserial arrangement of chambers forms a smooth conical figure, without any trace of the three flat faces so usual in this species. A similar condition, but depressed, is shown in *V. fusca*, Williamson, sp.

Valvulina conica, Parker and Jones, was described and figured in the Annals Nat. Hist. 2 ser. xix. p. 295, pl. 11. figs. 15, 16, but not named separately from the better developed type, which has a triangular apex. It is also figured by Dr. CARPENTER, *op. cit.* pl. 11. fig. 16. It occurs with the typical form, both in the fossil and the recent state (extremely large in sea-sands from Melbourne); it is rare and small in the mixed sands from Norway (MACANDREW and BARRETT). It lives also in the Mediterranean and on the Abrolhos Bank, South Atlantic.

The type, *V. triangularis*, D'Orb. (Modèles, No. 23; CARPENTER's 'Introd. Foram.' p. 146, pl. 11. fig. 15), though occurring of large size (with *V. conica*, also very large) in Australia, is usually rare; but it has been marvellously common and large in Tertiary times, as shown by specimens from Grignon and Hautville (France).

Lituola nautiloidea, Lamarck, Var. *Canariensis*, D'Orbigny, sp. Plate XV. figs. 45 *a*, 45 *b* (Arctic); Plate XVII. figs. 92–95 (North Atlantic).

Of the disco-spiral *Lituolæ* most are attached and therefore more or less plano-convex; when growing free, however, they attain the more symmetrical, somewhat biconvex, and nautiloïd shape of *L. Canariensis*, without attaining the outgrowing rectilinear series of chambers shown in LAMARCK's *L. nautiloidea*, and still more in *L. irregularis*, Rœmer, sp.

Lituola Canariensis, D'Orb., sp. (Foram. Canaries, p. 128, pl. 2. figs. 33, 34), has, like other *Lituolæ*, a rusty coloured shell-substance among the sand-grains that largely make up its shell. We have a few large specimens from Finmark (East of Rolfs Oe), 30 fathoms (MACANDREW and BARRETT); and some small specimens from the mixed sands from Norway. At the Hunde Islands (Dr. SUTHERLAND) it is large and common throughout; and in the sands from Baffin's Bay (PARRY) it is most common and sometimes large.

In the North Atlantic it is rare; on the Irish plateau it is small at 43 fathoms and middle-sized at 223 fathoms; and it is middle-sized at 1203 fathoms north of the Bank, and at 133 fathoms in Trinity Bay. The British coasts, Abrolhos Bank, Hobson's Bay (Australia), and Fiji are other localities for *L. Canariensis*.

Fig. 94 is probably not worth separating from *L. Canariensis*; its chambers are either imperfect or obsolete.

Lituola nautiloidea, Lamarck, Var. *globigeriniformis*, nov. Plate XV. figs. 46, 47 (Arctic); Plate XVII. figs. 96–98 (North Atlantic).

In this low form of *Lituola* the chambers are subglobular and agglomerated, presenting an isomorph of *Globigerina*; the somewhat scanty and rusty-red shell-substance cementing the sand-grains is characteristic, as in *Lituola nautiloidea* proper.

Lituola globigeriniformis is small and common at the Hunde Islands (Dr. Sutherland) from 30 to 70 fathoms. It is small also in Baffin's Bay; being common at 75° 10′ lat., 60° 12′ long., and rare at 75° 25′ lat., 60° long. (314 fathoms), and 75° lat., 59° 40′ long. (220 fathoms).

In the North Atlantic it is rare and middle-sized at 1660 fathoms in the "Boreal" portion of the abyss; and very rare and small north of the Bank at 145 and 954 fathoms. It is figured by Dr. Wallich in 'The North-Atlantic Sea-bed,' pl. 6. fig. 22.

L. globigeriniformis, Parker and Jones, is common, but small, in the Mediterranean; in our paper in the Quart. Journ. Geol. Soc. vol. xvi. Table, p. 302, it is referred to as "*L. pelagica*, D'Orb., sp.," as we then mistook the yellowish acerose *Globigerina* named "*Nonionina pelagica*" by D'Orbigny for our *Lituola*. It is present in the Red Sea, the Indian Ocean, and the South Atlantic.

Lituola nautiloidea, Lamarck, Var. *Scorpiurus*, Montfort, sp. Plate XV. figs. 48 *a*, 48 *b* (Arctic).

Lituola Scorpiurus, Montfort, sp., is a simple, linear, slightly curved, and, as it were, abortive variety of *L. nautiloidea*, Lamarck (see Ann. Nat. Hist. 3 ser. vol. v. p. 297; and Carpenter's 'Introd. Foram.' p. 143). It is of very common occurrence in shelly deposits, recent and fossil.

It is common and large at the Hunde Islands, 25 to 40 fathoms; common and middle-sized in Baffin's Bay, 75° 10′ lat., 60° 12′ long.; and rather common and very large at 150 fathoms, 76° 30′ lat., 77° 52′ long.

The late Mr. L. Barrett obtained large specimens of *L. Scorpiurus* in deep water off Jamaica, of very large size, labyrinthic, and passing into *L. Soldanii*, Parker and Jones. *L. Scorpiurus* lives also in the Adriatic, the North and South Atlantic, and in the Australian seas.

Genus Trochammina.

Trochammina squamata, Parker and Jones. Plate XV. figs. 30, 31 *a*, 31 *b*, 31 *c* (Arctic).

This is the subvesicular Rotaliform *Trochammina* (Quart. Journ. Geol. Soc. vol. xvi. p. 305), having lunate, flattened chambers, several in a whorl, and regularly increasing with the progress of growth; it much resembles those flatter varieties of *Discorbina Turbo* which are intermediate between *D. globularis* and *D. rosacea*, but it has an arenaceous shell; it is also like some little scale-like varieties of *Valvulina triangularis*; but the latter have only three chambers in a whorl, and are more coarsely sandy.

Trochammina squamata, the type of the species, is usually rare; it is small and rare at 360 fathoms off Crete (Captain SPRATT's soundings).

At the Hunde Islands (Dr. SUTHERLAND's dredgings) *Troch. squamata* is rare at 30 to 40 fathoms, common at 60 to 70 fathoms, but small throughout.

Trochammina squamata, Var. *gordialis*, Parker and Jones. Plate XV. fig. 32 (Arctic).

Trochammina gordialis, Parker and Jones (CARPENTER's 'Introd. Foram.' p. 141, pl. 11. fig. 4), presents sometimes an irregularly coiled tube, having but little segmentation; sometimes it presents long, inwound, tubular chambers.

It is common and small at 60 to 70 fathoms at the Hunde Islands, together with the type. It occurs in the Red Sea, and is found involutely coiled (commencing with a few irregularly segmented chambers, and continued as a long tube, turned and twisted on itself) in the Indian seas; the so-called *Serpula pusilla* of the Permian limestones is a very similar little Foraminifer.

Troch. incerta, D'Orb., sp., is discoidal, tubular, and without segments. The next stage beyond that seen in fig. 32 is that form of *Troch. squamata* shown by fig. 31.

Genus CORNUSPIRA.

Cornuspira foliacea, Philippi, sp. Plate XV. fig. 33 (Arctic).

The characters and relationships of this flat, spiral, non-segmented Milioline Foraminifer are treated of in CARPENTER's 'Introd. Foram.' p. 68. It inhabits the shallow sea-zones of every climate, and is found fossil (Tertiary).

We find it common in Dr. SUTHERLAND's dredgings from the Hunde Islands, where it is small at 60 to 70 fathoms, and of middle size at 25 to 30 fathoms. It is figured by Dr. WALLICH in 'The North-Atlantic Sea-bed,' pl. 5. fig. 12.

C. foliacea is extremely large (fossil) in the Crag of Sutton, Suffolk; in the recent state it is very large off Crete, and is found also living on the British coasts, in the Red Sea, the South Atlantic, and on the western and southern shores of Australia.

Genus MILIOLA.*

Miliola (*Spiroloculina*) *planulata*, Lamarck. Plate XVII. fig. 82 (North Atlantic).

The type of the symmetrical and flattened group of *Miliolæ*, *Spiroloculina planulata*, Lamarck, is often abundant in sea-sands and in Tertiary deposits.

In the North Atlantic it is rare; of middle size at 43 fathoms off Ireland; middle-sized at 2050 fathoms, and small at 2330 fathoms in the abyssal area. Dr. WALLICH figures it in 'The North-Atlantic Sea-bed,' pl. 5. fig. 13.

---

* For remarks on this genus (type, *M. Seminulum*), see CARPENTER's Introd. Foram. pp. 74, &c.

Miliola (Spiroloculina) limbata, D'Orbigny. Plate XVII. figs. 83 *a*, 83 *b* (North Atlantic).

Here the edges of the chambers are limbate, or thickened with shell-growth, a non-essential feature. It is figured by SOLDANI and named by D'ORBIGNY, Ann. Sci. Nat. vol. vii. p. 299, No. 12.

We have *Spiroloculina limbata* rare and small from the Irish marginal plateau of the North Atlantic, at 78 fathoms. It is not rare in the existing seas, and occurs in the Tertiary deposits.

Miliola (Biloculina) ringens, Lamarck. Plate XV. figs. 42–44 (Arctic).

Taking the Biloculine *Miliolæ* by themselves, this well-known common *Biloculina ringens*, Lamarck, is the type of a very variable group. Not only the degree of globosity of the chambers, but the amount of overlap at the sides or at the ends, constitute infinite variations, presented in all seas.

Large *Biloculinæ*, but subject to great differences in the points above alluded to, were found abundantly in nearly all the dredgings from Norway. Fig. 44 represents a highly globose and *striated* specimen from Norway. Dr. WALLICH figures *B. ringens* in 'The North-Atlantic Sea-bed,' pl. 5. figs. 1, 3, 4, 6.

Miliola (Biloculina) depressa, D'Orbigny. Plate XVII. figs. 89 *a*, 89 *b* (North Atlantic).

This depressed form of *Biloculina ringens* is not uncommon in both the recent and fossil (Tertiary) states. D'ORBIGNY illustrated it by his Modèle, No. 91.

It occurs in several soundings from the North Atlantic, though rare in each. It is small on the Irish plateau at 43 and 78 fathoms; small at 2176 fathoms, and middle-sized at 1450, 1660, and 2350 fathoms in the abyss. It is figured in Dr. WALLICH'S 'North-Atlantic Sea-bed,' pl. 5. figs. 2, 5, 8.

Miliola (Biloculina) elongata, D'Orbigny. Plate XVII. figs. 88, 90, 91 (North Atlantic).

Biloculina ringens contracted gives *B. elongata*, figured by SOLDANI and named by D'ORBIGNY, Ann. Sci. Nat. vol. vii. p. 298, No. 4, and not rare wherever other *Biloculinæ* exist.

We have *B. elongata* from the North Atlantic, small and rare in the deep, at 1950, 2050, and 2330 fathoms.

Miliola (Triloculina) tricarinata, D'Orbigny. Plate XV. fig. 40 (Arctic).

Triloculina tricarinata, D'Orb. (Modèles, No. 94) differs from *Tr. trigonula*, Lamarck, in having produced or keeled edges. Our figured specimen has rather flatter sides than are usual.

Tr. tricarinata, D'Orb., has a very wide distribution and, like *T. trigonula*, Lam., abounds in some Tertiary beds. The sea-sand near Melbourne, Australia, yields large specimens of *Tr. tricarinata*, together with striped *Tr. trigonula*. At the Hunde Islands *Tr. tricarinata* is small, common at 25 to 30 fathoms, rare at 60 to 70 fathoms.

Miliola (*Triloculina*) *cryptella*, D'Orbigny. Plate XV. fig. 39 (Arctic).

This is an extremely inflated and short Triloculine *Miliola*, its chambers overlapping so much more than in the symmetrical trigonal forms, that in some instances the antepenultimate chamber is but little exposed. It is not common.

Triloculina cryptella, D'Orb., For. Amér. Mér. p. 70, pl. 9. figs. 4, 5, approaches closely, in appearance, to *Biloculina sphæra*, D'Orb., *op. cit.* p. 66, pl. 8. figs. 13–16, with which it was found at the Falkland Islands. *B. sphæra* has its chambers so much overlapping that it scarcely shows the *penultimate* chamber (as characteristic of *Biloculina*), *Tr. cryptella* having so much overlap in its chambers that it scarcely shows the *antepenultimate* (as characteristic in *Triloculina*).

Tr. cryptella is a curious isomorph of *Sphæroidina* (p. 369), and might easily be mistaken for it, for both are white in colour; the texture, however, is hyaline in *Sphæroidina* (related to *Globigerina*), and opake in *Triloculina*, as in all *Miliolæ*.

We have *Triloculina cryptella* from Baffin's Bay, 75° 25′ lat., 60° long., where it is rather common and middle-sized at 314 fathoms.

Miliola (*Quinqueloculina*) *Seminulum*, Linnè, sp. Plate XV. figs. 35 *a*, 35 *b* (Arctic); Plate XVII. fig. 87 (North Atlantic).

Figs. 35 *a*, *b* represent a neat form of the typical and widely distributed *Miliola* (*M. Seminulum*, Linn., sp.), such as is common in deepish water, and well figured by D'ORBIGNY as *Quinqueloculina triangularis* (For. Foss. Vienn. p. 258, pl. 18. figs. 7–9). It is from Norway.

Fig. 87, from the North Atlantic, is a sandy specimen, but is not so coarsely built up as the variety known as *Q. agglutinans*, D'Orb. (Plate XV. fig. 37).

Q. Seminulum is common and large on the Norway coast; common and rather small at the Hunde Islands; rare and small at 220 fathoms in Baffin's Bay.

In the North Atlantic soundings it is small; common at 43 and 78 fathoms, and rare at 90 fathoms on the Irish plateau; rare at 2035, 2050, and 2350 fathoms in mid-ocean; and rare and of middle size at 954 fathoms north of the Bank.

In his 'North-Atlantic Sea-bed' Dr. WALLICH figures *Q. Seminulum*, pl. 5. figs. 9, 10, 15; and *Q. secans*, fig. 7.

Q. triangularis takes the place of the typical *Q. Seminulum* in many parts of the Mediterranean and Red Seas, and of the Indian, South Atlantic, and Pacific Oceans.

Miliola (*Quinqueloculina*) *agglutinans*, D'Orbigny. Plate XV. figs. 37 *a*, 37 *b* (Arctic).

Quinqueloculina agglutinans, D'Orb. (For. Cuba, p. 195, pl. 12. figs. 11–13), is a well-developed, often rusty-red, arenaceous *Miliola Seminulum*, of wide distribution, and varying much with the character of the sea-bed. The shell-substance cementing the grains of sand may be reddish in *Quinqueloculina*, though on white sand in Australia its shell becomes white, and on black sand at Orotava, Canaries, it is black.

We have *Q. agglutinans*, of middle size, from the Hunde Islands (Dr. SUTHERLAND), rare at 30 to 40 fathoms, common at 60 to 70 fathoms. Is rare and middle-sized in Baffin's Bay, 75° 10′ lat., 60° 12′ long. (PARRY).

Miliola (*Quinqueloculina*) *Ferussacii*, D'Orbigny. Plate XV. figs. 36 *a*, 36 *b*, 36 *c* (Arctic).

Quinqueloculina Ferussacii, D'Orb. (Modèles, No. 32), is a coarsely ribbed or plicated form of *Q. Seminulum* (the type of the *Miliolæ*); it is very variable, and is known by a host of names.

It is found in some abundance in the European and other seas, and also in the Tertiary deposits.

At the Hunde Islands it is common and middle-sized at from 30 to 70 fathoms.

Miliola (*Quinqueloculina*) *oblonga*, Montagu, sp. Plate XV. figs. 34, 41 (Arctic); Plate XVII. figs. 85 *a*, 85 *b*, 86 *a*, 86 *b* (North Atlantic).

When *Miliola Seminulum*, Linn., sp., is contracted in its growth, it produces very variable forms, in which the normal lateral exposure of the chambers does not take place; and somewhat elongate, oblong, Quinqueloculine and Triloculine forms are the result, such as *Q. oblonga*, Montagu, sp., which is often Triloculine in aspect, and has been registered as *Triloculina oblonga* by D'ORBIGNY and others (see Annals Nat. Hist. 2 ser. vol. xix. p. 300); but it often has indications of its being really a poorly developed Quinqueloculine *Miliola*. *Quinque-* and *Tri-loculinæ* are excessively variable shells, both as to shape and ornament, and are amongst the most common Foraminifers in all latitudes and depths. We have two genuine *Triloculinæ* in the Arctic dredgings (Hunde Islands); but the so-called *Triloculina oblonga* is an ill-grown *Quinqueloculina*. It usually abounds in company with the typical *Miliola Seminulum*; the largest specimens we know of are fossil in the Lower Crag of Sutton, Suffolk. It is one of the most abundant of the Quinqueloculine varieties.

This feeble *Quinqueloculina Seminulum*, with a Triloculine aspect, is common and large in most of the Norway dredgings (MACANDREW and BARRETT); common and small at the Hunde Islands (SUTHERLAND) at 25 to 30 fathoms.

We have it very rare and very small from 2330 fathoms in the North Atlantic. Figs. 14 & 16, in pl. 5 of Dr. WALLICH'S 'North-Atlantic Sea-bed,' also illustrate this variety.

Miliola (*Quinqueloculina*) *subrotunda*, Montagu, sp. Plate XV. figs. 38 *a*, 38 *b* (Arctic).

A small, roundish, biconvex variety of *Miliola Seminulum*, Linn., often accompanying other *Miliolæ*. It may be said to be a dwarf of the variety *Q. secans*, D'Orb., and is very widely distributed.

At the Hunde Islands (Dr. SUTHERLAND'S dredgings) it is common at 60 to 70 fathoms.

Miliola (*Quinqueloculina*) *tenuis*, Czjzek. Plate XVII. fig. 84 (North Atlantic).

A nearly complanate, but often curved, thin, more or less unsymmetrical Quinqueloculine *Miliola*, named *Quinqueloculina tenuis* by CZJZEK in his description of some fossil Foraminifera from the Vienna Basin, in HAIDINGER'S Abhandl. Wiss. vol. ii. p. 149, pl. 13. figs. 31–34.

This tiny shell, which presents an extreme enfeeblement of *Q. Seminulum*, Spiroloculine in aspect and twisted on itself, occurs at great depths in the Mediterranean and other seas. We find it fossil in the Lias clay of Stockton, Warwickshire.

In the North Atlantic *Q. tenuis* is small; rather common at 415 fathoms on the marginal plateau off Ireland; rare at 2050 fathoms in the abyss.

DESCRIPTION OF THE PLATES.

PLATE XII.

Map of the Deep-sea Soundings, in the North Atlantic, from Ireland to Newfoundland, by Lieut.-Commander J. DAYMAN, R.N., assisted by Mr. J. SCOTT, Master R.N., H.M.S. Cyclops, 1857. With a Section of the Bed of the Atlantic Ocean from Valentia to Trinity Bay. The soundings are given in fathoms. Vertical scale 2000 fathoms to 1 inch. Scales as 15 to 1. See APPENDIX VII.

This Map is copied from Commander DAYMAN'S Report on the Soundings (1858); indications of the Natural-History Provinces, and of the thirty-nine Soundings described in this memoir, being added.

NOTE.—In the 'Nautical Magazine,' vol. xxxi. No. 11, November 1862, was published "The Report on the Deep-sea Soundings to the Westward of Ireland, made in H.M.S. Porcupine, in June, July, and August 1862," by R. HOSKYN, Esq., R.N., with a Chart, showing the slope of the Eastern Plateau to be, in that line of soundings, at a less angle off Southern Ireland than Commander DAYMAN found it where he sounded.

PLATES XIII.–XIX. illustrating the Foraminifera from the Arctic and North Atlantic Oceans, and other Foraminifera from other parts of the Atlantic, the Pacific, and elsewhere.

PLATE XIII. (ARCTIC FORAMINIFERA.)

[Figs. 1–19 are magnified 12 diameters; figs. 20–58, 24 diameters.]

Fig. 1.	Glandulina lævigata, *D'Orbigny.*
Fig. 2, *a*, *b*. Fig. 3. Fig. 4, *a*, *b*. Fig. 5, *a*, *b*. Fig. 6. Fig. 7.	Nodosaria Radicula, *Linn.* Various individuals passing from *Glandulina lævigata*, through *Nodosaria humilis*, to *N. Radicula.*

Fig. 8.
Fig. 9. } Dentalina pauperata, *D'Orbigny*. Fragments.

Fig. 10. Dentalina communis, *D'Orbigny*.

Fig. 11. Dentalina guttifera, *D'Orbigny*. A fragment.

Fig. 12, *a*, *b*.
Fig. 13, *a*, *b*. } Vaginulina linearis, *Montagu*. Fragments.

Fig. 14, *a*, *b*. Marginulina Lituus, *D'Orbigny*.

Fig. 15.
Fig. 16, *a*, *b*. } Cristellaria Crepidula, *Fichtel and Moll*.

Fig. 17, *a*, *b*.
Fig. 18, *a*, *b*. } Cristellaria cultrata, *Montfort*.

Fig. 19, *a*, *b*. Cristellaria rotulata, *Lamarck*.

Fig. 20. Lagena distoma, *Parker and Jones*.

Fig. 21. Lagena distoma-polita, *Parker and Jones*.

Fig. 22. Lagena lævis, *Montagu*.

Fig. 23. Lagena semistriata, *Williamson*.

Fig. 24. Lagena sulcata, *Walker and Jacob*. With spiral narrow riblets.

Fig. 25.
Fig. 26. } Lagena striatopunctata, *Parker and Jones*.
Fig. 27.

Fig. 28, *a*, *b*.
Fig. 29, *a*, *b*. } Lagena sulcata, *Walker and Jacob*.
Fig. 30, *a*, *b*.
Fig. 31, *a*, *b*.

Fig. 32. Lagena sulcata, *Walker and Jacob*. Dwarf.

Fig. 33.
Fig. 34. } Lagena Melo, *D'Orbigny*.
Fig. 35.

Fig. 36. Lagena Melo, *D'Orbigny*. Double (monster).

Fig. 37, *a*, *b*. Lagena globosa, *Montagu*.

Fig. 38, *a*, *b*.
Fig. 39, *a*, *b*. } Lagena caudata, *D'Orbigny*. Smooth and entosolenian.

Fig. 40.
Fig. 41. } Lagena squamosa, *Montagu*.

Fig. 42, *a*, *b*.
Fig. 43, *a*, *b*. } Lagena marginata, *Montagu*.
Fig. 44.

Fig. 45, *a*, *b*.
Fig. 46, *a*, *b*. } Polymorphina lactea, *Walker and Jacob*.

Fig. 47, *a*, *b*.
Fig. 48, *a*, *b*.
Fig. 49. Polymorphina compressa, *D'Orbigny*.
Fig. 50.
Fig. 51.
Fig. 52, *a*, *b*, *c*, *d*. Polymorphina tubulosa, *D'Orbigny*.
Fig. 53, *a*, *b*.
Fig. 54, *a*, *b*.
Fig. 55. Uvigerina pygmæa, *D'Orbigny*.
Fig. 56.
Fig. 57.
Fig. 58, *a*, *b*. Uvigerina angulosa, *Williamson*.

PLATE XIV. (ARCTIC FORAMINIFERA.)

[Figs. 1, 2, 14–45 are magnified 12 diameters; figs. 3–13, 24 diameters.]

Fig. 1. Globigerina bulloides, *D'Orbigny*.
Fig. 2.
Fig. 3.
Fig. 4.
Fig. 5, *a*, *b*. Truncatulina lobatula, *Walker and Jacob*.
Fig. 6, *a*, *b*.
Fig. 7.
Fig. 8.
Fig. 9. Anomalina coronata, *Parker and Jones*.
Fig. 10.
Fig. 11, *a*, *b*.
Fig. 12. Pulvinulina punctulata, *D'Orbigny*.
Fig. 13, *a*, *b*.
Fig. 14. Pulvinulina Karsteni, *Reuss*.
Fig. 15, *a*, *b*.
Fig. 16, *a*, *b*. Pulvinulina Micheliniana, *D'Orbigny*.
Fig. 17. Pulvinulina Karsteni, *Reuss*.
Fig. 18. Discorbina obtusa, *D'Orbigny*.
Fig. 19, *a*, *b*.
Fig. 20.
Fig. 21. Discorbina globularis, *D'Orbigny*.
Fig. 22.
Fig. 23.
Fig. 24. Polystomella crispa, *Linn*.

Fig. 25.
Fig. 26.
Fig. 27. } Polystomella arctica, *Parker and Jones.*
Fig. 28.
Fig. 29.
Fig. 30.
Fig. 31.
Fig. 32. } Polystomella striatopunctata, *Fichtel and Moll.*
Fig. 33.
Fig. 34.
Fig. 35. Nonionina asterizans, *Fichtel and Moll.*
Fig. 36. Nonionina Faba, *Fichtel and Moll.*
Fig. 37. } Nonionina Scapha, *Fichtel and Moll.*
Fig. 38.
Fig. 39. Nonionina depressula, *Walker and Jacob.*
Fig. 40, *a*, *b*. } Nonionina stelligera, *D'Orbigny.*
Fig. 41, *a*, *b*.
Fig. 42, *a*, *b*. Nonionina umbilicatula, *Montagu.*
Fig. 43, *a*, *b*. Pullenia sphæroides, *D'Orbigny.*
Fig. 44, *a*, *b*. Operculina ammonoides, *Gronovius.*
Fig. 45, *a*, *b*. Nummulina planulata, *Lamarck.*

PLATE XV. (ARCTIC FORAMINIFERA.)

[Figs. 1–33, 36–41, 45–48 are magnified 24 diameters; figs. 34, 35, 42, 43, 44, 12 diameters.]

Fig. 1.
Fig. 2. } Cassidulina lævigata, *D'Orbigny.*
Fig. 3.
Fig. 4.
Fig. 5.
Fig. 6. } Cassidulina crassa, *D'Orbigny.*
Fig. 7.
Fig. 8. } Bulimina Pyrula, *D'Orbigny.*
Fig. 9, *a*, *b*.
Fig. 10, *a*, *b*. Bulimina marginata, *D'Orbigny.*
Fig. 11. Bulimina aculeata, *D'Orbigny.*
Fig. 12.
Fig. 13.
Fig. 14. } Bulimina elegantissima, *D'Orbigny.*
Fig. 15.
Fig. 16.
Fig. 17.

Fig. 18. Virgulina Schreibersii, *Czjzek.*

Fig. 19, *a*, *b*.} Virgulina squamosa, *D'Orbigny.*
Fig. 20, *a*, *b*.}

Fig. 21, *a*, *b*. Textularia agglutinans, *D'Orbigny.*

Fig. 22, *a*, *b*. Textularia Sagittula, *Defrance.*

Fig. 23, *a*, *b*.} Textularia biformis, *Parker and Jones.*
Fig. 24.}

Fig. 25. Bigenerina Nodosaria, *D'Orbigny.*

Fig. 26, *a*, *b*. Verneuilina polystropha, *Reuss.*

Fig. 27, *a*, *b*. Valvulina conica, *Parker and Jones.*

Fig. 28. Spirillina vivipara, *Ehrenberg.*

Fig. 29, *a*, *b*, *c*. Patellina corrugata, *Williamson.*

Fig. 30.} Trochammina squamata, *Parker and Jones.*
Fig. 31, *a*, *b*, *c*.}

Fig. 32. Trochammina gordialis, *Parker and Jones.*

Fig. 33, *a*, *b*. Cornuspira foliacea, *Philippi.*

Fig. 34. Quinqueloculina oblonga, *Montagu.*

Fig. 35, *a*, *b*. Quinqueloculina Seminulum, *Linnè* (Var. *triangularis*, D'Orbigny).

Fig. 36, *a*, *b*. Quinqueloculina Ferussacii, *D'Orbigny.*

Fig. 37, *a*, *b*. Quinqueloculina agglutinans, *D'Orbigny.*

Fig. 38, *a*, *b*. Quinqueloculina subrotunda, *Montagu.*

Fig. 39, *a*, *b*. Triloculina cryptella, *D'Orbigny.*

Fig. 40, *a*, *b*. Triloculina tricarinata, *D'Orbigny.*

Fig. 41, *a*, *b*. Quinqueloculina oblonga, *Montagu.*

Fig. 42, *a*, *b*.}
Fig. 43, *a*, *b*.} Biloculina ringens, *Lamarck.*
Fig. 44.}

Fig. 45, *a*, *b*. Lituola Canariensis, *D'Orbigny.*

Fig. 46.} Lituola globigeriniformis, *Parker and Jones.*
Fig. 47.}

Fig. 48, *a*, *b*. Lituola Scorpiurus, *Montfort.*

PLATE XVI. (NORTH ATLANTIC FORAMINIFERA).

[The figures are magnified 30 diameters.]

Fig. 1. Nodosaria Raphanus, *Linnè.* Dwarf.

Fig. 2, *a*, *b*, *c*. Nodosaria scalaris, *Batsch.*

Fig. 3. Dentalina consobrina, *D'Orbigny.* Fragment.

Fig. 4. Cristellaria Crepidula, *Fichtel and Moll.* Broken.

Fig. 5. Cristellaria cultrata, *Montfort.*

Fig. 6. Lagena sulcata, *Walker and Jacob.* Caudate variety.
Fig. 7. Lagena caudata, *D'Orbigny.* Striate.
Fig. 7, *a.* Lagena sulcata, *Walker and Jacob.*
Fig. 8. Lagena caudata, *D'Orbigny.* Smooth.
Fig. 9.
Fig. 9, *a.* Lagena lævis, *Montagu.*
Fig. 10, *a*, *b.* Lagena globosa, *Montagu.*
Fig. 11, *a*, *b.* Lagena squamosa, *Montagu.*
Fig. 12, *a*, *b.* Lagena marginata, *Montagu.*
Fig. 13. Orbulina universa, *D'Orbigny.*
Fig. 14.
Fig. 15. Globigerina bulloides, *D'Orbigny.*
Fig. 16. Globigerina inflata, *D'Orbigny.*
Fig. 17.
Fig. 18, edge view.
Fig. 19, upper view. Truncatulina lobatula, *Walker and Jacob.*
Fig. 20, lower view.
Fig. 21. Planorbulina Mediterranensis, *D'Orbigny.*
Fig. 22, *a*, *b.* Planorbulina Haidingerii, *D'Orbigny.*
Fig. 23, upper side.
Fig. 24, lower side. Planorbulina Ungeriana, *D'Orbigny.*
Fig. 25, edge.
Fig. 26, upper side. Discorbina Berthelotiana, *D'Orbigny.*
Fig. 27, lower side.
Fig. 28, *a*, upper side. Discorbina rosacea, *D'Orbigny.*
Fig. 28, *b*, edge.
Fig. 29, upper side. Rotalia Beccarii, *Linnè.*
Fig. 30, lower side.
Fig. 31, upper side.
Fig. 32, lower side. Rotalia Soldanii, *D'Orbigny.*
Fig. 33, edge.
Fig. 34, upper view. Rotalia orbicularis, *D'Orbigny.*
Fig. 35, upper view.
Fig. 36, lower view. Pulvinulina Menardii, *D'Orbigny.*
Fig. 37, edge.
Fig. 38, edge.
Fig. 39, upper side. Pulvinulina Karsteni, *Reuss.*
Fig. 40, lower side.
Fig. 41, lower side.
Fig. 42, edge. Pulvinulina Micheliniana, *D'Orbigny.*
Fig. 43, upper side.

Fig. 44, upper side.
Fig. 45, edge. } Pulvinulina elegans, *D'Orbigny*.
Fig. 46, lower side.
Fig. 47, lower side
Fig. 48, edge } Pulvinulina Canariensis, *D'Orbigny*.
Fig. 49, upper side
Fig. 50.
Fig. 51, *a*, *b* } Pulvinulina pauperata, *Parker and Jones*.
Fig. 52. Sphæroidina bulloides, *D'Orbigny*.

PLATE XVII. (NORTH ATLANTIC FORAMINIFERA.)

[The figures are magnified 30 diameters.]

Fig. 53. Pullenia sphæroides, *D'Orbigny*.
Fig. 54, *a*, *b*. Nonionina asterizans, *Fichtel and Moll*.
Fig. 55.
Fig. 56. } Nonionina Scapha, *Fichtel and Moll*.
Fig. 57, *a*, *b*, *c*. Nonionina turgida, *Williamson*.
Fig. 58.
Fig. 59. } Nonionina umbilicatula, *Montagu*.
Fig. 60, *a*, *b*. Nonionina striatopunctata, *Fichtel and Moll*.
Fig. 61, *a*, *b*. Polystomella crispa, *Linnè*.
Fig. 62.
Fig. 63. } Operculina ammonoides, *Gronovius*.
Fig. 64, *a*, *b*, *c*. Cassidulina lævigata, *D'Orbigny*.
Fig. 64, *d*. Cassidulina crassa, *D'Orbigny*.
Fig. 65, *a*, *b*. Uvigerina pygmæa, *D'Orbigny*.
Fig. 66, *a*, *b*. Uvigerina angulosa, *Williamson*.
Fig. 67, *a*, *b*. Bulimina ovata, *D'Orbigny*.
Fig. 68.
Fig. 69. } Bulimina aculeata, *D'Orbigny*.
Fig. 70, *a*, *b*, *c*. Bulimina marginata, *D'Orbigny*.
Fig. 71. Bulimina Buchiana, *D'Orbigny*.
Fig. 72.
Fig. 73. } Virgulina Schreibersii, *Czjzek*.
Fig. 74. Bolivina punctata, *D'Orbigny*.
Fig. 75. Bolivina costata, *D'Orbigny*.
Fig. 76, *a*, *b*. Textularia abbreviata, *D'Orbigny*.
Fig. 77, *a*, *b*. Textularia Sagittula, *Defrance*.
Fig. 78, *a*, *b*. Textularia pygmæa, *D'Orbigny*.
Fig. 79, *a*, *b*. Textularia carinata, *D'Orbigny*.

Fig. 80, *a*, *b*. Bigenerina Nodosaria, *D'Orbigny*.
Fig. 81. Bigenerina digitata, *D'Orbigny*.
Fig. 82. Spiroloculina planulata, *Lamarck*.
Fig. 83, *a*, *b*. Spiroloculina limbata, *D'Orbigny*.
Fig. 84. Quinqueloculina tenuis, *Czjzek*.
Fig. 85, *a*, *b*.
Fig. 86, *a*, *b*. } Quinqueloculina oblonga, *Montagu*.
Fig. 87. Quinqueloculina Seminulum, *Linnè*.
Fig. 88. Biloculina elongata, *D'Orbigny*.
Fig. 89, *a*, *b*. Biloculina depressa, *D'Orbigny*.
Fig. 90.
Fig. 91. } Biloculina elongata, *D'Orbigny*.
Fig. 92.
Fig. 93.
Fig. 94.
Fig. 95. } Lituola Canariensis, *D'Orbigny*.
Fig. 96.
Fig. 97.
Fig. 98. } Lituola globigeriniformis, *Parker and Jones*.

PLATE XVIII. (MISCELLANEOUS FORAMINIFERA.)

[Figures 15–18 are magnified 30 diameters; all the rest are magnified 60 diameters (excepting fig. 6 *b*, 200 diameters.)].

Fig. 1, *a*, *b*. Lagena trigono-marginata, *Parker and Jones*. A rare form, from the inside of an Eocene Tertiary shell from Grignon*. It is an isomorph of the trigonal *Nodosarinæ*. See page 348.

Fig. 2, *a*, *b*. Lagena squamoso-marginata, *Parker and Jones*. Living on the Coral-reefs of Australia (Jukes); fossil in the Middle Tertiary beds of San Domingo. See page 356.

Fig. 3, *a*, *b*. Lagena radiato-marginata, *Parker and Jones*. Rare. Recent, Australian Coral-reefs (Jukes); fossil, Middle Tertiary, Bordeaux. See page 355.

---

* This *Lagena*, as well as the other Grignon specimens on this plate, together with *Discorbina globigerinoïdes* on Plate XIX., and many other Foraminifera, were obtained from the inside of a *Cerithium giganteum*; and, as a group, they differ from those got by us from any other sample of the Calcaire grossier, in their extreme freshness and their minute size. The Australian seas supply a Foraminiferal fauna very analogous to that of Grignon (fossil); and that of the northern part of the Red Sea (300–600 fathoms) corresponds in many respects to that shown by the contents of the fossil shell referred to. The *Cerithium* itself would not, of course, indicate any such depth as that above mentioned; but the analogy of the fossil and recent faunæ under notice is certainly striking. Still, the smallness of some of the forms amongst those from the Red Sea, and the absence of *Polyzoa* and of small Gasteropods and Lamellibranchs in these soundings (replaced by abundance of small Pteropods), sufficiently separate the two.

Fig. 4, *a*, *b*. Lagena crenata, *Parker and Jones.* Rare. Recent, shore-sand at Swan River, Australia; fossil, Middle Tertiary of Bordeaux and Malaga. The figure well shows the characters of this pretty Lagena. Decanter-shaped; neck long and coiled; body gradually widening and smooth to the base, which for half its radius is widely and deeply crenate with broad radiating furrows; the centre of the base being smooth and gently convex.

Fig. 5. Lagena distoma-aculeata, *Parker and Jones.* Rare. Fossil at Grignon. Isomorphous of prickly *Nodosarinæ.* See page 348.

Fig. 6, *a*, *b*. Lagena distoma-margaritifera, *Parker and Jones.* Recent, from the surf-washed sponges at Melbourne, Australia. See page 357.

Fig. 7, *a*, *b*. Lagena tubifero-squamosa, *Parker and Jones.* Fossil at Grignon. This very large globular *Lagena*, with a distinct and ramifying neck, has shallow honeycombings and a very thick shell, the outer layers of which decaying leave a very smooth, thin *Lagena*, ordinary-looking except for its neck. See page 354.

Fig. 8. Lagena distoma-polita, *Parker and Jones.* A large, smooth, two-mouthed, fusiform *Lagena*, from the Red Sea and Australia. See page 357.

Fig. 9, *a*, *b*. Lagena lævis, *Montagu.* A double individual (monster). Fig. 9 *b* is a section. Rare. Recent, from the English Channel at Eastbourne. See page 353.

Figs. 10, 11. Lagena lævis, *Montagu.* Monstrous *Lagenæ*, double by lateral growth. Fossil, Grignon. See page 353.

Fig. 12, *a*, *b*. Lagena lævis, *Montagu.* Monstrous bilobed specimen. Fossil: Grignon. See page 353.

Fig. 13. Nodosaria scalaris, *Batsch.* For comparison with figs. 9 *a*, 9 *b*. See pages 340 and 353.

Fig. 14, *a*, *b*. Lagena tretagona, *Parker and Jones.* A rare, delicate, feeble form of *L. striatopunctata* with four ridges and surfaces. Fossil: Grignon. See page 350.

Fig. 15. Uvigerina (Sagrina) nodosa, *D'Orbigny.* See page 363.

Figs. 16, *a*, *b*, & 17. Uvigerina (Sagrina) Raphanus, *Parker and Jones.* Recent: West Indies, Panama, India (on Clam-shell), Bombay Harbour (anchor-mud), Hong Kong (anchor-mud), Australian Coral-reefs (17 fathoms). See page 364.

Fig. 18. Uvigerina (Sagrina) dimorpha, *Parker and Jones.* Recent: Red Sea (near the Isle of Shadwan, at 372 fathoms), Abrohlos Bank (260 fathoms), Australian Coral-reefs (17 fathoms). See page 364.

Fig. 19. Textularia Folium, *Parker and Jones.* A very thin *Textularia*, with linear chambers, usually very unequal in their length, and forming a flat, pectinated, irregularly triangular or subrhomboidal shell, seldom so symmetrical in shape as the figured specimen. Shore-sand near Melbourne. See page 370.

PLATE XIX. (MISCELLANEOUS FORAMINIFERA.)

[Figures 2 & 3 are magnified 15 diameters; figs. 1, 4–13, 25 diameters (excepting fig. 5 *c*, 25 diameters.)].

Fig. 1. Planorbulina Culter, *Parker and Jones.* Very rare. Tropical Atlantic (1080 fathoms). A neat, discoidal, biconvex, trochiform *Planorbulina*, showing on its upper face about twenty-five (often more) neatly set chambers in a compact spire, bordered with a thin keel, as wide as a whorl of the chambers. It is an extreme varietal condition of the subsymmetrical form, imitating *Pulvinulina*, and ought to have been noticed at page 379, as a starved *Pl. Ungeriana.*

Fig. 2. Planorbulina retinaculata, *Parker and Jones.* Parasitic on Shells, East and West Indies. See page 380.

Fig. 3, *a*, *b*. Planorbulina larvata, *Parker and Jones.* Indian Sea. See page 380.

Fig. 4, *a*, *b*. Pullenia obliquiloculata, *Parker and Jones.* Abrohlos Bank (260 fathoms), Tropical Atlantic (1080 fathoms), Indian Ocean (2200 fathoms). See page 368.

Fig. 5, *a*, *b*, *c*. Sphæroidina dehiscens, *Parker and Jones.* Fig. 5 *c*, fragment of shell-wall more highly magnified. Tropical Atlantic (1080 fathoms) and Indian Ocean (2200 fathoms). See page 369.

Fig. 6, *a*, *b*, *c*. Discorbina rimosa, *Parker and Jones.* Recent: India (on Clam-shell). Fossil: Tertiary, at Grignon, Hautville, Freville, La-Fosse-de-Launy, &c. (Sir C. Lyell's Collection). This is smaller than *D. vesicularis*, and close to it and *D. elegans* in alliance; somewhat oval in shape; shell-substance thick, pores large; septal plane notched for aperture; chambers very much larger in the newer than in the older part of the shell, and discrete; and on the upper side several of the newer chambers are separated by chinks. On the under side there are secondary chambers over the umbilicus, perfect, large, and astral, with chinks at their periphery. See page 385.

Fig. 7, *a*, *b*, *c*. Discorbina globigerinoides, *Parker and Jones.* Common in the Calcaire grossier of Grignon. This *Discorbina* equals in size fine Tropical *Globigerinæ*, and reminds one of their form. It is also isomorphous with *Cymbalopora bulloides*, D'Orb., sp. In appearance it is the very opposite of its real ally *D. Parisiensis*; but it has much the same kind of septal face, the inner two-thirds of which are thickly covered with sinuous wrinkles and granules of exogenous shell-matter, having large pores opening out of them, and thus presenting a rudiment of the canal-system. A similar thickened surface, but formed of radiating granules, on the under side of the shell, is seen in *D. obtusa*, D'Orb., and *D. Parisiensis*, D'Orb. The astral processes in *D. globigerinoides* are abortive. See page 385.

Fig. 8, *a*, *b*, *c*. Discorbina polystomelloides, *Parker and Jones.* From the Australian Coral-reefs (Jukes's dredgings). This may be said to be a granulose form of, *D. rimosa*; but it is larger, more symmetrical, and extremely rough; and the chinks between the chambers are partly bridged over, so as to form a rough canal-system, as in some of the *Polystomellæ*.

Fig. 9, *a*, *b*, *c*. Discorbina dimidiata, *Parker and Jones*. Large and profusely abundant among the surf-washed Sponges on the Melbourne coast. This is merely *D. vesicularis* modified by being sharp-edged, and flat, and even scooped on the under face (opposite to that which is flat in *Truncatulina*). The astral flaps or valves are strongly marked over the umbilicus. See page 385.

Fig. 10, *a*, *b*, *c*. Discorbina biconcava, *Parker and Jones*. Shore-sand, Melbourne. A very small isomorph of *Planulina Ariminensis*. It is a hyaline, thick, limbate, square-edged, biconcave *Discorbina*, most concave on the umbilical face (as usual with the genus). Its astral flaps are feeble. See page 385.

Fig. 11, *a*, *b*, *c*. Rotalia annectens, *Parker and Jones*. Hong Kong (anchor-mud) and Fiji (coral-reef). A well developed Conus-shaped *Rotalia*, which has, on its under or umbilical surface, partially formed secondary chambers, owing to angular processes of the septa nipping the umbilical lobes. It is thus a passage-form between *R. Schrœteriana*, P. & J., and *R.* (*Asterigerina*) *lobata*, D'Orb. See page 387.

Fig. 12, *a*, *b*, *c*. Rotalia craticulata, *Parker and Jones*. Fiji. This Polystomelloid *Rotalia* is noticed by Dr. CARPENTER, Introd. Study Foram. p. 213. See page 387.

Fig. 13, *a*, *b*, *c*. Rotalia dentata, *Parker and Jones*. Bombay Harbour (anchor-mud). A well-grown, biconvex *Rotalia*, with numerous subquadrate chambers, thickened and raised septal edges, rowelled margin, and massive umbilicus. See page 387.

APPENDIX I.—*Additional North Atlantic Foraminifera.*

The Rev. J. S. TUTE, of Markington, has shown us a set of carefully executed drawings of minute Foraminifera from 67 fathoms, Atlantic Soundings, belonging to the Rev. W. FOWLER, of Cleckheaton. These comprise

Globigerina bulloides.
Spirillina vivipara.
Planorbulina lobatula.
—— Ungeriana.
Textularia pygmæa.
Miliola (young).

Also

Pteropoda (*Cuvieria*? and *Limacina*?).

Among the above, *Spirillina vivipara* is additional to our list of Foraminifera from the Atlantic Soundings. See also page 368.

With reference to *very minute* Foraminifera, such as are here referred to, it may be

observed that wherever Foraminifera are abundant small individuals are plentiful, but they very rarely represent other types than those to which the larger specimens are referable.

APPENDIX II.—*Professor* J. W. BAILEY'S *Researches on the "Virginian" Foraminifera of the North Atlantic.*

"Microscopical Examination of Soundings made by the U. S. Coast-survey off the Atlantic Coast of the U. S. By Professor J. W. BAILEY," Smithsonian Contributions to Knowledge, vol. ii. 1861, Article III. *

The examination was made and reported in 1848. The soundings were taken off the coast of New Jersey and Delaware, from lat. 50° to lat. 38° N., varying in depth from 10 to 105 fathoms. In the deeper soundings Professor BAILEY found "a truly wonderful development of minute organic forms, consisting chiefly of Polythalamia" (Foraminifera). He also remarked that these deep soundings were from a sea-bed under the influence, more or less, of the Gulf-stream; and that probably this might cause an immense development of organic life—giving rise to a "milky way of Polythalamia." Professor BAILEY also noticed that Foraminifera abundant in deep water would necessarily there make extensive calcareous deposits, contrasting with the quartzose and felspathic sands and muds of the coast.

We will, in the first place, give abridged notices of those soundings which were found to contain *Foraminifera*; and afterwards we will offer some remarks on Prof. BAILEY'S specific determinations, adapting them to the nomenclature used in this monograph. and so make them available for comparison with our "Celtic" forms.

E. No. 37. About South-east of Montauk Point; lat. 40° 59′ 55″, long. 71° 48′ 55″: 19 fathoms. Coarse gravel. mingled with ash-coloured mud. With a few small *Foraminifera*, chiefly *Rotalina*; a small bivalve Crustacean, *Diatomaceæ*, and Sponge-spicules.

E. No. 9. Lat. 40° 21′ 54″, long. 70° 55′ 35″: 51 fathoms. Greenish-grey mud or fine sand, with a few bits of shells, and a considerable number of *Foraminifera*, among which were *Marginulina Bachei*, Bailey (fig. 5, not abundant), *Robulina D'Orbignii*, Bailey (figs. 9 & 10), and *Bulimina auriculata*, Bailey (figs. 25–27).

F. No. 27. About South-east of Fire Island Inlet; lat 40° 14′ 13″, long. 72° 21′ 30″: 20 fathoms [material not described]. One specimen of *Quinqueloculina occidentalis*, Bailey (figs. 46–48); with a spine of *Echinus* and small plates of an Echinoderm.

F. No. 24. Lat. 39° 52′ 40″, long. 72° 14′: 49 fathoms. Greenish grey, rather coarse sand, mixed with some mud. *Foraminifera* rather abundant, comprising *Marginulina Bachei*, Bailey (fig. 5, rather common), *Orbulina universa*, D'Orb. (fig. 1, rare), a small *Bulimina*, a few small specimens of *Globigerina*; also a few Sponge-spicules, a small Cypridiform Crustacean shell, and a spine of *Echinus*.

---

* As this memoir is referred to by Professor BAILEY in the Am. Journ. Sc. Arts, March 1854, it was in print long before 1861.

F. No. 25. Lat. 39° 41′ 10″, long. 71° 43′: 105 fathoms. Fine greyish-green sand, very rich in *Foraminifera*, especially in *Globigerina* (figs. 20–22, *Gl. inflata*, D'Orb.), with *Marginulina Bachei*, Bailey (fig. 5, rare), and *Textularia Atlantica*, Bailey (figs. 11–13, common); also Sponge-spicules and *Diatomaceæ*.

G. No. 27. About East from Little Egg Harbour; lat. 38° 41′, long. 76° 6′: 20 fathoms. Fine-grained sand with black specks. A few fragments of bivalve and univalve Shells, small spines and numerous plates of an Echinoderm, and some *Foraminifera*: *Triloculina Brongniartiana*, D'Orb. (figs. 44, 45), *Robulina D'Orbignii*, Bailey (figs. 9, 10, rather common), and several specimens of a minute species of *Rotalina* (?); also *Diatomaceæ*.

G. No. 31. Lat. 39° 20′ 38″, long. 72° 44′ 35″: 50 fathoms. Fine-grained greyish sand with much mud. A considerable number of *Foraminifera*, including *Marginulina Bachei*, Bailey (rather common), *Robulina D'Orbignii*, Bailey (figs. 9, 10), and *Globigerina rubra*, D'Orb. (common; but not so common as in F. No. 25); also *Diatomaceæ* and some Sponge-spicules.

G. No. 8. Lat. 39° 31′, long. 72° 11′ 20″: 89 fathoms. Sand, coarser than the last, not so muddy, and about the same colour. Abounding in *Textularia Atlantica*, Bailey (figs. 38–43), and in *Globigerinæ* (figs. 20–24, *Gl. inflata* and *Gl. bulloides*), and also containing *Marginulina Bachei*, Bailey, *Robulina D'Orbignii*, Bailey, and *Orbulina universa*, D'Orb., together with a few *Diatomaceæ* and Sponge-spicules.

H. No. 2. South-east from Cape Henlopen; lat. 38° 46′ 40″, long. 75° 00′ 30″: 10 fathoms. Fine sand, slightly muddy. One specimen of *Triloculina* and a few minute nautiloid *Foraminifera*; together with a great variety of *Diatomaceæ*, some Sponge-spicules, and a few small spines of an Echinoderm.

H. No. 17. Lat. 38° 29′ 56″, long. 74° 38′ 4″: 20 fathoms. Clean quartzose sand, coarser than the last, white and yellow, with black specks. Many *Diatomaceæ*, but no evidences of *Foraminifera* except their soft parts, retaining the form of the chambers.

H. No. 67. Lat. 38° 9′ 25″, long. 74° 4′ 5″: 50 fathoms. Clean greyish sand, containing a few minute *Globigerinæ* and *Rotalinæ*; also *Diatomaceæ*.

H. No. 1. Lat. 38° 4′ 40′, long. 73° 56′ 47″: 90 fathoms. A rather coarse grey sand, with some mud, containing a few *Diatomaceæ* and a vast number of *Foraminifera*, "particularly *Globigerina*, many thousands of which must exist in every inch of the sea-bottom at this locality." The following were also common here:—*Orbulina universa*, D'Orb. (fig. 1), *Marginulina Bachei*, Bailey (figs. 2–6), *Robulina D'Orbignii*, Bailey (figs. 9, 10), *Rotalina Ehrenbergii*, Bailey (figs. 11–13).

Professor Bailey described and figured nearly, if not quite, all the different forms of Foraminifera that he met with in his examination of these soundings,—also some of the Diatoms and Sponge-spicules, as well as some minute spherical calcareous bodies, occurring either singly or united in strings and bunches (transparent when mounted in balsam), which he thought might possibly be ova of Foraminifera, but which we believe to be little inorganic crystalline globules of calcite, common in many sea-beds. The calcareous granules he found abundantly at 90 fathoms, and at 105, 89, and 20 fathoms.

The allusions to the Foraminifera in the Soundings "E. No. 37," "H. No. 2," and "H. No. 67," are not precise enough for the determination of the species found therein; and even with the notes appended to the account of the Species, we cannot make a very exact table of the distribution.

In Professor BAILEY'S plate illustrating his memoir, we have

1. *Orbulina universa*, D'Orb., fig. 1.

2. *Nodosaria*, a fragment, fig. 8. With almost cylindrical chambers, as in some subvarieties of *N. Pyrula*, D'Orb. Several fragments in the deeper soundings are said to have occurred.

3. *Dentalina mutabilis*, Bailey, fig. 7. This fragment might well belong to such a subvariety of *Dentalina communis* as *D. pauperata*, D'Orb. Several fragments were found in "H. No. 1."

4. *Marginulina Bachei*, Bailey, figs. 2–6. Figs. 2–4 are the same as *M. similis*, D'Orb., and *M. pedum*, D'Orb., all of these being dimorphous or Marginuline modifications of *Nodosaria Radicula*, Linn., sp.; and figs. 5, 6 represent a larger individual of the same form, such as has been named *Marginulina regularis* by D'ORBIGNY in his 'Foram. Foss. Bassin Vienne,' where the others are figured.

5. *Robulina D'Orbignii*, Bailey, figs. 9, 10. This is the common *Cristellaria cultrata*, Montfort, sp. The figured specimen has its last few chambers keelless, and trying, as it were, to leave the discoidal plan of growth, each having its septal aperture almost free. This is said to accompany the foregoing, which was in considerable numbers in all except the shallow soundings.

6. *Rotalina Ehrenbergii*, Bailey, figs. 11–13. This is *Planorbulina Haidingerii*, D'Orb., sp. (a variety of *Pl. farcta*, Fichtel and Moll, sp.), and occurred in "F. No. 25," and in several of the deeper soundings. Professor BAILEY thought it to be near *Rotalia Soldanii*, D'Orb.; and in truth *Pl. Haidingerii* does resemble that form,—but as an isomorph, not as a relative: so also it is an isomorph of *Pulvinulina truncatulinoides*, D'Orb.

7. *Rotalina cultrata?*, D'Orb., figs. 14–16. This is the common *Pulvinulina Menardii*, D'Orb., a variety of *P. repanda*, Fichtel and Moll, sp. Referred to as common in the deeper soundings.

8. *Rotalina semipunctata*, Bailey, figs. 17–19. The same as *Planorbulina Ungeriana*, D'Orb., sp. (*Pl. farcta*, var.).

9. *Globigerina rubra*, D'Orb., figs. 20–24. Professor BAILEY rightly considered figs. 20–22 to represent a separate form; it is *Gl. inflata*, D'Orb., a variety of *Gl. bulloides*, D'Orb., to which all must be referred specifically, D'ORBIGNY'S *Gl. rubra* being so named on account of the ruddiness of its shell, which is not dependent on the sarcode for its pink colour. *Gl. inflata* is specially noticed as occurring at 105 fathoms. Vast numbers of *Globigerinæ* occurred in the deeper soundings, especially the deepest; whilst they were but few and small at 49 fathoms. "The abundance in which the species of *Globigerina* occur in the deep soundings G. No. 31 and H. No. 1 gives to these green muds a most striking resemblance to the green Tertiary marls perforated by the artesian wells

at Charleston, S. C. This similarity appears to indicate that the Charleston beds were a deep-sea deposit, perhaps made under the influence of an ancient Gulf-stream" (p. 11).

10. *Bulimina auriculata*, Bailey, figs. 25–27. This is *B. Pyrula*, D'Orb. Several found at 51 fathoms.

11. *Bulimina turgida*, Bailey, figs. 28–31. A slight modification of *B. Pyrula*, D'Orb., the newer chambers being proportionally large and overlapping. It occurred with the foregoing, and at 49 fathoms.

12. *Bulimina serrata*, Bailey, figs. 32–34. The very small *Bulimina* (*Virgulina*) *Schreibersii*, Czjzek.

13. *Bulimina compressa*, Bailey, figs. 35–37. The same as *B.* (*Virgulina*) *squamosa*, D'Orb.

14. *Textularia Atlantica*, Bailey, figs. 38–43. This is the *Textularia* (*Verneuilina*) *triquetra*, Münster (*Verneuilina tricarinata*, D'Orb.). Found by Professor BAILEY only in the deeper soundings; especially abundant at 89 fathoms ("G. No. 38"). (Judging from our own specimens, we think that in these figured specimens the aperture of the shell is drawn too smoothly.)

15. *Triloculina Brongniartii*, D'Orb., figs. 44, 45.

16. *Quinqueloculina occidentalis*, Bailey, figs. 46–48. This fair typical form of *Miliola* (*Quinqueloculina*) *Seminulum*, Linn., sp., is said by Professor BAILEY to occur "not uncommonly in the sands along the western shores of the Atlantic,"—as indeed it does along many coasts.

In presenting the annexed bathymetrical Table (No. VIII.) of Professor BAILEY'S Foraminifera, we must express a hope that some day a fuller Synopsis of this marginal Fauna of the "Virginian Province" will be produced by the Transatlantic naturalists from more ample materials than Professor BAILEY had to work on; for we cannot think that this Fauna is fully represented by the present list.

TABLE VIII.—Table of the Foraminifera of the "Virginian Province." (After Professor BAILEY; with Nomenclature corrected.)

GENERA, SPECIES, AND VARIETIES.	1	2	3	4	5	6	7	8	9	10	11
	H. 2*.	E. 37†.	F. 27.	G. 27‡.	F. 24.	G. 31.	H. 67§.	E. 9.	G. 8.	H. 1.	F. 25.
	Lat. 38° 46′ 40″ Long. 75° 0′ 30″	Lat. 40° 59′ 50″ Long. 71° 48′ 55″	Lat. 40° 14′ 13″ Long. 72° 21′ 30″	Lat. 38° 41′ 0″ Long. 76° 6′ 0″	Lat. 39° 52′ 40″ Long. 72° 14′ 0″	Lat. 39° 20′ 38″ Long. 72° 44′ 35″	Lat. 38° 9′ 25″ Long. 74° 4′ 5″	Lat. 40° 21′ 54″ Long. 70° 55′ 35″	Lat. 39° 31′ 0″ Long. 72° 11′ 20″	Lat. 38° 4′ 40″ Long. 73° 56′ 47″	Lat. 39° 41′ 10″ Long. 71° 43′ 0″
Fathoms . .	10.	19.	20.	20.	49.	50.	50.	51.	89.	90.	105.
Orbulina universa, D'O.	..	..	..	..	*	..	..	..	*	*	
Nodosaria Pyrula, D'O.	..	..	..	..	?	?	..	?	?	?	?
Dentalina communis (pauperata), D'O.	..	..	..	..	..	..	.	..	..	*	
Marginulina regularis, D'O.	..	..	..	..	*	*	..	*	*	*	*
Cristellaria cultrata, Montf.	..	..	..	*	?	*	..	*	*	*	?
Planorbulina Haidingerii, D'O.	..	..	..	..	*	?	?	?	?	*	
—— Ungeriana, D'O.	..	..	..	..	..	..	..	..	*		
Pulvinulina Menardii, D'O.	..	..	..	..	?	?	..	?	?	?	?
Globigerina bulloides, D'O.	..	..	..	..	*	*	*	..	*	*	*
Bulimina Pyrula, D'O.	..	..	..	..	*	..	..	*			
—— (Virgulina) Schreibersii, Czjzek	..	..	..	..	..	*	..	*			
—— (——) squamosa, D'O.	..	..	..	..	*	*	..	..	..	..	*
Textularia (Verneuilina) triquetra, Münst.	..	..	..	..	..	?	..	..	*	*	*
Triloculina Brongniartii, D'O.	*	..	..	*							
Quinqueloculina Seminulum, Linn.	..	..	*								
Diatomaceæ	*	*	..	*	..	*	*	..	*	*	*
Sponge-spicules	*	*	..	..	*	*	..	..	*	..	*
Echinodermata	*	..	*	*	*						
Mollusca (fragments of shells)	..	..	..	*	..	..	*				
Bivalved Entomostraca	..	*	..	..	*						

* Containing "a few minute nautiloid Foraminifera" besides the *Triloculinæ*.

† A few small Foraminifera, chiefly "Rotalina," were found in this sounding.

‡ Also containing "a minute species of Rotalina."

§ Containing a few minute *Globigerinæ* and *Rotalinæ*.

Appendix III.—*Further Researches by Professor* J. W. Bailey.

"Examination of Deep Soundings from the Atlantic Ocean." By Professor J. W. Bailey, of West Point, New York," American Journal of Science and Arts, 2 ser. vol. xvii. p. 176, &c. 1854.

In this memoir Professor Bailey describes the results of his examination of five deep-sea soundings, from the Atlantic, given him by Lieut. Maury, and of one sounding, of less depth, made by Lieut. Berryman.

I. Lieutenant Maury's Soundings.

	Fathoms.				
1.	1800	Lat. 42° 04′	Long. 29° 00′	N. of the Azores.	"Lusitanian Province."
2.	1360	Lat. 44° 41′	Long. 24° 35′	N.E. of the Azores.	
3.	1580	Lat. 49° 56′ 30″	Long. 13° 13′ 45″	S.W. of Ireland.	"Celtic Province."
4.	1800	Lat. 47° 38′	Long. 09° 08′	Off the mouth of English Channel.	
5.	2000	Lat. 54° 17′	Long. 22° 33′	W. of Ireland.	

These soundings contained no gravel, sand, or other recognizable inorganic mineral matter, but consisted of *Foraminifera* and calcareous mud derived from their disintegrated shells. *Globigerinæ* greatly predominated; and *Orbulinæ* were in immense numbers in some, especially in the sounding from 1800 fathoms. They all contained *Diatomaceæ*, Sponge-spicules, and *Polycystinæ*. Professor Bailey remarked that *Agathistegia* (*Miliola*, &c.) were absent, as well as *Marginulina*, *Textularia*, and other forms that he had met with in shallower soundings.

II. Lieutenant Berryman's Sounding.

Fathoms.		
175.	Lat. 42° 53′ 30″ N. Long. 50° 05′ 45″ W.	S.S.E. of Newfoundland. On northern border of the "Virginian Province" (the western extension of the "Celtic Province").

The sea-bed off Newfoundland is here destitute of Foraminifera as far as this sounding shows; the quartzose sand, with a few grains of hornblende, being barren of shells or other organic remains.

Professor Bailey's results in these examinations are therefore very similar to those obtained by ourselves from similar parts of the Atlantic bed.

APPENDIX IV.—*Researches on the North Atlantic Foraminifera,*
by F. L. POURTALES, *Esq.*

"Examination (by F. L. POURTALES, Esq., Assistant in the United States Coast-survey) of Specimens of Bottom obtained in the Exploration of the Gulf-stream, by Lieutenants Commanding, T. A. M. CRAVEN and J. N. MAFFITT, United States Navy," Report of the Superintendent of the United States Coast-survey for 1853; Appendix, No. 30, pages 82*, 83*, 1854.

From fourteen soundings off the eastern coast of Florida, and three off Georgia (all belonging to the "Caribbæan Province"), Count F. POURTALES obtained results similar in a great degree to those of Professor BAILEY's examination of the soundings off New Jersey and Delaware (see above, page 423); and having soundings from much greater depths (150 to 1050 fathoms), he met with a greater predominance of *Globigerinæ*, forming, with other Foraminifera, the white mud of the sea-bed; in one instance *Globigerinæ* and the minute green stony casts of these shells entirely formed the bed† (at 150 fathoms, lat. 31° 2′, long. 79° 35′). At 1050 fathoms (lat. 28° 24′, long. 79° 13′) he found *Globigerina* and *Orbulina*, and the so-called *Rotalina cultrata*, *R. Ehrenbergii*, and *R. Bayleyi*, with fragments of Molluscan Shells, of Corals, and of *Anatifer*, as well as some *Pteropoda*; and only about 1 or 2 per cent. of fine sand in the Foraminiferal mud.

As these soundings are beyond the limits of the "Provinces" that we have to do with in the foregoing memoir, we omit the details of the other specimens of the "Caribbæan" sea-bed; but we remark that the author of this notice refers to former Reports (and Proc. Amer. Assoc. Charleston) in which he had intimated that "with the increase in depth—in the greater depths—the number of individuals [of *Foraminifera*, especially *Globigerina*] appeared to increase," having then seen a sounding from 267 fathoms where the sand contained 50 *per cent.* of *Foraminifera*; whilst now he found at upwards of 1000 fathoms *Foraminifera* with little or no sand. The extension of life to greater depths than 300 fathoms (E. FORBES, Ægean, Brit. Assoc. Rep. 1843) is also noticed by the author; but his suggestion, that *Globigerina* would be found to decrease gradually "for a considerable depth before it should cease to appear," does not appear to be as yet substantiated, since *Globigerina* holds its own at the greatest depth (2700 fathoms, South Atlantic) hitherto experimented upon. He remarks that the *Foraminifera* appear to be fresh in the deep-sea soundings, and probably live at the great depths from which they are brought up.

Note.—MAURY has already observed that the bed of the Atlantic at more than two miles depth has no sand nor gravel, but consists chiefly of *Foraminifera* and a small number of *Diatomaceæ* (siliceous).—"Sailing Directions," &c., 6th edit. 1864.

---

† To this Professor BAILEY refers in his interesting paper "On the Origin of Greensand and its formation in the Oceans of the present Epoch," Quart. Journ. Microscop. Science, vol. v. pp. 83–87; 1857.

APPENDIX V.—*The Foraminifera of the "Celtic and Virginian" Provinces of the North Atlantic, as a Fauna.*

The accompanying Table (No. IX.), already alluded to at p. 332, gives us a synoptical view of the Foraminifera of the "Celtic Province," including its western or "Virginian" portion. Excepting that further research will enrich the "Virginian" columns (Coralline and Coral zones of the American side of the Province), the Table comprises a complete Foraminiferal Fauna; and we believe that, by careful condensation of the multitudinous varietal forms under specific heads, we have fairly indicated the range and relative abundance of the members of a natural-history-group under such local conditions as naturalists have determined, chiefly by the aid of Mollusca and other marine animals, to belong to a more or less uniform zoological area.

Professor WILLIAMSON'S 'Monograph of the British Recent Foraminifera' has (with corrections of nomenclature) supplied the first column, for the Littoral, Laminarian, Coralline, and Coral zones; Mr. H. B. BRADY'S researches in the Shetland and other British Foraminifera give us the second column; the next four columns refer to the different parts of the North Atlantic from whence we have many of the Foraminifera described in this memoir; and the last two columns comprise what we know of the "Virginian" Foraminifera, to which the Appendices Nos. II., III., & IV. have reference.

TABLE IX.—Table of the Foraminifera of the "Celtic Province," including the North-American or "Virginian" portion of that Province.

Note.—Mr. H. B. BRADY has kindly aided us in making the first two Columns as complete as possible.

GENERA, SPECIES, AND VARIETIES.	1	2	3	4	5	6	7	8
							"Virginian Province" (after Bailey).	
	Littoral, Laminarian, Coralline, and Coral (?)-zones of British Isles (chiefly after Williamson).	Coral-zone [50–100 fms.] of British Isles; represented by the Shetland Fauna (Mr. H. B. Brady, F.L.S.) of 60–80 fms.	North Atlantic [Coral-zone] (43–90 fms.); off the Irish coast.	North Atlantic; Deep Water of the Eastern Plateau (200–400 fms.).	North Atlantic; Abyssal Depths (1750–2176 fms.).	North Atlantic; Abyssal Depths (1450–2350 fms.), "Boreal Province."	[Coralline zone] (10–20 fms.).	[Coral-zone] (49–105 fms.).
Fathoms ...	0–80.		43–90.	200–400.	1750–2176.	1450–2350.	10–20.	49–105.
					1450–2350.			
Lagena sulcata .... type	*	*	*	*	..	*		
—— lævis	*	*	..	*				
—— semistriata	*	*						
—— striata	*	*						
—— distoma .... subtype	*	*						
—— (Entosolenia) globosa	*	*	..	*				
—— (——) caudata	*	*	..	..	*	*		
—— (——) marginata	*	*	**	*	..	**		
—— (——) squamosa	*	*	..	..	..	*		
—— (——) Melo	*	*						
Nodosarina (Glandulina) lævigata	..	*						
—— (Nodosaria) scalaris	*	*	*	*				
—— (——) Pyrula	*	*	..	..	..	..	..	*
—— (——) Raphanus .... subtype	*	*	*					
—— (Lingulina) carinata	*	*						
—— (Dentalina) communis (and subvarieties)	*	*	..	..	*	..	..	*
—— (——) Acicula	*	*						
—— (Vaginulina) linearis	*	*						
—— (Cristellaria) cultrata and rotulata	*	*	*	..	..	..	*	*
—— (——) Crepidula	*	*	*					
—— (——) Italica	*							
—— (Marginulina) Lituus	*							
—— (——) regularis	..	..	..	..	..	...	..	*
Polymorphina lactea .... type	*	*						
—— compressa	*	*						
—— tubulosa	*	*						
—— concava	*	*						
—— myristiformis	*	*						
Uvigerina pygmæa .... type	*	*	**	**				
—— angulosa	*	*	*	*				
—— irregularis	*							
Orbulina universa .... type	*	*	*	**	**	**	?	*
Globigerina bulloides .... type	*	*	**	***	***	***	?	***
Sphæroidina bulloides	..	..	..	*	...	*		
Pullenia sphæroides	..	..	..	*	*	*		
Textularia Sagittula	*	*	*					
—— Trochus	*	*						
—— variabilis	*	*						
—— abbreviata	..	..	**	*				
—— pygmæa	*	*	**	*	..	*		

Table IX. (continued).

Genera, Species, and Varieties.	1	2	3	4	5	6	7	8
	Littoral, Laminarian, Coralline, and Coral(?)-zones of British Isles (chiefly after Williamson).	Coral-zone [50–100 fms.] of British Isles; represented by the Shetland Fauna (Mr. H. B. Brady, F.L.S.) of 60–80 fms.	North Atlantic [Coral-zone] (43–90 fms.); off the Irish coast.	North Atlantic; Deep Water of the Eastern Plateau (200–400 fms.).	North Atlantic; Abyssal Depths (1750–2176 fms.).	North Atlantic; Abyssal Depths (1450–2350 fms.), "Boreal Province."	"Virginian Province" (after Bailey). [Coralline zone] (10–20 fms.).	 [Coral-zone] (49–105 fms.).
Fathoms ...	0–80.		43–90.	200–400.	1750–2176.	1450–2350.	10–20.	49–105.
					1450–2350.			
Textularia carinata	..	..	**					
—— difformis	*	*						
—— complexa	*							
—— (Verneuilina) polystropha	*	*						
—— (——) triquetra	..	..	..	..	..	..	..	**
—— (Bigenerina) digitata	..	*	*					
—— (——) Nodosaria	..	*	**					
Bulimina pupoides	*	*						
—— Pyrula	..	..	..	..	..	..	..	*
—— Buchiana	..	..	*					
—— marginata	*	*	**	*				
—— aculeata	*	*	..	*				
—— ovata	*	*	*	..	*			
—— convoluta	*	*						
—— (Robertina) elegantissima	*	*						
—— (Virgulina) Schreibersii	*	*	..	..	*	*	..	*
—— (——) squamosa	..	..	..	..	..	..	..	*
—— (Bolivina) punctata	..	*	*	*				
—— (——) costata	*	..	..	*				
Cassidulina lævigata ... type	*	*	**	*	*			
—— crassa	*	*	..	*				
Spirillina vivipara ... type	*	*						
—— margaritifera	*							
Discorbina rosacea	*	*	*					
—— ochracea	*	*						
—— globularis	*	*						
—— Berthelotiana	..	*	*					
Planorbulina Mediterranensis	*	*	*					
—— Haidingerii	..	*	..	..	*	..	..	*
—— Ungeriana	..	*	*	*	*	*	..	*
—— (Truncatulina) lobatula	*	*	**	*	*			
—— (——) refulgens	*	*						
—— (Anomalina) coronata	..	*						
Pulvinulina repanda ... type	*	*						
—— Auricula	*	*						
—— Karsteni	..	*						
—— concentrica	..	*						
—— elegans	*	..	*	..	..	*		
—— Menardii ... subtype	*	..	*	*	**	**	..	*
—— Canariensis	..	..	*	*	**	**		
—— pauperata	..	..	..	..	*	*		
—— Micheliniana	..	..	*	**	**	*		
Rotalia Beccarii ... type	*	*	*					
—— nitida	*	*						

TABLE IX. (continued).

Genera, Species, and Varieties.	1	2	3	4	5	6	7	8
	Littoral, Laminarian, Coralline, and Coral(?)-zones of British Isles (chiefly after Williamson).	Coral-zone [50–100 fms.] of British Isles; represented by the Shetland Fauna (Mr. H. B. Brady, F.L.S.) of 60–80 fms.	North Atlantic [Coral-zone] (43–90 fms.); off the Irish coast.	North Atlantic; Deep Water of the Eastern Plateau (200–400 fms.).	North Atlantic; Abyssal Depths (1750–2176 fms.).	North Atlantic; Abyssal Depths (1450–2350 fms.), “Boreal Province.”	“Virginian Province” (after Bailey).	
							[Coralline zone] (10–20 fms.).	[Coral-zone] (49–105 fms.).
Fathoms ...	0–80.		43–90.	200–400.	1750–2176.	1450–2350.	10–20.	49–105.
					1450–2350.			
Rotalia Soldanii	..	..	*	*	*	*		
—— orbicularis	*	*	..	..	*			
Tinoporus lævis	*	*						
Patellina corrugata	*	*						
Nummulina radiata	*							
—— (Operculina) ammonoides	*	*	**	**				
Polystomella crispa ... type	*	*						
—— striatopunctata	*	*	*	*				
—— Arctica	..	*						
—— (Nonionina) umbilicatula	*	*	**	**	*	*		
—— (——) depressula	*	*						
—— (——) turgida	*	*	*	*				
—— (——) Scapha	*	..	..	*				
—— (——) stelligera	..	*						
—— (——) asterizans ... subtype	*	*						
Valvulina Austriaca	*	*						
Lituola nautiloidea ... type	*							
—— Canariensis	*	*	*	*				
—— Scorpiurus	*?	*						
Trochammina inflata	*	*						
—— incerta	*	*						
Cornuspira foliacea ... type	*	*						
Miliola (Quinqueloculina) Seminulum .. type	**	**	**	*	*	*	*	
—— (——) agglutinans	*	..	..	**	*	*		
—— (——) secans	*	*						
—— (——) bicornis	*	*						
—— (——) Ferussacii	*	*						
—— (——) pulchella	..	*						
—— (——) subrotunda	*							
—— (——) tenuis	..	..	..	*	..	*		
—— (Triloculina) oblonga	*	*	*	..	..	*		
—— (——) Brongniartii	*	..	..	..	..	..	*	
—— (——) trigonula ... subtype	*	*						
—— (——) tricarinata	..	*						
—— (Biloculina) ringens ... subtype	*	*						
—— (——) compressa	*	*						
—— (——) depressa	*	..		..	*	*		
—— (——) elongata	*	*	..	..	*	*		
—— (——) Sphæra	*	*						
—— (——) contraria	..	*						
—— (Spiroloculina) planulata ... subtype	*	*	*	..	*	*		
—— (——) limbata	*	*	*					
—— (——) excavata	*							
—— (——) canaliculata	*	*	*					

APPENDIX VI.—*General Distribution of Foraminifera.*

For the comparison of the Arctic and North-Atlantic Foraminifera with those of other seas, we selected twenty-nine sets of specimens from different parts of the Atlantic, Mediterranean, Red Sea, Indian Ocean, and Pacific, and showed in Table VII. the relative distribution of such of them as we have obtained from the Arctic and North-Atlantic sea-beds. Most of the localities, however, yielded other forms, the enumeration of which will complete what we know of the Foraminiferal fauna of each of the places quoted in Table VII.; and, as the proportional size and occurrence can also be indicated, so many complete lists will furnish material help in the study of representative groups of Foraminifera, as to their distribution and habits.

TABLE X.—Showing the Foraminifera belonging to the several Dredgings and Soundings indicated in Table VII., but omitted there as not being known in the Arctic and North-Atlantic Sea-beds. (The materials of this Table and of Table VII., taken together, supply perfect lists of the Foraminiferal Fauna for the several localities. Columns Nos. 5, 11, 12, 13, & 25 of Table VII. are complete in themselves.)

vl. Very large. *l.* Large. *rl.* Rather large. *m.* Middle-sized. *s.* Small. *vs.* Very small.
VC. Very common. C. Common. RC. Rather common. RR. Rather rare.
R. Rare. VR. Very rare.

ADDITIONAL GENERA, SPECIES, AND VARIETIES.	
FOR COLUMN No. 1.	
Trochammina inflata, Montag.	*vl* VC
FOR COLUMN No. 2.	
Uvigerina aculeata, D'O.	*vs* R
Textularia variabilis, Will.	*vs* VC
Verneuilina pygmæa, Egger	*vs* C
Trochammina inflata, Montag.	*vl* VC
Lituola agglutinans, D'O.	*s* R
FOR COLUMN No. 3.	
Nodosaria aculeata, D'O.	*vs* VR
Textularia variabilis, Will.	*s* C
FOR COLUMN No. 4.	
Nonionina granosa, D'O.	*vs* C
Bulimina pupoides, D'O.	*rs* C
Textularia variabilis, Will.	*s* C
FOR COLUMN No. 6.	
Polystomella strigillata, β, F. & M.	*rs* RR
Bulimina pupoides, D'O.	*m* R
Textularia variabilis, Will.	*s* R
Trochammina inflata, Montag.	*m* VC
Triloculina Brongniartii, D'O.	*m* C
FOR COLUMN No. 7.	
Polystomella strigillata, β, F. & M.	*m* C
Tinoporus lævis, P. & J.	*s* R
Spiroloculina excavata, D'O.	*m* C
Quinqueloculina secans, D'O.	*rl* C
—— pulchella, D'O.	*rl* C
Triloculina trigonula, Lam.	*m* C
—— Brongniartii, D'O.	*rl* C
FOR COLUMN No. 8.	
Quinqueloculina secans, D'O.	*l* VC

TABLE X. (continued).

ADDITIONAL GENERA, SPECIES, AND VARIETIES.	
FOR COLUMN No. 9.	
Polystomella strigillata, β, F. & M.	*vs* C
Bulimina pupoides, D'O.	*vs* RC
Textularia variabilis, Will.	*s* RC
Quinqueloculina secans, D'O.	*l* RR
FOR COLUMN No. 10.	
Polystomella strigillata, β, F. & M.	*m* C
Bulimina pupoides, D'O.	*m* RR
Trochammina inflata, Montag.	*m* C
Quinqueloculina secans, D'O.	*m* RC
—— pulchella, D'O.	*m* C
Triloculina Brongniartii, D'O.	*m* C
FOR COLUMN No. 14.	
Lingulina carinata, D'O.	*l* C
Dentalina brevis, D'O.	*m* C
—— elegans, D'O.	*l* C
—— Acicula, Lam.	*m* C
Vaginulina Badenensis, D'O.	*m* C
Rimulina glabra, D'O.	*rl* RC
Marginulina tuberosa, D'O.	*m* C
—— Falx, P. & J.	*m* C
—— elongata, D'O.	*m* C
Cristellaria Calcar, Linn.	*rl* C
—— Italica, Defr.	*m* RC
—— Vortex, F. & M.	*m* RC
Uvigerina aculeata, D'O.	*m* C
Globigerina hirsuta, D'O.	*m* R
—— helicina, D'O.	*m* RC
Planulina Ariminensis, D'O.	*m* VC
Planorbulina reticulata, Czjzek	*m* C
Pulvinulina repanda, F. & M.	*l* C
Cassidulina oblonga, Rss.	*m* RR
Bolivina Triticum, nov.	*m* RR
Textularia carinata, D'O.	*m* RC
—— conica, D'O.	*m* RC
Bigenerina rugosa, D'O.	*vl* C
Verneuilina triquetra, Münst.	*m* RC
Clavulina communis, D'O.	*l* VC
Webbina* clavata, P. & J.	*l* VC
Trochammina incerta, D'O.	*m* RC
—— charoides, P. & J.	*m* RC
Spiroloculina abortiva, nov.	*s* R
—— canaliculata, D'O.	*m* C
Biloculina Sphæra, D'O.	*m* C
Lituola Cenomana, D'O.	*vs* R

ADDITIONAL GENERA, SPECIES, AND VARIETIES.	
FOR COLUMN No. 15.	
Rotalia ornata, D'O.	*m* C
Calcarina rarispina, Desh.	*s* C
—— Defrancii, D'O.	*s* C
Cymbalopora Poeyi, D'O.	*m* C
Pulvinulina Schreibersii, D'O.*	*m* C
—— Auricula, F. & M.	*m* C
Cassidulina serrata, Rss.	*m* R
Polystomella discoidalis, D'O.	*m* C
Amphistegina vulgaris, D'O.	*m* C
Bolivina plicata, D'O.	*m* C
Verneuilina spinulosa, Rss.	*m* RC
Textularia Partschii, Czjzek	*m* C
—— pectinata, Rss.	*m* RC
—— Trochus, D'O.	*m* RC
—— Candeiana, D'O.	*m* RC
Spiroloculina alata, nov.	*l* C
Quinqueloculina Sagra, D'O.	*l* R
—— pulchella, D'O.	*m* C
Biloculina Sphæra, D'O.	*s* R
FOR COLUMN No. 16.	
Dentalina elegantissima, D'O.	*s* R
Uvigerina aculeata, D'O.	*s* C
Sagrina dimorpha, P. & J.	*s* R
Globigerina helicina, D'O.	*m* C
Rotalia ornata, D'O.	*s* R
Cymbalopora Poyei, D'O.	*m* C
Planorbulina ammonoides, Rss.	*s* RC
Pulvinulina pulchella, D'O.	*m* C
—— Auricula, F. & M.	*m* C
—— excavata, D'O.	*rs* RC
—— Schreibersii, D'O.	*m* RR
Amphistegina vulgaris, D'O.	*vs* RC
Cassidulina oblonga, Rss.	*vs* RC
Bolivina dilatata, Rss.	*m* C
—— plicata, D'O.	*m* C
—— Triticum, nov.	*s* R
Textularia Candeiana, D'O.	*m* C
—— prælonga, Rss.	*s* RC
—— pectinata, Rss.	*m* RC
Vertebralina inæqualis, Gm.	*vs* C
—— alata, nov.	*vs* C
Spiroloculina alata, nov.	*s* RR
Orbitolites complanatus, Lam.	*vs* RR

---

* We retain D'ORBIGNY'S term *Webbina* for the subtype of *Trochammina* which he named *Webbina irregularis*, with its varieties *W. clavata*, &c.

TABLE X. (continued.)

ADDITIONAL GENERA, SPECIES, AND VARIETIES.	
FOR COLUMN No. 17.	
Uvigerina aculeata, D'O.	*s* C
Globigerina hirsuta, D'O.	*l* VC
—— helicina, D'O.	*l* VC
Planorbulina ammonoides, Rss.	*m* RC
Pulvinulina excavata, D'O.	*s* R
Bolivina hyalina, nov.	*s* C
—— dilatata, Rss.	*s* RR
Textularia variabilis, Will.	*m* RC
—— Candeiana, D'O.	*m* RR
—— prælonga, Rss.	*m* RC
Spiroloculina alata, nov.	*m* R
FOR COLUMN No. 18.	
Uvigerina aculeata, D'O.	*s* C
Globigerina hirsuta, D'O.	*l* VC
—— helicina, D'O.	*l* VC
Planorbulina ammonoides, Rss.	*s* R
Bolivina dilatata, Rss.	*s* RR
Textularia Candeiana, D'O.	*m* R
Trochammina charoides, P. & J.	*s* R
Spiroloculina alata, nov.	*m* C
Peneroplis pertusus, Forsk.	*s* R
FOR COLUMN No. 19.	
Uvigerina aculeata, D'O.	*s* RC
Sagrina Raphanus, P. & J.	*m* RC
Rotalia dentata, P. & J.	*m* VC
—— ornata, D'O.	*m* VC
Planorbulina ammonoides, Rss.	*s* RR
Pulvinulina Auricula, F. & M.	*s* C
—— pulchella, D'O.	*m* C
Polystomella Sagra, D'O.	*m* RC
Bulimina pupoides, D'O.	*s* VC
Bolivina hyalina, nov.	*s* VC
Verneuilina spinulosa, Rss.	*s* R
Textularia variabilis, Will.	*s* RR
Quinqueloculina dilatata, D'O.	*s* R
Peneroplis pertusus, Forsk.	*s* C
FOR COLUMN No. 20.	
Sagrina Raphanus, P. & J.	*m* RR
Rotalia Schrœteriana, P. & J.	*l* RR
—— annectens, P. & J.	*l* C
Planulina Ariminensis, D'O.	*s* RC
Planorbulina ammonoides, Rss.	*s* RC
Cymbalopora Poeyi, D'O.	*m* RC
Pulvinulina Auricula, F. & M.	*s* RC

ADDITIONAL GENERA, SPECIES, AND VARIETIES.	
Polystomella Sagra, D'O.	*m* VC
—— discoidalis, D'O.	*m* VC
Bolivina Triticum, nov.	*m* RC
Verneuilina spinulosa, Rss.	*m* C
Textularia Candeiana, D'O.	*m* C
Spiroloculina canaliculata, D'O.	*m* RC
Quinqueloculina Sagra, D'O.	*s* RC
—— pulchella, D'O.	*s* RC
Triloculina trigonula, Lam.	*m* RC
FOR COLUMN No. 21.	
Marginulina tuberosa, D'O.	*s* R
Uvigerina aculeata, D'O.	*l* C
Globigerina helicina, D'O.	*l* VC
Anomalina variolaria, D'O.	*rl* R
Planorbulina Culter, P. & J.	*m* RC
—— Clementiana, D'O.	*rl* R
Pulvinulina crassa, D'O.	*l* VC
—— cuneiformis, nov.	*l* VC
Sphæroidina dehiscens, P. & J.	*vl* VC
Pullenia obliquiloculata, P. & J.	*vl* VC
Cassidulina oblonga, Rss.	*vl* C
—— serrata, Rss.	*l* C
Verneuilina spinulosa, Rss.	*rl* R
Textularia variabilis, Will.	*s* C
FOR COLUMN No. 22.	
Planulina Ariminensis, D'O.	*m* C
Pulvinulina pulchella, D'O.	*m* C
—— Schreibersii, D'O.	*rl* C
Verneuilina spinulosa, Rss.	*m* RC
Lituola Soldanii, P. & J.	*l* C
FOR COLUMN No. 23.	
Nodosaria hirsuta, D'O.	*m* C
Uvigerina aculeata, D'O.	*m* C
Sagrina dimorpha, P. & J.	*m* C
Planulina Ariminensis, D'O.	*m* C
Planorbulina ammonoides, Rss.	*m* RC
—— reticulata, Czjzek	*l* C
Pulvinulina crassa, D'O.	*m* C
Pullenia obliquiloculata, P. & J.	*m* RC
—— quinqueloba, Rss.	*s* RC
Bolivina plicata, D'O.	*m* RC
—— dilatata, Rss.	*l* C
—— Triticum, nov.	*s* C
Verneuilina spinulosa, Rss.	*s* R
Gaudryina Badenensis, Rss.	*m* C
Textularia prælonga, Rss.	*m* RC
Trochammina inflata, Montag.	*s* RR

TABLE X. (continued.)

ADDITIONAL GENERA, SPECIES, AND VARIETIES.	
Trochammina charoides, P. & J.	*l* RC
Webbina clavata, P. & J.	*s* R
Quinqueloculina pulchella, D'O.	*s* RR
Biloculina Sphæra, D'O.	*s* RR
FOR COLUMN No. 24.	
Planulina Ariminensis, D'O.	*s* R
Planorbulina ammonoides, Rss.	*l* RC
Cymbalopora Poeyi, D'O.	*s* C
Pulvinulina crassa, D'O.	*m* RR
Amphistegina vulgaris, D'O.	*s* C
Textularia variabilis, Will.	*s* C
Spiroloculina alata, nov.	*m* C
Quinqueloculina pulchella, D'O.	*s* R
Biloculina contraria, D'O.	*m* R
Vertebralina Cassis, D'O.	*s* R
—— inæqualis, Gm.	*s* C
—— conico-articulata, Batsch.	*s* RR
FOR COLUMN No. 26.	
Globigerina hirsuta, D'O.	*l* C
—— helicina, D'O.	*l* C
Pulvinulina cuneiformis, nov.	*l* RC
Sphæroidina dehiscens, P. & J.	*vl* VC
Pullenia obliquiloculata, P. & J.	*vl* VC
FOR COLUMN No. 27.	
Dentalina Acicula, Lam.	*l* R
Vaginulina Badenensis, D'O.	*s* R
Uvigerina aculeata, D'O.	*m* RR
Globigerina hirsuta, D'O.	*m* C
Planorbulina farcta, F. & M.	*m* C
Pulvinulina crassa, D'O.	*m* VC
Cassidulina oblonga, Rss.	*m* C
Bolivina dilatata, Rss.	*m* C
Verneuilina pygmæa, Egger	*m* R
Gaudryina Badenensis, Rss.	*s* R
Textularia variabilis, Will.	*m* RC
FOR COLUMN No. 28.	
Lagena squamoso-marginata, P. & J.	*m* C
Rotalia ornata, D'O.	*l* C
Planorbulina vulgaris, D'O.	*m* RC
Pulvinulina pulchella, D'O.	*m* RC
—— Auricula, F. & M.	*m* C
Cymbalopora Poeyi, D'O.	*s* RC
—— squamosa, D'O.	*l* R
Calcarina Spengleri, Gm.	*m* RR
Polystomella craticulata, F. & M.	*vl* VC
Amphistegina vulgaris, D'O.	*m* VC
Bulimina convoluta, Will.	*s* R
Bolivina Triticum, nov.	*s* R
—— dilatata, Rss.	*m* R
Verneuilina spinulosa, Rss.	*s* R
Textularia Partschii, Czjzek.	*l* VC
—— Trochus, D'O.	*l* VC
—— Candeiana, D'O.	*l* VC
—— prælonga, Rss.	*vl* C
Valvulina Parisiensis, D'O.	*s* RR
—— angularis, D'O.	*m* C
Tinoporus vesicularis, P. & J.	*m* R
Spiroloculina rugoso-depressa, nov.	*l* C
—— striata, D'O.	*m* RC
Quinqueloculina Sagra, D'O.	*l* VC
—— pulchella, D'O.	*m* VC
—— Inca, D'O.	*l* C
—— rugoso-saxorum, nov.	*l* C
Triloculina trigonula, Lam.	*s* R
Hauerina plicata*, P. & J.	*m* C
—— complanata, nov.	*m* C
Vertebralina Cassis, D'O.	*m* C
—— conico-articulata, Batsch.	*m* C
Alveolina sabulosa, Montf.	*m* C
Alveolina Quoyii, D'O.	*m* C
Orbitolites complanatus, Lam.	*m* VC
Peneroplis pertusus, Forsk.	*l* VC
Dendritina Arbuscula, D'O.	*l* VC
Spirolina Lituus, Gm.	*s* RR
Dactylopora Eruca, P. & J.	*m* R
FOR COLUMN No. 29.	
Discorbina vesicularis, Lam.	*m* RC
—— Turbo, D'O.	*rl* C
Polystomella craticulata, F. & M.	*m* C
Bolivina plicata, D'O.	*m* RR
Textularia Candeiana, D'O.	*m* C
Valvulina Polystoma†, P. & J.	*m* C
—— Parisiensis, D'O.	*m* C
—— angulosa, D'O.	*m* C
Spiroloculina striata, D'O.	*l* C
Quinqueloculina tricarinata, D'O.	*vl* RC
—— Sagra, D'O.	*l* RC
Triloculina trigonula, Lam.	*s* C
Vertebralina Cassis, D'O.	*rl* C
—— striata, D'O.	*l* VC
—— inæqualis, Gm.	*m* R
Orbitolites complanatus, Lam.	*m* VC
Peneroplis pertusus, Forsk.	*m* VC
Spirolina Lituus, Gm.	*m* RC
Nubecularia lucifuga, Defr.	*m* RC

---

* CARPENTER's Introd. Foram. pl. 6. fig. 35.

† Ibid. pl. 11. figs. 21 & 24.

TABLE X. (continued.)

ADDITIONAL GENERA, SPECIES, AND VARIETIES.		ADDITIONAL GENERA, SPECIES, AND VARIETIES.	
FOR COLUMN No. 30.		FOR COLUMN No. 31.	
Lagena distoma-margaritifera, P. & J.	*l* C	Polymorphina Thouini, D'O.	*m* C
Dentalina brevis, D'O.	*rs* VR	Uvigerina aculeata, D'O.	*vs* R
Vaginulina Badenensis, D'O.	*s* R	Polystomella discoidalis, D'O.	*m* C
Polymorphina Thouini, D'O.	*m* C	Bulimina pupoides, D'O.	*m* C
—— elegantissima, nov.	*m* RC	Bolivina plicata, D'O.	*m* C
Planorbulina vulgaris, D'O.	*vl* VC	—— hyalina, nov.	*m* R
—— ammonoides, Rss.	*s* RR	Textularia variabilis, Will.	*s* RC
Discorbina vesicularis, Lam.	*vl* VC		
—— dimidiata, P. & J.	*vl* VC		
—— biconcava, P. & J.	*s* C	FOR COLUMN No. 32.	
—— Turbo, D'O.	*vl* VC		
—— Cora, D'O.	*m* R	Calcarina Spengleri, Gm.	*m* C
Polystomella macella, F. & M.	*l* C	—— Defrancii, D'O.	*s* RR
—— strigillata β, F. & M.	*s* RC	Rotalia annectens, P. & J.	*m* C
Textularia variabilis, Will.	*l* RC	—— craticulata, P. & J.	*m* C
—— Folium, P. & J.	*s* RC	Planorbulina farcta, F. & M.	*m* RC
Valvulina Parisiensis, D'O.	*l* C	Cymbalopora Poeyi, D'O.	*m* C
—— angularis, D'O.	*l* C	Discorbina Turbo, D'O.	*m* C
—— mixta*, P. & J.	*l* VC	—— Pileolus, D'O.	*m* RC
—— Polystoma, P. & J.	*l* C	Polystomella craticulata, F. & M.	*m* C
—— triangularis, D'O.	*m* VC	—— macella, F. & M.	*m* C
Patellina annularis, P. & J.	*m* C	Heterostegina depressa, D'O.	*m* C
—— simplex, P. & J.	*s* R	Amphistegina vulgaris, D'O.	*m* C
Spiroculina striata, D'O.	*l* VC	Textularia conica, D'O.	*m* RC
Quinqueloculina tricarinata, D'O.	*vl* RC	Tinoporus lævis, P. & J.	*l* C
—— pulchella, D'O.	*l* C	—— sphærulo-lineatus†, P. & J.	*l* C
—— secans, D'O.	*m* C	Polytrema miniaceum, Esper.	*l* RC
—— dilatata, D'O.	*l* VC	Spiroloculina striata, D'O.	*m* RC
Triloculina striato-trigonula, nov.	*l* VC	Quinqueloculina tricarinata, D'O.	*l* C
Vertebralina striata, D'O.	*l* C	Triloculina reticulata‡, D'O.	*l* RC
Peneroplis pertusus, Forsk.	*l* RC	Peneroplis pertusus, Forsk.	*m* VC
Spirolina Lituus, Gm.	*m* RC	Orbitolites complanatus, Lam.	*vl* VC
		Alveolina Quoyii, D'O.	*vl* VC

In these Tables (VII. & X.) we have materials for a conspectus of nearly all the Foraminiferal Genera (of which few, if any, can be said to have more than one true species), as represented by one form or another, type or subtype, species or variety, in widely distant parts of the world, under very different conditions of climate, depth, and sea-bottom.

It is probable that, in some of the instances tabulated, the smallness of the quantity of sand, clay, or ooze manipulated has limited the catalogue of forms, and therefore that further observation is necessary; nevertheless, the freedom with which some genera range over the globe, whilst others are limited to narrow areas, or rather to special conditions, is readily apparent. Table XI. exemplifies this.

* CARPENTER's Introd. Foram. pl. 11. figs. 19, 20, 25, 26.

† Ibid. pl. 15. fig. 1.

‡ Ibid. pl. 6. fig. 13.

Table XI.—Showing the distribution of the Genera of Foraminifera in Thirty-two Gatherings from the Atlantic, Mediterranean, Red Sea, Indian Ocean, and Pacific.

Genera of Foraminifera (represented by species or varieties).	1 Sub-recent. Peterborough Fen, 1 mile from its western boundary (sandy clay).	2 Sub-recent. Peterborough Fen, 2 miles from its western boundary (sandy clay).	3 Sub-recent (clay). Boston, Lincolnshire.	4 Sub-recent (clay). Wisbech, Cambridgeshire (Valley of the Nene).	5 Mouth of Thames, Southend (shallow-water sands).	6 Pegwell Bay, near Margate, Kent (muddy shore-sand).	7 Isle of Arran, N.B. (muddy sands from shallow water).	8 Douglas, Isle of Man (shallow-water sands).	9 Eastbourne, Sussex (mud from shallow water).	10 Colne (tidal) River, Essex (oyster-beds).	11 Greenland (Arctic). See Table IV.	12 Norway (Arctic). See Table IV.	13 North Atlantic, Ireland to Newfoundland, thirty-nine casts, from 43 to 2350 fms. See Table V.	14 Mediterranean. Galita Island, S. 32° W., 32 miles. Lat. 38° 00′ N., Long. 9° 13′ E., 320 fms.	15 Red Sea. Gulf of Suez (muddy, shelly sand). 30 fms. Lat. 28° 38′ N., Long. 33° 9′ E.	16 Red Sea, close to Island Shadwan, off S.E. point, entrance of Jubal Strait, 372 fms. (light-yellow clay).	17 Red Sea, 557 fms. Lat. 17° 49′ N., Long. 40° 2′ E. (various-coloured mud).	18 Red Sea, 678 fms. Lat. 23° 30′ N., Long. 36° 58′ E. (pale clay).	19 Black Anchor-mud, Bombay Harbour.	20 Dark Anchor-mud, Hong Kong, 8 or 9 fms.	21 Tropical Atlantic, 1080 fms. Lat. 2° 20′ N., Long. 28° 44′ W. (almost entirely organic).	22 South Atlantic, Abrolhos Bank, 47 fms. Lat. 23° 02′ S., Long. 41° 02′ W. (sand).	23 South Atlantic, Abrolhos Bank, 260 fms. Lat. 22° 54′ S., Long. 40° 37′, W. (dark mud).	24 South Atlantic, Abrolhos Bank, 940 fms. Lat. 19° 32′ S., Long. 37° 51¼′ W. (whitish mud).	25 South Atlantic, 2700 fms. Lat. 26° 45′ S. Long. 32° 52′ W. (pale mud).	26 Indian Ocean, 2200 fms. Lat. 5° 37′ S. Long. 61° 33′ E. (fine white calcareous mud, with Polycystinea).	27 Indian Ocean, two casts near each other, 900 and 1120 fms. Lat. 36° 58′, Long. 51° 49′ E. (pale mud.)	28 Australia, Coral-reef, 17 fms. (white shelly mud).	29 Swan River, Australia, 7 or 8 fms. (white shelly mud).	30 Melbourne, Australia, Coast-sand (coarse quartz sand, full of shells, zoophytes, sponges, and algæ).	31 Black Anchor-mud, Hobson's Bay, Australia.	32 Fiji, coral-reef, and adherent to a hydroid polype.
Lagena	..	*	*	*	*	*	*	*	*	..	*	*	*	*	*	*	*	*	*	*	*	..	*	*	*	*	*	*	*	*	*	
Nodosarina	..	*	*	..	..	*	*	..	..	..	*	*	*	*	*	*	*	..	*	*	*	*	*	*	..	..	*	*	*	*	*	
Polymorphina	..	*	*	..	..	*	*	*	*	..	*	*	..	..	..	*	..	..	..	*	..	..	..	..	..	..	..	*	..	*	*	
Uvigerina	..	*	..	..	..	..	..	..	..	..	*	*	*	*	..	*	*	*	*	*	*	..	*	..	..	..	*	..	*	*	*	
Orbulina	..	..	..	..	..	..	..	..	..	..	..	..	*	*	*	*	*	*	..	..	*	..	*	*	..	*	*					
Spirillina	..	*	..	..	..	..	..	..	..	..	*	*	*	..	..	*	*	*	..	..	..	..	..	..	..	..	..	..	*	*		
Globigerina	..	*	..	..	..	..	..	..	..	..	*	*	*	*	*	*	*	*	*	..	*	*	*	*	*	*	*	*				
Pullenia	..	..	..	..	..	..	..	..	..	..	..	*	*	*	..	..	..	..	..	..	*	*	*	..	..	*						
Sphæroidina	..	..	..	..	..	..	..	..	..	..	..	..	*	*	..	*	..	..	..	..	*	..	*	*	..	*						
Textularia	..	*	*	*	..	*	*	..	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	..	..	*	*	*	*	*	*
Bulimina	..	*	..	*	..	*	..	..	*	*	*	*	*	*	*	*	*	*	*	*	*	..	*	*	..	..	..	*	*	*	*	
Cassidulina	..	..	..	..	..	..	..	..	..	..	*	*	*	*	..	*	..	..	*	..	*	..	..	*	..	..	*					
Discorbina	..	*	..	*	..	..	*	..	*	..	*	*	*	*	*	*	..	..	*	*	..	*	*	..	..	..	..	*	*	*	*	*
Planorbulina	..	..	*	*	..	..	*	*	*	..	*	*	*	*	*	*	*	*	*	*	*	*	*	*	..	..	*	*	*	*	*	*
Pulvinulina	..	*	*	*	..	..	..	..	*	..	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*				
Rotalia	*	*	*	*	*	*	*	*	*	*	..	..	*	*	*	*	*	*	*	*	..	..	..	..	..	..	..	*	*	*	*	*
Cymbalopora	..	..	..	..	..	..	..	..	..	..	..	..	..	..	*	*	..	..	..	*	..	..	..	*	..	..	..	*	..	..	..	*
Calcarina	..	..	..	..	..	..	..	..	..	..	..	..	..	..	*	..	..	..	..	..	..	..	..	..	..	..	..	*	..	..	..	*
Tinoporus	..	..	..	..	..	..	*	..	..	..	..	..	..	*	..	*	..	*	..	..	..	..	..	..	..	..	..	*	..	*	..	*
Patellina	..	*	..	..	..	..	..	..	..	..	*	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	*		
Polytrema	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	*
Amphistegina	..	..	..	..	..	..	..	..	..	..	..	..	..	..	*	*	..	..	..	..	..	..	..	*	..	..	..	*	..	..	..	*
Nummulina	..	..	..	..	..	..	..	..	..	..	*	*	*	*	*	..	..	..	*	..	..	*	..	..	..	..	..	*	..	..	..	*
Polystomella	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	..	..	*	*	..	..	*	*	*	*	*	*
Heterostegina	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	*
Valvulina	..	..	..	..	..	..	..	..	..	..	..	*	..	*	..	..	..	..	..	..	..	*	..	..	..	..	..	*	*	*		
Lituola	*	*	..	..	..	*	..	..	..	*	*	*	*	*	..	..	*	*	..	..	..	*	*	*	..	*	..	*	..	..	*	*
Trochammina	*	*	..	..	..	*	..	..	..	*	*	..	..	*	..	..	*	*	..	..	..	..	*									
Cornuspira	..	*	*	*	..	..	..	*	*	*	*	..	..	..	..	*	..	..	..	..	..	..	*	*	..	..	..	..	*	*		
Nubecularia	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	*	*		
Vertebralina	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	*	..	..	..	..	..	..	..	*	..	..	..	*	*	*		
Miliola	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*
Peneroplis	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	*	*	..	..	..	..	..	..	..	..	*	*	*	..	*
Alveolina	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	*	..	..	..	*
Orbitolites	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	*	..	..	..	..	..	..	..	..	..	..	..	*	*	..	..	*
Dactylopora	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	*				
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32

APPENDIX VII.—*The North-Atlantic Soundings.*

Owing to our having taken the positions of the soundings from the MS. labels, we find in some instances discrepancies as to the depths and positions given in the published Report, arising probably from corrections of the observations in some cases, and from errors of copying and printing in others. Some, also, of our specimens are not noted in the Report, as, for instance, Nos. 15, 25, 31, 34, 35, & 36; and Nos. 4 & 33 can be only doubtfully recognized. No. 21 (80) has 1405 instead of 1450 fathoms; No. 26 (22) has 2250 instead of 1660 fathoms; and No. 28 (86) has 2050 instead of 1950 fathoms; and there are minor discrepancies of depth and position, as the annexed Table indicates. These we point out now, to save any waste of labour to those who wish to verify our work.

In consequence of the differences in some of the manuscript and printed positions, the vertical lines drawn over the reduced copy of Commander DAYMAN'S Chart (Plate XII.) are often merely approximative; and the Section of the Sea-bed is not quite correct at Soundings No. 21 (80) & 26 (22).

TABLE XII.—The Thirty-nine Soundings described in the foregoing Memoir; with their positions and depths, as indicated by the MS. Labels and by the printed Report.

Nos. in Table V.	From the Labels.			From the Admiralty Report.					REMARKS.
	Nos.*	Fms.	Lat. N. & Long. W.	Nos.*	Fms.	Lat. N. & Long. W.	Materials †	Page.	
1	53	195	Lt. 48° 0′ 30″ Ln. 53 27 35	..	195	Lt. 48° 0′ 30″ Ln. 53 27 45	Mud.	56	
2	49	129	Lt. 48 0 10 Ln. 53 26 36	..	129	Lt. 48 8 10 Ln. 53 22 36	Stones, mud.	56	
3	47	190	Lt. 48 9 0 Ln. 53 15 0	..	190	Lt. 48 9 5 Ln. 53 15 0	Mud.	56	
4	39	124	Lt. 48 15 30 Ln. 53 13 0	?	125	Lt. 48 15 15 Ln. 53 9 0	Blue mud.	56	Possibly the same Soundings.
5	45	150	Lt. 48 9 45 Ln. 53 10 50	..	150	Lt. 48 9 54 Ln. 53 10 50	Blue mud.	56	
6	41	129	Lt. 48 11 0 Ln. 53 7 50	..	129	Lt. 48 12 0 Ln. 53 7 55	Mud.	56	
7	61	167	Lt. 48 14 22 Ln. 53 1 0	..	167	Lt. 48 14 22 Ln. 53 1 0	Dark mud.	56	
8	59	133	Lt. 48 18 0 Ln. 52 56 0	..	133	Lt. 48 17 55 Ln. 52 45 50	Dark mud.	56	
9	55	112	Lt. 48 21 0 Ln. 52 44 0	..	112	Lt. 48 21 0 Ln. 52 42 40	Mud, stones.	56	
10	65	102	Lt. 48 28 30 Ln. 52 19 30	..	102	Lt. 48 28 30 Ln. 52 19 30	Stone, clay.	56	
11	69	146	Lt. 48 40 0 Ln. 51 45 0	..	146	Lt. 48 40 0 Ln. 51 45 0	Mud, stone.	56	
12	63	145	Lt. 47 57 20 Ln. 51 31 30	..	145	Lt. 47 57 20 Ln. 53 31 30	Mud.	56	
13	73	161	Lt. 49 0 0 Ln. 50 48 30	..	161	Lt. 49 0 0 Ln. 50 48 30	Mud.	56	

---

* These numbers refer to the compartments of the box containing the specimens.

† See also Table VI.

TABLE XII. (continued).

Nos. in Table V.	From the Labels.			From the Admiralty Report.					Remarks.
	Nos.*	Fms.	Lat. N. & Long. W.	Nos.*	Fms.	Lat. N. & Long. W.	Materials †.	Page.	
14	33	405	Lt. 49° 2′ 0″ Ln. 50 14 30	..	405	Lt. 49° 5′ 0″ Ln. 53 3 0	Mud.	55 & 59	
15	77	221	Lt. 49 23 30 Ln. 49 55 0						
16	78	329	Lt. 49 26 0 Ln. 49 48 0	..	330	Lt. 49 25 0 Ln. 49 48 0	Sand, mud.	55 & 59	331 fathoms, at p. 59.
17	32	740	Lt. 49 16 30 Ln. 49 17 0	32	742	Lt. 49 12 0 Ln. 49 35 0	Mud.	54 & 59	
18	79	725	Lt. 49 18 0 Ln. 49 12 0	79	725	Lt. 49 18 0 Ln. 49 12 0	Mud, with a Worm.	53 & 59	
19	31	954	Lt. 49 23 0 Ln. 48 48 0	..	954	Lt. 49 24 0 Ln. 48 48 0	Ooze ‡.	59	Our specimen of this Sound was a sandy mud. See TABLE VI.
20	30	1203	Lt. 49 33 0 Ln. 48 5 0	30	1203	Lt. 49 32 0 Ln. 48 4 0	Blue mud, with "remains of Bones, &c."	51 & 59	Lat. 49° 32′ 30″, at p. 59.
21	80	1450	Lt. 50 6 0 Ln. 45 45 0	80	1405	Lt. 50 6 0 Ln. 45 45 0	Ooze, "full of Foraminifera, when seen in the Microscope."	47 & 59	Lat. 50° 6′ 30″, at p. 59.
22	26	2330	Lt. 50 25 0 Ln. 44 19 0	26	2330	Lt. 50 25 0 Ln. 44 19 0	Ooze.	44 & 59	
23	25	2250	Lt. 50 46 0 Ln. 42 20 0	25	2050	Lt. 50 49 0 Ln. 42 26 0	Ooze, with Foraminifera.	42 & 59	
24	19	2035	Lt. 52 11 0 Ln. 31 29 0	19	2030	Lt. 52 11 0 Ln. 31 27 0	Ooze.	29 & 58	2030 fms. is the *corrected* depth. Long. 31° 27′ 30″, at p. 58.
25	81	2350	Lt. 51 29 0 Ln. 38 1 0	..	....	....	....	....	Compare No. 26 (22).
26	22	1660	Lt. 51 30 0 Ln. 38 0 0	22	2250	Lt. 51 29 0 Ln. 38 0 0	Ooze.	37 & 58	Compare No. 25, 2350 fathoms.
27	85	2176	Lt. 52 16 30 Ln. 29 28 30	85	2176	Lt. 52 16 30 Ln. 29 28 0	Ooze.	27 & 58	Long. 29° 28′ 30″, at p. 58.
28	86	1950	Lt. 52 25 0 Ln. 28 10 0	86	2050	Lt. 52 26 0 Ln. 28 10 0	Ooze.	26 & 58	
29	15	1776	Lt. 52 33 30 Ln. 21 16 0	15	1800?	Lt. 52 46 0 Ln. 21 20 0	Ooze, with Foraminifera and Diatomaceæ.	21 & 58	
30	90	2050	Lt. 52 16 0 Ln. 16 46 0	90	2050	Lt. 52 16 30 Ln. 16 46 0	Ooze.	16 & 58	
31	13	2050	Lt. 52 16 0 Ln. 16 42 0	..	....	....	....	....	Compare No. 30 (90), 2050 fms.
32	12	1750	Lt. 52 21 30 Ln. 15 6 0	12	1750	Lt. 52 21 30 Ln. 15 6 0	Ooze.	14 & 58	Lat. 52° 21′ 40″, at p. 58.
33	93	200	Lt. 52 16 0 Ln. 14 30 0	?	240	Lt. 52 17 0 Ln. 14 30 0	Fine sand.	13 & 58	Possibly the same Soundings.
34	95	223	Lt. 52 11 0 Ln. 13 45 0		....	....	....	....	Not noticed, but intermediate to others mentioned at pp. 13 & 58.
35	98	415	Lt. 52 8 30 Ln. 12 31 0						
36	7	338	Lt. 52 0 30 Ln. 12 7 30						
37	99	90	Lt. 52 1 0 Ln. 11 14 40	..	90	Lt. 52 1 0 Ln. 11 15 0	Sand.	13 & 58	Long. 11° 14′ 40″, at p. 58.
38	100	78	Lt. 51 59 0 Ln. 11 0 0	..	78	Lt. 51 59 0 Ln. 11 0 0	Fine sand.	13 & 58	
39	102	43	Lt. 51 57 0 Ln. 10 30 30	..	43	Lt. 51 57 0 Ln. 10 30 0	Fine sand.	13 & 58	

---

* These numbers refer to the compartments of the box containing the specimens. † See also Table VI.

‡ Described as "a light-coloured fine mud;" "a soft mealy substance;" "sticky."

In one of the above-mentioned Soundings from the Abyssal ooze-floor of the North-Atlantic (Nos. 19–32), Commander Dayman observed "remains of bones" (No. 20); and other rare extraneous objects were noticed by him in some of the deep soundings not included in the foregoing Table. As the presence of Molluscan Shells and of stones at such depths, and so far from land, are of great interest, we append an abstract of such occurrences.

Nos.*	Fms.	Lat. N. & Lon. W.	Materials.	Report, page.	Remarks.
103	1950	Lat. 52° 37′ Long. 17 39	One small stone.	17 & 58	
88	2100	Lat. 52 30 Long. 19 10	Ooze, full of Foraminifera and Diatomaceæ.	19 & 58	
87	2400	Lat. 52 29 Long. 26 14	Ooze.	25 & 58	The deepest sounding showing bottom: but a deeper (2424 fms.) was exactly measured Lat. 51°9′N., Long. 40° 3′ W.
18	1675	Lat. 52 14 Long. 30 45	Broken Shells.	9, 28, 58	1765 fathoms, at p. 9.
	1600	Lat. 51 52 Long. 33 21	Two small stones.	9, 31, 32, 58	Marked "oz." on the Chart by mistake.
27	2225	Lat. 50 14 Long. 45 23	Ooze and stones.	46 & 59	
28	1450	Lat. 50 9 Long. 46 15	Ooze and stones.	49 & 59	
29	1495	Lat. 49 47 Long. 46 52	All stones, at p. 50.	50 & 59	Lat. 49° 47′ 30″ Long. 41 51 0 } and Ooze, at p. 59.

* In the box of specimens.

VII. *New Observations upon the Minute Anatomy of the Papillæ of the Frog's Tongue.*

By Lionel S. Beale, *M.B., F.R.S., Fellow of the Royal College of Physicians, Professor of Physiology and of General and Morbid Anatomy in King's College, London; Physician to King's College Hospital, &c.*

Received June 16,—Read June 16, 1864.

In this paper I propose to give the results of some recent investigations upon the minute anatomy of the beautiful fungiform papillæ of the tongue of the little green tree-frog (*Hyla arborea*). The specimens have been prepared according to the principles laid down in former communications. The success I have met with in this and other minute anatomical inquiries is, I believe, almost entirely due to the process of investigation which I have adopted for some years past, and which enables me to render specimens very transparent, and to demonstrate all the tissues in one specimen. By this plan sections are obtained so exceedingly thin, without the destruction even of the most delicate tissues, that they may be examined under the highest powers which it is possible to obtain ($\frac{1}{26}$ magnifying 1700 linear, and $\frac{1}{50}$ magnifying about 3000 linear).

The following are among the most recent contributions to the anatomy of the papillæ of the frog's tongue:—

Waller: "Minute structure of the Papillæ and Nerves of the Tongue of the Frog and Toad," Philosophical Transactions, 1847.

Billroth: "Ueber die Epithelzellen der frosch-zunge, sowie über den Bau der cylinder-und flimmerepithelien und ihr Verhältniss zum bindegewebe," Archiv für Anat. Physiologie, 1858, S. 163.

Hoyer: "Mikroskopiche Untersuchungen über die zunge des Frosches," Archiv für Anat. Phys. 1859, S. 488.

Axel Key: "Ueber d. Endigungen d. Geschmäcksnerven in der zunge Frosches," Müller's Archiv, 1861, S. 329.

Hartmann: "Ueber die Endigungsweise der nerven in den Papillæ-fungiformes der Froschzunge," Archiv für Anat. Phys. 1863, S. 634.

Although the views of Axel Key are supported by schematic figures which do not accurately represent the real arrangement of the tissues, they approach much nearer to the truth than those of other observers. He describes two kinds of cells at the summit of the papilla, epithelial and special cells concerned in taste. I have not been able to verify his statements in this particular. He has not demonstrated the peculiar network at the summit of the papilla which is seen so distinctly in my specimens, and his delineations of the prolongation of the axis-cylinder alone, and its division into fibres far too fine to be visible by the magnifying powers employed, and the abrupt cessation of the white substance delineated by him, are evidently schematic,—indeed he does not pretend that the figures referred to are copies from nature. Still his inferences regarding the division of the nerve-fibres into very fine fibres which pass into the epithelium-like tissue at the summit of the papilla, approach much nearer to the actual arrangement than those of any other observers with which I am acquainted.

The latest researches upon the mode of termination of the nerves are by Dr. HARTMANN. These are concluded in the Number of REICHERT and DU BOIS-REYMOND'S 'Archiv' for 1863, which has only just been received in this country (June 1864). The drawings of the papillæ accompanying this memoir, especially fig. 65, plate 18, form an excellent illustration of how most beautiful and well-defined structures may be rendered quite invisible by being soaked in aqueous solution of bichromate of potash for three days, one day in carmine solution, and then in caustic soda!

In order that I may not express myself against the mode of preparation followed by this and many other observers in Germany in the present day more strongly than is justified by the results obtained as shown by their own drawings, I would refer to HARTMANN'S figure. Of this drawing it is not too much to say that it represents nothing sufficiently definite to enable any one to form an idea of the structure of the part. The drawing, and I conclude the preparation from which it was taken, are far behind the day; and it seems to me most remarkable that after all the anatomical research of the last twenty years an observer should publish such a figure as this as a representation of natural structure. The nerve-fibres are completely altered by the mode of investigation followed, and the finer fibres are of course destroyed or rendered invisible. Nor can I admit that the epithelium upon the summit of the papillæ represented in his fig. 64 gives a correct idea of this structure.

It may be proved conclusively by experiments that soaking delicate animal tissues in dilute aqueous solution of bichromate of potash renders invisible and destroys structures which can be demonstrated by other means. Inquiries conducted by the aid of such plans of preparation retard rather than advance anatomical inquiry, for some of the most important anatomical characters are rendered completely invisible. The very conflicting opinions now entertained by observers in Germany upon the structure of these papillæ, render it important that they should be studied again with the advantage of the highest powers, and the most advantageous methods of preparation which we now possess.

In this communication I shall only attempt to describe briefly those points which I believe to be new, and which are I conceive demonstrated in my specimens for the first time. Most of the points described in this paper were demonstrated more than eighteen months ago, and during this period the specimens have been repeatedly studied and shown to other observers. The points described can still be demonstrated in the same specimens (June 1864).

The structures entering into the formation of the papilla are the following:—

1. The connective tissue which forms the body of the papilla.
2. The "epithelium."
3. The nerve-fibres in the body of the papilla, and the fibres prolonged from them which form a plexus upon its summit.
4. Nerve-fibres ramifying in the connective tissue, upon the capillary vessels and amongst the muscular fibres.
5. The muscular fibres.
6. The vessels.

The Connective Tissue.

The nerves, vessels, and muscular fibres are imbedded in a very transparent basis-substance which exhibits a slightly striated or fibrous appearance when stretched, but this structure in all the papillæ of the Hyla is exceedingly delicate and transparent.

The great majority of the nuclei seen in this basis or connective substance are undoubtedly connected with the nerves, vessels, and muscular fibres, but there are a few which seem to belong to the connective substance alone, and may therefore be called "*connective-tissue corpuscles.*" It is possible that these at an earlier period may have been connected with nerves or muscles; they have descended from the same nuclei or masses of germinal matter as the nuclei taking part in the production of these tissues.

I consider that indefinite connective tissue of this kind results principally from the accumulation of the remains of higher structures, especially nerve-fibres, which were in a state of functional activity at an earlier period of life. At an early period of development nuclei (masses of germinal matter) can alone be detected. As development proceeds, tissue is formed by these nuclei, and increases as age advances. The large and fully-formed fungiform papillæ have twice as many nerve-fibres as smaller and younger ones. During the development of such an organ as one of these papillæ many changes occur, and much texture is produced and removed before the papilla attains its fully developed state. That passive substance called connective tissue which remains and occupies the intervals between the higher tissues, which possess active and special endowments, slowly accumulates, but undergoes condensation as the organ advances in age. Amongst this are a few nuclei which can no longer produce anything but indefinite "connective tissue" of the same character. In Plate XXI. fig. 9 it would have been impossible, had the specimen been prepared in the usual manner, to have determined if the nuclei marked *a*, *b* were nuclei of the muscle concerned in producing muscle, or connective-tissue corpuscles concerned in the formation of connective tissue only. This question requires restudy from a new point of view. It is quite certain that many of the nuclei figured in all my drawings in connexion with *nerves*, *vessels*, *muscles*, and *other tissues*, would, if the specimens had been prepared in a different manner, so that their connexions were not so very distinctly seen, have been called "CONNECTIVE-TISSUE CORPUSCLES."

The drawings accompanying my paper explain the relation which I believe the essential structures entering into the formation of the papilla bear to the indefinite connective tissue in which they lie imbedded.

Epithelium.

The so-called epithelium upon the summit of the papilla of the frog's tongue (Plate XXI. fig. 1, *a*) differs from the epithelium attached to its sides (*b*), that covering the simple papillæ (*c*), and that on the surface of the tongue generally, in many important characters. As is well known it is *not ciliated.* The cells differ from the ciliated cells in several points. They are smaller than these. The nucleus is very large in proportion to the entire cell. The cells are not easily separated from one another, as is the case with the ciliated epithelium. These cells form a compact mass, the upper surface of which is convex. This is adherent by its lower surface to the summit of the papilla, and it is not detached without employing force. The cells do not separate one by one, as occurs with the ordinary epithelium, but the whole collection is usually detached entire, and it is I believe *torn away.*

Although some observers would assert that the two or three layers of cells represented in my drawings do not exist, but that the appearance is produced by the cells of a single layer being pushed over one another by pressure, I am convinced that in this mass upon the summit of the papilla of the Hyla there is more than the single layer of cells represented by HARTMANN, who is the latest observer on this point.

HARTMANN'S representation (*l. c.*) of this very same structure from the summit of the papilla of the Hyla is very different from my drawings. Not only do we represent these same cells of very different shapes, but the nucleus in my specimens is three or four times as large in proportion to the cell as represented by him.

The general outline of the free surface is convex (*a, a, a,* fig. 1, Plate XXI.), and the tissue which intervenes between the nuclei appears very transparent and projects a little, so as to give the convex summit a honeycombed appearance (Plate XXI. fig. 7).

The under concave surface of this hemispheroidal mass which adheres to the summit of the papilla of the Hyla's tongue, corresponds to the exact area over which the nerve-fibres of the papilla are distributed, as will presently be shown. The shape of these cell-like bodies, of which the mass is composed, and their connexion with fibres is shown in Plate XXI. fig. 3, and in the very highly magnified specimen represented in fig. 2. After the examination of a vast number of specimens I think these figures represent the actual arrangement, but this point is most difficult of investigation. In the intervals between what would be called, if they were capable of complete separation from one another, the individual cells, fibres are seen. These fibres do not I think arise simply from the pressure to which the masses have been subjected. I have represented the arrangement as I believe it to be in Plate XXI. fig. 6, from the central part of one of the hemispheroidal masses. I regard the entire hemispheroidal mass as resembling in its essential structure the network I have described at the summit of the papilla, but the masses of germinal matter are so very close together and the fibres so much interlaced with one another, that it is most difficult to unravel the mass without destroying it. The arrangement at the surface is seen in Plate XXI. fig. 7.

The epithelium of the tongue generally is easily removed, but many of these hemispheroidal masses remain connected with the summits of the papillæ to which they belong. From what I have stated, it will I think be admitted that the constituent parts of the mass at the summit of the papilla could not be properly called epithelial cells, so that, with reference to the termination of the nerves in the papilla, I think it is more correct to say that nerves may be traced to special bodies or cells which form a hemispheroidal mass attached to the summit of the papilla, than to assert that the separate bodies, which compose the mass in which nerves terminate, are actual epithelial cells.

In the simple papillæ (Plate XXI. fig. 1, *d*) of the frog's tongue, a "nucleus" of a nerve sometimes projects beyond the outline of the papilla and lies amongst the epithelium. This nucleus, however, adheres to the papilla when all the epithelial cells have been detached. It might from its position be easily mistaken for an epithelial cell, but it is no more really related to this structure than is a ganglion-cell, or a caudate nerve-cell of the spinal cord. The cells of the ciliated epithelium of the frog's tongue are not in any instance, as far as I am able to observe, connected with the nerve-fibres. It is probable that the opposite inference, which is still held by many observers, has resulted from the observation of such a nucleus as is represented in Plate XXI. fig. 1, *d* projecting beyond and adherent to the surface of the papilla. It is really continuous with the delicate nerve-fibres (*e*) ramifying in the substance of the papilla, but it is not an epithelial cell, and remains adherent after every particle of epithelium has been removed.

The nervous tissue is in all cases structurally distinct from every other tissue, in every part of its distribution. It never blends with epithelium any more than it blends with fibrous tissue, cartilage, bone, or muscle. If nerves exert any direct influenee upon the nutrition of any of these tissues, the influence must be exerted through some distance. The results of anatomical research render any physiological doctrine which maintains that nerves act through their structural continuity with other tissues untenable. My own observations lead me to conclude that nerves do not directly influence the processes of nutrition, growth, or development at all. They act only indirectly, and affect the supply of nutrient matter distributed by modifying the calibre of the vessels, and hence regulate the supply of blood which passes to the capillaries. The nerves I believe really exert their influence upon the contractile muscular coat of the small arteries and veins alone, and do not act directly upon any other tissues.

The Nerves.

With regard to the trunks of the nerves, I remark the following facts of importance:—

1. That the bundle of nerve-fibres distributed to a papilla always divides into two bundles which pursue opposite directions. The division of the bundle may take place just at the base of the papilla, or at some distance from it, but it always occurs (Plate XXI. fig. 1).

2. Fine pale nerve-fibres pass from the same trunk of dark-bordered fibres as that which gives off the bundle of nerves to the papilla. The fine fibres ramify—

a. Amongst the muscular fibres of the tongue (Plate XXI. figs. 1, 9).

b. Upon the vessels (Plate XXI. fig. 1, *i*, *i*, *i*).

c. In the connective tissue of the tongue generally, and also in the simple papillæ (Plate XXI. fig. 1, *d*, *e*).

The division of the bundle at the base of a papilla is shown in Plate XXI. fig. 1, and in Plate XXII. fig. 10 is a diagram to indicate the manner in which the nerve-plexuses at the summits of the papillæ are connected together by commissural fibres. Thus in action the papillæ may be associated together. The bearing of this arrangement upon the existence of complete nervous circuits is discussed in my 'Archives,' vol. iv. The bundle in the central part of the papilla consists of dark-bordered fibres, which frequently cross and interlace with one another in this part of their course. They vary much in diameter, some being so fine as scarcely to be visible.

As the bundle passes towards the summit of the papilla, the individual fibres divide and subdivide into finer branches. Now, as I have before remarked, nerves so near their distribution as these do not usually possess an axis-cylinder as a structure distinct from the white substance. The white substance does not abruptly cease, while the axis-cylinder is alone prolonged onwards by itself as is often described, but the entire fibre divides and subdivides. In fact dark-bordered nerve-fibres, near their ultimate ramifications, always consist of fatty albuminous material imbedded in a transparent matrix of connective tissue. The "tubular membrane," "white substance," and "axis-cylinder" can never be demonstrated as distinct structures near the peripheral distribution of nerves. The "tubular membrane" is nothing more than the transparent matrix in which one or more nerve-fibres are imbedded.

The dark-bordered fibres divide into finer fibres about the level of the ring or half-ring of capillaries at the summit of the papilla. As the fibres are exceedingly transparent, they are usually lost from view about this point. For example, HARTMANN's figures convey the idea that distinct dark-bordered fibres can be followed as high as this point, but that they cannot be traced further. Above this spot the papilla is a little thickened and the tissue more granular, and hence it is not to be wondered at that great difficulty should have been experienced in demonstrating the further course of the nerves, or that many different views should be entertained upon the oft debated question of the mode of ending of nerves in this situation; but it is most certain that the fibres do divide and subdivide into finer and much more transparent fibres at this point, and that these again divide and subdivide and form an elaborate plexus in the summit of the papilla, which has not been before described.

By reference to the figure, the arrangement, which is not easily described with accuracy, will be at once understood, so that a minute description of it would be superfluous.

Above the plexus *c* (Plate XXI. fig. 3), and below the epithelium-like organ at the summit of the papilla (*a*), is a layer (*b*) which appears to be composed of granular matter. In my most perfect specimens, however, this "granular layer," when examined by very high powers under the influence of a good light, is seen to consist of a plexus of extremely fine fibres which interlace with one another in every direction, but which pass from the plexus above to the epithelium-like nervous (?) organ upon the summit of the papilla (Plate XXI. fig. 2). I believe this granular appearance to result from the extreme delicacy and fineness of the nerve-fibres at this part of their course. In like manner the "granular matter" seen in the grey matter of the cerebral convolutions and that of the retina, results mainly from the breaking down of very fine and delicate nerve-fibres, which undergo disintegration very soon after death, unless they are subjected to special methods of preparation.

Of the existence of the elaborate network of nerve-fibres with the large nuclei, represented in Plate XXI. fig. 3 *c*, there can be no question whatever; but there may be some difference of opinion regarding the exact relation of the very fine nerve-fibres at the summit of the papilla, to the peculiar cells which surmount it, and the nature of the granular matter just described. However, there are but two possible arrangements:—

1. That the nerves form a network of exceedingly fine fibres upon the summit of the papilla, upon which the bases of the epithelium-like cells impinge.

2. That the very fine nerve-fibres are really continuous with the peculiar and epithelium-like cells; in which case these bodies must be regarded as part of the nervous apparatus.

There seems to me to be so much strong evidence in favour of the last view, that I venture to express a decided opinion that this is the truth. In many specimens I have seen, and most distinctly, the delicate network of fibres represented in Plate XXI. fig. 3 continuous with the fine nerve-fibres in the summit of the papilla, and I have demonstrated the continuity of these fine fibres with the matter of which the outer part of these peculiar cells consists (Plate XXI. figs. 2, 3, 6). I have also seen what I consider to be nerve-fibres in the intervals between some of these cells (Plate XXI. fig. 7). Upon the whole I am justified in the inference that there is a structural continuity between the matter which intervenes between the masses of germinal matter at the summit of the papilla and the nerve-fibres in its axis, and I consider that an impression produced upon the surface of these peculiar cells may be conducted by *con-*

tinuity of tissue to the bundle of nerve-fibres in the body of the papilla. These peculiar cells in the summit of the papilla cannot therefore be regarded as epithelium, and the mass constitutes a peculiar organ which belongs not to epithelial structures, but to the nervous system.

Although there can be no doubt whatever as to the existence of an intricate and exceedingly delicate nervous network or plexus at the summit of every papilla, such a plexus might be connected with the nerves according to one of two very different arrangements:—

1. The plexus might be formed at the extremity of a nerve or nerves, as represented in diagram (Plate XXII. fig. 17).
2. The plexus might form a part of the course of a nerve or nerves, as represented in diagram (Plate XXII. fig. 18).

If the first be true, the network must be terminal, and impressions must be conveyed along the fibre, of which the plexus is but the terminal expansion, direct from periphery to centre. If the second arrangement is correct, the network forms a part of a continuous circuit or of continuous circuits. I believe the division of the nerves at the base of the papilla, already adverted to, is alone sufficient to justify us in accepting the second conclusion as the more probable; but when this fact is considered with reference to those which I have adduced in my paper published in the 'Transactions' for 1863, and that in the 'Proceedings' for June 1864, and the observations published in several papers in vols. ii., iii., and iv. of my 'Archives,' and in the Croonian Lecture for 1865, I think the general view in favour of complete circuits is the only one which the anatomical facts render tenable. The mode of branching and division of trunks and individual fibres is represented in Plate XXII. figs. 20, 21, 22, 23.

From the number and size of the nerve-fibres constituting the bundle in the centre of the papilla, we should infer that the finest ramifications resulting from the subdivision of these branches would be very numerous, since it has been shown that the fine fibres resulting from the subdivision of a *single* dark-bordered fibre in the frog's bladder, palate, skin, and muscle, constitute plexuses or networks which pass over a very extended area. The mode of formation of a nerve-plexus is represented in Plate XXII. figs. 11 & 14. In these beautiful little organs the numerous fibres resulting from the subdivision of the dark-bordered fibres are distributed over a comparatively small extent of tissue, forming the summit of the papilla. Still we have the same formation of plexuses, the constant change in the direction taken by fibres, and the same crossing and intercrossing which have been noticed in other situations. In fact the nervous distribution in these organs presents the same typical arrangement as is met with in other tissues, but it is compressed into a very small space.

Now with regard to the epithelium-like structure upon the summit, it has been shown that the nerve-fibres are probably continuous with the material lying between the large nuclei. In fact if the interpretation of the appearances which I have given be correct, the arrangement may be expressed thus:—

The material marked *a* (Plate XXI. fig. 2) is a continuation of the nervous structure or tissue, while the matter marked *b* bears the same relation to this as the so-called nucleus of a nerve bears to its fibre, of an epithelial cell to its wall. If this be so, the matter which is freely exposed at the very summit of the papilla is at least structurally continuous with nerve-tissue, if it is not to be regarded as nerve itself. My own opinion is that it is just as much nerve-tissue as a fine nerve-fibre is nerve-tissue, or the caudate process of a nerve-cell is nerve-tissue. The formed matter is produced by the large masses of germinal matter which are so very numerous, just as the formed matter of a central nerve-cell results from changes occurring in its germinal matter.

It may not be out of place here to consider how the elaborate organ connected with the bundle of nerve-fibres of the papilla may act during life. As already stated, the free surface is uneven, and the arrangement is such that there are many elevations projecting, like fibres, by slightly varying distances, from the general surface. Now from the intricate interlacement of the nerve-fibres in the summit of a papilla, as well as at the point between this and the peculiar organ (Plate XXI. fig. 3, *b*), it is obvious that a fibre given off from one coming from the extreme left of the papilla, for example, may be situated a very short distance from a fibre coming from the opposite side. Any object, therefore, which connects the exposed projections would produce a temporary disturbance in the nerve-currents which are traversing these fibres, and this alteration in the current would of course produce a change in the cell or cells which form part of the same circuit in the nerve-centre. Any strong pressure would influence all the fibres distributed to this delicate nervous organ.

The supposed mode of action is explained by the plan (Plate XXI. fig. 4).

Nerve-fibres ramifying upon the capillary vessels, in the connective tissue, and upon the muscular fibres.

Many of the so-called connective-tissue corpuscles, with their anastomosing processes or "*tubes*," are really nerve-nuclei and very fine pale nerve-fibres, as has already been shown in observations upon the frog's bladder. In the tongue I have followed these fine fibres in very many specimens. They can only be seen and traced in specimens prepared in syrup, glycerine, or other viscid medium miscible in all proportions with water.

In Plate XXI. fig. 1, *f*, and in fig. 8, one of these fine branches, coming off from a bundle of dark-bordered fibres, is represented. Now, if examined by a low power, this might be mistaken for a fibre of connective tissue; but it really consists of several very fine fibres, which in their arrangement exhibit the same peculiarities observed in nerves ramifying in larger trunks (Plate XXII. figs. 20, 23). The fine branches divide and subdivide, and the delicate fibres resulting from their division can be followed for a very long distance. The finest are composed of several finer fibres, and they form networks or plexuses, the meshes of which vary much in size.

The branches which are distributed around the capillaries, in the connective tissue, and to the musular fibres, seem to result from the division and subdivision of the same fibres (Plate XXI. fig. 1).

Nerves which are constantly distributed external to the capillary vessels and in the connective tissue have been demonstrated by me (Plate XXII. fig. 15) (see Archives, vol. iv. page 19). I consider these fibres as the afferent fibres through which an impression conveyed from the surface or from the tissues around capillaries, influences the motor nerves distributed to the small arteries from which the capillaries are derived. It is probable that these nerve-fibres pass to the very same set of central cells as that from which the vaso-motor fibres take their rise. It is through these fibres that changes in the nutrition of the tissues may affect the circulation in the neighbouring vessels.

In these fungiform papillæ, then, there are

1. The bundle of nerve-fibres which is distributed to the sensitive nervous organ at the summit.
2. Delicate fibres which may be traced to fibres running in the same bundles as purely sensitive fibres. These delicate fibres are distributed
    - *a.* Around the capillaries of the papilla (Plate XXI. fig. 1, *i*). See also Plate XXII. fig. 15.
    - *b.* Some fibres ramify in the connective tissue of the simple papillæ (Plate XXI. fig. 1).
    - *c.* Some are distributed to the muscular fibres (Plate XXI. figs. 1 & 9).

Now the first and second fibres are probably sensitive, excitor, or afferent, whilst the last must be motor. From this observation it follows that certain afferent and motor fibres are intimately related at their distribution, a conclusion already arrived at in my investigations upon the distribution of the nerves to the frog's bladder, the palate, and pharynx. Moreover I think that fine fibres passing from the plexus of sensitive fibres at the summit of the papilla, establish here and there a structural continuity between these and the fibres ramifying in the connective tissue and around the capillary vessels. It is very difficult to obtain a specimen which renders this perfectly certain, but I have been led to a similar conclusion in investigations upon the corpuscula tactûs of the human subject. The physiological interest and importance of this branch of anatomical inquiry are so great, and it promises to lead to such important results, that it cannot be too minutely or too patiently worked out.

Of the Muscles.

The muscular fibres of the papilla (Plate XXI. fig. 1) are the continuations of muscular fibres in the substance of the tongue. They are excellent examples of branching striped muscle. The finest branches are less than $\frac{1}{50,000}$th of an inch in diameter, but these exhibit the most distinct transverse markings. The markings, however, gradually cease, and the fibre becomes a mere line, which is lost in the connective tissue at the summit of the papilla. The arrangement will be understood if Plate XXI. figs. 1 & 9 be referred to.

The so-called nuclei or masses of germinal matter in connexion with these fine muscular fibres present several points which will well repay attentive study. These masses of germinal matter are sometimes twice or three times the width of the fibre with which they are connected. In a paper published in Part XIV. of my 'Archives,' I have adduced facts which render it probable that these nuclei or masses of germinal matter change their position in a very remarkable manner during life.

The conclusions I have arrived at upon this point are as follows:—

The masses of germinal matter appear to move along the surface of the already-formed muscular tissue, and as they move part of their substance becomes converted into muscle (Plate XXII. fig. 13). It is in this way that new muscle is formed and new muscular tissue is added to that already produced. The germinal matter itself does not diminish in size, because it absorbs as much pabulum as will compensate for what it loses of its own substance by conversion into tissue. In the young muscle the nucleus increases in size.

From what I have observed, I think that these oval masses of germinal matter move in different directions, but always in a line with the fibrillated structure, so that in a muscle some will be moving upwards, some downwards; and when the nuclei are arranged in rows or straight lines, the nuclei lying in adjacent lines will be moving in opposite directions. During the formation of a muscle these masses undergo division in two directions, longitudinally and transversely. The two masses which result from the division of one will pass in opposite directions.

As is well known, the position of these nuclei with respect to the formed muscular tissue is very different in different cases. Sometimes they are in the very centre of the elementary fibre, as in the constantly-growing fibres of the heart, sometimes upon its surface only, sometimes distributed at very equal distances throughout its substance. Wherever these nuclei are situated new muscular tissue may be produced, and it is only in these situations that muscular tissue ever is produced; so that by the position of the nuclei we learn the seat of formation of new muscle at different periods of life.

The facts which I regard as favourable to the view above expressed concerning the movements of the masses of germinal matter of muscle, are derived from many sources, but I will refer to some observed in the case of the muscles of the papillæ of the tongue. Here the muscular fibre is very thin and delicate, and very favourable for observation. The mass of germinal matter is very much wider than the muscle. Often three or four of these masses are seen close together (Plate XXI. fig. 9), while for some distance above and below the muscular fibre is destitute of nuclei. The narrowest extremity of the oval mass is directed in some cases towards the terminal extremity of the muscle, in others towards its base. There are often three or four fine fibres branching off from one stem, and gradually tapering into fine threads towards their insertion at the summit of the papilla (Plate XXI. figs. 1 & 9). The nuclei are three or four times as wide as these fibres. The greatest difference is observed in the distance between contiguous nuclei connected with the very same fibre. If the muscle had gone on growing uniformly in all parts since the earliest period of its development, the nuclei would be separated from one another by equal distances, or by distances gradually but regularly increasing or diminishing from one extremity of the fibre towards the other.

I think the irregular arrangement of the nuclei in these muscular fibres of the tongue is to be accounted for by their movements. Perhaps, of a collection of these nuclei close together, two may be moving upwards towards the narrow extremity of the muscle which is inserted into the connective tissue, while the third may be moving in the opposite direction.

In some instances a "fault" is observed in the production of the muscular tissue, as if the nucleus had bridged over a space and formed a thin layer or band of muscular tissue, which, when fully formed, was separated by a narrow space or interval from the rest of the muscle. See Plate XXII. fig. 12.

In cases in which the nuclei are distributed at intervals throughout the muscular tissue, as in the large elementary fibres of the muscles of the frog, the formation of the contractile material gradually ceases as the elementary fibre attains its full size. When this point has been reached some of the nuclei gradually diminish in size, and their original seat is marked by a collection of granules. These granules are sometimes absorbed, and the seat of the original nucleus is marked by a short line which gradually tapers at the two extremities until it is lost.

It is almost needless to say that no alteration produced by the different contractions of the muscle in different parts, would account for the position of the nuclei observed in the fine fibres of the papillæ of the frog's tongue.

These views, it need scarcely be said, differ entirely from those generally entertained upon the development and formation of muscular tissue. They are supported by detailed observations made in all classes of animals, and in the same species at different periods of age. There are some facts in connexion with the changes occurring in disease which afford support to this view, which involves three positions. That in the nutrition of muscle the pabulum invariably becomes converted into germinal matter; that the latter undergoes change, and gradually becomes contractile tissue; and that all the contractile material of muscle was once in the state of the material of which the nuclei or masses of germinal matter are composed. It is not deposited from the blood, nor produced by the action of the nuclei at a distance, but it results from a change in the very matter of the nucleus itself. The manner in which this occurs has been already discussed in the paper above referred to (Archives, No. XIV.). It was shown that the oval nucleus could be followed into a very fine band of contractile tissue or fibrilla (Plate XXII. fig. 13). We pass from the matter of the nucleus into very transparent imperfectly-formed tissue in which no transverse lines are perceptible, and from this into fully-developed contractile material in which the characteristic transverse markings are fully developed.

Of the Capillaries.

The capillaries of the papilla of the frog's tongue are remarkable for their large size. In the common frog there is a complete vascular ring at the summit of the papilla, through which the bundle of nerve-fibres distributed to this part pass. In the Hyla the same is observed in some of the papillæ, but the more common arrangement may be described as a half ring or a simple loop, bent upon one side at its upper part (Plate XXI. fig. 1).

When the large capillaries of the papilla are distended with transparent Prussian-blue injection, their walls are seen to be of extreme tenuity and transparency. Connected with the transparent tissue are numerous oval masses of germinal matter (nuclei), which are separated from one another by very short intervals. Some of these masses project slightly from the inner surface of the vessel into its interior, but the majority seem to be upon its external surface. Of an oval form, many of these latter gradually taper into thin fibres which are continuous with the tissue of which the vascular wall is constituted. The delicate membrane constituting the vascular wall exhibits longitudinal striæ, which are probably produced in its formation, and by its external surface is connected with the delicate connective tissue which forms, as it were, the basis-substance of the papilla, and intervenes between all the important tissues which are found in it. This is proved by the fact that the vessel is moved when the transparent connective tissue at some distance from it is drawn in a direction from the vessel.

The most interesting point I have observed in connexion with the anatomy of these vessels, is the existence of very fine nerve-fibres. These form a lax network around the capillary. I have traced these fine fibres continuous with undoubted nerve-trunks in many instances, and have followed the latter into the trunks of dark-bordered fibres, from which the bundle in the papilla is derived. A similar arrangement of fine nerve-fibres has been demonstrated in connexion with other capillary vessels of the frog. These fine nerve-fibres are very distinct in several of my specimens.

I have indeed observed, in my paper published in the Transactions for 1863, contrary to the statements of most anatomists, that capillary vessels generally are freely supplied with nerves, but the latter and their nuclei have been regarded as connective-tissue fibres and connective-tissue corpuscles; I have shown in certain specimens that, of the two fibres resulting from the subdivision of a dark-bordered fibre, one was distributed to the fibres of voluntary muscle, while the other ramified over the vessels supplying the muscle (Plate XXII. fig. 15). These facts, it need scarcely be said, are of great importance with reference to the mechanism of nervous action.

I have not succeeded in demonstrating lymphatic vessels in the papillæ of the frog's tongue.

Besides the various nuclei described, there are several round, oval, and variously-shaped bodies, about the size of a frog's blood-corpuscle, which are composed principally of minute oil-globules and granules. These are not coloured by carmine. Many contain a small mass of germinal matter (nucleus) in the centre, which is of course coloured. In some of the smaller ones this mass of germinal matter is much larger in proportion to the entire "cell." These bodies resemble many of the fat-cells of the frog, and I think it probable they are of this nature. It is, however, possible that these masses may be altered lymph-corpuscles. The Hylæ which I examined had been for some time in confinement, and contained very little adipose tissue.

Conclusions.

1. That fine nerve-fibres ramify in the connective tissue of which the simple papillæ are composed, and that connected with these nerve-fibres are oval masses of germinal matter or nuclei, which are usually regarded as "connective-tissue corpuscles."

2. That neither the epithelial cells of the frog's tongue generally, nor those covering the simple papillæ, are connected with nerve-fibres.

3. That the mass consisting of epithelium-like cells upon the summit of the fungiform papilla, is connected with the nerve-fibres, but it is not an epithelial structure.

4. That the dark-bordered sensitive fibres constituting the bundle of nerves in the axis of the papilla divide near its summit into numerous very fine branches, with which nuclei are connected. Thus is formed a plexus or network of exceedingly fine fibres upon the summit of each papilla; from this network numerous fine fibres may be traced into the special *nervous* organ, composed of epithelium-like cells upon the summit, with every one of which nerve-fibres appear to be connected.

5. That the bundle of nerve-fibres distributed to a papilla always divides into two bundles which pursue opposite directions. The division of the bundle may take place just at the base of the papilla, or at some distance from it, but it always occurs.

6. That fine pale nerve-fibres pass from the same trunk of dark-bordered fibres as that which gives off the bundle of nerves to the papilla. The fine fibres ramify—

   a. Amongst the muscular fibres of the tongue.
   b. Upon the vessels.
   c. In the connective tissue of the tongue generally, and also in the simple papillæ.

7. That the fine nucleated nerve-fibres ramify freely amongst the delicate branching muscular fibres of the papillæ, and form plexuses or networks which exhibit no nerve-ends or terminations, nor in any case does a nerve-fibre penetrate through the sarcolemma or investing tissue of the fibre, or connect itself with the nuclei of the muscle. As many of the muscular fibres are so very fine and narrow, the distribution of the nerves, and their exact relation to the contractile tissue, can be demonstrated very distinctly in the case of the muscles of the papillæ of the frog's tongue.

Description of the Plates.

The figures represented in Plate XXI. illustrate the structure of the papillæ of the frog's tongue. In fig. 1 an entire fungiform papilla only in part finished is delineated. A portion of every tissue entering into its formation is however represented. The structure of this papilla is most interesting, because in a very small space we have *epithelium, muscle, connective tissue, nerves of special sensation, motor nerves, distributed to the branching muscular fibres, and nerves distributed to the capillary vessels and connective tissue which are probably afferent.* In the other figures the most important structures entering into the formation of the papilla are represented very highly magnified. Many of the preparations from which these drawings have been taken are in my possession, and can be examined by any one desirous of seeing them. The mode of preparation adopted is special, and has been referred to very generally in previous papers. It is described in detail in 'How to Work with the Microscope.' Each figure is fully explained in the text beneath it, so that it is unnecessary to give a more minute description of the illustrations in this or the following Plate.

VIII. *A Dynamical Theory of the Electromagnetic Field.* *By* J. CLERK MAXWELL, *F.R.S.*

Received October 27,—Read December 8, 1864.

PART I.—INTRODUCTORY.

(1) THE most obvious mechanical phenomenon in electrical and magnetical experiments is the mutual action by which bodies in certain states set each other in motion while still at a sensible distance from each other. The first step, therefore, in reducing these phenomena into scientific form, is to ascertain the magnitude and direction of the force acting between the bodies, and when it is found that this force depends in a certain way upon the relative position of the bodies and on their electric or magnetic condition, it seems at first sight natural to explain the facts by assuming the existence of something either at rest or in motion in each body, constituting its electric or magnetic state, and capable of acting at a distance according to mathematical laws.

In this way mathematical theories of statical electricity, of magnetism, of the mechanical action between conductors carrying currents, and of the induction of currents have been formed. In these theories the force acting between the two bodies is treated with reference only to the condition of the bodies and their relative position, and without any express consideration of the surrounding medium.

These theories assume, more or less explicitly, the existence of substances the particles of which have the property of acting on one another at a distance by attraction or repulsion. The most complete development of a theory of this kind is that of M. W. WEBER[*], who has made the same theory include electrostatic and electromagnetic phenomena.

In doing so, however, he has found it necessary to assume that the force between two electric particles depends on their relative velocity, as well as on their distance.

This theory, as developed by MM. W. WEBER and C. NEUMANN[†], is exceedingly ingenious, and wonderfully comprehensive in its application to the phenomena of statical electricity, electromagnetic attractions, induction of currents and diamagnetic phenomena; and it comes to us with the more authority, as it has served to guide the speculations of one who has made so great an advance in the practical part of electric science, both by introducing a consistent system of units in electrical measurement, and by actually determining electrical quantities with an accuracy hitherto unknown.

---

[*] Electrodynamische Maassbestimmungen. Leipzic Trans. vol. i. 1849, and TAYLOR's Scientific Memoirs, vol. v. art. xiv.

[†] "Explicare tentatur quomodo fiat ut lucis planum polarizationis per vires electricas vel magneticas declinetur."—Halis Saxonum, 1858.

(2) The mechanical difficulties, however, which are involved in the assumption of particles acting at a distance with forces which depend on their velocities are such as to prevent me from considering this theory as an ultimate one, though it may have been, and may yet be useful in leading to the coordination of phenomena.

I have therefore preferred to seek an explanation of the fact in another direction, by supposing them to be produced by actions which go on in the surrounding medium as well as in the excited bodies, and endeavouring to explain the action between distant bodies without assuming the existence of forces capable of acting directly at sensible distances.

(3) The theory I propose may therefore be called a theory of the *Electromagnetic Field*, because it has to do with the space in the neighbourhood of the electric or magnetic bodies, and it may be called a *Dynamical* Theory, because it assumes that in that space there is matter in motion, by which the observed electromagnetic phenomena are produced.

(4) The electromagnetic field is that part of space which contains and surrounds bodies in electric or magnetic conditions.

It may be filled with any kind of matter, or we may endeavour to render it empty of all gross matter, as in the case of GEISSLER's tubes and other so-called vacua.

There is always, however, enough of matter left to receive and transmit the undulations of light and heat, and it is because the transmission of these radiations is not greatly altered when transparent bodies of measurable density are substituted for the so-called vacuum, that we are obliged to admit that the undulations are those of an æthereal substance, and not of the gross matter, the presence of which merely modifies in some way the motion of the æther.

We have therefore some reason to believe, from the phenomena of light and heat, that there is an æthereal medium filling space and permeating bodies, capable of being set in motion and of transmitting that motion from one part to another, and of communicating that motion to gross matter so as to heat it and affect it in various ways.

(5) Now the energy communicated to the body in heating it must have formerly existed in the moving medium, for the undulations had left the source of heat some time before they reached the body, and during that time the energy must have been half in the form of motion of the medium and half in the form of elastic resilience. From these considerations Professor W. THOMSON has argued*, that the medium must have a density capable of comparison with that of gross matter, and has even assigned an inferior limit to that density.

(6) We may therefore receive, as a datum derived from a branch of science independent of that with which we have to deal, the existence of a pervading medium, of small but real density, capable of being set in motion, and of transmitting motion from one part to another with great, but not infinite, velocity.

Hence the parts of this medium must be so connected that the motion of one part depends in some way on the motion of the rest; and at the same time these connexions must be capable of a certain kind of elastic yielding, since the communication of motion is not instantaneous, but occupies time.

---

* "On the Possible Density of the Luminiferous Medium, and on the Mechanical Value of a Cubic Mile of Sunlight," Transactions of the Royal Society of Edinburgh (1854), p. 57.

The medium is therefore capable of receiving and storing up two kinds of energy, namely, the "actual" energy depending on the motions of its parts, and "potential" energy, consisting of the work which the medium will do in recovering from displacement in virtue of its elasticity.

The propagation of undulations consists in the continual transformation of one of these forms of energy into the other alternately, and at any instant the amount of energy in the whole medium is equally divided, so that half is energy of motion, and half is elastic resilience.

(7) A medium having such a constitution may be capable of other kinds of motion and displacement than those which produce the phenomena of light and heat, and some of these may be of such a kind that they may be evidenced to our senses by the phenomena they produce.

(8) Now we know that the luminiferous medium is in certain cases acted on by magnetism; for FARADAY* discovered that when a plane polarized ray traverses a transparent diamagnetic medium in the direction of the lines of magnetic force produced by magnets or currents in the neighbourhood, the plane of polarization is caused to rotate.

This rotation is always in the direction in which positive electricity must be carried round the diamagnetic body in order to produce the actual magnetization of the field.

M. VERDET† has since discovered that if a paramagnetic body, such as solution of perchloride of iron in ether, be substituted for the diamagnetic body, the rotation is in the opposite direction.

Now Professor W. THOMSON‡ has pointed out that no distribution of forces acting between the parts of a medium whose only motion is that of the luminous vibrations, is sufficient to account for the phenomena, but that we must admit the existence of a motion in the medium depending on the magnetization, in addition to the vibratory motion which constitutes light.

It is true that the rotation by magnetism of the plane of polarization has been observed only in media of considerable density; but the properties of the magnetic field are not so much altered by the substitution of one medium for another, or for a vacuum, as to allow us to suppose that the dense medium does anything more than merely modify the motion of the ether. We have therefore warrantable grounds for inquiring whether there may not be a motion of the ethereal medium going on wherever magnetic effects are observed, and we have some reason to suppose that this motion is one of rotation, having the direction of the magnetic force as its axis.

(9) We may now consider another phenomenon observed in the electromagnetic

---

* Experimental Researches, Series 19.

† Comptes Rendus (1856, second half year, p. 529, and 1857, first half year, p. 1209).

‡ Proceedings of the Royal Society, June 1856 and June 1861.

field. When a body is moved across the lines of magnetic force it experiences what is called an electromotive force; the two extremities of the body tend to become oppositely electrified, and an electric current tends to flow through the body. When the electromotive force is sufficiently powerful, and is made to act on certain compound bodies, it decomposes them, and causes one of their components to pass towards one extremity of the body, and the other in the opposite direction.

Here we have evidence of a force causing an electric current in spite of resistance; electrifying the extremities of a body in opposite ways, a condition which is sustained only by the action of the electromotive force, and which, as soon as that force is removed, tends, with an equal and opposite force, to produce a counter current through the body and to restore the original electrical state of the body; and finally, if strong enough, tearing to pieces chemical compounds and carrying their components in opposite directions, while their natural tendency is to combine, and to combine with a force which can generate an electromotive force in the reverse direction.

This, then, is a force acting on a body caused by its motion through the electromagnetic field, or by changes occurring in that field itself; and the effect of the force is either to produce a current and heat the body, or to decompose the body, or, when it can do neither, to put the body in a state of electric polarization,—a state of constraint in which opposite extremities are oppositely electrified, and from which the body tends to relieve itself as soon as the disturbing force is removed.

(10) According to the theory which I propose to explain, this "electromotive force" is the force called into play during the communication of motion from one part of the medium to another, and it is by means of this force that the motion of one part causes motion in another part. When electromotive force acts on a conducting circuit, it produces a current, which, as it meets with resistance, occasions a continual transformation of electrical energy into heat, which is incapable of being restored again to the form of electrical energy by any reversal of the process.

(11) But when electromotive force acts on a dielectric it produces a state of polarization of its parts similar in distribution to the polarity of the parts of a mass of iron under the influence of a magnet, and like the magnetic polarization, capable of being described as a state in which every particle has its opposite poles in opposite conditions*.

In a dielectric under the action of electromotive force, we may conceive that the electricity in each molecule is so displaced that one side is rendered positively and the other negatively electrical, but that the electricity remains entirely connected with the molecule, and does not pass from one molecule to another. The effect of this action on the whole dielectric mass is to produce a general displacement of electricity in a certain direction. This displacement does not amount to a current, because when it has attained to a certain value it remains constant, but it is the commencement of a current, and its variations constitute currents in the positive or the negative direction according as the displacement is increasing or decreasing. In the interior of the dielectric there is no indication of electrification, because the electrification of the surface of any molecule is neutralized by the opposite electrification of the surface of the molecules in contact with it; but at the bounding surface of the dielectric, where the electrification is not neutralized, we find the phenomena which indicate positive or negative electrification.

---

* FARADAY, Exp. Res. Series XI.; MOSSOTTI, Mem. della Soc. Italiana (Modena), vol. xxiv. part 2. p. 49.

The relation between the electromotive force and the amount of electric displacement it produces depends on the nature of the dielectric, the same electromotive force producing generally a greater electric displacement in solid dielectrics, such as glass or sulphur, than in air.

(12) Here, then, we perceive another effect of electromotive force, namely, electric displacement, which according to our theory is a kind of elastic yielding to the action of the force, similar to that which takes place in structures and machines owing to the want of perfect rigidity of the connexions.

(13) The practical investigation of the inductive capacity of dielectrics is rendered difficult on account of two disturbing phenomena. The first is the conductivity of the dielectric, which, though in many cases exceedingly small, is not altogether insensible. The second is the phenomenon called electric absorption*, in virtue of which, when the dielectric is exposed to electromotive force, the electric displacement gradually increases, and when the electromotive force is removed, the dielectric does not instantly return to its primitive state, but only discharges a portion of its electrification, and when left to itself gradually acquires electrification on its surface, as the interior gradually becomes depolarized. Almost all solid dielectrics exhibit this phenomenon, which gives rise to the residual charge in the Leyden jar, and to several phenomena of electric cables described by Mr. F. JENKIN†.

(14) We have here two other kinds of yielding besides the yielding of the perfect dielectric, which we have compared to a perfectly elastic body. The yielding due to conductivity may be compared to that of a viscous fluid (that is to say, a fluid having great internal friction), or a soft solid on which the smallest force produces a permanent alteration of figure increasing with the time during which the force acts. The yielding due to electric absorption may be compared to that of a cellular elastic body containing a thick fluid in its cavities. Such a body, when subjected to pressure, is compressed by degrees on account of the gradual yielding of the thick fluid; and when the pressure is removed it does not at once recover its figure, because the elasticity of the substance of the body has gradually to overcome the tenacity of the fluid before it can regain complete equilibrium.

Several solid bodies in which no such structure as we have supposed can be found, seem to possess a mechanical property of this kind‡; and it seems probable that the substances, if dielectrics, may possess the analogous electrical property, and if magnetic, may have corresponding properties relating to the acquisition, retention, and loss of magnetic polarity.

---

* FARADAY, Exp. Res. 1233–1250.

† Reports of British Association, 1859, p. 248; and Report of Committee of Board of Trade on Submarine Cables, pp. 136 & 464.

‡ As, for instance, the composition of glue, treacle, &c., of which small plastic figures are made, which after being distorted gradually recover their shape.

(15) It appears therefore that certain phenomena in electricity and magnetism lead to the same conclusion as those of optics, namely, that there is an æthereal medium pervading all bodies, and modified only in degree by their presence; that the parts of this medium are capable of being set in motion by electric currents and magnets; that this motion is communicated from one part of the medium to another by forces arising from the connexions of those parts; that under the action of these forces there is a certain yielding depending on the elasticity of these connexions; and that therefore energy in two different forms may exist in the medium, the one form being the actual energy of motion of its parts, and the other being the potential energy stored up in the connexions, in virtue of their elasticity.

(16) Thus, then, we are led to the conception of a complicated mechanism capable of a vast variety of motion, but at the same time so connected that the motion of one part depends, according to definite relations, on the motion of other parts, these motions being communicated by forces arising from the relative displacement of the connected parts, in virtue of their elasticity. Such a mechanism must be subject to the general laws of Dynamics, and we ought to be able to work out all the consequences of its motion, provided we know the form of the relation between the motions of the parts.

(17) We know that when an electric current is established in a conducting circuit, the neighbouring part of the field is characterized by certain magnetic properties, and that if two circuits are in the field, the magnetic properties of the field due to the two currents are combined. Thus each part of the field is in connexion with both currents, and the two currents are put in connexion with each other in virtue of their connexion with the magnetization of the field. The first result of this connexion that I propose to examine, is the induction of one current by another, and by the motion of conductors in the field.

The second result, which is deduced from this, is the mechanical action between conductors carrying currents. The phenomenon of the induction of currents has been deduced from their mechanical action by HELMHOLTZ* and THOMSON†. I have followed the reverse order, and deduced the mechanical action from the laws of induction. I have then described experimental methods of determining the quantities L, M, N, on which these phenomena depend.

(18) I then apply the phenomena of induction and attraction of currents to the exploration of the electromagnetic field, and the laying down systems of lines of magnetic force which indicate its magnetic properties. By exploring the same field with a magnet, I show the distribution of its equipotential magnetic surfaces, cutting the lines of force at right angles.

---

* "Conservation of Force," Physical Society of Berlin, 1847; and TAYLOR's Scientific Memoirs, 1853, p. 114.

† Reports of the British Association, 1848; Philosophical Magazine, Dec. 1851.

In order to bring these results within the power of symbolical calculation, I then express them in the form of the General Equations of the Electromagnetic Field. These equations express—

(A) The relation between electric displacement, true conduction, and the total current, compounded of both.

(B) The relation between the lines of magnetic force and the inductive coefficients of a circuit, as already deduced from the laws of induction.

(C) The relation between the strength of a current and its magnetic effects, according to the electromagnetic system of measurement.

(D) The value of the electromotive force in a body, as arising from the motion of the body in the field, the alteration of the field itself, and the variation of electric potential from one part of the field to another.

(E) The relation between electric displacement, and the electromotive force which produces it.

(F) The relation between an electric current, and the electromotive force which produces it.

(G) The relation between the amount of free electricity at any point, and the electric displacements in the neighbourhood.

(H) The relation between the increase or diminution of free electricity and the electric currents in the neighbourhood.

There are twenty of these equations in all, involving twenty variable quantities.

(19) I then express in terms of these quantities the intrinsic energy of the Electromagnetic Field as depending partly on its magnetic and partly on its electric polarization at every point.

From this I determine the mechanical force acting, 1st, on a moveable conductor carrying an electric current; 2ndly, on a magnetic pole; 3rdly, on an electrified body.

The last result, namely, the mechanical force acting on an electrified body, gives rise to an independent method of electrical measurement founded on its electrostatic effects. The relation between the units employed in the two methods is shown to depend on what I have called the "electric elasticity" of the medium, and to be a velocity, which has been experimentally determined by MM. WEBER and KOHLRAUSCH.

I then show how to calculate the electrostatic capacity of a condenser, and the specific inductive capacity of a dielectric.

The case of a condenser composed of parallel layers of substances of different electric resistances and inductive capacities is next examined, and it is shown that the phenomenon called electric absorption will generally occur, that is, the condenser, when suddenly discharged, will after a short time show signs of a *residual* charge.

(20) The general equations are next applied to the case of a magnetic disturbance propagated through a non-conducting field, and it is shown that the only disturbances which can be so propagated are those which are transverse to the direction of propagation, and that the velocity of propagation is the velocity v, found from experiments such

as those of **Weber**, which expresses the number of electrostatic units of electricity which are contained in one electromagnetic unit.

This velocity is so nearly that of light, that it seems we have strong reason to conclude that light itself (including radiant heat, and other radiations if any) is an electromagnetic disturbance in the form of waves propagated through the electromagnetic field according to electromagnetic laws. If so, the agreement between the elasticity of the medium as calculated from the rapid alternations of luminous vibrations, and as found by the slow processes of electrical experiments, shows how perfect and regular the elastic properties of the medium must be when not encumbered with any matter denser than air. If the same character of the elasticity is retained in dense transparent bodies, it appears that the square of the index of refraction is equal to the product of the specific dielectric capacity and the specific magnetic capacity. Conducting media are shown to absorb such radiations rapidly, and therefore to be generally opaque.

The conception of the propagation of transverse magnetic disturbances to the exclusion of normal ones is distinctly set forth by Professor **Faraday**[*] in his "Thoughts on Ray Vibrations." The electromagnetic theory of light, as proposed by him, is the same in substance as that which I have begun to develope in this paper, except that in 1846 there were no data to calculate the velocity of propagation.

(21) The general equations are then applied to the calculation of the coefficients of mutual induction of two circular currents and the coefficient of self-induction in a coil. The want of uniformity of the current in the different parts of the section of a wire at the commencement of the current is investigated, I believe for the first time, and the consequent correction of the coefficient of self-induction is found.

These results are applied to the calculation of the self-induction of the coil used in the experiments of the Committee of the British Association on Standards of Electric Resistance, and the value compared with that deduced from the experiments.

PART II.—ON ELECTROMAGNETIC INDUCTION.

Electromagnetic Momentum of a Current.

(22) We may begin by considering the state of the field in the neighbourhood of an electric current. We know that magnetic forces are excited in the field, their direction and magnitude depending according to known laws upon the form of the conductor carrying the current. When the strength of the current is increased, all the magnetic effects are increased in the same proportion. Now, if the magnetic state of the field depends on motions of the medium, a certain force must be exerted in order to increase or diminish these motions, and when the motions are excited they continue, so that the effect of the connexion between the current and the electromagnetic field surrounding it, is to endow the current with a kind of momentum, just as the connexion between the driving-point of a machine and a fly-wheel endows the driving-point with an addi-

---

[*] Philosophical Magazine, May 1846, or Experimental Researches, iii. p. 447.

tional momentum, which may be called the momentum of the fly-wheel reduced to the driving-point. The unbalanced force acting on the driving-point increases this momentum, and is measured by the rate of its increase.

In the case of electric currents, the resistance to sudden increase or diminution of strength produces effects exactly like those of momentum, but the amount of this momentum depends on the shape of the conductor and the relative position of its different parts.

Mutual Action of two Currents.

(23) If there are two electric currents in the field, the magnetic force at any point is that compounded of the forces due to each current separately, and since the two currents are in connexion with every point of the field, they will be in connexion with each other, so that any increase or diminution of the one will produce a force acting with or contrary to the other.

Dynamical Illustration of Reduced Momentum.

(24) As a dynamical illustration, let us suppose a body C so connected with two independent driving-points A and B that its velocity is p times that of A together with q times that of B. Let u be the velocity of A, v that of B, and w that of C, and let δx, δy, δz be their simultaneous displacements, then by the general equation of dynamics*,

$$\mathrm{C}\frac{dw}{dt}\delta z=\mathrm{X}\delta x+\mathrm{Y}\delta y,$$

where X and Y are the forces acting at A and B.

But

$$\frac{dw}{dt}=p\frac{du}{dt}+q\frac{dv}{dt},$$

and

$$\delta z=p\delta x+q\delta y.$$

Substituting, and remembering that δx and δy are independent,

$$\left.\begin{aligned}\mathrm{X}&=\frac{d}{dt}(\mathrm{C}p^2u+\mathrm{C}pqv),\\ \mathrm{Y}&=\frac{d}{dt}(\mathrm{C}pqu+\mathrm{C}q^2v).\end{aligned}\right\}\quad\ldots\ldots\ldots\ldots\quad(1)$$

We may call $\mathrm{C}p^2u+\mathrm{C}pqv$ the momentum of C referred to A, and $\mathrm{C}pqu+\mathrm{C}q^2v$ its momentum referred to B; then we may say that the effect of the force X is to increase the momentum of C referred to A, and that of Y to increase its momentum referred to B.

If there are many bodies connected with A and B in a similar way but with different values of p and q, we may treat the question in the same way by assuming

$$\mathrm{L}=\Sigma(\mathrm{C}p^2),\quad \mathrm{M}=\Sigma(\mathrm{C}pq),\quad \text{and } \mathrm{N}=\Sigma(\mathrm{C}q^2),$$

---

* LAGRANGE, Méc. Anal. ii. 2. § 5.

where the summation is extended to all the bodies with their proper values of C, p, and q. Then the momentum of the system referred to A is

$$Lu+Mv,$$

and referred to B,

$$Mu+Nv,$$

and we shall have

$$\left.\begin{aligned} X&=\frac{d}{dt}(Lu+Mv),\\ Y&=\frac{d}{dt}(Mu+Nv), \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where X and Y are the external forces acting on A and B.

(25) To make the illustration more complete we have only to suppose that the motion of A is resisted by a force proportional to its velocity, which we may call Ru, and that of B by a similar force, which we may call Sv, R and S being coefficients of resistance. Then if ξ and η are the forces on A and B

$$\left.\begin{aligned} \xi&=X+Ru=Ru+\frac{d}{dt}(Lu+Mv),\\ \eta&=Y+Sv\ =Sv+\frac{d}{dt}(Mu+Nv) \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

If the velocity of A be increased at the rate $\frac{du}{dt}$, then in order to prevent B from moving a force, $\eta=\frac{d}{dt}(Mu)$ must be applied to it.

This effect on B, due to an increase of the velocity of A, corresponds to the electromotive force on one circuit arising from an increase in the strength of a neighbouring circuit.

This dynamical illustration is to be considered merely as assisting the reader to understand what is meant in mechanics by Reduced Momentum. The facts of the induction of currents as depending on the variations of the quantity called Electromagnetic Momentum, or Electrotonic State, rest on the experiments of FARADAY*, FELICI†, &c.

Coefficients of Induction for Two Circuits.

(26) In the electromagnetic field the values of L, M, N depend on the distribution of the magnetic effects due to the two circuits, and this distribution depends only on the form and relative position of the circuits. Hence L, M, N are quantities depending on the form and relative position of the circuits, and are subject to variation with the motion of the conductors. It will be presently seen that L, M, N are geometrical quantities of the nature of lines, that is, of one dimension in space; L depends on the form of the first conductor, which we shall call A, N on that of the second, which we shall call B, and M on the relative position of A and B.

(27) Let ξ be the electromotive force acting on A, x the strength of the current, and

---

[*] Experimental Researches, Series I., IX.

[†] Annales de Chimie, sér. 3. xxxiv. (1852) p. 64.

R the resistance, then Rx will be the resisting force. In steady currents the electromotive force just balances the resisting force, but in variable currents the resultant force $\xi = Rx$ is expended in increasing the "electromagnetic momentum," using the word momentum merely to express that which is generated by a force acting during a time, that is, a velocity existing in a body.

In the case of electric currents, the force in action is not ordinary mechanical force, at least we are not as yet able to measure it as common force, but we call it electromotive force, and the body moved is not merely the electricity in the conductor, but something outside the conductor, and capable of being affected by other conductors in the neighbourhood carrying currents. In this it resembles rather the reduced momentum of a driving-point of a machine as influenced by its mechanical connexions, than that of a simple moving body like a cannon ball, or water in a tube.

Electromagnetic Relations of two Conducting Circuits.

(28.) In the case of two conducting circuits, A and B, we shall assume that the electromagnetic momentum belonging to A is

$$Lx + My,$$

and that belonging to B,

$$Mx + Ny,$$

where L, M, N correspond to the same quantities in the dynamical illustration, except that they are supposed to be capable of variation when the conductors A or B are moved.

Then the equation of the current x in A will be

$$\xi = Rx + \frac{d}{dt}(Lx + My), \quad \dots \quad (4)$$

and that of y in B

$$\eta = Sy + \frac{d}{dt}(Mx + Ny), \quad \dots \quad (5)$$

where ξ and η are the electromotive forces, x and y the currents, and R and S the resistances in A and B respectively.

Induction of one Current by another.

(29) Case 1st. Let there be no electromotive force on B, except that which arises from the action of A, and let the current of A increase from 0 to the value x, then

$$Sy + \frac{d}{dt}(Mx + Ny) = 0,$$

whence . . . .

$$Y = \int_0^t y\,dt = -\frac{M}{S}x,$$

that is, a quantity of electricity Y, being the total induced current, will flow through B when x rises from 0 to x. This is induction by variation of the current in the primary conductor. When M is positive, the induced current due to increase of the primary current is negative.

Induction by Motion of Conductor.

(30) Case 2nd. Let x remain constant, and let M change from M to M′, then

$$\mathrm{Y}=-\frac{\mathrm{M}'-\mathrm{M}}{\mathrm{S}}x;$$

so that if M is increased, which it will be by the primary and secondary circuits approaching each other, there will be a negative induced current, the total quantity of electricity passed through B being Y.

This is induction by the relative motion of the primary and secondary conductors.

Equation of Work and Energy.

(31) To form the equation between work done and energy produced, multiply (1) by x and (2) by y, and add

$$\xi x+\eta y=\mathrm{R}x^2+\mathrm{S}y^2+x\frac{d}{dt}(\mathrm{L}x+\mathrm{M}y)+y\frac{d}{dt}(\mathrm{M}x+\mathrm{N}y). \quad . \quad . \quad . \quad . \quad (8)$$

Here ξx is the work done in unit of time by the electromotive force ξ acting on the current x and maintaining it, and ηy is the work done by the electromotive force η. Hence the left-hand side of the equation represents the work done by the electromotive forces in unit of time.

Heat produced by the Current.

(32) On the other side of the equation we have, first,

$$\mathrm{R}x^2+\mathrm{S}y^2=\mathrm{H}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

which represents the work done in overcoming the resistance of the circuits in unit of time. This is converted into heat. The remaining terms represent work not converted into heat. They may be written

$$\tfrac{1}{2}\frac{d}{dt}(\mathrm{L}x^2+2\mathrm{M}xy+\mathrm{N}y^2)+\tfrac{1}{2}\frac{d\mathrm{L}}{dt}x^2+\frac{d\mathrm{M}}{dt}xy+\tfrac{1}{2}\frac{d\mathrm{N}}{dt}y^2.$$

Intrinsic Energy of the Currents.

(33) If L, M, N are constant, the whole work of the electromotive forces which is not spent against resistance will be devoted to the development of the currents. The whole intrinsic energy of the currents is therefore

$$\tfrac{1}{2}\mathrm{L}x^2+\mathrm{M}xy+\tfrac{1}{2}\mathrm{N}y^2=\mathrm{E}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

This energy exists in a form imperceptible to our senses, probably as actual motion, the seat of this motion being not merely the conducting circuits, but the space surrounding them.

Mechanical Action between Conductors.

(34) The remaining terms,

$$\tfrac{1}{2}\frac{dL}{dt}x^2+\frac{dM}{dt}xy+\tfrac{1}{2}\frac{dN}{dt}y^2=W \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (11)$$

represent the work done in unit of time arising from the variations of L, M, and N, or, what is the same thing, alterations in the form and position of the conducting circuits A and B.

Now if work is done when a body is moved, it must arise from ordinary mechanical force acting on the body while it is moved. Hence this part of the expression shows that there is a mechanical force urging every part of the conductors themselves in that direction in which L, M, and N will be most increased.

The existence of the electromagnetic force between conductors carrying currents is therefore a direct consequence of the joint and independent action of each current on the electromagnetic field. If A and B are allowed to approach a distance *ds*, so as to increase M from M to M′ while the currents are x and y, then the work done will be

$$(M'-M)xy,$$

and the force in the direction of *ds* will be

$$\frac{dM}{ds}xy, \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (12)$$

and this will be an attraction if x and y are of the same sign, and if M is increased as A and B approach.

It appears, therefore, that if we admit that the unresisted part of electromotive force goes on as long as it acts, generating a self-persistent state of the current, which we may call (from mechanical analogy) its electromagnetic momentum, and that this momentum depends on circumstances external to the conductor, then both induction of currents and electromagnetic attractions may be proved by mechanical reasoning.

What I have called electromagnetic momentum is the same quantity which is called by FARADAY* the electrotonic state of the circuit, every change of which involves the action of an electromotive force, just as change of momentum involves the action of mechanical force.

If, therefore, the phenomena described by FARADAY in the Ninth Series of his Experimental Researches were the only known facts about electric currents, the laws of AMPÈRE relating to the attraction of conductors carrying currents, as well as those of FARADAY about the mutual induction of currents, might be deduced by mechanical reasoning.

In order to bring these results within the range of experimental verification, I shall next investigate the case of a single current, of two currents, and of the six currents in the electric balance, so as to enable the experimenter to determine the values of L, M, N.

---

* Experimental Researches, Series I. 60, &c.

Case of a single Circuit.

(35) The equation of the current x in a circuit whose resistance is R, and whose coefficient of self-induction is L, acted on by an external electromotive force ξ, is

$$\xi - \mathrm{R}x = \frac{d}{dt}\mathrm{L}x. \quad \ldots \ldots \quad (13)$$

When ξ is constant, the solution is of the form

$$x = b + (a-b)e^{-\frac{\mathrm{R}}{\mathrm{L}}t},$$

where a is the value of the current at the commencement, and b is its final value.

The total quantity of electricity which passes in time t, where t is great, is

$$\int_0^t x\,dt = bt + (a-b)\frac{\mathrm{L}}{\mathrm{R}}. \quad \ldots \ldots \quad (14)$$

The value of the integral of x^2 with respect to the time is

$$\int_0^t x^2 dt = b^2 t + (a-b)\frac{\mathrm{L}}{\mathrm{R}}\left(\frac{3b+a}{2}\right). \quad \ldots \ldots \quad (15)$$

The actual current changes gradually from the initial value a to the final value b, but the values of the integrals of x and x^2 are the same as if a steady current of intensity $\frac{1}{2}(a+b)$ were to flow for a time $2\frac{\mathrm{L}}{\mathrm{R}}$, and were then succeeded by the steady current b. The time $2\frac{\mathrm{L}}{\mathrm{R}}$ is generally so minute a fraction of a second, that the effects on the galvanometer and dynamometer may be calculated as if the impulse were instantaneous.

If the circuit consists of a battery and a coil, then, when the circuit is first completed, the effects are the same as if the current had only half its final strength during the time $2\frac{\mathrm{L}}{\mathrm{R}}$. This diminution of the current, due to induction, is sometimes called the counter-current.

(36) If an additional resistance r is suddenly thrown into the circuit, as by breaking contact, so as to force the current to pass through a thin wire of resistance r, then the original current is $a=\frac{\xi}{\mathrm{R}}$, and the final current is $b=\frac{\xi}{\mathrm{R}+r}$.

The current of induction is then $\frac{1}{2}\xi\frac{2\mathrm{R}+r}{\mathrm{R}(\mathrm{R}+r)}$, and continues for a time $2\frac{\mathrm{L}}{\mathrm{R}+r}$. This current is greater than that which the battery can maintain in the two wires R and r, and may be sufficient to ignite the thin wire r.

When contact is broken by separating the wires in air, this additional resistance is given by the interposed air, and since the electromotive force across the new resistance is very great, a spark will be forced across.

If the electromotive force is of the form $E\sin pt$, as in the case of a coil revolving in a magnetic field, then

$$x=\frac{E}{\varrho}\sin(pt-\alpha),$$

where $\varrho^2=R^2+L^2p^2$, and $\tan\alpha=\frac{Lp}{R}$.

Case of two Circuits.

(37) Let R be the primary circuit and S the secondary circuit, then we have a case similar to that of the induction coil.

The equations of currents are those marked A and B, and we may here assume L, M, N as constant because there is no motion of the conductors. The equations then become

$$\left.\begin{aligned} Rx+L\frac{dx}{dt}+M\frac{dy}{dt}&=\xi,\\ Sy+M\frac{dx}{dt}+N\frac{dy}{dt}&=0.\end{aligned}\right\} \quad \text{. . . . . (13*)}$$

To find the total quantity of electricity which passes, we have only to integrate these equations with respect to t; then if x_0, y_0 be the strengths of the currents at time 0, and x_1, y_1 at time t, and if X, Y be the quantities of electricity passed through each circuit during time t,

$$\left.\begin{aligned} X&=\frac{1}{R}\{\xi t+L(x_0-x_1)+M(y_0-y_1)\},\\ Y&=\frac{1}{S}\{M(x_0-x_1)+N(y_0-y_1)\}.\end{aligned}\right\} \quad \text{. . . . . . . (14*)}$$

When the circuit R is completed, then the total currents up to time t, when t is great, are found by making

$$x_0=0,\quad x_1=\frac{\xi}{R},\quad y_0=0,\quad y_1=0;$$

then

$$X=x_1\left(t-\frac{L}{R}\right),\quad Y=-\frac{M}{S}x_1. \quad \text{. . . . . (15*)}$$

The value of the total counter-current in R is therefore independent of the secondary circuit, and the induction current in the secondary circuit depends only on M, the coefficient of induction between the coils, S the resistance of the secondary coil, and x_1 the final strength of the current in R.

When the electromotive force ξ ceases to act, there is an extra current in the primary circuit, and a positive induced current in the secondary circuit, whose values are equal and opposite to those produced on making contact.

(38) All questions relating to the total quantity of transient currents, as measured by the impulse given to the magnet of the galvanometer, may be solved in this way without the necessity of a complete solution of the equations. The heating effect of

the current, and the impulse it gives to the suspended coil of WEBER's dynamometer, depend on the square of the current at every instant during the short time it lasts. Hence we must obtain the solution of the equations, and from the solution we may find the effects both on the galvanometer and dynamometer; and we may then make use of the method of WEBER for estimating the intensity and duration of a current uniform while it lasts which would produce the same effects.

(39) Let n_1, n_2 be the roots of the equation

$$(\mathrm{LN}-\mathrm{M}^2)n^2+(\mathrm{RN}+\mathrm{LS})n+\mathrm{RS}=0, \quad . \; . \; . \; . \; . \; . \; . \quad (16)$$

and let the primary coil be acted on by a constant electromotive force Rc, so that c is the constant current it could maintain; then the complete solution of the equations for making contact is

$$x=\frac{c}{\mathrm{S}}\frac{n_1n_2}{n_1-n_2}\left\{\left(\frac{\mathrm{S}}{n_1}+\mathrm{N}\right)e^{n_1t}-\left(\frac{\mathrm{S}}{n_2}+\mathrm{N}\right)e^{n_2t}+\mathrm{S}\frac{n_1-n_2}{n_1n_2}\right\}, \quad . \; . \; . \; . \quad (17)$$

$$y=\frac{c\mathrm{M}}{\mathrm{S}}\frac{n_1n_2}{n_1-n_2}\{e^{n_1t}-e^{n_2t}\}. \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (18)$$

From these we obtain for calculating the impulse on the dynamometer,

$$\int x^2dt=c^2\left\{t-\tfrac{3}{2}\frac{\mathrm{L}}{\mathrm{R}}-\tfrac{1}{2}\frac{\mathrm{M}^2}{\mathrm{RN}+\mathrm{LS}}\right\}, \quad . \; . \; . \; . \; . \; . \; . \; . \quad (19)$$

$$\int y^2dt=c^2\tfrac{1}{2}\frac{\mathrm{M}^2\mathrm{R}}{\mathrm{S}(\mathrm{RN}+\mathrm{LS})}. \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (20)$$

The effects of the current in the secondary coil on the galvanometer and dynamometer are the same as those of a uniform current

$$-\tfrac{1}{2}c\frac{\mathrm{MR}}{\mathrm{RN}+\mathrm{LS}}$$

for a time

$$2\left(\frac{\mathrm{L}}{\mathrm{R}}+\frac{\mathrm{N}}{\mathrm{S}}\right).$$

(40) The equation between work and energy may be easily verified. The work done by the electromotive force is

$$\xi\int x\,dt=c^2(\mathrm{R}t-\mathrm{L}).$$

Work done in overcoming resistance and producing heat,

$$\mathrm{R}\int x^2dt+\mathrm{S}\int y^2dt=c^2(\mathrm{R}t-\tfrac{3}{2}\mathrm{L}).$$

Energy remaining in the system,

$$=\tfrac{1}{2}c^2\mathrm{L}.$$

(41) If the circuit R is suddenly and completely interrupted while carrying a current c, then the equation of the current in the secondary coil would be

$$y=c\frac{\mathrm{M}}{\mathrm{N}}e^{-\frac{\mathrm{S}}{\mathrm{N}}t}.$$

This current begins with a value $c\dfrac{\mathrm{M}}{\mathrm{N}}$, and gradually disappears.

The total quantity of electricity is $c\frac{\mathrm{M}}{\mathrm{S}}$, and the value of $\int y^2 dt$ is $c^2\frac{\mathrm{M}^2}{2\mathrm{SN}}$.

The effects on the galvanometer and dynamometer are equal to those of a uniform current $\frac{1}{2}c\frac{\mathrm{M}}{\mathrm{N}}$ for a time $2\frac{\mathrm{N}}{\mathrm{S}}$.

The heating effect is therefore greater than that of the current on making contact.

(42) If an electromotive force of the form $\xi=\mathrm{E}\cos pt$ acts on the circuit R, then if the circuit S is removed, the value of x will be

$$x=\frac{\mathrm{E}}{\mathrm{A}}\sin(pt-\alpha),$$

where

$$\mathrm{A}^2=\mathrm{R}^2+\mathrm{L}^2p^2,$$

and

$$\tan\alpha=\frac{\mathrm{L}p}{\mathrm{R}}.$$

The effect of the presence of the circuit S in the neighbourhood is to alter the value of A and α, to that which they would be if R become

$$\mathrm{R}+p^2\frac{\mathrm{MS}}{\mathrm{S}^2+p^2\mathrm{N}^2},$$

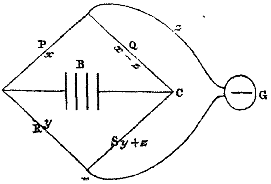
and L became

$$\mathrm{L}-p^2\frac{\mathrm{MN}}{\mathrm{S}^2+p^2\mathrm{N}^2}.$$

Hence the effect of the presence of the circuit S is to increase the apparent resistance and diminish the apparent self-induction of the circuit R.

On the Determination of Coefficients of Induction by the Electric Balance.

(43) The electric balance consists of six conductors joining four points, A C D E, two and two. One pair, A C, of these points is connected through the battery B. The opposite pair, D E, is connected through the galvanometer G. Then if the resistances of the four remaining conductors are represented by P, Q, R, S, and the currents in them by x, $x-z$, y, and $y+z$, the current through G will be z. Let the potentials at the four points be A, C, D, E. Then the conditions of steady currents may be found from the equations



$$\left.\begin{array}{ll} \mathrm{P}x=\mathrm{A}-\mathrm{D} & \mathrm{Q}(x-z)=\mathrm{D}-\mathrm{C}, \\ \mathrm{R}y=\mathrm{A}-\mathrm{E} & \mathrm{S}(y+z)=\mathrm{E}-\mathrm{C}, \\ \mathrm{G}z=\mathrm{D}-\mathrm{E} & \mathrm{B}(x+y)=-\mathrm{A}+\mathrm{C}+\mathrm{F}. \end{array}\right\} \quad \ldots\ldots\ldots\ldots \quad (21)$$

Solving these equations for z, we find

$$z\left\{\frac{1}{\mathrm{P}}+\frac{1}{\mathrm{Q}}+\frac{1}{\mathrm{R}}+\frac{1}{\mathrm{S}}+\mathrm{B}\left(\frac{1}{\mathrm{P}}+\frac{1}{\mathrm{R}}\right)\left(\frac{1}{\mathrm{Q}}+\frac{1}{\mathrm{S}}\right)+\mathrm{G}\left(\frac{1}{\mathrm{P}}+\frac{1}{\mathrm{Q}}\right)\left(\frac{1}{\mathrm{R}}+\frac{1}{\mathrm{S}}\right)+\frac{\mathrm{BG}}{\mathrm{PQRS}}(\mathrm{P}+\mathrm{Q}+\mathrm{R}+\mathrm{S})\right\}=\mathrm{F}\left(\frac{1}{\mathrm{PS}}-\frac{1}{\mathrm{QR}}\right). \quad (2$$

In this expression F is the electromotive force of the battery, z the current through the galvanometer when it has become steady. P, Q, R, S the resistances in the four arms. B that of the battery and electrodes, and G that of the galvanometer.

(44) If PS=QR, then $z=0$, and there will be no steady current, but a transient current through the galvanometer may be produced on making or breaking circuit on account of induction, and the indications of the galvanometer may be used to determine the coefficients of induction, provided we understand the actions which take place.

We shall suppose PS=QR, so that the current z vanishes when sufficient time is allowed, and

$$x(\mathrm{P}+\mathrm{Q})=y(\mathrm{R}+\mathrm{S})=\frac{\mathrm{F}(\mathrm{P}+\mathrm{Q})(\mathrm{R}+\mathrm{S})}{(\mathrm{P}+\mathrm{Q})(\mathrm{R}+\mathrm{S})+\mathrm{B}(\mathrm{P}+\mathrm{Q})(\mathrm{R}+\mathrm{S})}.$$

Let the induction coefficients between P, Q, R S, be given by the following Table, the coefficient of induction of P on itself being p, between P and Q, h, and so on.

	P	Q	R	S
P	p	h	k	l
Q	h	q	m	n
R	k	m	r	o
S	l	n	o	s

Let g be the coefficient of induction of the galvanometer on itself, and let it be out of the reach of the inductive influence of P, Q, R, S (as it must be in order to avoid direct action of P, Q, R, S on the needle). Let X, Y, Z be the integrals of x, y, z with respect to t. At making contact x, y, z are zero. After a time z disappears, and x and y reach constant values. The equations for each conductor will therefore be

$$\left.\begin{aligned}&\mathrm{PX}+(p+h)x+(k+l)y=\int\mathrm{A}dt-\int\mathrm{D}dt,\\&\mathrm{Q}(\mathrm{X}-\mathrm{Z})+(h+q)x+(m+n)y=\int\mathrm{D}dt-\int\mathrm{C}dt,\\&\mathrm{RY}+(k+m)x+(r+o)y=\int\mathrm{A}dt-\int\mathrm{E}dt,\\&\mathrm{S}(\mathrm{Y}+\mathrm{Z})+(l+n)x+(o+s)y=\int\mathrm{E}dt-\int\mathrm{C}dt,\\&\mathrm{GZ}=\int\mathrm{D}td-\int\mathrm{E}dt.\end{aligned}\right\}\quad\dots\dots\quad(24)$$

Solving these equations for Z, we find

$$\left.\begin{aligned}&\mathrm{Z}\left\{\frac{1}{\mathrm{P}}+\frac{1}{\mathrm{Q}}+\frac{1}{\mathrm{R}}+\frac{1}{\mathrm{S}}+\mathrm{B}\left(\frac{1}{\mathrm{P}}+\frac{1}{\mathrm{R}}\right)\left(\frac{1}{\mathrm{Q}}+\frac{1}{\mathrm{S}}\right)+\mathrm{G}\left(\frac{1}{\mathrm{P}}+\frac{1}{\mathrm{Q}}\right)\left(\frac{1}{\mathrm{R}}+\frac{1}{\mathrm{S}}\right)+\frac{\mathrm{BG}}{\mathrm{PQRS}}(\mathrm{P}+\mathrm{Q}+\mathrm{R}+\mathrm{S})\right\}\\&\quad=-\mathrm{F}\frac{1}{\mathrm{PS}}\left\{\frac{p}{\mathrm{P}}-\frac{q}{\mathrm{Q}}-\frac{r}{\mathrm{R}}+\frac{s}{\mathrm{S}}+h\left(\frac{1}{\mathrm{P}}-\frac{1}{\mathrm{Q}}\right)+k\left(\frac{1}{\mathrm{R}}-\frac{1}{\mathrm{P}}\right)+l\left(\frac{1}{\mathrm{R}}+\frac{1}{\mathrm{Q}}\right)-m\left(\frac{1}{\mathrm{P}}+\frac{1}{\mathrm{S}}\right)\right.\\&\quad\left.+n\left(\frac{1}{\mathrm{Q}}-\frac{1}{\mathrm{S}}\right)+o\left(\frac{1}{\mathrm{S}}-\frac{1}{\mathrm{R}}\right)\right\}.\end{aligned}\right\}\quad(25)$$

(45) Now let the deflection of the galvanometer by the instantaneous current whose intensity is Z be α.

Let the permanent deflection produced by making the ratio of PS to QR, ρ instead of unity, be θ.

Also let the time of vibration of the galvanometer needle from rest to rest be T.

Then calling the quantity

$$\frac{p}{\mathrm{P}}-\frac{q}{\mathrm{Q}}-\frac{r}{\mathrm{R}}+\frac{s}{\mathrm{S}}+h\left(\frac{1}{\mathrm{P}}-\frac{1}{\mathrm{Q}}\right)+k\left(\frac{1}{\mathrm{R}}-\frac{1}{\mathrm{P}}\right)+l\left(\frac{1}{\mathrm{R}}+\frac{1}{\mathrm{Q}}\right)-m\left(\frac{1}{\mathrm{P}}+\frac{1}{\mathrm{S}}\right)+n\left(\frac{1}{\mathrm{Q}}-\frac{1}{\mathrm{S}}\right)+o\left(\frac{1}{\mathrm{S}}-\frac{1}{\mathrm{R}}\right)=\tau, \quad (26)$$

we find

$$\frac{\mathrm{Z}}{z}=\frac{2\sin\frac{1}{2}\alpha}{\tan\theta}\,\frac{\mathrm{T}}{\pi}=\frac{\tau}{1-\varrho}. \quad (27)$$

In determining τ by experiment, it is best to make the alteration of resistance in one of the arms by means of the arrangement described by Mr. JENKIN in the Report of the British Association for 1863, by which any value of ϱ from 1 to 1·01 can be accurately measured.

We observe (α) the greatest deflection due to the impulse of induction when the galvanometer is in circuit, when the connexions are made, and when the resistances are so adjusted as to give no permanent current.

We then observe (β) the greatest deflection produced by the permanent current when the resistance of one of the arms is increased in the ratio of 1 to ϱ, the galvanometer not being in circuit till a little while after the connexion is made with the battery.

In order to eliminate the effects of resistance of the air, it is best to vary ϱ till $\beta=2\alpha$ nearly; then

$$\tau=\mathrm{T}\frac{1}{\pi}(1-\varrho)\frac{2\sin\frac{1}{2}\alpha}{\tan\frac{1}{2}\beta}. \quad (28)$$

If all the arms of the balance except P consist of resistance coils of very fine wire of no great length and doubled before being coiled, the induction coefficients belonging to these coils will be insensible, and τ will be reduced to $\frac{p}{\mathrm{P}}$. The electric balance therefore affords the means of measuring the self-induction of any circuit whose resistance is known.

(46) It may also be used to determine the coefficient of induction between two circuits, as for instance, that between P and S which we have called m; but it would be more convenient to measure this by directly measuring the current, as in (37), without using the balance. We may also ascertain the equality of $\frac{p}{\mathrm{P}}$ and $\frac{q}{\mathrm{Q}}$ by there being no current of induction, and thus, when we know the value of p, we may determine that of q by a more perfect method than the comparison of deflections.

Exploration of the Electromagnetic Field.

(47) Let us now suppose the primary circuit A to be of invariable form, and let us explore the electromagnetic field by means of the secondary circuit B, which we shall suppose to be variable in form and position.

We may begin by supposing B to consist of a short straight conductor with its extremities sliding on two parallel conducting rails, which are put in connexion at some distance from the sliding-piece.

Then, if sliding the moveable conductor in a given direction increases the value of M, a negative electromotive force will act in the circuit B, tending to produce a negative current in B during the motion of the sliding-piece.

If a current be kept up in the circuit B, then the sliding-piece will itself tend to move in that direction, which causes M to increase. At every point of the field there will always be a certain direction such that a conductor moved in that direction does not experience any electromotive force in whatever direction its extremities are turned. A conductor carrying a current will experience no mechanical force urging it in that direction or the opposite.

This direction is called the direction of the line of magnetic force through that point.

Motion of a conductor across such a line produces electromotive force in a direction perpendicular to the line and to the direction of motion, and a conductor carrying a current is urged in a direction perpendicular to the line and to the direction of the current.

(48) We may next suppose B to consist of a very small plane circuit capable of being placed in any position and of having its plane turned in any direction. The value of M will be greatest when the plane of the circuit is perpendicular to the line of magnetic force. Hence if a current is maintained in B it will tend to set itself in this position, and will of itself indicate, like a magnet, the direction of the magnetic force.

On Lines of Magnetic Force.

(49) Let any surface be drawn, cutting the lines of magnetic force, and on this surface let any system of lines be drawn at small intervals, so as to lie side by side without cutting each other. Next, let any line be drawn on the surface cutting all these lines, and let a second line be drawn near it, its distance from the first being such that the value of M for each of the small spaces enclosed between these two lines and the lines of the first system is equal to unity.

In this way let more lines be drawn so as to form a second system, so that the value of M for every reticulation formed by the intersection of the two systems of lines is unity.

Finally, from every point of intersection of these reticulations let a line be drawn through the field, always coinciding in direction with the direction of magnetic force.

(50) In this way the whole field will be filled with lines of magnetic force at regular intervals, and the properties of the electromagnetic field will be completely expressed by them.

For, 1st, If any closed curve be drawn in the field, the value of M for that curve will be expressed by the *number* of lines of force which *pass through* that closed curve.

2ndly. If this curve be a conducting circuit and be moved through the field, an electromotive force will act in it, represented by the rate of decrease of the number of lines passing through the curve.

3rdly. If a current be maintained in the circuit, the conductor will be acted on by forces tending to move it so as to increase the number of lines passing through it, and the amount of work done by these forces is equal to the current in the circuit multiplied by the number of additional lines.

4thly. If a small plane circuit be placed in the field, and be free to turn, it will place its plane perpendicular to the lines of force. A small magnet will place itself with its axis in the direction of the lines of force.

5thly. If a long uniformly magnetized bar is placed in the field, each pole will be acted on by a force in the direction of the lines of force. The number of lines of force passing through unit of area is equal to the force acting on a unit pole multiplied by a coefficient depending on the magnetic nature of the medium, and called the coefficient of magnetic induction.

In fluids and isotropic solids the value of this coefficient μ is the same in whatever direction the lines of force pass through the substance, but in crystallized, strained, and organized solids the value of μ may depend on the direction of the lines of force with respect to the axes of crystallization, strain, or growth.

In all bodies μ is affected by temperature, and in iron it appears to diminish as the intensity of the magnetization increases.

On Magnetic Equipotential Surfaces.

(51) If we explore the field with a uniformly magnetized bar, so long that one of its poles is in a very weak part of the magnetic field, then the magnetic forces will perform work on the other pole as it moves about the field.

If we start from a given point, and move this pole from it to any other point, the work performed will be independent of the path of the pole between the two points; provided that no electric current passes between the different paths pursued by the pole.

Hence, when there are no electric currents but only magnets in the field, we may draw a series of surfaces such that the work done in passing from one to another shall be constant whatever be the path pursued between them. Such surfaces are called Equipotential Surfaces, and in ordinary cases are perpendicular to the Lines of magnetic force.

If these surfaces are so drawn that, when a unit pole passes from any one to the next in order, unity of work is done, then the work done in any motion of a magnetic pole will be measured by the strength of the pole multiplied by the number of surfaces which it has passed through in the positive direction.

(52) If there are circuits carrying electric currents in the field, then there will still be equipotential surfaces in the parts of the field external to the conductors carrying the currents, but the work done on a unit pole in passing from one to another will depend on the number of times which the path of the pole circulates round any of these currents. Hence the potential in each surface will have a series of values in arithmetical progression, differing by the work done in passing completely round one of the currents in the field.

The equipotential surfaces will not be continuous closed surfaces, but some of them will be limited sheets, terminating in the electric circuit as their common edge or boundary. The number of these will be equal to the amount of work done on a unit pole in going round the current, and this by the ordinary measurement $=4\pi\gamma$, where γ is the value of the current.

These surfaces, therefore, are connected with the electric current as soap-bubbles are connected with a ring in M. PLATEAU'S experiments. Every current γ has $4\pi\gamma$ surfaces attached to it. These surfaces have the current for their common edge, and meet it at equal angles. The form of the surfaces in other parts depends on the presence of other currents and magnets, as well as on the shape of the circuit to which they belong.

PART III.—GENERAL EQUATIONS OF THE ELECTROMAGNETIC FIELD.

(53.) Let us assume three rectangular directions in space as the axes of x, y, and z, and let all quantities having direction be expressed by their components in these three directions.

Electrical Currents (p, q, r).

(54) An electrical current consists in the transmission of electricity from one part of a body to another. Let the quantity of electricity transmitted in unit of time across unit of area perpendicular to the axis of x be called p, then p is the component of the current at that place in the direction of x.

We shall use the letters p, q, r to denote the components of the current per unit of area in the directions of x, y, z.

Electrical Displacements (f, g, h).

(55) Electrical displacement consists in the opposite electrification of the sides of a molecule or particle of a body which may or may not be accompanied with transmission through the body. Let the quantity of electricity which would appear on the faces $dy.dz$ of an element dx, dy, dz cut from the body be $f.dy.dz$, then f is the component of electric displacement parallel to x. We shall use f, g, h to denote the electric displacements parallel to x, y, z respectively.

The variations of the electrical displacement must be added to the currents p, q, r to get the total motion of electricity, which we may call p', q', r', so that

$$\left.\begin{aligned} p'&=p+\frac{df}{dt}, \\ q'&=q+\frac{dg}{dt}, \\ r'&=r+\frac{dh}{dt}, \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad \text{(A)}$$

Electromotive Force (P, Q, R).

(56) Let P, Q, R represent the components of the electromotive force at any point.

Then P represents the difference of potential per unit of length in a conductor placed in the direction of x at the given point. We may suppose an indefinitely short wire placed parallel to x at a given point and touched, during the action of the force P, by two small conductors, which are then insulated and removed from the influence of the electromotive force. The value of P might then be ascertained by measuring the charge of the conductors.

Thus if l be the length of the wire, the difference of potential at its ends will be Pl, and if C be the capacity of each of the small conductors the charge on each will be $\frac{1}{2}CPl$. Since the capacities of moderately large conductors, measured on the electromagnetic system, are exceedingly small, ordinary electromotive forces arising from electromagnetic actions could hardly be measured in this way. In practice such measurements are always made with long conductors, forming closed or nearly closed circuits.

Electromagnetic Momentum (F, G, H).

(57) Let F, G, H represent the components of electromagnetic momentum at any point of the field, due to any system of magnets or currents.

Then F is the total impulse of the electromotive force in the direction of x that would be generated by the removal of these magnets or currents from the field, that is, if P be the electromotive force at any instant during the removal of the system

$$F=\int P\,dt.$$

Hence the part of the electromotive force which depends on the motion of magnets or currents in the field, or their alteration of intensity, is

$$P=-\frac{dF}{dt},\quad Q=-\frac{dG}{dt},\quad R=-\frac{dH}{dt}. \qquad (29)$$

Electromagnetic Momentum of a Circuit.

(58) Let s be the length of the circuit, then if we integrate

$$\int\left(F\frac{dx}{ds}+G\frac{dy}{ds}+H\frac{dz}{ds}\right)ds \qquad (30)$$

round the circuit, we shall get the total electromagnetic momentum of the circuit, or the number of lines of magnetic force which pass through it, the variations of which measure the total electromotive force in the circuit. This electromagnetic momentum is the same thing to which Professor FARADAY has applied the name of the Electrotonic State.

If the circuit be the boundary of the elementary area $dy\,dz$, then its electromagnetic momentum is

$$\left(\frac{dH}{dy}-\frac{dG}{dz}\right)dy\,dz,$$

and this is the number of lines of magnetic force which pass through the area $dy\,dz$.

Magnetic Force (α, β, γ).

(59) Let α, β, γ represent the force acting on a unit magnetic pole placed at the given point resolved in the directions of x, y, and z.

Coefficient of Magnetic Induction (μ).

(60) Let μ be the ratio of the magnetic induction in a given medium to that in air under an equal magnetizing force, then the number of lines of force in unit of area perpendicular to x will be $\mu\alpha$ (μ is a quantity depending on the nature of the medium, its temperature, the amount of magnetization already produced, and in crystalline bodies varying with the direction).

(61) Expressing the electric momentum of small circuits perpendicular to the three axes in this notation, we obtain the following

Equations of Magnetic Force.

$$\left.\begin{aligned} \mu\alpha &= \frac{dH}{dy} - \frac{dG}{dz}, \\ \mu\beta &= \frac{dF}{dz} - \frac{dH}{dx}, \\ \mu\gamma &= \frac{dG}{dx} - \frac{dF}{dy}. \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (B)$$

Equations of Currents.

(62) It is known from experiment that the motion of a magnetic pole in the electromagnetic field in a closed circuit cannot generate work unless the circuit which the pole describes passes round an electric current. Hence, except in the space occupied by the electric currents,

$$\alpha dx + \beta dy + \gamma dz = d\varphi \quad \ldots\ldots\ldots\ldots \quad (31)$$

a complete differential of φ, the magnetic potential.

The quantity φ may be susceptible of an indefinite number of distinct values, according to the number of times that the exploring point passes round electric currents in its course, the difference between successive values of φ corresponding to a passage completely round a current of strength c being $4\pi c$.

Hence if there is no electric current,

$${}_{dy} - \frac{d\beta}{dz} = 0\,;$$

but if there is a current p',

Similarly,

$$\left.\begin{aligned} \frac{d\gamma}{dy} - \frac{d\beta}{dz} &= 4\pi p'. \\ \frac{d\alpha}{dz} - \frac{d\gamma}{dx} &= 4\pi q', \\ \frac{d\beta}{dx} - \frac{d\alpha}{dy} &= 4\pi r'. \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (C)$$

We may call these the Equations of Currents.

Electromotive Force in a Circuit.

(63) Let ξ be the electromotive force acting round the circuit A, then

$$\xi=\int\left(P\frac{dx}{ds}+Q\frac{dy}{ds}+R\frac{dz}{ds}\right)ds, \quad \ldots \quad (32)$$

where ds is the element of length, and the integration is performed round the circuit.

Let the forces in the field be those due to the circuits A and B, then the electromagnetic momentum of A is

$$\int\left(F\frac{dx}{ds}+G\frac{dy}{ds}+H\frac{dz}{ds}\right)ds=Lu+Mv, \quad \ldots \quad (33)$$

where u and v are the currents in A and B, and

$$\xi=-\frac{d}{dt}(Lu+Mv). \quad \ldots \quad (34)$$

Hence, if there is no motion of the circuit A,

$$\left.\begin{aligned} P&=-\frac{dF}{dt}-\frac{d\Psi}{dx}, \\ Q&=-\frac{dG}{dt}-\frac{d\Psi}{dy}, \\ R&=-\frac{dH}{dt}-\frac{d\Psi}{dz}, \end{aligned}\right\} \quad \ldots \quad (35)$$

where Ψ is a function of x, y, z, and t, which is indeterminate as far as regards the solution of the above equations, because the terms depending on it will disappear on integrating round the circuit. The quantity Ψ can always, however, be determined in any particular case when we know the actual conditions of the question. The physical interpretation of Ψ is, that it represents the *electric potential* at each point of space.

Electromotive Force on a Moving Conductor.

(64) Let a short straight conductor of length a, parallel to the axis of x, move with a velocity whose components are $\frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}$, and let its extremities slide along two parallel conductors with a velocity $\frac{ds}{dt}$. Let us find the alteration of the electromagnetic momentum of the circuit of which this arrangement forms a part.

In unit of time the moving conductor has travelled distances $\frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}$ along the directions of the three axes, and at the same time the lengths of the parallel conductors included in the circuit have each been increased by $\frac{ds}{dt}$.

Hence the quantity

$$\int\left(F\frac{dx}{ds}+G\frac{dy}{ds}+H\frac{dz}{ds}\right)ds$$

will be increased by the following increments,

$$a\left(\frac{dF}{dx}\frac{dx}{dt}+\frac{dF}{dy}\frac{dy}{dt}+\frac{dF}{dz}\frac{dz}{dt}\right), \text{ due to motion of conductor,}$$

$$-a\frac{ds}{dt}\left(\frac{dF}{dx}\frac{dx}{ds}+\frac{dG}{dx}\frac{dy}{ds}+\frac{dH}{dx}\frac{dz}{ds}\right), \text{ due to lengthening of circuit.}$$

The total increment will therefore be

$$a\left(\frac{dF}{dy}-\frac{dG}{dx}\right)\frac{dy}{dt}-a\left(\frac{dH}{dx}-\frac{dF}{dz}\right)\frac{dz}{dt};$$

or, by the equations of Magnetic Force (8),

$$-a\left(\mu\gamma\frac{dy}{dt}-\mu\beta\frac{dz}{dt}\right).$$

If P is the electromotive force in the moving conductor parallel to x referred to unit of length, then the actual electromotive force is Pa; and since this is measured by the decrement of the electromagnetic momentum of the circuit, the electromotive force due to motion will be

$$P=\mu\gamma\frac{dy}{dt}-\mu\beta\frac{dz}{dt}. \qquad \text{(36)}$$

(65) The complete equations of electromotive force on a moving conductor may now be written as follows:—

Equations of Electromotive Force.

$$\left.\begin{aligned} P&=\mu\left(\gamma\frac{dy}{dt}-\beta\frac{dz}{dt}\right)-\frac{dF}{dt}-\frac{d\Psi}{dx}, \\ Q&=\mu\left(\alpha\frac{dz}{dt}-\gamma\frac{dx}{dt}\right)-\frac{dG}{dt}-\frac{d\Psi}{dy}, \\ R&=\mu\left(\beta\frac{dx}{dt}-\alpha\frac{dy}{dt}\right)-\frac{dH}{dt}-\frac{d\Psi}{dz}. \end{aligned}\right\} \qquad \text{(D)}$$

The first term on the right-hand side of each equation represents the electromotive force arising from the motion of the conductor itself. This electromotive force is perpendicular to the direction of motion and to the lines of magnetic force; and if a parallelogram be drawn whose sides represent in direction and magnitude the velocity of the conductor and the magnetic induction at that point of the field, then the area of the parallelogram will represent the electromotive force due to the motion of the conductor, and the direction of the force is perpendicular to the plane of the parallelogram.

The second term in each equation indicates the effect of changes in the position or strength of magnets or currents in the field.

The third term shows the effect of the electric potential Ψ. It has no effect in causing a circulating current in a closed circuit. It indicates the existence of a force urging the electricity to or from certain definite points in the field.

Electric Elasticity.

(66) When an electromotive force acts on a dielectric, it puts every part of the dielectric into a polarized condition, in which its opposite sides are oppositely electrified. The amount of this electrification depends on the electromotive force and on the nature of the substance, and, in solids having a structure defined by axes, on the direction of the electromotive force with respect to these axes. In isotropic substances, if k is the ratio of the electromotive force to the electric displacement, we may write the

Equations of Electric Elasticity,

$$\left.\begin{aligned}P&=kf,\\ Q&=kg,\\ R&=kh.\end{aligned}\right\} \quad \ldots\ldots\ldots \quad (E)$$

Electric Resistance.

(67) When an electromotive force acts on a conductor it produces a current of electricity through it. This effect is additional to the electric displacement already considered. In solids of complex structure, the relation between the electromotive force and the current depends on their direction through the solid. In isotropic substances, which alone we shall here consider, if ϱ is the specific resistance referred to unit of volume, we may write the

Equations of Electric Resistance,

$$\left.\begin{aligned}P&=-\varrho p,\\ Q&=-\varrho q,\\ R&=-\varrho r.\end{aligned}\right\} \quad \ldots\ldots\ldots \quad (F)$$

Electric Quantity.

(68) Let e represent the quantity of free positive electricity contained in unit of volume at any part of the field, then, since this arises from the electrification of the different parts of the field not neutralizing each other, we may write the

Equation of Free Electricity,

$$e+\frac{df}{dx}+\frac{dg}{dy}+\frac{dh}{dz}=0. \quad \ldots\ldots\ldots \quad (G)$$

(69) If the medium conducts electricity, then we shall have another condition, which may be called, as in hydrodynamics, the

Equation of Continuity,

$$\frac{de}{dt}+\frac{dp}{dx}+\frac{dq}{dy}+\frac{dr}{dz}=0. \quad \ldots\ldots\ldots \quad (H)$$

(70) In these equations of the electromagnetic field we have assumed twenty variable

quantities, namely,

For Electromagnetic Momentum	F	G	H
,, Magnetic Intensity	α	β	γ
,, Electromotive Force	P	Q	R
,, Current due to true conduction	p	q	r
,, Electric Displacement	f	g	h
,, Total Current (including variation of displacement)	p'	q'	r'
,, Quantity of free Electricity	e		
,, Electric Potential	Ψ		

Between these twenty quantities we have found twenty equations, viz.

Three equations of Magnetic Force	(B)
,, Electric Currents	(C)
,, Electromotive Force	(D)
,, Electric Elasticity	(E)
,, Electric Resistance	(F)
,, Total Currents	(A)
One equation of Free Electricity	(G)
,, Continuity	(H)

These equations are therefore sufficient to determine all the quantities which occur in them, provided we know the conditions of the problem. In many questions, however, only a few of the equations are required.

Intrinsic Energy of the Electromagnetic Field.

(71) We have seen (33) that the intrinsic energy of any system of currents is found by multiplying half the current in each circuit into its electromagnetic momentum. This is equivalent to finding the integral

$$E=\tfrac{1}{2}\Sigma(\mathrm{F}p'+\mathrm{G}q'+\mathrm{H}r')d\mathrm{V} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (37)$$

over all the space occupied by currents, where p, q, r are the components of currents, and F, G, H the components of electromagnetic momentum.

Substituting the values of p', q', r' from the equations of Currents (C), this becomes

$$\frac{1}{8\pi}\Sigma\left\{\mathrm{F}\left(\frac{d\gamma}{dy}-\frac{d\beta}{dz}\right)+\mathrm{G}\left(\frac{d\alpha}{dz}-\frac{d\gamma}{dx}\right)+\mathrm{H}\left(\frac{d\beta}{dx}-\frac{d\alpha}{dy}\right)\right\}d\mathrm{V}.$$

Integrating by parts, and remembering that α, β, γ vanish at an infinite distance, the expression becomes

$$\frac{1}{8\pi}\Sigma\left\{\alpha\left(\frac{d\mathrm{H}}{dy}-\frac{d\mathrm{G}}{dz}\right)+\beta\left(\frac{d\mathrm{F}}{dz}-\frac{d\mathrm{H}}{dx}\right)+\gamma\left(\frac{d\mathrm{G}}{dx}-\frac{d\mathrm{F}}{dy}\right)\right\}d\mathrm{V},$$

where the integration is to be extended over all space. Referring to the equations of Magnetic Force (B), p. 482, this becomes

$$E=\frac{1}{8\pi}\Sigma\{\alpha.\mu\alpha+\beta.\mu\beta+\gamma.\mu\gamma\}d\mathrm{V}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (38)$$

where α, β, γ are the components of magnetic intensity or the force on a unit magnetic pole, and $\mu\alpha$, $\mu\beta$, $\mu\gamma$ are the components of the quantity of magnetic induction, or the number of lines of force in unit of area.

In isotropic media the value of μ is the same in all directions, and we may express the result more simply by saying that the intrinsic energy of any part of the magnetic field arising from its magnetization is

$$\frac{\mu}{8\pi}\mathrm{I}^2$$

per unit of volume, where I is the magnetic intensity.

(72) Energy may be stored up in the field in a different way, namely, by the action of electromotive force in producing electric displacement. The work done by a variable electromotive force, P, in producing a variable displacement, f, is got by integrating

$$\int \mathrm{P}df$$

from $\mathrm{P}=0$ to the given value of P.

Since $\mathrm{P}=kf$, equation (E), this quantity becomes

$$\int kfdf=\tfrac{1}{2}kf^2=\tfrac{1}{2}\mathrm{P}f.$$

Hence the intrinsic energy of any part of the field, as existing in the form of electric displacement, is

$$\tfrac{1}{2}\Sigma(\mathrm{P}f+\mathrm{Q}g+\mathrm{R}h)d\mathrm{V}.$$

The total energy existing in the field is therefore

$$\mathrm{E}=\Sigma\left\{\frac{1}{8\pi}(\alpha\mu\alpha+\beta\mu\beta+\gamma\mu\gamma)+\tfrac{1}{2}(\mathrm{P}f+\mathrm{Q}g+\mathrm{R}h)\right\}d\mathrm{V}. \quad . \quad . \quad . \quad . \quad (\mathrm{I})$$

The first term of this expression depends on the magnetization of the field, and is explained on our theory by actual motion of some kind. The second term depends on the electric polarization of the field, and is explained on our theory by strain of some kind in an elastic medium.

(73) I have on a former occasion* attempted to describe a particular kind of motion and a particular kind of strain, so arranged as to account for the phenomena. In the present paper I avoid any hypothesis of this kind; and in using such words as electric momentum and electric elasticity in reference to the known phenomena of the induction of currents and the polarization of dielectrics, I wish merely to direct the mind of the reader to mechanical phenomena which will assist him in understanding the electrical ones. All such phrases in the present paper are to be considered as illustrative, not as explanatory.

(74) In speaking of the Energy of the field, however, I wish to be understood literally. All energy is the same as mechanical energy, whether it exists in the form of motion or in that of elasticity, or in any other form. The energy in electromagnetic phenomena is mechanical energy. The only question is, Where does it reside? On the old theories

---

* "On Physical Lines of Force," Philosophical Magazine, 1861-62.

it resides in the electrified bodies, conducting circuits, and magnets, in the form of an unknown quality called potential energy, or the power of producing certain effects at a distance. On our theory it resides in the electromagnetic field, in the space surrounding the electrified and magnetic bodies, as well as in those bodies themselves, and is in two different forms, which may be described without hypothesis as magnetic polarization and electric polarization, or, according to a very probable hypothesis, as the motion and the strain of one and the same medium.

(75) The conclusions arrived at in the present paper are independent of this hypothesis, being deduced from experimental facts of three kinds:—

1. The induction of electric currents by the increase or diminution of neighbouring currents according to the changes in the lines of force passing through the circuit.

2. The distribution of magnetic intensity according to the variations of a magnetic potential.

3. The induction (or influence) of statical electricity through dielectrics.

We may now proceed to demonstrate from these principles the existence and laws of the mechanical forces which act upon electric currents, magnets, and electrified bodies placed in the electromagnetic field.

PART IV.—MECHANICAL ACTIONS IN THE FIELD.

Mechanical Force on a Moveable Conductor.

(76) We have shown (§§ 34 & 35) that the work done by the electromagnetic forces in aiding the motion of a conductor is equal to the product of the current in the conductor multiplied by the increment of the electromagnetic momentum due to the motion.

Let a short straight conductor of length a move parallel to itself in the direction of x, with its extremities on two parallel conductors. Then the increment of the electromagnetic momentum due to the motion of a will be

$$a\left(\frac{dF}{dx}\frac{dx}{ds}+\frac{dG}{dx}\frac{dy}{ds}+\frac{dH}{dx}\frac{dz}{ds}\right)\delta x.$$

That due to the lengthening of the circuit by increasing the length of the parallel conductors will be

$$-a\left(\frac{dF}{dx}\frac{dx}{ds}+\frac{dF}{dy}\frac{dy}{ds}+\frac{dF}{dz}\frac{dz}{ds}\right)\delta x.$$

The total increment is

$$a\delta x\left\{\frac{dy}{ds}\left(\frac{dG}{dx}-\frac{dF}{dy}\right)-\frac{dz}{ds}\left(\frac{dF}{dz}-\frac{dH}{dx}\right)\right\},$$

which is by the equations of Magnetic Force (B), p. 482,

$$a\delta x\left(\frac{dy}{ds}\mu\gamma-\frac{dz}{ds}\mu\beta\right).$$

Let X be the force acting along the direction of x per unit of length of the conductor, then the work done is $\text{X}a\delta x$.

Let C be the current in the conductor, and let p', q', r' be its components, then

$$\mathrm{X}a\delta=\mathrm{C}a\delta x x\left(\frac{dy}{ds}\mu\gamma-\frac{dz}{ds}\mu\beta\right),$$

or

Similarly,

$$\left.\begin{aligned}\mathrm{X}&=\mu\gamma q'-\mu\beta r'.\\ \mathrm{Y}&=\mu\alpha r'-\mu\gamma p',\\ \mathrm{Z}&=\mu\beta p'-\mu\alpha q'.\end{aligned}\right\}\quad\ldots\ldots\ldots\ldots\quad(\mathrm{J})$$

These are the equations which determine the mechanical force acting on a conductor carrying a current. The force is perpendicular to the current and to the lines of force, and is measured by the area of the parallelogram formed by lines parallel to the current and lines of force, and proportional to their intensities.

Mechanical Force on a Magnet.

(77) In any part of the field not traversed by electric currents the distribution of magnetic intensity may be represented by the differential coefficients of a function which may be called the magnetic potential. When there are no currents in the field, this quantity has a single value for each point. When there are currents, the potential has a series of values at each point, but its differential coefficients have only one value, namely,

$$\frac{d\phi}{dx}=\alpha,\quad \frac{d\phi}{dy}=\beta,\quad \frac{d\phi}{dz}=\gamma.$$

Substituting these values of α, β, γ in the expression (equation 38) for the intrinsic energy of the field, and integrating by parts, it becomes

$$-\Sigma\left\{\phi\,\frac{1}{8\pi}\left(\frac{d\mu\alpha}{dx}+\frac{d\mu\beta}{dy}+\frac{d\mu\gamma}{dz}\right)\right\}d\mathrm{V}.$$

The expression

$$\Sigma\left(\frac{d\mu\alpha}{dx}+\frac{d\mu\beta}{dy}+\frac{d\mu\gamma}{dz}\right)d\mathrm{V}=\Sigma m d\mathrm{V}\quad\ldots\ldots\quad(39)$$

indicates the number of lines of magnetic force which have their origin within the space V. Now a magnetic pole is known to us only as the origin or termination of lines of magnetic force, and a unit pole is one which has 4π lines belonging to it, since it produces unit of magnetic intensity at unit of distance over a sphere whose surface is 4π.

Hence if m is the amount of free positive magnetism in unit of volume, the above expression may be written $4\pi m$, and the expression for the energy of the field becomes

$$\mathrm{E}=-\Sigma(\tfrac{1}{2}\phi m)d\mathrm{V}.\quad\ldots\ldots\ldots\ldots\quad(40)$$

If there are two magnetic poles m_1 and m_2 producing potentials ϕ_1 and ϕ_2 in the field, then if m_2 is moved a distance dx, and is urged in that direction by a force X, then the work done is $\mathrm{X}dx$, and the decrease of energy in the field is

$$d\left(\tfrac{1}{2}(\phi_1+\phi_2)(m_1+m_2)\right),$$

and these must be equal by the principle of Conservation of Energy.

Since the distribution ϕ_1 is determined by m_1, and ϕ_2 by m_2, the quantities $\phi_1 m_1$ and $\phi_2 m_2$ will remain constant.

It can be shown also, as GREEN has proved (Essay, p. 10), that

$$m_1\phi_2 = m_2\phi_1,$$

so that we get

$$\mathrm{X}dx = d(m_2\phi_1),$$

or

$$\left.\begin{array}{ll} & \mathrm{X} = m_2\dfrac{d\phi_1}{dx} = m_2\alpha_1, \\ \text{where } \alpha_1 \text{ represents the magnetic intensity due to } m_1. & \\ \text{Similarly,} & \mathrm{Y} = m_2\beta_1, \\ & \mathrm{Z} = m_2\gamma_1. \end{array}\right\} \quad \ldots\ldots\ldots \quad (\mathrm{K})$$

So that a magnetic pole is urged in the direction of the lines of magnetic force with a force equal to the product of the strength of the pole and the magnetic intensity.

(78) If a single magnetic pole, that is one pole of a very long magnet, be placed in the field, the only solution of ϕ is

$$\phi_1 = -\frac{m_1}{\mu}\frac{1}{r}, \quad \ldots\ldots\ldots\ldots \quad (41)$$

where m_1 is the strength of the pole and r the distance from it.

The repulsion between two poles of strength m_1 and m_2 is

$$m_2\frac{d\phi_1}{dr} = \frac{m_1 m_2}{\mu r^2}. \quad \ldots\ldots\ldots\ldots \quad (42)$$

In air or any medium in which $\mu=1$ this is simply $\frac{m_1 m_2}{r^2}$, but in other media the force acting between two given magnetic poles is inversely proportional to the coefficient of magnetic induction for the medium. This may be explained by the magnetization of the medium induced by the action of the poles.

Mechanical Force on an Electrified Body.

(79) If there is no motion or change of strength of currents or magnets in the field, the electromotive force is entirely due to variation of electric potential, and we shall have (§ 65)

$$\mathrm{P} = -\frac{d\Psi}{dx}, \quad \mathrm{Q} = -\frac{d\Psi}{dy}, \quad \mathrm{R} = -\frac{d\Psi}{dz}.$$

Integrating by parts the expression (I) for the energy due to electric displacement, and remembering that P, Q, R vanish at an infinite distance, it becomes

$$\tfrac{1}{2}\Sigma\left\{\Psi\left(\frac{df}{dx}+\frac{dg}{dy}+\frac{dh}{dz}\right)\right\}d\mathrm{V},$$

or by the equation of Free Electricity (G), p. 485,

$$-\tfrac{1}{2}\Sigma(\Psi e)d\mathrm{V}.$$

By the same demonstration as was used in the case of the mechanical action on a magnet, it may be shown that the mechanical force on a small body containing a quantity e_2 of free electricity placed in a field whose potential arising from other electrified bodies is Ψ_1, has for components

$$\left.\begin{aligned} X&=e_2\frac{d\Psi_1}{dx}=-P_1e_2,\\ Y&=e_2\frac{d\Psi_1}{dy}=-Q_1e_2,\\ Z&=e_3\frac{d\Psi_1}{dz}=-R_1e_2. \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad \text{(D)}$$

So that an electrified body is urged in the direction of the electromotive force with a force equal to the product of the quantity of free electricity and the electromotive force.

If the electrification of the field arises from the presence of a small electrified body containing e_1 of free electrity, the only solution of Ψ_1 is

$$\Psi_1=\frac{k}{4\pi}\frac{e_1}{r}, \quad \ldots\ldots\ldots\ldots \quad (43)$$

where r is the distance from the electrified body.

The repulsion between two electrified bodies e_1, e_2 is therefore

$$e_2\frac{d\Psi_1}{dr}=\frac{k}{4\pi}\frac{e_1e_2}{r^2}. \quad \ldots\ldots\ldots\ldots \quad (44)$$

Measurement of Electrical Phenomena by Electrostatic Effects.

(80) The quantities with which we have had to do have been hitherto expressed in terms of the Electromagnetic System of measurement, which is founded on the mechanical action between currents. The electrostatic system of measurement is founded on the mechanical action between electrified bodies, and is independent of, and incompatible with, the electromagnetic system; so that the units of the different kinds of quantity have different values according to the system we adopt, and to pass from the one system to the other, a reduction of all the quantities is required.

According to the electrostatic system, the repulsion between two small bodies charged with quantities η_1, η_2 of electricity is

$$\frac{\eta_1\eta_2}{r^2},$$

where r is the distance between them.

Let the relation of the two systems be such that one electromagnetic unit of electricity contains v electrostatic units; then $\eta_1=ve_1$ and $\eta_2=ve_2$, and this repulsion becomes

$$v^2\frac{e_1e_2}{r^2}=\frac{k}{4\pi}\frac{e_1e_2}{r^2} \text{ by equation (44)}, \quad \ldots\ldots\ldots \quad (45)$$

whence k, the coefficient of "electric elasticity" in the medium in which the experiments are made, *i. e.* common air, is related to v, the number of electrostatic units in one electromagnetic unit, by the equation

$$k=4\pi v^2. \quad \ldots\ldots\ldots\ldots \quad (46)$$

The quantity v may be determined by experiment in several ways. According to the experiments of MM. WEBER and KOHLRAUSCH,

$$v=310{,}740{,}000 \text{ metres per second.}$$

(81) It appears from this investigation, that if we assume that the medium which constitutes the electromagnetic field is, when dielectric, capable of receiving in every part of it an electric polarization, in which the opposite sides of every element into which we may conceive the medium divided are oppositely electrified, and if we also assume that this polarization or electric displacement is proportional to the electromotive force which produces or maintains it, then we can show that electrified bodies in a dielectric medium will act on one another with forces obeying the same laws as are established by experiment.

The energy, by the expenditure of which electrical attractions and repulsions are produced, we suppose to be stored up in the dielectric medium which surrounds the electrified bodies, and not on the surface of those bodies themselves, which on our theory are merely the bounding surfaces of the air or other dielectric in which the true springs of action are to be sought.

Note on the Attraction of Gravitation.

(82) After tracing to the action of the surrounding medium both the magnetic and the electric attractions and repulsions, and finding them to depend on the inverse square of the distance, we are naturally led to inquire whether the attraction of gravitation, which follows the same law of the distance, is not also traceable to the action of a surrounding medium.

Gravitation differs from magnetism and electricity in this; that the bodies concerned are all of the same kind, instead of being of opposite signs, like magnetic poles and electrified bodies, and that the force between these bodies is an attraction and not a repulsion, as is the case between like electric and magnetic bodies.

The lines of gravitating force near two dense bodies are exactly of the same form as the lines of magnetic force near two poles of the same name; but whereas the poles are repelled, the bodies are attracted. Let E be the intrinsic energy of the field surrounding two gravitating bodies M_1, M_2, and let E′ be the intrinsic energy of the field surrounding two magnetic poles m_1, m_2, equal in numerical value to M_1, M_2, and let X be the gravitating force acting during the displacement δx, and X′ the magnetic force,

$$X\delta x=\delta E, \qquad X'\delta x=\delta E';$$

now X and X′ are equal in numerical value, but of opposite signs; so that

$$\delta E=-\delta E',$$

or

$$\begin{aligned} E&=C-E' \\ &=C-\Sigma\frac{1}{8\pi}(\alpha^2+\beta^2+\gamma^2)dV, \end{aligned}$$

where α, β, γ are the components of magnetic intensity. If R be the resultant gravitating force, and R′ the resultant magnetic force at a corresponding part of the field,

$$R = -R', \text{ and } \alpha^2+\beta^2+\gamma^2 = R^2 = R'^2.$$

Hence

$$E = C - \Sigma \frac{1}{8\pi} R^2 dV. \quad \ldots \ldots \ldots \ldots \quad (47)$$

The intrinsic energy of the field of gravitation must therefore be less wherever there is a resultant gravitating force.

As energy is essentially positive, it is impossible for any part of space to have negative intrinsic energy. Hence those parts of space in which there is no resultant force, such as the points of equilibrium in the space between the different bodies of a system, and within the substance of each body, must have an intrinsic energy per unit of volume greater than

$$\frac{1}{8\pi} R^2,$$

where R is the greatest possible value of the intensity of gravitating force in any part of the universe.

The assumption, therefore, that gravitation arises from the action of the surrounding medium in the way pointed out, leads to the conclusion that every part of this medium possesses, when undisturbed, an enormous intrinsic energy, and that the presence of dense bodies influences the medium so as to diminish this energy wherever there is a resultant attraction.

As I am unable to understand in what way a medium can possess such properties, I cannot go any further in this direction in searching for the cause of gravitation.

PART V.—THEORY OF CONDENSERS.

Capacity of a Condenser.

(83) The simplest form of condenser consists of a uniform layer of insulating matter bounded by two conducting surfaces, and its capacity is measured by the quantity of electricity on either surface when the difference of potentials is unity.

Let S be the area of either surface, a the thickness of the dielectric, and k its coefficient of electric elasticity; then on one side of the condenser the potential is Ψ_1, and on the other side Ψ_1+1, and within its substance

$$\frac{d\Psi}{dx} = \frac{1}{a} = kf. \quad \ldots \ldots \ldots \ldots \quad (48)$$

Since $\frac{d\Psi}{dx}$ and therefore f is zero outside the condenser, the quantity of electricity on its first surface $= -Sf$, and on the second $+Sf$. The capacity of the condenser is therefore $Sf = \frac{S}{ak}$ in electromagnetic measure.

Specific Capacity of Electric Induction (D).

(84) If the dielectric of the condenser be air, then its capacity in electrostatic measure is $\frac{S}{4\pi a}$ (neglecting corrections arising from the conditions to be fulfilled at the edges). If the dielectric have a capacity whose ratio to that of air is D, then the capacity of the condenser will be $\frac{DS}{4\pi a}$.

Hence $$D=\frac{k_0}{k}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (49)$$ where k_0 is the value of k in air, which is taken for unity.

Electric Absorption.

(85) When the dielectric of which the condenser is formed is not a perfect insulator, the phenomena of conduction are combined with those of electric displacement. The condenser, when left charged, gradually loses its charge, and in some cases, after being discharged completely, it gradually acquires a new charge of the same sign as the original charge, and this finally disappears. These phenomena have been described by Professor FARADAY (Experimental Researches, Series XI.) and by Mr. F. JENKIN (Report of Committee of Board of Trade on Submarine Cables), and may be classed under the name of "Electric Absorption."

(86) We shall take the case of a condenser composed of any number of parallel layers of different materials. If a constant difference of potentials between its extreme surfaces is kept up for a sufficient time till a condition of permanent steady flow of electricity is established, then each bounding surface will have a charge of electricity depending on the nature of the substances on each side of it. If the extreme surfaces be now discharged, these internal charges will gradually be dissipated, and a certain charge may reappear on the extreme surfaces if they are insulated, or, if they are connected by a conductor, a certain quantity of electricity may be urged through the conductor during the reestablishment of equilibrium.

Let the thickness of the several layers of the condenser be a_1, a_2, &c.

Let the values of k for these layers be respectively k_1, k_2, k_3, and let $$a_1k_2+a_2k_2+\&c.=ak, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (50)$$ where k is the "electric elasticity" of air, and a is the thickness of an equivalent condenser of air.

Let the resistances of the layers be respectively r_1, r_2, &c., and let r_1+r_2+ &c. $=r$ be the resistance of the whole condenser, to a steady current through it per unit of surface.

Let the electric displacement in each layer be f_1, f_2, &c.

Let the electric current in each layer be p_1, p_2, &c.

Let the potential on the first surface be Ψ_1, and the electricity per unit of surface e_1.

Let the corresponding quantities at the boundary of the first and second surface be Ψ_2 and e_2, and so on. Then by equations (G) and (H),

$$\left.\begin{array}{ll} e_1=-f_1, & \frac{de_1}{dt}=-p_1, \\ e_2=f_1-f_2, & \frac{de_2}{dt}=p_1-p_2, \\ \&c. & \&c. \end{array}\right\} \quad \dots\dots\dots \quad (51)$$

But by equations (E) and (F),

$$\left.\begin{array}{l} \Psi_1-\Psi_2=a_1k_1f_1=-r_1p_1, \\ \Psi_2-\Psi_3=a_2k_2f_2=-r_2p_2, \\ \quad \&c. \qquad \&c. \qquad \&c. \end{array}\right\}. \quad \dots\dots\dots \quad (52)$$

After the electromotive force has been kept up for a sufficient time the current becomes the same in each layer, and

$$p_1=p_2=\&c.=p=\frac{\Psi}{r},$$

where Ψ is the total difference of potentials between the extreme layers. We have then

$$\text{and} \quad \left.\begin{array}{ll} f_1=-\frac{\Psi}{r}\frac{r_1}{a_1k_1}, & f_2=-\frac{\Psi}{r}\frac{r_2}{a_2k_2}, \ \&c. \\ e_1=\frac{\Psi}{r}\frac{r_1}{a_1k_1}, & e_2=\frac{\Psi}{r}\left(\frac{r_2}{a_2k_2}-\frac{r_1}{a\kappa_1}\right), \ \&c. \end{array}\right\} \quad \dots\dots \quad (53)$$

These are the quantities of electricity on the different surfaces.

(87) Now let the condenser be discharged by connecting the extreme surfaces through a perfect conductor so that their potentials are instantly rendered equal, then the electricity on the extreme surfaces will be altered, but that on the internal surfaces will not have time to escape. The total difference of potentials is now

$$\Psi'=a_1k_1e'_1+a_2k_2(e'_1+e_2)+a_3k_3(e'_1+e_2+e_3), \ \&c.=0, \quad \dots\dots \quad (54)$$

whence if e'_1 is what e_1 becomes at the instant of discharge,

$$e'_1=\frac{\Psi}{r}\frac{r_1}{a_1k_1}-\frac{\Psi}{ak}=e_1-\frac{\Psi}{ak}. \quad \dots\dots\dots \quad (55)$$

The instantaneous discharge is therefore $\frac{\Psi}{ak}$, or the quantity which would be discharged by a condenser of air of the equivalent thickness a, and it is unaffected by the want of perfect insulation.

(88) Now let us suppose the connexion between the extreme surfaces broken, and the condenser left to itself, and let us consider the gradual dissipation of the internal charges. Let Ψ' be the difference of potential of the extreme surfaces at any time t; then

$$\Psi'=a_1k_1f_1+a_2k_2f_2+\&c.; \quad \dots\dots\dots \quad (56)$$

but

$$a_1k_1f_1=-r_1\frac{df_1}{dt},$$

$$a_2k_2f_2=-r_2\frac{df_2}{dt}.$$

Hence $f_1 = \mathrm{A}_1 e^{-\frac{a_1 k_1}{r_1} t}$, $f_2 = \mathrm{A}_2 e^{-\frac{a_2 k_2}{r_2} t}$, &c.; and by referring to the values of e'_1, e_2, &c., we find

$$\left.\begin{aligned} \mathrm{A}_1 &= \frac{\Psi}{r} \frac{r_1}{a_1 k_1} - \frac{\Psi}{ak}, \\ \mathrm{A}_2 &= \frac{\Psi}{r} \frac{r_2}{a_2 k_2} - \frac{\Psi}{ak}, \\ &\text{\&c.}; \end{aligned}\right\} \quad \text{. . . . .} \quad (57)$$

so that we find for the difference of extreme potentials at any time,

$$\Psi' = \Psi \left\{ \left(\frac{r_1}{r} - \frac{a_1 k_1}{ak} \right) e^{-\frac{a_1 k_1}{r_1} t} + \left(\frac{r_2}{r} - \frac{a_2 k_2}{ak} \right) e^{-\frac{a_2 k_2}{r_2} t} + \text{\&c.} \right\}. \quad \text{. . . . .} \quad (58)$$

(89) It appears from this result that if all the layers are made of the same substance, Ψ' will be zero always. If they are of different substances, the order in which they are placed is indifferent, and the effect will be the same whether each substance consists of one layer, or is divided into any number of thin layers and arranged in any order among thin layers of the other substances. Any substance, therefore, the parts of which are not mathematically homogeneous, though they may be apparently so, may exhibit phenomena of absorption. Also, since the order of magnitude of the coefficients is the same as that of the indices, the value of Ψ' can never change sign, but must start from zero, become positive, and finally disappear.

(90) Let us next consider the total amount of electricity which would pass from the first surface to the second, if the condenser, after being thoroughly saturated by the current and then discharged, has its extreme surfaces connected by a conductor of resistance R. Let p be the current in this conductor; then, during the discharge,

$$\Psi' = p_1 r_1 + p_2 r_2 + \text{\&c.} = p\mathrm{R}. \quad \text{. . . . .} \quad (59)$$

Integrating with respect to the time, and calling q_1, q_2, q the quantities of electricity which traverse the different conductors,

$$q_1 r_1 + q_2 r_2 + \text{\&c.} = q\mathrm{R}. \quad \text{. . . . .} \quad (60)$$

The quantities of electricity on the several surfaces will be

$$\begin{aligned} &e'_1 - q - q_1, \\ &e_2 + q_1 - q_2, \\ &\text{\&c.}; \end{aligned}$$

and since at last all these quantities vanish, we find

$$\begin{aligned} q_1 &= e'_1 - q, \\ q_2 &= e'_1 + e_2 - q; \end{aligned}$$

whence

$$q\mathrm{R} = \frac{\Psi}{r} \left(\frac{r_1^2}{a_1 k_1} + \frac{r_2^2}{a_2 k_2} + \text{\&c.} \right) - \frac{\Psi r}{ak},$$

or

$$q = \frac{\Psi}{akr\mathrm{R}} \left\{ a_1 k_1 a_2 k_2 \left(\frac{r_1}{a_1 k_1} - \frac{r_2}{a_2 k_2} \right)^2 + a_2 k_2 a_3 k_3 \left(\frac{r_2}{a_2 k_2} - \frac{r_3}{a_3 k_3} \right)^2 + \text{\&c.} \right\}, \quad \text{. . .} \quad (61)$$

a quantity essentially positive; so that, when the primary electrification is in one direction, the secondary discharge is always in the same direction as the primary discharge*.

PART VI.—ELECTROMAGNETIC THEORY OF LIGHT.

(91) At the commencement of this paper we made use of the optical hypothesis of an elastic medium through which the vibrations of light are propagated, in order to show that we have warrantable grounds for seeking, in the same medium, the cause of other phenomena as well as those of light. We then examined electromagnetic phenomena, seeking for their explanation in the properties of the field which surrounds the electrified or magnetic bodies. In this way we arrived at certain equations expressing certain properties of the electromagnetic field. We now proceed to investigate whether these properties of that which constitutes the electromagnetic field, deduced from electromagnetic phenomena alone, are sufficient to explain the propagation of light through the same substance.

(92) Let us suppose that a plane wave whose direction cosines are l, m, n is propagated through the field with a velocity V. Then all the electromagnetic functions will be functions of

$$w=lx+my+nz-\mathrm{V}t.$$

The equations of Magnetic Force (B), p. 482, will become

$$\mu\alpha=m\frac{d\mathrm{H}}{dw}-n\frac{d\mathrm{G}}{dw},$$

$$\mu\beta=n\frac{d\mathrm{F}}{dw}-l\frac{d\mathrm{H}}{dw},$$

$$\mu\gamma=l\frac{d\mathrm{G}}{dw}-m\frac{d\mathrm{F}}{dw}.$$

If we multiply these equations respectively by l, m, n, and add, we find

$$l\mu\alpha+m\mu\beta+n\mu\gamma=0, \quad \dots\dots\dots\dots \quad (62)$$

which shows that the direction of the magnetization must be in the plane of the wave.

(93) If we combine the equations of Magnetic Force (B) with those of Electric Currents (C), and put for brevity

$$\frac{d\mathrm{F}}{dx}+\frac{d\mathrm{G}}{dy}+\frac{d\mathrm{H}}{dz}=\mathrm{J}, \text{ and } \frac{d^2}{dx^2}+\frac{d^2}{dy^2}+\frac{d^2}{dz^2}=\nabla^2, \quad \dots\dots \quad (63)$$

$$\left.\begin{aligned} 4\pi\mu p' &= \frac{d\mathrm{J}}{dx}-\nabla^2\mathrm{F}, \\ 4\pi\mu q' &= \frac{d\mathrm{J}}{dy}-\nabla^2\mathrm{G}, \\ 4\pi\mu r' &= \frac{d\mathrm{J}}{dz}-\nabla^2\mathrm{H}. \end{aligned}\right\} \quad \dots\dots\dots\dots \quad (64)$$

---

* Since this paper was communicated to the Royal Society, I have seen a paper by M. Gaugain in the Annales de Chimie for 1864, in which he has deduced the phenomena of electric absorption and secondary discharge from the theory of compound condensers.

If the medium in the field is a perfect dielectric there is no true conduction, and the currents p', q', r' are only variations in the electric displacement, or, by the equations of Total Currents (A),

$$p'=\frac{df}{dt}, \qquad q'=\frac{dg}{dt}, \qquad r'=\frac{dh}{dt}. \quad \ldots\ldots \quad (65)$$

But these electric displacements are caused by electromotive forces, and by the equations of Electric Elasticity (E),

$$P=kf, \qquad Q=kg, \qquad R=kh. \quad \ldots\ldots \quad (66)$$

These electromotive forces are due to the variations either of the electromagnetic or the electrostatic functions, as there is no motion of conductors in the field; so that the equations of electromotive force (D) are

$$\left.\begin{aligned} P&=-\frac{dF}{dt}-\frac{d\Psi}{dx}, \\ Q&=-\frac{dG}{dt}-\frac{d\Psi}{dy}, \\ R&=-\frac{dH}{dt}-\frac{d\Psi}{dz}. \end{aligned}\right\} \quad \ldots\ldots \quad (67)$$

(94) Combining these equations, we obtain the following:—

$$\left.\begin{aligned} k\left(\frac{dJ}{dx}-\nabla^2F\right)+4\pi\mu\left(\frac{d^2F}{dt^2}+\frac{d^2\Psi}{dxdt}\right)&=0, \\ k\left(\frac{dJ}{dy}-\nabla^2G\right)+4\pi\mu\left(\frac{d^2G}{dt^2}+\frac{d^2\Psi}{dydt}\right)&=0, \\ k\left(\frac{dJ}{dz}-\nabla^2H\right)+4\pi\mu\left(\frac{d^2H}{dt^2}+\frac{d^2\Psi}{dzdt}\right)&=0. \end{aligned}\right\} \quad \ldots\ldots \quad (68)$$

If we differentiate the third of these equations with respect to y, and the second with respect to z, and subtract, J and Ψ disappear, and by remembering the equations (B) of magnetic force, the results may be written

$$\left.\begin{aligned} k\nabla^2\mu\alpha&=4\pi\mu\frac{d^2}{dt^2}\mu\alpha, \\ k\nabla^2\mu\beta&=4\pi\mu\frac{d^2}{dt^2}\mu\beta, \\ k\nabla^2\mu\gamma&=4\pi\mu\frac{d^2}{d^2t}\mu\gamma. \end{aligned}\right\} \quad \ldots\ldots \quad (69)$$

(95) If we assume that α, β, γ are functions of $lx+my+nz-Vt=w$, the first equation becomes

$$k\mu\frac{d^2\alpha}{dw^2}=4\pi\mu^2V^2\frac{d^2\alpha}{dw^2}, \quad \ldots\ldots \quad (70)$$

or

$$V=\pm\sqrt{\frac{k}{4\pi\mu}}. \quad \ldots\ldots \quad (71)$$

The other equations give the same value for V, so that the wave is propagated in either direction with a velocity V.

This wave consists entirely of magnetic disturbances, the direction of magnetization being in the plane of the wave. No magnetic disturbance whose direction of magnetization is not in the plane of the wave can be propagated as a plane wave at all.

Hence magnetic disturbances propagated through the electromagnetic field agree with light in this, that the disturbance at any point is transverse to the direction of propagation, and such waves may have all the properties of polarized light.

(96) The only medium in which experiments have been made to determine the value of k is air, in which $\mu=1$, and therefore, by equation (46),

$$V=v. \qquad \qquad (72)$$

By the electromagnetic experiments of MM. Weber and Kohlrausch[*],

$$v=310,740,000 \text{ metres per second}$$

is the number of electrostatic units in one electromagnetic unit of electricity, and this, according to our result, should be equal to the velocity of light in air or vacuum.

The velocity of light in air, by M. Fizeau's[†] experiments, is

$$V=314,858,000;$$

according to the more accurate experiments of M. Foucault[‡],

$$V=298,000,000.$$

The velocity of light in the space surrounding the earth, deduced from the coefficient of aberration and the received value of the radius of the earth's orbit, is

$$V=308,000,000.$$

(97) Hence the velocity of light deduced from experiment agrees sufficiently well with the value of v deduced from the only set of experiments we as yet possess. The value of v was determined by measuring the electromotive force with which a condenser of known capacity was charged, and then discharging the condenser through a galvanometer, so as to measure the quantity of electricity in it in electromagnetic measure. The only use made of light in the experiment was to see the instruments. The value of V found by M. Foucault was obtained by determining the angle through which a revolving mirror turned, while the light reflected from it went and returned along a measured course. No use whatever was made of electricity or magnetism.

The agreement of the results seems to show that light and magnetism are affections of the same substance, and that light is an electromagnetic disturbance propagated through the field according to electromagnetic laws.

(98) Let us now go back upon the equations in (94), in which the quantities J and Ψ occur, to see whether any other kind of disturbance can be propagated through the medium depending on these quantities which disappeared from the final equations.

---

[*] Leipzig Transactions, vol. v. (1857), p. 260, or Poggendorff's 'Annalen,' Aug. 1856, p. 10.

[†] Comptes Rendus, vol. xxix. (1849), p. 90.

[‡] Ibid. vol. lv. (1862), pp. 501, 792.

If we determine χ from the equation

$$\nabla^2\chi=\frac{d^2\chi}{dx^2}+\frac{d^2\chi}{dy^2}+\frac{d^2\chi}{dz^2}=\mathrm{J}, \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \tag{73}$$

and F′, G′, H′ from the equations

$$\mathrm{F}'=\mathrm{F}-\frac{d\chi}{dx}, \quad \mathrm{G}'=\mathrm{G}-\frac{d\chi}{dy}, \quad \mathrm{H}'=\mathrm{H}-\frac{d\chi}{dz}, \quad . \; . \; . \; . \; . \tag{74}$$

then

$$\frac{d\mathrm{F}'}{dx}+\frac{d\mathrm{G}'}{dy}+\frac{d\mathrm{H}'}{dz}=0, \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \tag{75}$$

and the equations in (94) become of the form

$$k\nabla^2\mathrm{F}'=4\pi\mu\left(\frac{d^2\mathrm{F}'}{dt^2}+\frac{d}{dxdt}\left(\Psi+\frac{d\chi}{dt}\right)\right). \quad . \; . \; . \; . \; . \; . \; . \tag{76}$$

Differentiating the three equations with respect to x, y, and z, and adding, we find that

$$\Psi=-\frac{d\chi}{dt}+\phi(x, y, z), \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \tag{77}$$

and that

$$\left.\begin{aligned} k\nabla^2\mathrm{F}' &=4\pi\mu\frac{d^2\mathrm{F}'}{dt^2}, \\ k\nabla^2\mathrm{G}' &=4\pi\mu\frac{d^2\mathrm{G}'}{dt^2}, \\ k\nabla^2\mathrm{H}' &=4\pi\mu\frac{d^2\mathrm{H}'}{dt^2}. \end{aligned}\right\} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \tag{78}$$

Hence the disturbances indicated by F′, G′, H′ are propagated with the velocity $\mathrm{V}=\sqrt{\frac{k}{4\pi\mu}}$ through the field; and since

$$\frac{d\mathrm{F}'}{dx}+\frac{d\mathrm{G}'}{dy}+\frac{d\mathrm{H}'}{dx}=0,$$

the resultant of these disturbances is in the plane of the wave.

(99) The remaining part of the total disturbances F, G, H being the part depending on χ, is subject to no condition except that expressed in the equation

$$\frac{d\Psi}{dt}+\frac{d^2\chi}{dt^2}=0.$$

If we perform the operation ∇^2 on this equation, it becomes

$$ke=\frac{d\mathrm{J}}{dt}-k\nabla^2\phi(x, y, z). \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \tag{79}$$

Since the medium is a perfect insulator, e, the free electricity, is immoveable, and therefore $\frac{d\mathrm{J}}{dt}$ is a function of x, y, z, and the value of J is either constant or zero, or uniformly increasing or diminishing with the time; so that no disturbance depending on J can be propagated as a wave.

(100) The equations of the electromagnetic field, deduced from purely experimental evidence, show that transversal vibrations only can be propagated. If we were to go beyond our experimental knowledge and to assign a definite density to a substance which

we should call the electric fluid, and select either vitreous or resinous electricity as the representative of that fluid, then we might have normal vibrations propagated with a velocity depending on this density. We have, however, no evidence as to the density of electricity, as we do not even know whether to consider vitreous electricity as a substance or as the absence of a substance.

Hence electromagnetic science leads to exactly the same conclusions as optical science with respect to the direction of the disturbances which can be propagated through the field; both affirm the propagation of transverse vibrations, and both give the same velocity of propagation. On the other hand, both sciences are at a loss when called on to affirm or deny the existence of normal vibrations.

Relation between the Index of Refraction and the Electromagnetic Character of the substance.

(101) The velocity of light in a medium, according to the Undulatory Theory, is

$$\frac{1}{i}V_0,$$

where i is the index of refraction and V_0 is the velocity in vacuum. The velocity, according to the Electromagnetic Theory, is

$$\sqrt{\frac{k}{4\pi\mu}},$$

where, by equations (49) and (71), $k=\frac{1}{D}k_0$, and $k_0=4\pi V_0^2$.

Hence $$D=\frac{i^2}{\mu}, \quad \dots\dots\dots\dots \quad (80)$$

or the Specific Inductive Capacity is equal to the square of the index of refraction divided by the coefficient of magnetic induction.

Propagation of Electromagnetic Disturbances in a Crystallized Medium.

(102) Let us now calculate the conditions of propagation of a plane wave in a medium for which the values of k and μ are different in different directions. As we do not propose to give a complete investigation of the question in the present imperfect state of the theory as extended to disturbances of short period, we shall assume that the axes of magnetic induction coincide in direction with those of electric elasticity.

(103) Let the values of the magnetic coefficient for the three axes be λ, μ, ν, then the equations of magnetic force (B) become

$$\left.\begin{aligned} \lambda\alpha &= \frac{dH}{dy}-\frac{dG}{dz}, \\ \mu\beta &= \frac{dF}{dz}-\frac{dH}{dx}, \\ \nu\gamma &= \frac{dG}{dx}-\frac{dF}{dy}. \end{aligned}\right\} \quad \dots\dots\dots\dots \quad (81)$$

The equations of electric currents (C) remain as before.

The equations of electric elasticity (E) will be

$$\left.\begin{aligned} P&=4\pi a^2 f,\\ Q&=4\pi b^2 g,\\ R&=4\pi c^2 h, \end{aligned}\right\} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (82)$$

where $4\pi a^2$, $4\pi b^2$, and $4\pi c^2$ are the values of k for the axes of x, y, z.

Combining these equations with (A) and (D), we get equations of the form

$$\frac{1}{\mu\nu}\left(\lambda\frac{d^2F}{dx^2}+\mu\frac{d^2F}{dy^2}+\nu\frac{d^2F}{dz^2}\right)-\frac{1}{\mu\nu}\frac{d}{dx}\left(\lambda\frac{dF}{dx}+\mu\frac{dG}{dy}+\nu\frac{dH}{dz}\right)=\frac{1}{a^2}\left(\frac{d^2F}{dt^2}+\frac{d^2\Psi}{dxdt}\right). \quad . \quad (83)$$

(104) If l, m, n are the direction-cosines of the wave, and V its velocity, and if

$$lx+my+nz-Vt=w, \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (84)$$

then F, G, H, and Ψ will be functions of w; and if we put F', G', H', Ψ' for the second differentials of these quantities with respect to w, the equations will be

$$\left.\begin{aligned} &\left(V^2-a^2\left(\frac{m^2}{\nu}+\frac{n^2}{\mu}\right)\right)F'+\frac{a^2lm}{\nu}G'+\frac{a^2ln}{\mu}H'-lV\Psi'=0,\\ &\left(V^2-b^2\left(\frac{n^2}{\lambda}+\frac{l^2}{\nu}\right)\right)G'+\frac{b^2mn}{\lambda}H'+\frac{b^2ml}{\nu}F'-mV\Psi'=0,\\ &\left(V^2-c^2\left(\frac{l^2}{\mu}+\frac{m^2}{\lambda}\right)\right)H'+\frac{c^2nl}{\mu}F'+\frac{c^2nm}{\lambda}G'-nV\Psi'=0. \end{aligned}\right\} . \; . \; . \; . \; . \quad (85)$$

If we now put

$$\left.\begin{aligned} V^4-V^2\frac{1}{\lambda\mu\nu}&\left\{l^2\lambda(b^2\mu+c^2\nu)+m^2\mu(c^2\nu+a^2\lambda)+n^2\nu(a^2\lambda+b^2\mu)\right\}\\ &+\frac{a^2b^2c^2}{\lambda\mu\nu}\left(\frac{l^2}{a^2}+\frac{m^2}{b^2}+\frac{n^2}{c^2}\right)(l^2\lambda+m^2\mu+n^2\nu)=U, \end{aligned}\right\} \quad . \; . \; . \; . \quad (86)$$

we shall find

$$F'V^2U-l\Psi'VU=0, \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (87)$$

with two similar equations for G' and H'. Hence either

$$V=0, \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (88)$$

$$U=0, \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (89)$$

or

$$VF'=l\Psi',\ VG'=m\Psi' \text{ and } VH'=n\Psi'. \quad . \; . \; . \; . \quad (90)$$

The third supposition indicates that the resultant of F', G', H' is in the direction normal to the plane of the wave; but the equations do not indicate that such a disturbance, if possible, could be propagated, as we have no other relation between Ψ' and F', G', H'.

The solution $V=0$ refers to a case in which there is no propagation.

The solution $U=0$ gives two values for V^2 corresponding to values of F', G', H', which are given by the equations

$$\frac{l}{a^2}F'+\frac{m}{b^2}G'+\frac{n}{c^2}H'=0, \quad \ldots \ldots \quad (91)$$

$$\frac{a^2l\lambda}{F'}(b^2\mu-c^2\nu)+\frac{b^2m\mu}{G'}(c^2\nu-a^2\lambda)+\frac{c^2n\nu}{H'}(a^2\lambda-b^2\mu)=0, \quad \ldots \quad (92)$$

(105) The velocities along the axes are as follows:—

Direction of propagation		x	y	z
Direction of the electric displacements	x		$\frac{a^2}{\nu}$	$\frac{a^2}{\mu}$
	y	$\frac{b^2}{\nu}$		$\frac{b^2}{\lambda}$
	z	$\frac{c^2}{\mu}$	$\frac{c^2}{\lambda}$	

Now we know that in each principal plane of a crystal the ray polarized in that plane obeys the ordinary law of refraction, and therefore its velocity is the same in whatever direction in that plane it is propagated.

If polarized light consists of electromagnetic disturbances in which the electric displacement is in the plane of polarization, then

$$a^2=b^2=c^2. \quad \ldots \ldots \quad (93)$$

If, on the contrary, the electric displacements are perpendicular to the plane of polarization,

$$\lambda=\mu=\nu. \quad \ldots \ldots \quad (94)$$

We know, from the magnetic experiments of FARADAY, PLÜCKER, &c., that in many crystals λ, μ, ν are unequal.

The experiments of KNOBLAUCH* on electric induction through crystals seem to show that a, b and c, may be different.

The inequality, however, of λ, μ, ν is so small that great magnetic forces are required to indicate their difference, and the differences do not seem of sufficient magnitude to account for the double refraction of the crystals.

On the other hand, experiments on electric induction are liable to error on account of minute flaws, or portions of conducting matter in the crystal.

Further experiments on the magnetic and dielectric properties of crystals are required before we can decide whether the relation of these bodies to magnetic and electric forces is the same, when these forces are permanent as when they are alternating with the rapidity of the vibrations of light.

---

* Philosophical Magazine, 1852.

Relation between Electric Resistance and Transparency.

(106) If the medium, instead of being a perfect insulator, is a conductor whose resistance per unit of volume is ϱ, then there will be not only electric displacements, but true currents of conduction in which electrical energy is transformed into heat, and the undulation is thereby weakened. To determine the coefficient of absorption, let us investigate the propagation along the axis of x of the transverse disturbance G.

By the former equations

$$\frac{d^2G}{dx^2}=-4\pi\mu(q')$$

$$=-4\pi\mu\left(\frac{df}{dt}+q\right) \text{ by (A),}$$

$$\frac{d^2G}{dx^2}=+4\pi\mu\left(\frac{1}{k}\frac{d^2G}{dt^2}-\frac{1}{\varrho}\frac{dG}{dt}\right) \text{ by (E) and (F).} \quad \dots\dots \quad (95)$$

If G is of the form

$$G=e^{-px}\cos(qx+nt), \quad \dots\dots \quad (96)$$

we find that

$$p=\frac{2\pi\mu}{\varrho}\frac{n}{q}=\frac{2\pi\mu}{\varrho}\frac{V}{i}, \quad \dots\dots \quad (97)$$

where V is the velocity of light in air, and i is the index of refraction. The proportion of incident light transmitted through the thickness x is

$$e^{-2px}. \quad \dots\dots \quad (98)$$

Let R be the resistance in electromagnetic measure of a plate of the substance whose thickness is x, breadth b, and length l, then

$$R=\frac{l\varrho}{bx},$$

$$2px=4\pi\mu\frac{V}{i}\frac{l}{bR}. \quad \dots\dots \quad (99)$$

(107) Most transparent solid bodies are good insulators, whereas all good conductors are very opaque.

Electrolytes allow a current to pass easily and yet are often very transparent. We may suppose, however, that in the rapidly alternating vibrations of light, the electromotive forces act for so short a time that they are unable to effect a complete separation between the particles in combination, so that when the force is reversed the particles oscillate into their former position without loss of energy.

Gold, silver, and platinum are good conductors, and yet when reduced to sufficiently thin plates they allow light to pass through them. If the resistance of gold is the same for electromotive forces of short period as for those with which we make experiments, the amount of light which passes through a piece of gold-leaf, of which the resistance was determined by Mr. C. HOCKIN, would be only 10^{-50} of the incident light, a totally imperceptible quantity. I find that between $\frac{1}{500}$ and $\frac{1}{1000}$ of green light gets through such gold-leaf. Much of this is transmitted through holes and cracks; there is enough, however, transmitted through the gold itself to give a strong green hue to the transmitted light. This result cannot be reconciled with the electromagnetic theory of light, unless we suppose that there is less loss of energy when the electromotive forces are reversed with the rapidity of the vibrations of light than when they act for sensible times, as in our experiments.

Absolute Values of the Electromotive and Magnetic Forces called into play in the Propagation of Light.

(108) If the equation of propagation of light is

$$F=A\cos\frac{2\pi}{\lambda}(z-Vt),$$

the electromotive force will be

$$P=-A\frac{2\pi}{\lambda}V\sin\frac{2\pi}{\lambda}(z-Vt);$$

and the energy per unit of volume will be

$$\frac{P^2}{8\pi\mu V^2},$$

where P represents the greatest value of the electromotive force. Half of this consists of magnetic and half of electric energy.

The energy passing through a unit of area is

$$W=\frac{P^2}{8\pi\mu V};$$

so that

$$P=\sqrt{8\pi\mu VW},$$

where V is the velocity of light, and W is the energy communicated to unit of area by the light in a second.

According to POUILLET'S data, as calculated by Professor W. THOMSON[*], the mechanical value of direct sunlight at the Earth is

83·4 foot-pounds per second per square foot.

This gives the maximum value of P in direct sunlight at the Earth's distance from the Sun,

$$P=60{,}000{,}000,$$

or about 600 DANIELL'S cells per metre.

At the Sun's surface the value of P would be about

13,000 DANIELL'S cells per metre.

At the Earth the maximum magnetic force would be ·193[†].

At the Sun it would be 4·13.

These electromotive and magnetic forces must be conceived to be reversed twice in every vibration of light; that is, more than a thousand million million times in a second.

---

[*] Transactions of the Royal Society of Edinburgh, 1854 ("Mechanical Energies of the Solar System").

[†] The horizontal magnetic force at Kew is about 1·76 in metrical units.

PART VII.—CALCULATION OF THE COEFFICIENTS OF ELECTROMAGNETIC INDUCTION.

General Methods.

(109) The electromagnetic relations between two conducting circuits, A and B, depend upon a function M of their form and relative position, as has been already shown.

M may be calculated in several different ways, which must of course all lead to the same result.

First Method. M is the electromagnetic momentum of the circuit B when A carries a unit current, or

$$M=\int\left(F\frac{dx}{ds'}+G\frac{dy}{ds'}+H\frac{dz}{ds'}\right)ds',$$

where F, G, H are the components of electromagnetic momentnm due to a unit current in A, and ds' is an element of length of B, and the integration is performed round the circuit of B.

To find F, G, H, we observe that by (B) and (C)

$$\frac{d^2F}{dx^2}+\frac{d^2F}{dy^2}+\frac{d^2F}{dz^2}=-4\pi\mu p',$$

with corresponding equations for G and H, p', q', and r' being the components of the current in A.

Now if we consider only a single element ds of A, we shall have

$$p'=\frac{dx}{ds}ds,\qquad q'=\frac{dy}{ds}ds,\qquad r'=\frac{dz}{ds}ds,$$

and the solution of the equation gives

$$F=\frac{\mu}{\varrho}\frac{dx}{ds}ds,\qquad G=\frac{\mu}{\varrho}\frac{dy}{ds}ds,\qquad H=\frac{\mu}{\varrho}\frac{dz}{ds}ds,$$

where ϱ is the distance of any point from ds. Hence

$$\begin{aligned}M&=\iint\frac{\mu}{\varrho}\left(\frac{dx}{ds}\frac{dx}{ds'}+\frac{dy}{ds}\frac{dy}{ds'}+\frac{dz}{ds}\frac{dz}{ds'}\right)ds\,ds'\\&=\iint\frac{\mu}{\varrho}\cos\theta\,ds\,ds',\end{aligned}$$

where θ is the angle between the directions of the two elements ds, ds', and ϱ is the distance between them, and the integration is performed round both circuits.

In this method we confine our attention during integration to the two linear circuits alone.

(110) Second Method. M is the number of lines of magnetic force which pass through the circuit B when A carries a unit current, or

$$M=\Sigma(\mu\alpha l+\mu\beta m+\mu\gamma n)dS',$$

where $\mu\alpha$, $\mu\beta$, $\mu\gamma$ are the components of magnetic induction due to unit current in A, S′ is a surface bounded by the current B, and l, m, n are the direction-cosines of the normal to the surface, the integration being extended over the surface.

We may express this in the form

$$\mathrm{M}=\mu\Sigma\frac{1}{\varrho^2}\sin\theta\sin\theta'\sin\phi d\mathrm{S}'ds,$$

where $d\mathrm{S}'$ is an element of the surface bounded by B, ds is an element of the circuit A, ϱ is the distance between them, θ and θ' are the angles between ϱ and ds and between ϱ and the normal to $d\mathrm{S}'$ respectively, and ϕ is the angle between the planes in which θ and θ' are measured. The integration is performed round the circuit A and over the surface bounded by B.

This method is most convenient in the case of circuits lying in one plane, in which case $\sin\theta=1$, and $\sin\phi=1$.

111. Third Method. M is that part of the intrinsic magnetic energy of the whole field which depends on the product of the currents in the two circuits, each current being unity.

Let α, β, γ be the components of magnetic intensity at any point due to the first circuit, α', β', γ' the same for the second circuit; then the intrinsic energy of the element of volume $d\mathrm{V}$ of the field is

$$\frac{\mu}{8\pi}\Big((\alpha+\alpha')^2+(\beta+\beta')^2+(\gamma+\gamma')^2\Big)d\mathrm{V}.$$

The part which depends on the product of the currents is

$$\frac{\mu}{4\pi}(\alpha\alpha'+\beta\beta'+\gamma\gamma')d\mathrm{V}.$$

Hence if we know the magnetic intensities I and I′ due to unit current in each circuit, we may obtain M by integrating

$$\frac{\mu}{4\pi}\Sigma\mu\mathrm{I}\,\mathrm{I}'\cos\theta d\mathrm{V}$$

over all space, where θ is the angle between the directions of I and I′.

Application to a Coil.

(112) To find the coefficient (M) of mutual induction between two circular linear conductors in parallel planes, the distance between the curves being everywhere the same, and small compared with the radius of either.

If r be the distance between the curves, and a the radius of either, then when r is very small compared with a, we find by the second method, as a first approximation,

$$\mathrm{M}=4\pi a\left(\log_e\frac{8a}{r}-2\right).$$

To approximate more closely to the value of M, let a and a_1 be the radii of the circles, and b the distance between their planes; then

$$r^2=(a-a_1)^2+b^2.$$

We obtain M by considering the following conditions:—

1st. M must fulfil the differential equation

$$\frac{d^2\mathrm{M}}{da^2}+\frac{d^2\mathrm{M}}{db^2}+\frac{1}{a}\frac{d\mathrm{M}}{da}=0.$$

This equation being true for any magnetic field symmetrical with respect to the common axis of the circles, cannot of itself lead to the determination of M as a function of a, a_1, and b. We therefore make use of other conditions.

2ndly. The value of M must remain the same when a and a_1 are exchanged.

3rdly. The first two terms of M must be the same as those given above.

M may thus be expanded in the following series:—

$$\begin{aligned}\mathrm{M}=4\pi a\log\frac{8a}{r}&\left\{1+\frac{1}{2}\frac{a-a_1}{a}+\frac{1}{16}\frac{3b^2+(a_1-a)^2}{a^2}-\frac{1}{32}\frac{\left(3b^2+(a-a_1)^2\right)(a-a_1)}{a^3}+\&\mathrm{c.}\right\}\\ &-4\pi a\left\{2+\frac{1}{2}\frac{a-a_1}{a}+\frac{1}{16}\frac{b^2-3(a-a_1{}^2)}{a^2}-\frac{1}{48}\frac{\left(6b^2-(a-a_1)^2\right)(a-a_1)}{a^3}+\&\mathrm{c.}\right\}.\end{aligned}$$

(113) We may apply this result to find the coefficient of self-induction (L) of a circular coil of wire whose section is small compared with the radius of the circle.

Let the section of the coil be a rectangle, the breadth in the plane of the circle being c, and the depth perpendicular to the plane of the circle being b.

Let the mean radius of the coil be a, and the number of windings n; then we find, by integrating,

$$\mathrm{L}=\frac{n^2}{b^2c^2}\iiiint\mathrm{M}(xy\,x'y')dx\,dy\,dx'\,dy',$$

where $\mathrm{M}(xy\,x'y')$ means the value of M for the two windings whose coordinates are xy and $x'y'$ respectively; and the integration is performed first with respect to x and y over the rectangular section, and then with respect to x' and y' over the same space.

$$\begin{aligned}\mathrm{L}=4\pi n^2a&\left\{\log_e\frac{8a}{r}+\frac{1}{12}-\frac{4}{3}\left(\theta-\frac{\pi}{4}\right)\cot 2\theta-\frac{\pi}{3}\cos 2\theta-\frac{1}{6}\cot^2\theta\log\cos\theta-\frac{1}{6}\tan^2\theta\log\sin\theta\right\}\\ &+\frac{\pi n^2r^2}{24a}\left\{\log\frac{8a}{r}(2\sin^2\theta+1)+3{\cdot}45+27{\cdot}475\cos^2\theta-3{\cdot}2\left(\frac{\pi}{2}-\theta\right)\frac{\sin^3\theta}{\cos\theta}+\frac{1}{5}\frac{\cos^4\theta}{\sin^2\theta}\log\cos\theta\right.\\ &\left.\qquad+\frac{13}{3}\frac{\sin^4\theta}{\cos^2\theta}\log\sin\theta\right\}+\&\mathrm{c.}\end{aligned}$$

Here $a=$ mean radius of the coil.

,, $r=$ diagonal of the rectangular section $=\sqrt{b^2+c^2}$.

,, $\theta=$ angle between r and the plane of the circle.

,, $n=$ number of windings.

The logarithms are Napierian, and the angles are in circular measure.

In the experiments made by the Committee of the British Association for determining a standard of Electrical Resistance, a double coil was used, consisting of two nearly equal coils of rectangular section, placed parallel to each other, with a small interval between them.

The value of L for this coil was found in the following way.

The value of L was calculated by the preceding formula for six different cases, in which the rectangular section considered has always the same breadth, while the depth was

$$A,\quad B,\quad C,\quad A+B,\quad B+C,\quad A+B+C,$$

and $n=1$ in each case.

Calling the results

$$L(A),\quad L(B),\quad L(C), \&c.,$$

we calculate the coefficient of mutual induction M(AC) of the two coils thus,

$$2ACM(AC)=(A+B+C)^2L(A+B+C)-(A+B)^2L(A+B)-(B+C)^2L(B+C)+B^2L(B).$$

Then if n_1 is the number of windings in the coil A and n_2 in the coil B, the coefficient of self-induction of the two coils together is

$$L=n_1^2L(A)+2n_1n_2L(AC)+n_2^2L(B).$$

(114) These values of L are calculated on the supposition that the windings of the wire are evenly distributed so as to fill up exactly the whole section. This, however, is not the case, as the wire is generally circular and covered with insulating material. Hence the current in the wire is more concentrated than it would have been if it had been distributed uniformly over the section, and the currents in the neighbouring wires do not act on it exactly as such a uniform current would do.

The corrections arising from these considerations may be expressed as numerical quantities, by which we must multiply the length of the wire, and they are the same whatever be the form of the coil.

Let the distance between each wire and the next, on the supposition that they are arranged in square order, be D, and let the diameter of the wire be d, then the correction for diameter of wire is

$$+2\left(\log\frac{D}{d}+\frac{4}{3}\log 2+\frac{\pi}{3}-\frac{11}{6}\right).$$

The correction for the eight nearest wires is

$$+0\cdot0236.$$

For the sixteen in the next row

$$+0\cdot00083.$$

These corrections being multiplied by the length of wire and added to the former result, give the true value of L, considered as the measure of the potential of the coil on itself for unit current in the wire when that current has been established for some time, and is uniformly distributed through the section of the wire.

(115) But at the commencement of a current and during its variation the current is not uniform throughout the section of the wire, because the inductive action between different portions of the current tends to make the current stronger at one part of the section than at another. When a uniform electromotive force P arising from any cause

acts on a cylindrical wire of specific resistance ϱ, we have

$$p\varrho=\mathrm{P}-\frac{d\mathrm{F}}{dt},$$

where F is got from the equation

$$\frac{d^2\mathrm{F}}{dr^2}+\frac{1}{r}\frac{d\mathrm{F}}{dr}=-4\pi\mu p,$$

r being the distance from the axis of the cylinder.

Let one term of the value of F be of the form $\mathrm{T}r^n$, where T is a function of the time, then the term of p which produced it is of the form

$$-\frac{1}{4\pi\mu}n^2\mathrm{T}r^{n-2}.$$

Hence if we write

$$\mathrm{F}=\mathrm{T}+\frac{\mu\pi}{\varrho}\left(-\mathrm{P}+\frac{d\mathrm{T}}{dt}\right)r^2+\left.\overline{\frac{\mu\pi}{\varrho}}\right|^2\frac{1}{1^2.2^2}\frac{d\mathrm{T}^2}{dt^2}r^4+\&\mathrm{c}.$$

$$p\varrho=\left(\mathrm{P}+\frac{d\mathrm{T}}{dt}\right)-\frac{\mu\pi}{\varrho}\frac{d^2\mathrm{T}}{dt^2}r^2-\left.\overline{\frac{\mu\pi}{\varrho}}\right|^2\frac{1}{1^2.2^2}\frac{d^3\mathrm{T}}{dt^3}r^4-\&\mathrm{c}.$$

The total counter current of self-induction at any point is

$$\int\left(\frac{\mathrm{P}}{\varrho}-p\right)dt=\frac{1}{\varrho}\mathrm{T}+\frac{\mu\pi}{\varrho^2}\frac{d\mathrm{T}}{dt}r^2+\frac{\mu^2\pi^2}{\varrho^3}\frac{1}{1^2 2^2}\frac{d^2\mathrm{T}}{dt^2}r^4+\&\mathrm{c}.$$

from $t=0$ to $t=\infty$.

When $t=0$, $p=0$, $\quad\therefore\left(\frac{d\mathrm{T}}{dt}\right)_0=\mathrm{P}$, $\left(\frac{d^2\mathrm{T}}{dt^2}\right)_0=0$, &c.

When $t=\infty$, $p=\frac{\mathrm{P}}{\varrho}$, $\quad\therefore\left(\frac{d\mathrm{T}}{dt}\right)_\infty=0$, $\left(\frac{d^2\mathrm{T}}{dt^2}\right)_\infty=0$, &c.

$$\int_0^\infty\int_0^r 2\pi\left(\frac{\mathrm{P}}{\varrho}-p\right)rdrdt=\frac{1}{\varrho}\mathrm{T}\pi r^2+\frac{1}{2}\frac{\mu\pi^2}{\varrho^2}\frac{d\mathrm{T}}{dt}r^4+\frac{\mu^2\pi^3}{\varrho^3}\frac{1}{1^2.2^2.3}\frac{d^2\mathrm{T}}{dt^2}r^6+\&\mathrm{c}.$$

from t=0 to $=\infty$.

When $t=0$, $p=0$ throughout the section, $\therefore\left(\frac{d\mathrm{T}}{dt}\right)_0=\mathrm{P}$, $\left(\frac{d^2\mathrm{T}}{dt^2}\right)_0=0$, &c.

When $t=\infty$, $p=0$ throughout . . . . . $\therefore\left(\frac{d\mathrm{T}}{dt}\right)_\infty=0$, $\left(\frac{d^2\mathrm{T}}{dt^2}\right)_\infty=0$, &c.

Also if l be the length of the wire, and R its resistance,

$$\mathrm{R}=\frac{\varrho l}{\pi r^2};$$

and if C be the current when established in the wire, $\mathrm{C}=\frac{\mathrm{P}l}{\mathrm{R}}$.

The total counter current may be written

$$\frac{l}{\mathrm{R}}(\mathrm{T}_\infty-\mathrm{T}_0)-\frac{1}{2}\mu\frac{l}{\mathrm{R}}\mathrm{C}=-\frac{\mathrm{LC}}{\mathrm{R}}\text{ by § (35).}$$

Now if the current instead of being variable from the centre to the circumference of the section of the wire had been the same throughout, the value of F would have been

$$F=T+\mu\gamma\left(1-\frac{r^2}{r_0^2}\right),$$

where γ is the current in the wire at any instant, and the total countercurrent would have been

$$\int_0^\infty\int_0^r \frac{1}{\varrho}\frac{dF}{dt}2\pi r dr=\frac{l}{R}(T_\infty-T_0)-\frac{3}{4}\mu\frac{l}{R}C=-\frac{L'C}{R}, \text{ say.}$$

Hence

$$L=L'-\tfrac{1}{4}\mu l,$$

or the value of L which must be used in calculating the self-induction of a wire for variable currents is less than that which is deduced from the supposition of the current being constant throughout the section of the wire by $\frac{1}{4}\mu l$, where l is the length of the wire, and μ is the coefficient of magnetic induction for the substance of the wire.

(116) The dimensions of the coil used by the Committee of the British Association in their experiments at King's College in 1864 were as follows:—

	metre.
Mean radius	$=a=\cdot158194$
Depth of each coil	$=b=\cdot01608$
Breadth of each coil	$=c=\cdot01841$
Distance between the coils. . .	$=\cdot02010$
Number of windings	$n=313$
Diameter of wire	$=\cdot00126$

The value of L derived from the first term of the expression is 437440 metres.

The correction depending on the radius not being infinitely great compared with the section of the coil as found from the second term is −7345 metres.

The correction depending on the diameter of the wire is per unit of length	+·44997
Correction of eight neighbouring wires.	+·0236
For sixteen wires next to these	+·0008
Correction for variation of current in different parts of section	−·2500
Total correction per unit of length	·22437
Length	311·236 metres.
Sum of corrections of this kind	70 „
Final value of L by calculation	430165 „

This value of L was employed in reducing the observations, according to the method explained in the Report of the Committee*. The correction depending on L varies as the square of the velocity. The results of sixteen experiments to which this correction had been applied, and in which the velocity varied from 100 revolutions in seventeen seconds to 100 in seventy-seven seconds, were compared by the method of least squares to determine what further correction depending on the square of the velocity should be applied to make the outstanding errors a minimum.

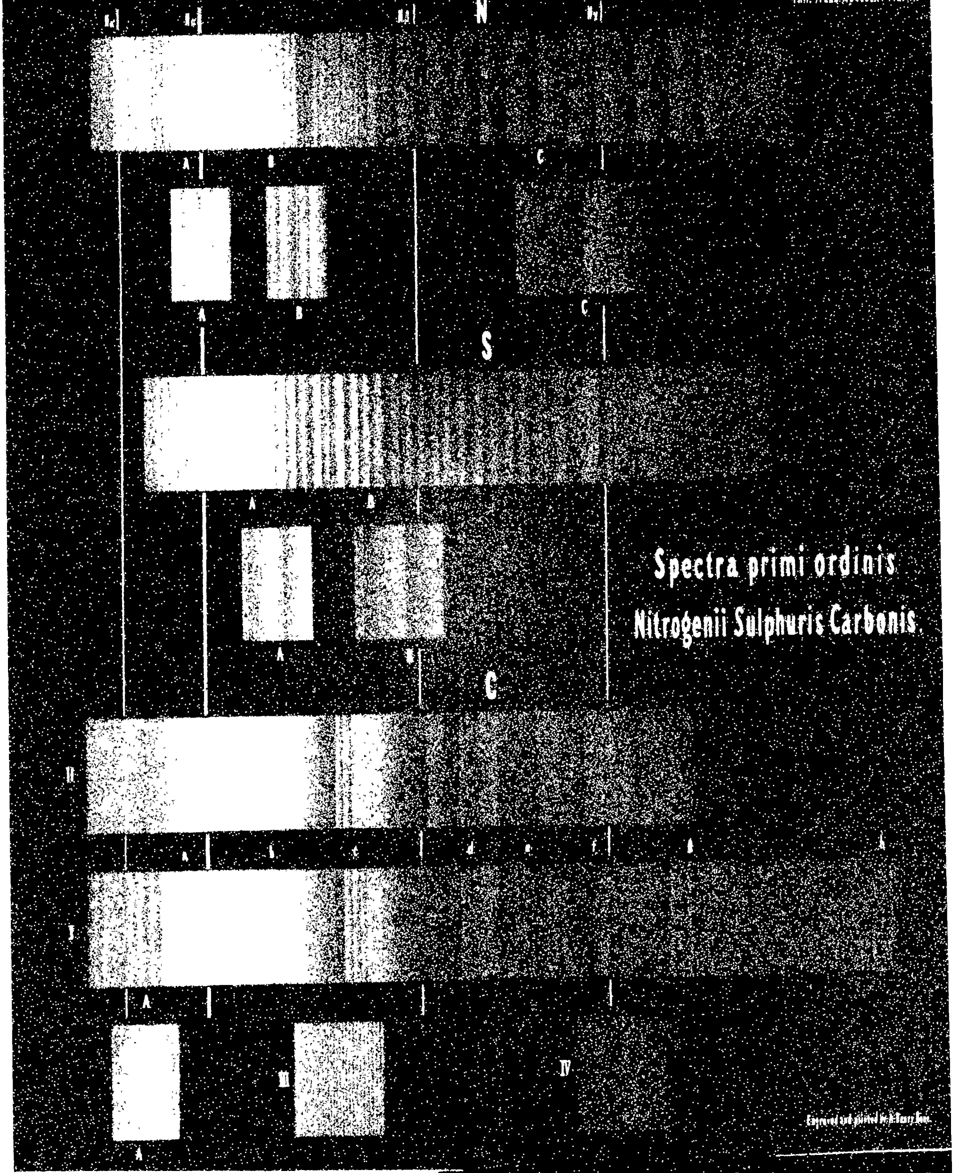
---

* British Association Reports, 1863, p. 169.

The result of this examination showed that the calculated value of L should be multiplied by 1·0618 to obtain the value of L, which would give the most consistent results.

We have therefore L by calculation	430165 metres.
Probable value of L by method of least squares	456748 ,,
Result of rough experiment with the Electric Balance (see § 46)	410000 ,,

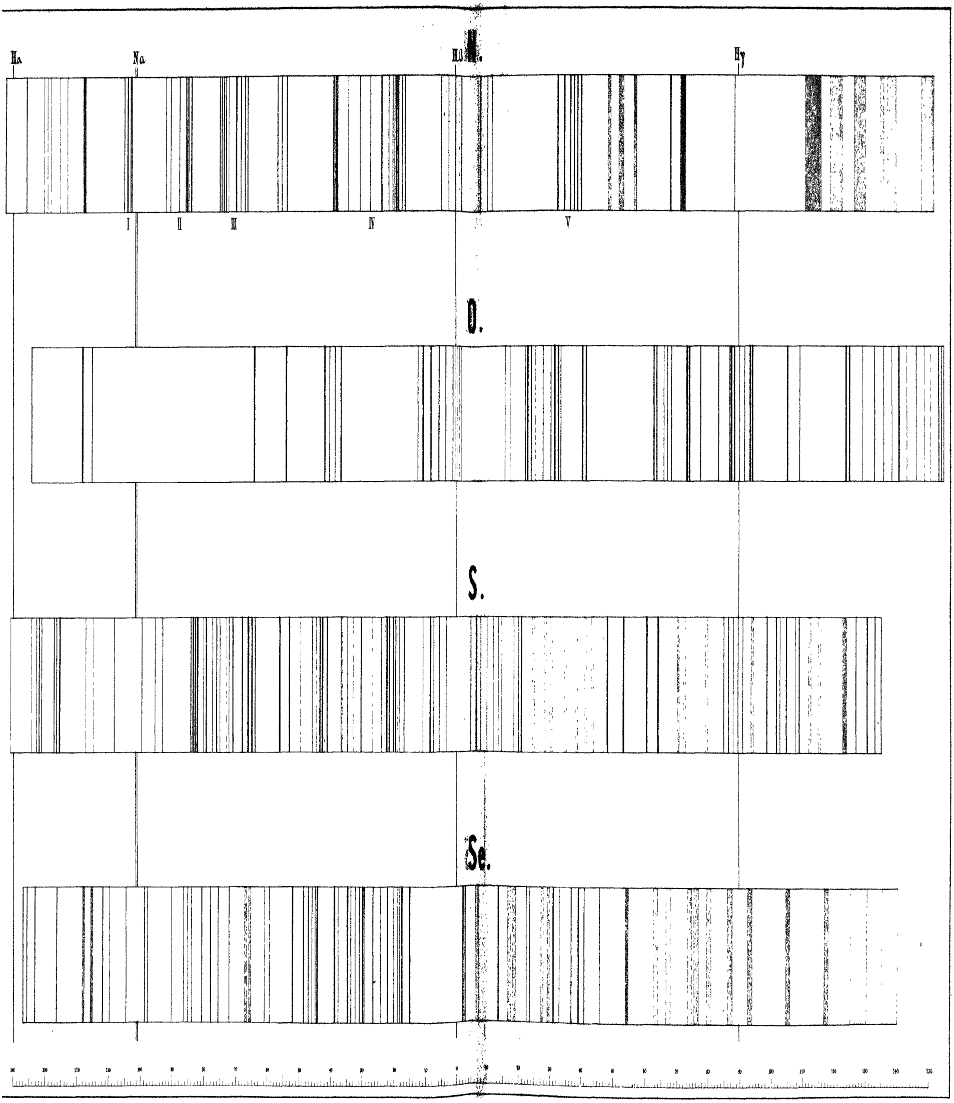
The value of L calculated from the dimensions of the coil is probably much more accurate than either of the other determinations.



Spectra primi ordinis
Nitrogenii Sulphuris Carbonis.

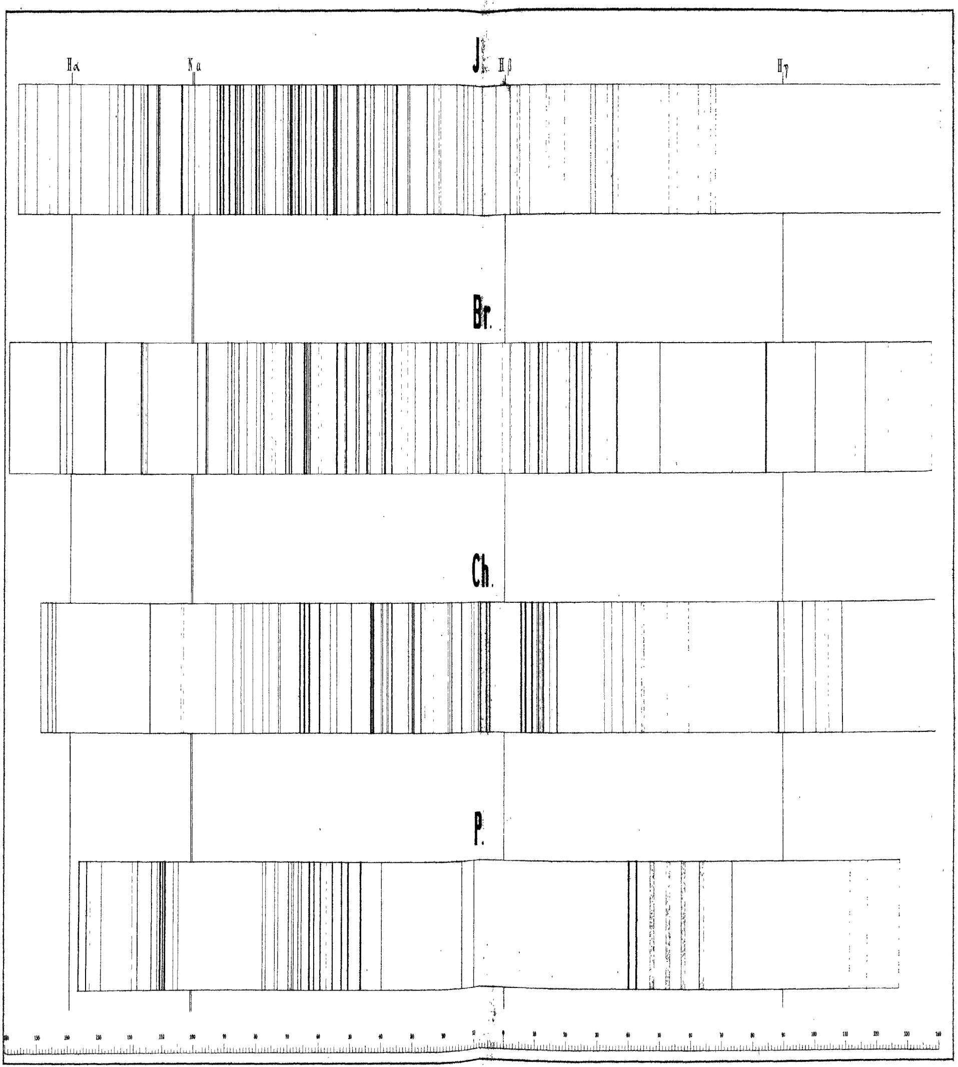
Engraved and printed by [illegible].

Spectra secundi Ordinis Nitrogenii Oxygenii Sulphuris Selenii.

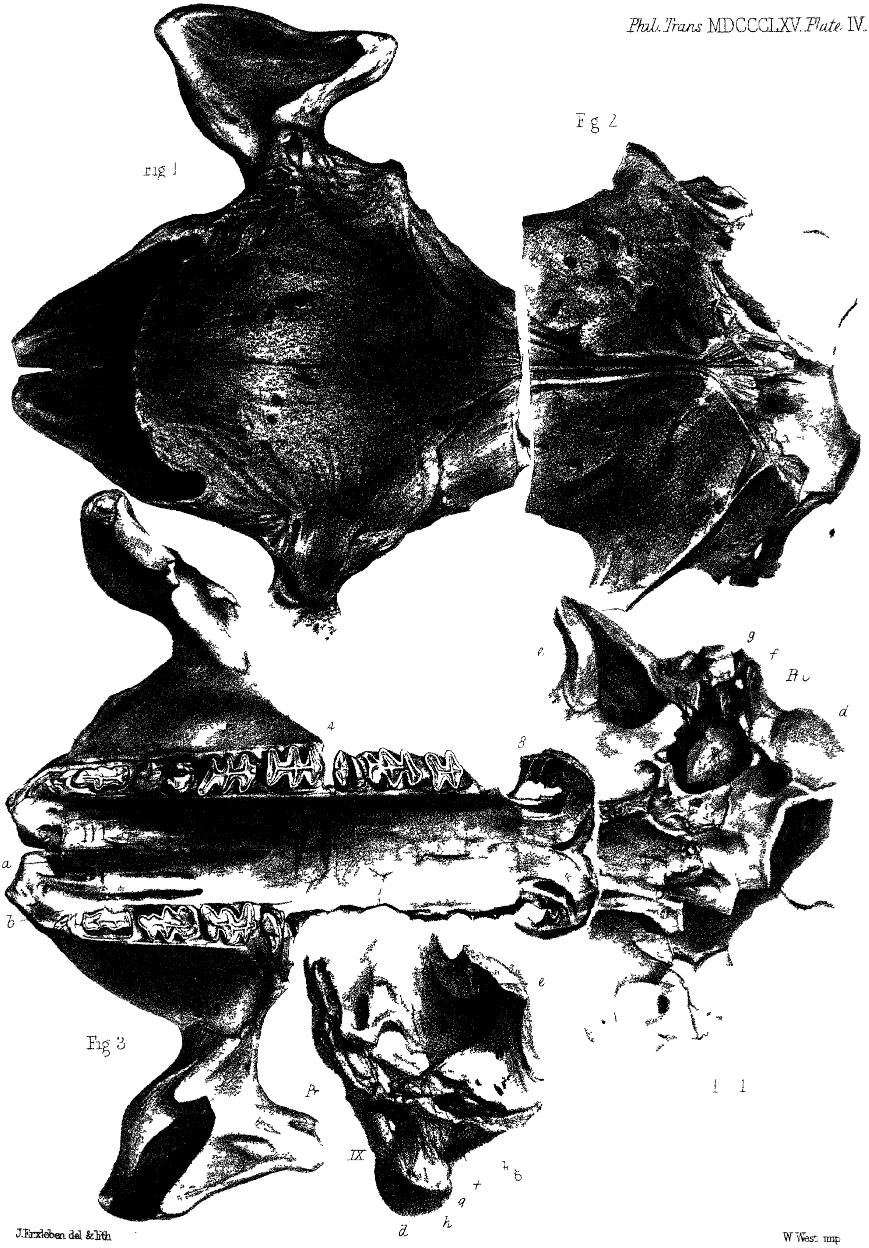


Engraved and printed by A Henry. Bonn.

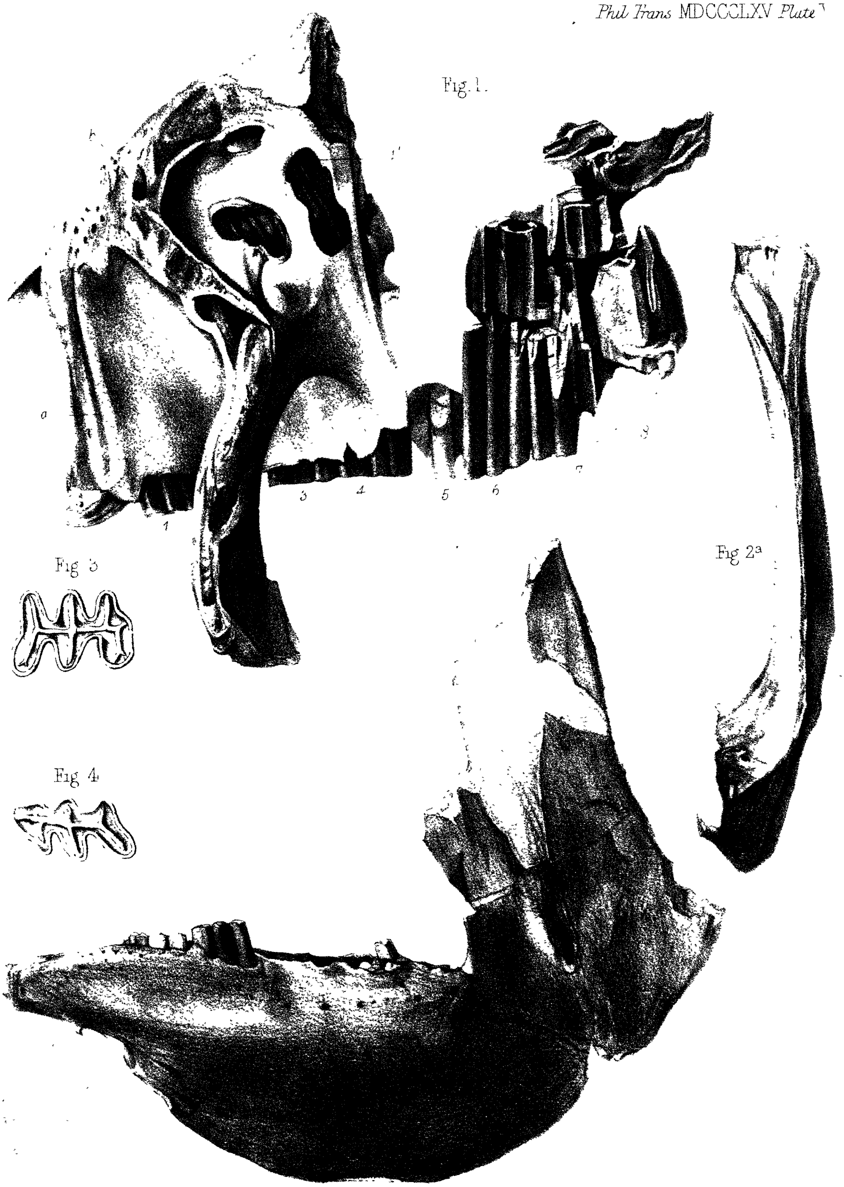
Spectra secundi ordinis Jodi Bromi Chlori Phosphori.



Engraved and printed by A. Henry Bonn.



J. Erxleben del. & lith
W West imp



J Erxleben del & lith

W West imp

Phil Trans MDCCCLXV. *Plate* VI

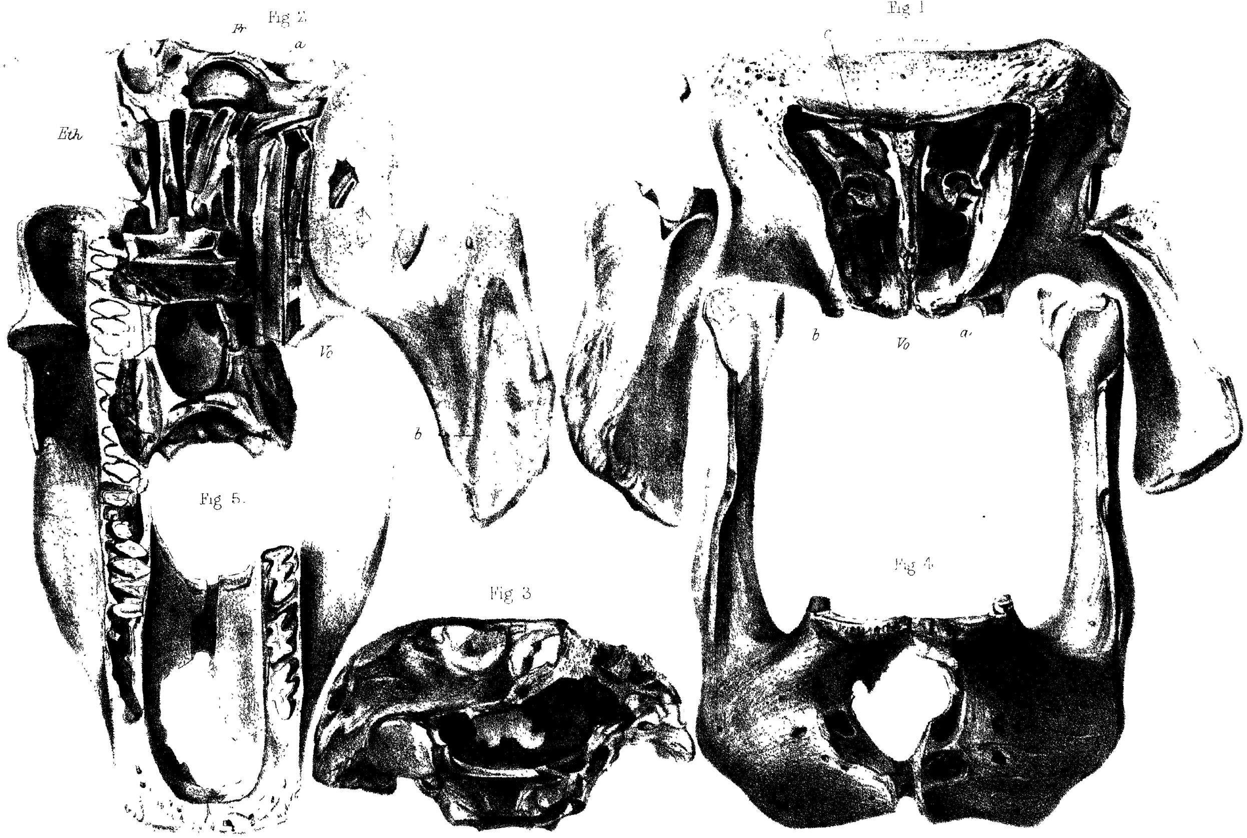


Fig 1
Fig 2
Fig 3
Fig 4
Fig 5.

J Erxleben del & lith
W West imp.

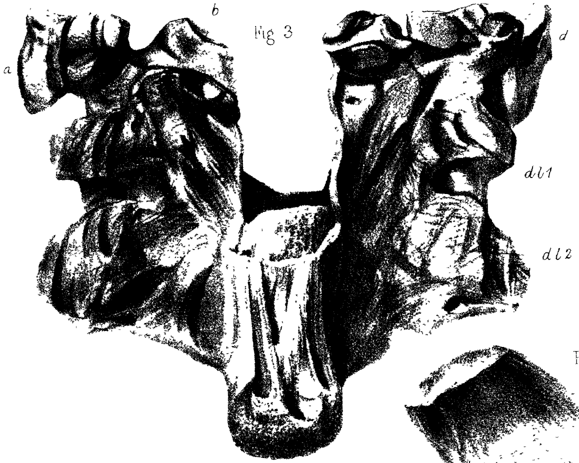


Fig 3

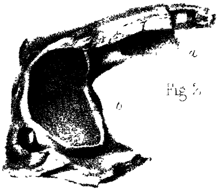
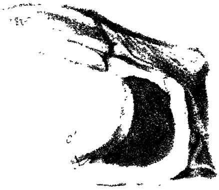


Fig 2



Fig

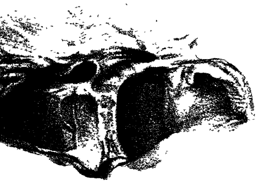


Fig 6

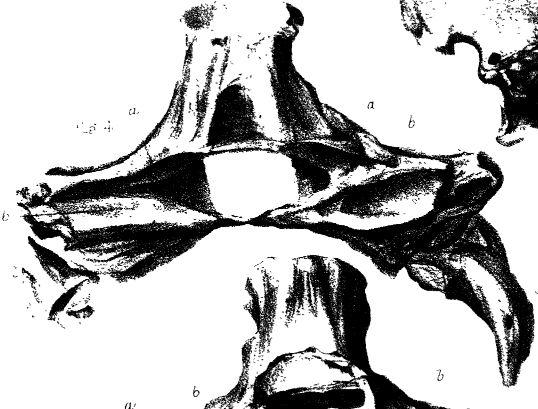


Fig 4

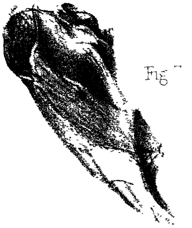


Fig 7

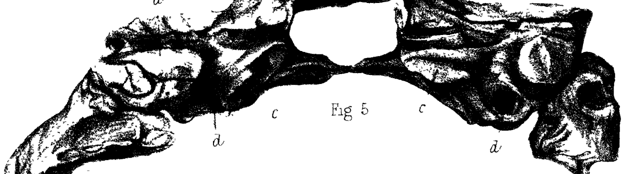


Fig 5

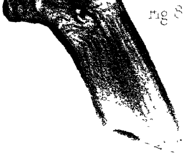


Fig 8

Fig 9

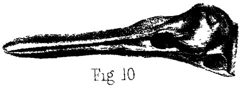


Fig 10

J Erxleben del & lith

W West imp

Phil. Trans. MDCCCLXV Pl. VIII

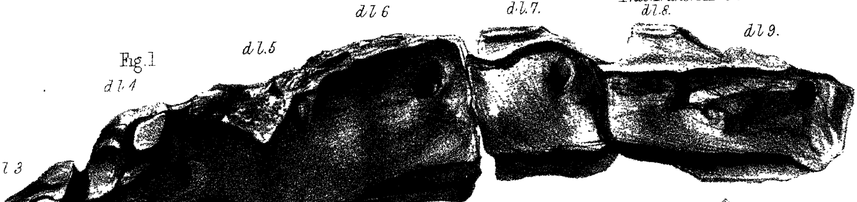
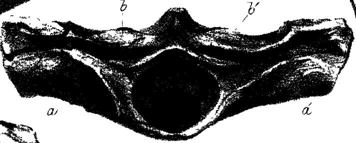


Fig. 1

Fig. 3



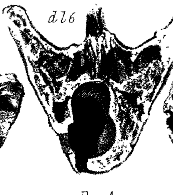


Fig 4

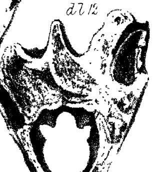


Fig 5

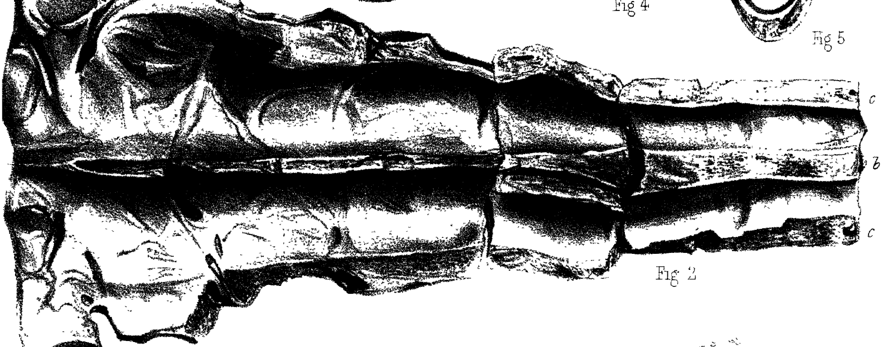


Fig 2

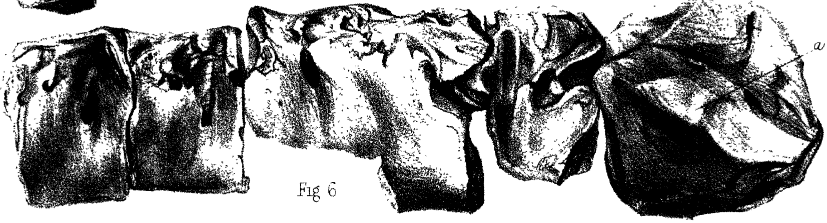


Fig 6

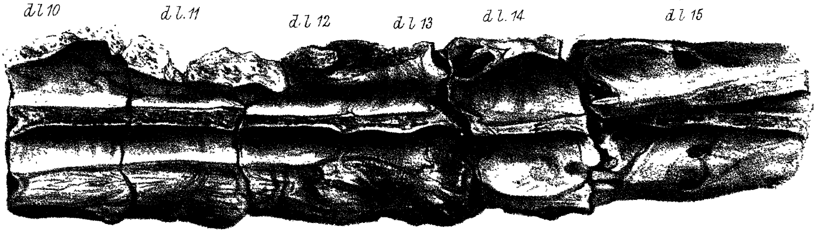


Fig 7

J Erxleben del. & lith
W. West imp.

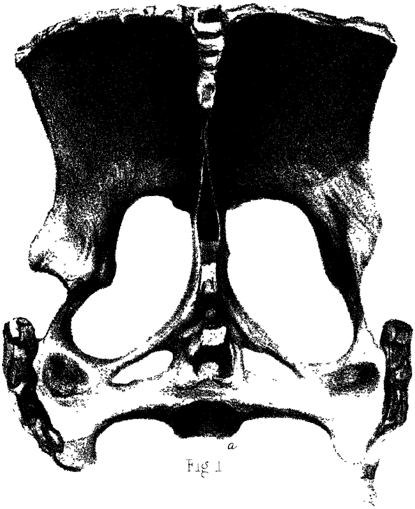


Fig 1

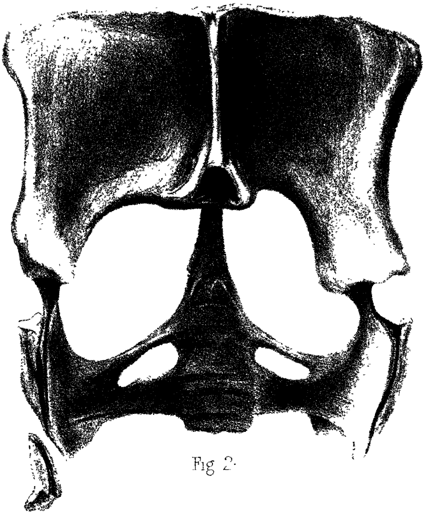


Fig 2

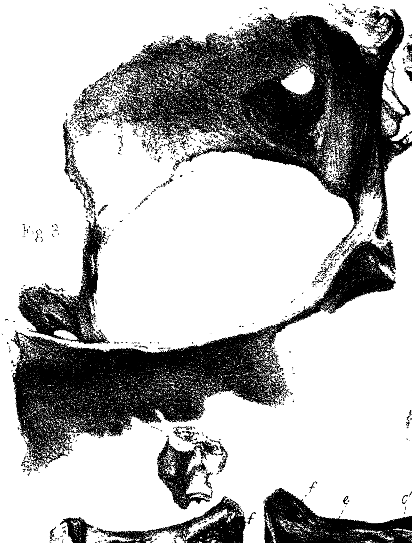


Fig 3

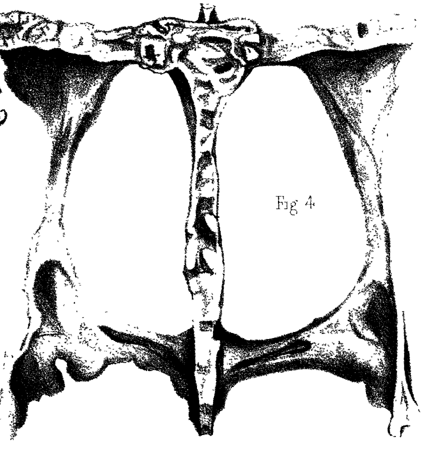


Fig 4

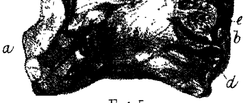


Fig 5

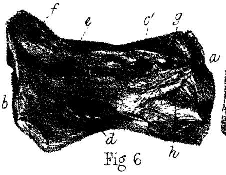


Fig 6

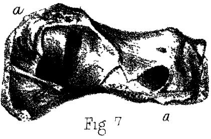


Fig 7

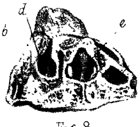
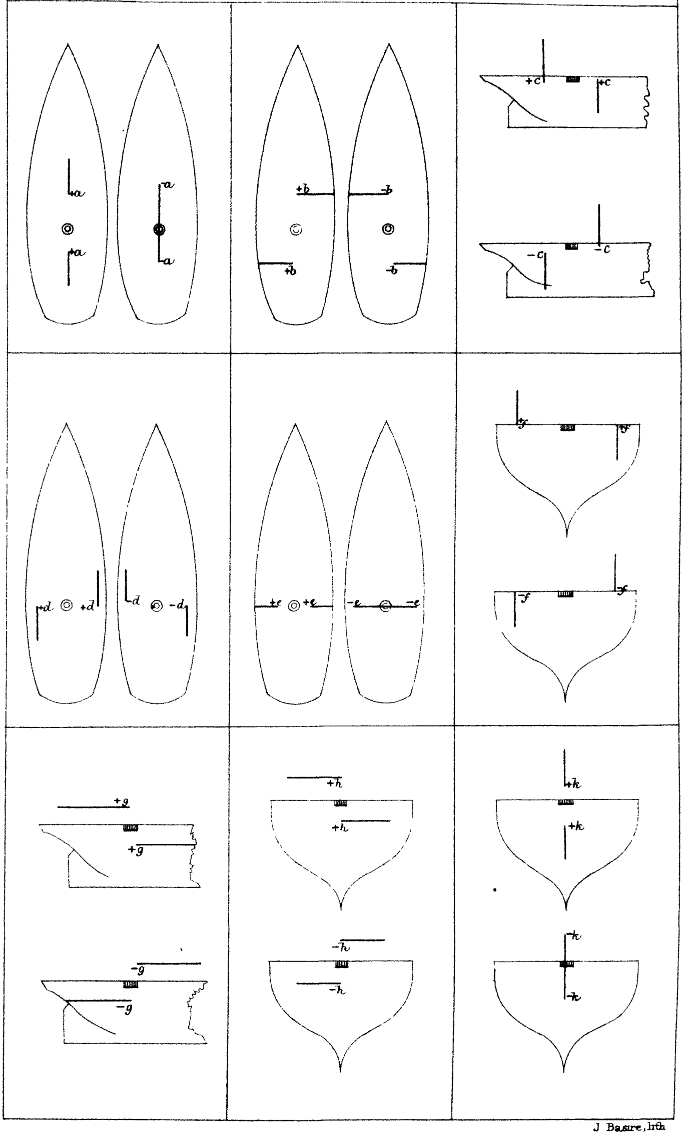


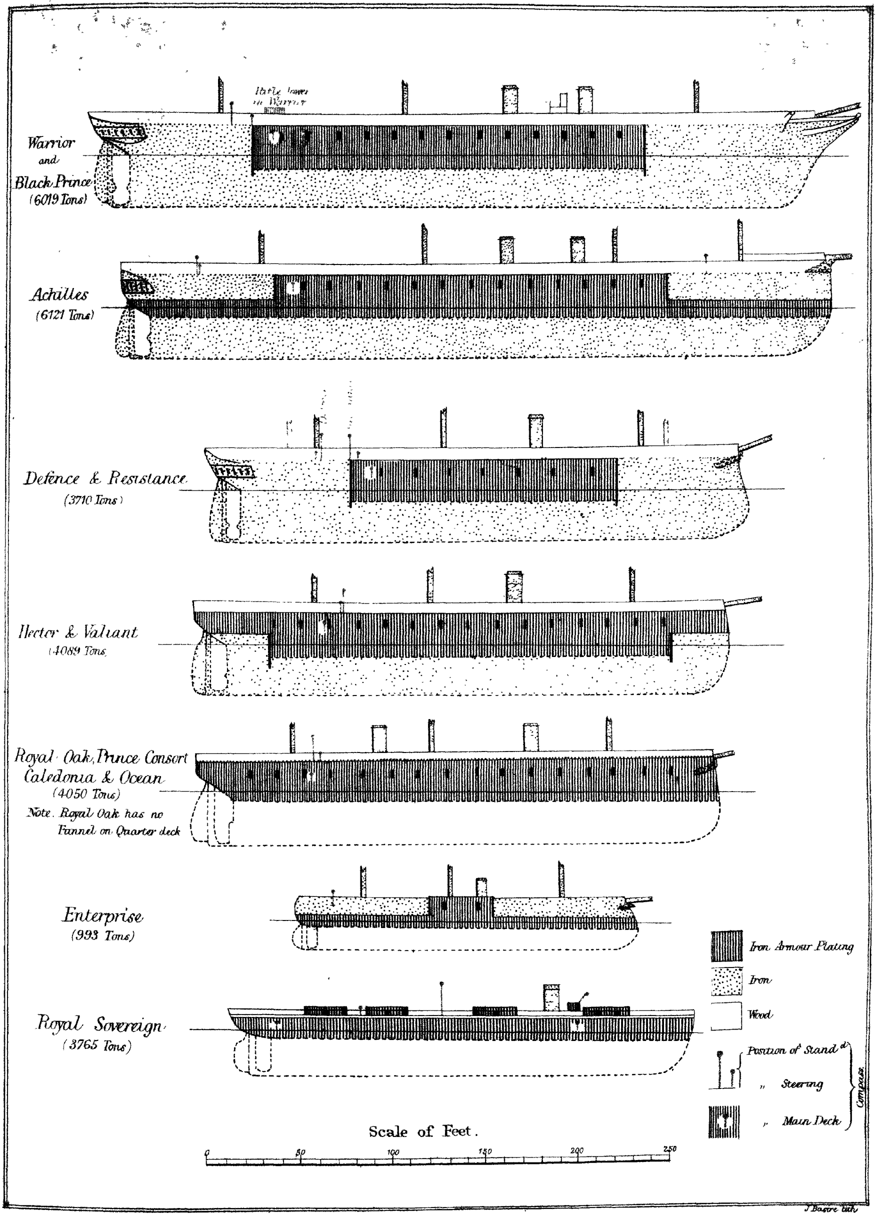
Fig 8

J Erxleben del & lith
W West imp

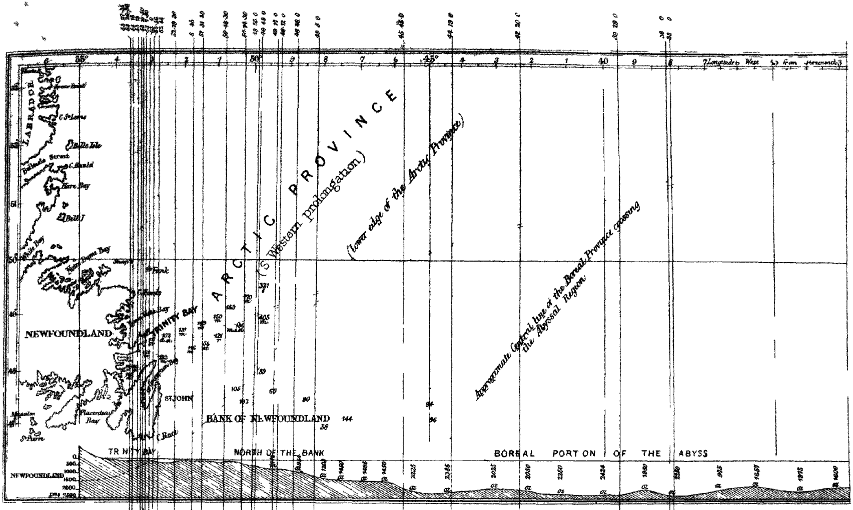
Diagram showing the positions of the nine soft iron rods which represent the whole of the soft iron of a ship as regards its action on the compass.



J Basire, lith


Battle tower in Warrior
Warrior and Black Prince (6019 Tons)
Achilles (6121 Tons)
Defence & Resistance (3710 Tons)
Hector & Valiant (4089 Tons)
Royal Oak, Prince Consort Caledonia & Ocean (4050 Tons)
Note. Royal Oak has no Funnel on Quarter deck
Enterprise (993 Tons)
Royal Sovereign (3765 Tons)
Iron Armour Plating
Iron
Wood
Position of Stand^d
" Steering
" Main Deck
Compass.
Scale of Feet.
0
50
100
150
200
250

J Basire lith.



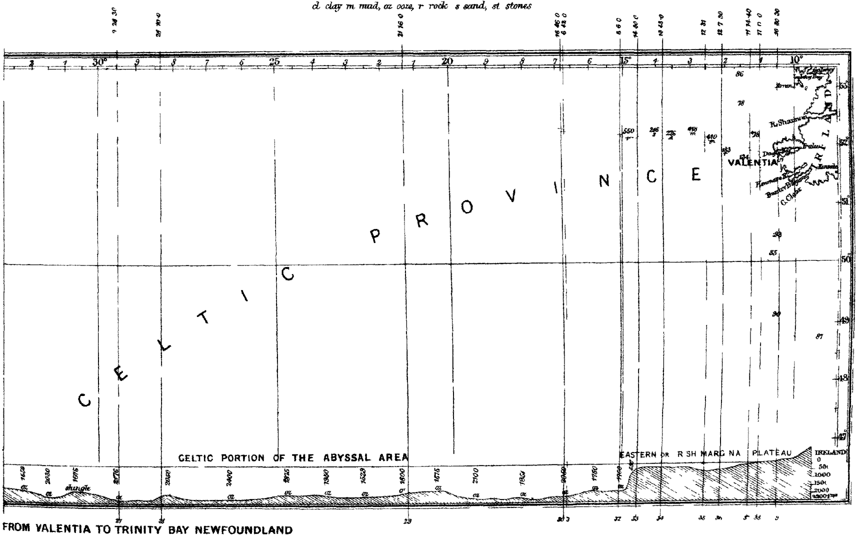
SECTION OF THE BED OF THE ATLANTIC OCEAN

DEEP-SEA-SOUNDINGS IN THE NORTH ATLANTIC FROM IRELAND TO NEWFOUNDLAND,

By Lieut. J. Dayman R.N. *assisted by Mr. J. Scott Mastr. R.N.* H.M.S. Cyclops 1857

SOUNDINGS IN FATHOMS

cl. clay m. mud, oz. ooze, r. rock s. sand, st. stones

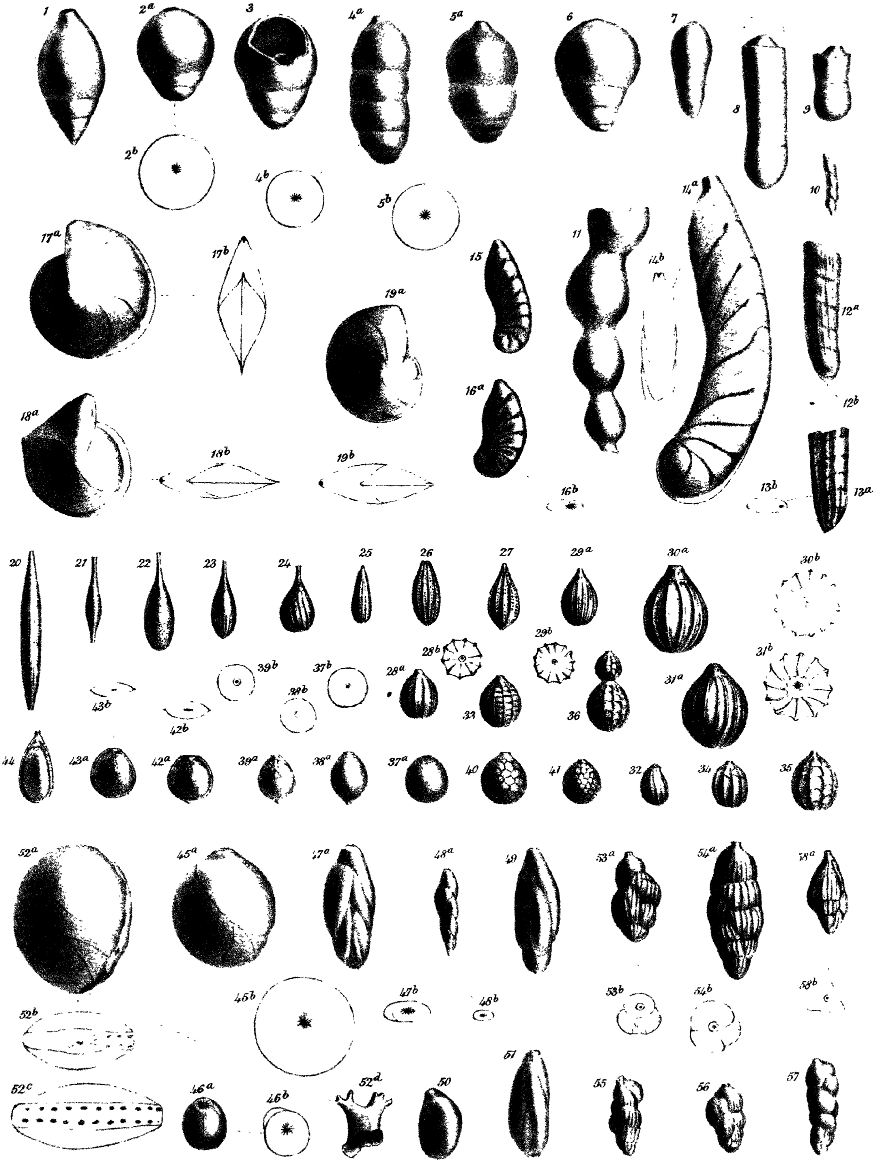


FROM VALENTIA TO TRINITY BAY NEWFOUNDLAND

N.B. The vertical lines numbered 1—39 & bearing measures of Longitude, cross the spots where the 39 Soundings were taken

Vertical Scale 2,000 fms. 1 Inch. Scales as 15 to 1

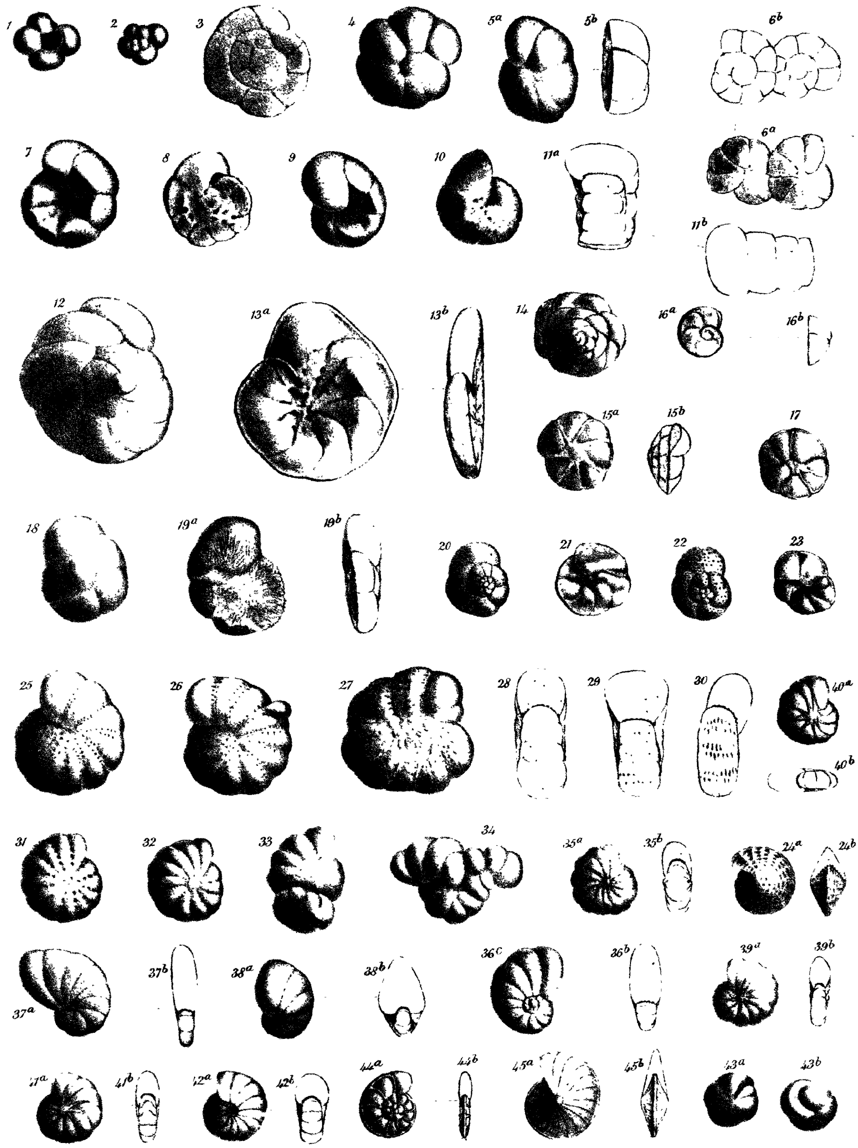
Basire lith.



G.West del

W West imp

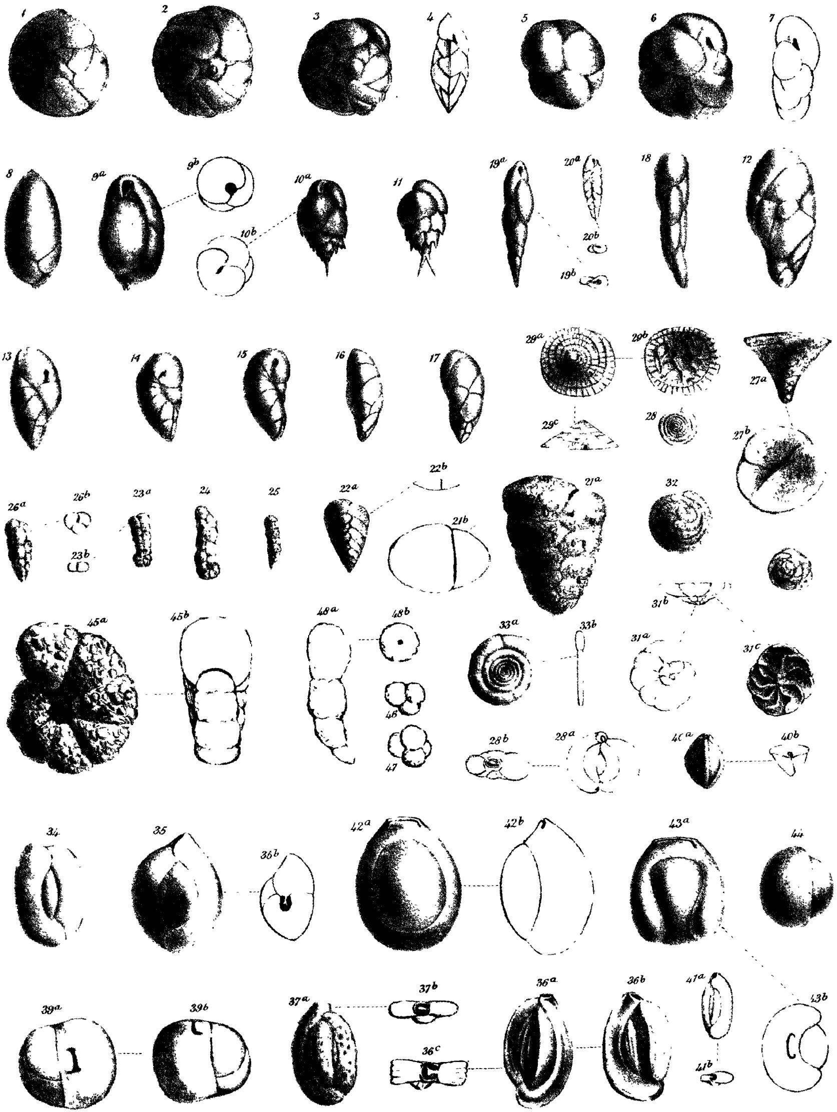
ARCTIC FORAMINIFERA



G West del

W West imp

ARCTIC FORAMINIFERA

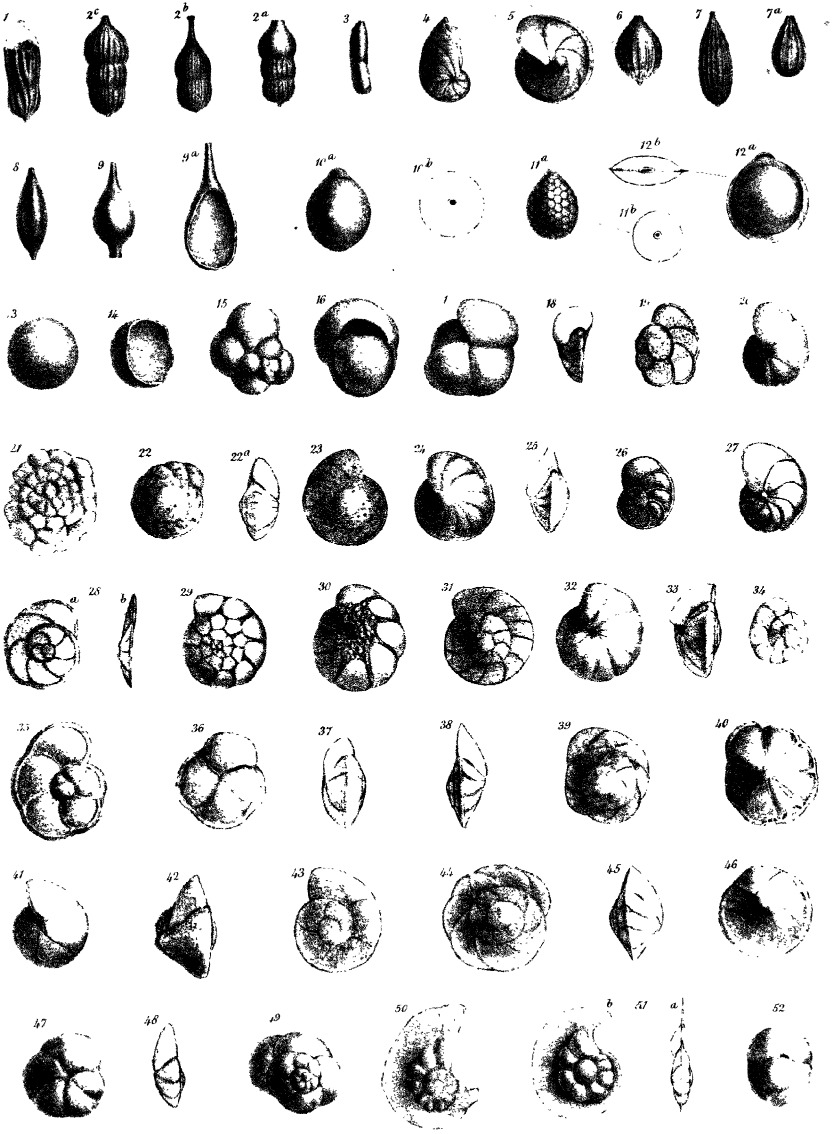


G West del

W West imp

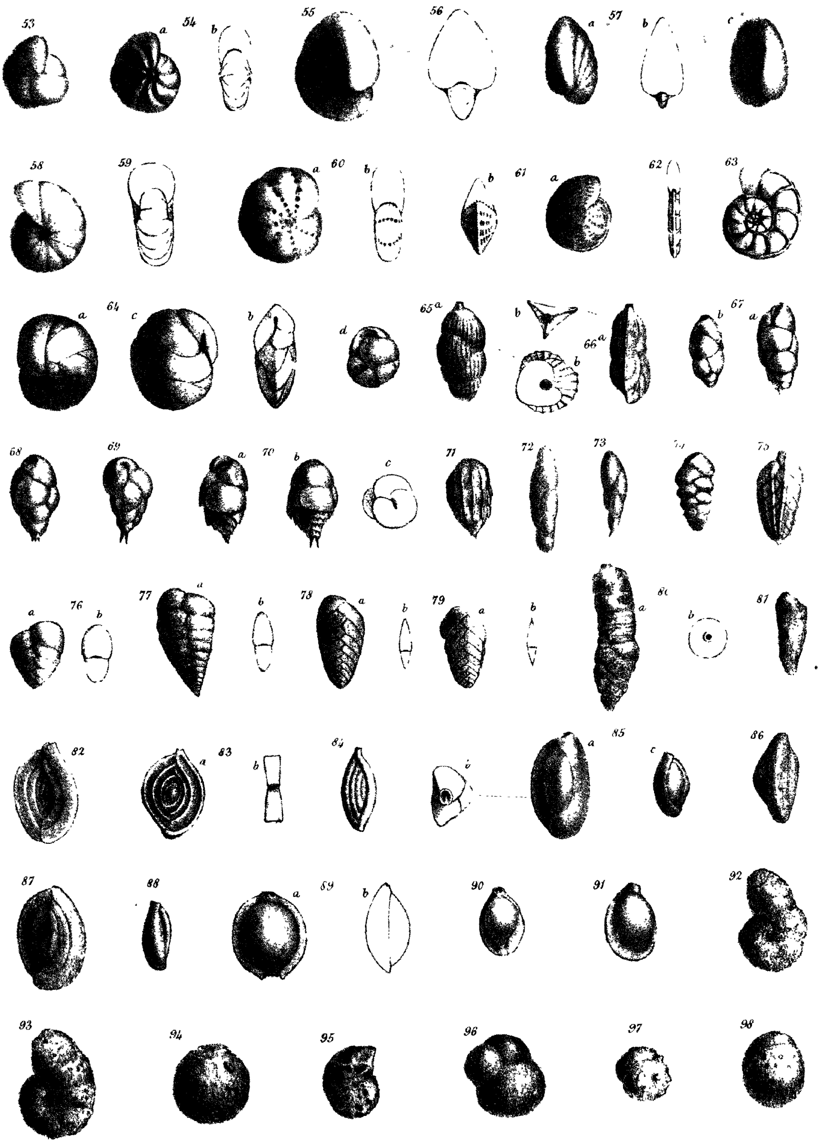
ARCTIC FORAMINIFERA

Phil. Trans. MDCCCLXV Pl. XVI



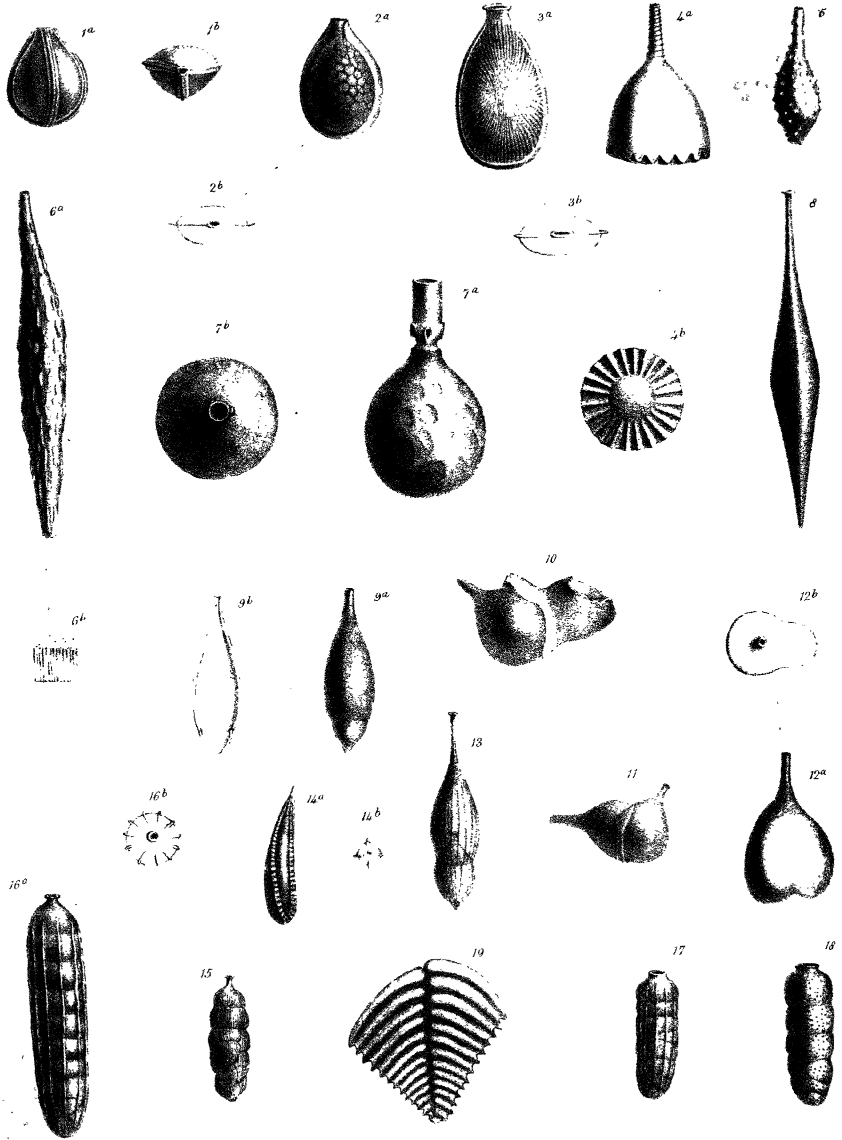
G West del.

NORTH-ATLANTIC FORAMINIFERA



G West del

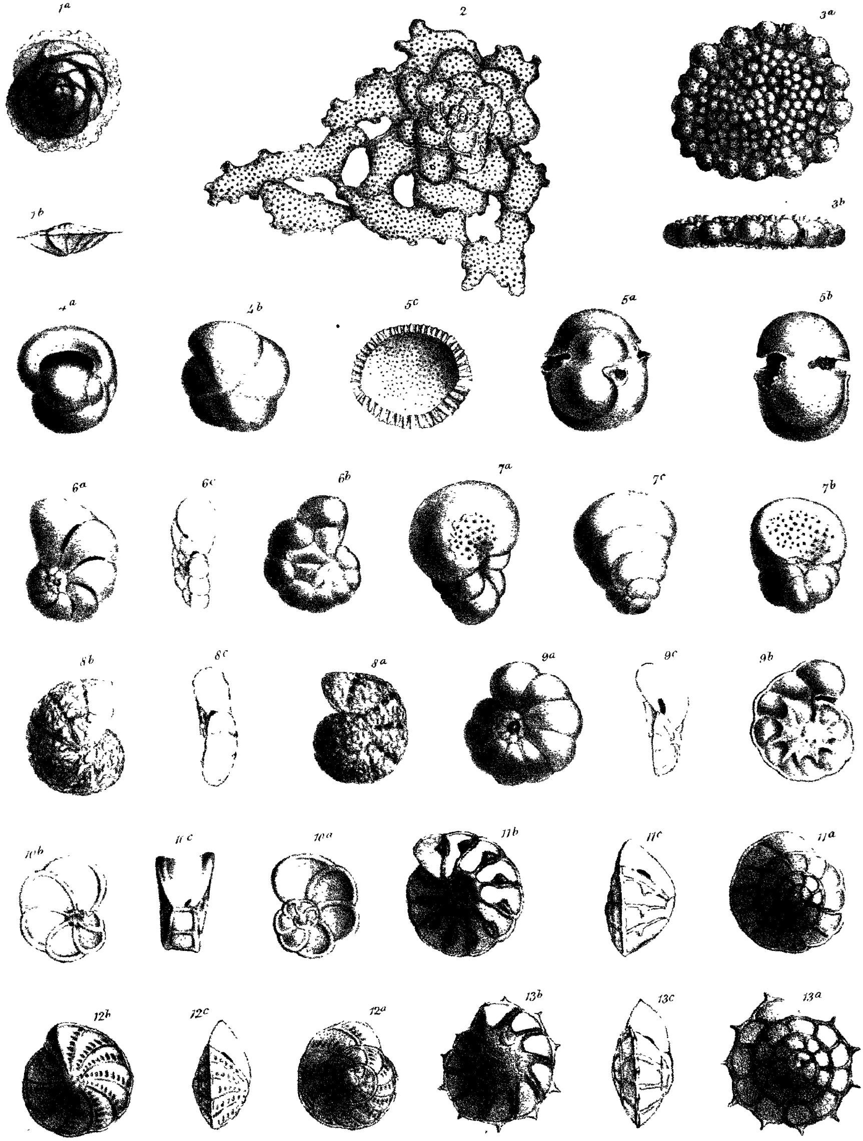
NORTH-ATLANTIC FORAMINIFERA



C West del.

W West imp

FORAMINIFERA



G. West del.

W. West imp.

FORAMINIFERA

Fig 3

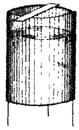


Fig 4

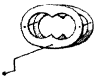


Fig. 5.

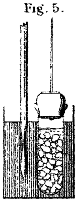


Fig 6

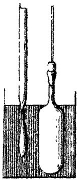


Fig 8

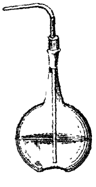


Fig 7

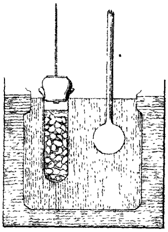


Fig 1

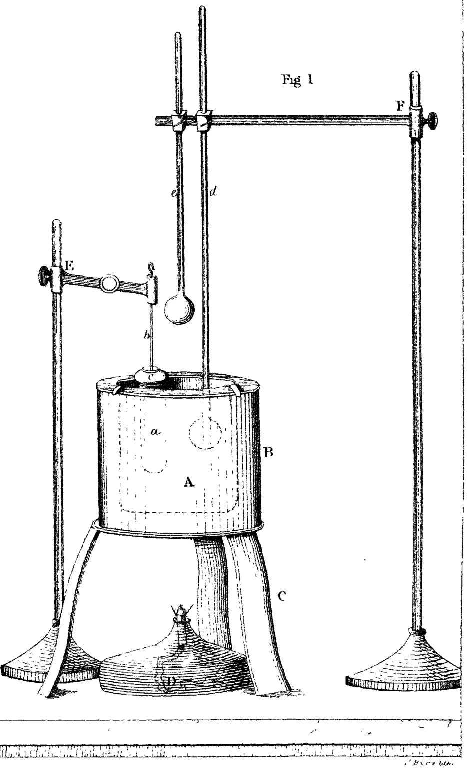


Fig 2

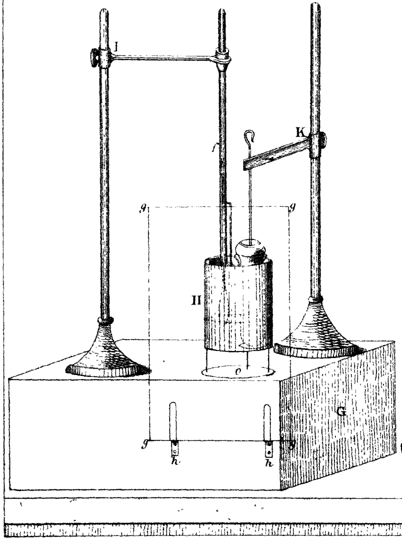
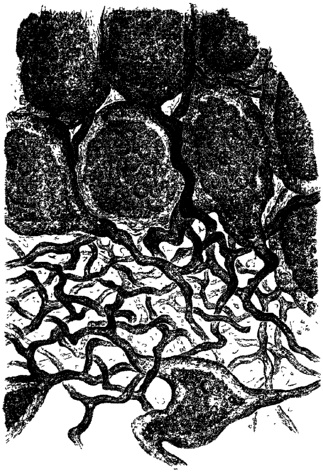
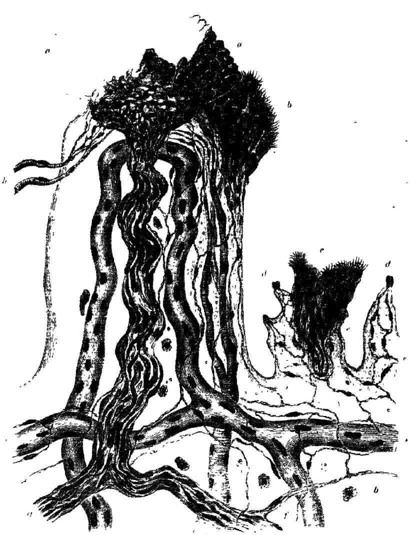


Fig 2.



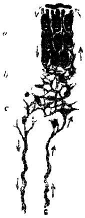
Small portion of the plexus of nerve fibres at the summit of the papilla showing [illegible] with the cells × [illegible] *a* Epithelium [illegible] *d* Triangular cells [illegible] *b* [illegible] matter [illegible] epithelium [illegible] *e* [illegible] corresponding [illegible] *b* [illegible]

Fig 1.



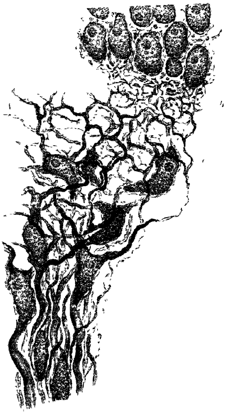
Summit [illegible] and simple papillae [illegible] of the [illegible] *a* Epithelium-like mass at summit of papilla *b* [illegible] of the papilla *c* [illegible] simple papillae *dd* [illegible] *e* with nuclei connected with the nerves [illegible] *e* [illegible] *f* [illegible] *g* *h* Muscular fibres [illegible] *g* [illegible] the finest [illegible] into the [illegible] × [illegible]

Fig. 3

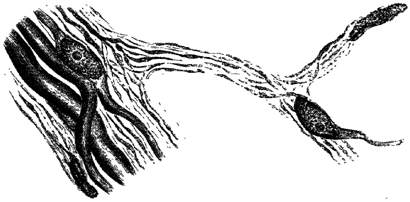


[illegible] arrangement [illegible] the [illegible] summit [illegible]

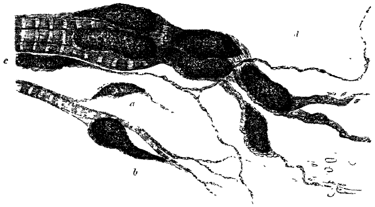
Fig. 4



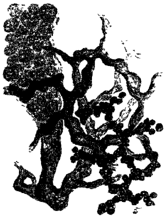
[illegible] arrangement of nerve fibres [illegible] at the top of the papilla [illegible] *a* [illegible] the summit of the [illegible] *b* [illegible] fibres [illegible] *e* [illegible] nerve fibres [illegible] × 1700



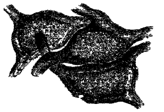
[illegible] distribution of [illegible] muscles [illegible] papilla [illegible] *f* Fig 1 × [illegible]



Muscular fibres at the summit of the papilla showing the relation of the germinal matter to the contractile tissue and the mode of formation of the latter by the masses of germinal matter *a* Germinal matter, or nucleus-forming fibrillæ *b* Another nucleus or mass of germinal matter connected with muscular tissue *c* Germinal matter or nucleus of fine nerve fibre distributed to the muscle near the summit of the papilla *d* Fine nerve fibre × 1800

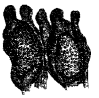


A portion of one of the triangular cells or nuclei connected with the fine nerve fibres, forming the plexus, at the top of the papilla, as seen after the removal of the epithelium-like mass from the summit The fine fibres upon the surface are those which pass to the epithelium like mass on the summit × 1800



Thin cells from the epithelium like mass upon the summit of the papilla × 700

Fig 7



Free surface of the cells of the epithelium like mass at the summit of the papilla × 700

Fig. 10

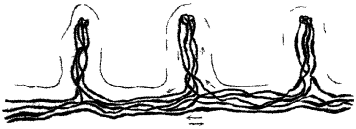
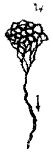
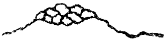


Diagram of three papillæ from the frog's tongue to show the arrangement of the nerve fibres

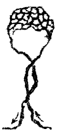


e wh h 4 n fib s pp nma d g to m h

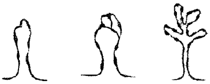
Fig. 1



[illegible] mation of the network finest nerve fib



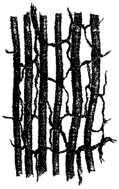
he a th a an m nt fibres an non o h nal n ex w k fin u fibres



[illegible] terminal pa fibre in papillæ

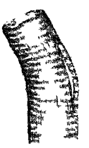
nerve

Fig. 11



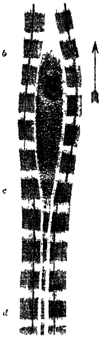
Muscular fibres from tongue w n r fib s ramifying am s hem

Fig. 1



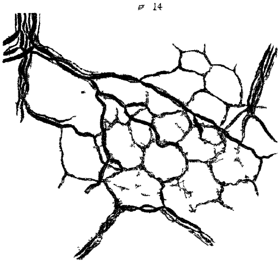
muscular fibre show a a 400

Fig. 1

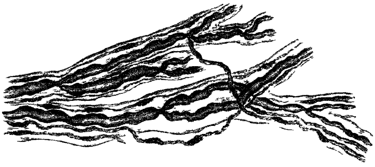


w h mod n wh n s pa h rma o h fib m s b arrow show n vh h the n s ace occ he n an d a mo b

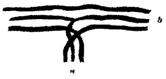
Fig. 14



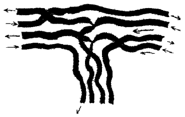
Draw w s L k bres n d d v fi d d v u lo a n c exu is r a n w arran d



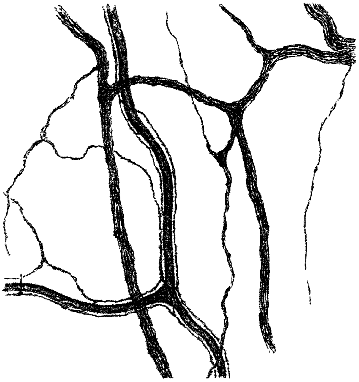
n a k e x fi es ram n h B m g × 400



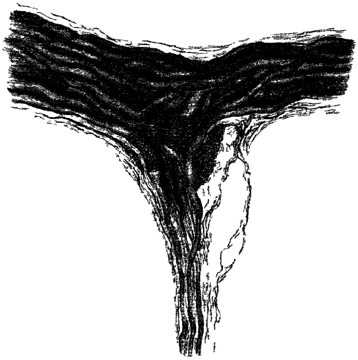
a



n b art a b



From the mylohyoid muscle of the Hyla. Trunks o ns da k bo dered nerve fibres with fine fibres coming from them some o wh h may be raced to the vessels while others are distributed to the muscular fibres wh h are no represented in the drawing. The arrangement o the nerves supplying the capillary es e are we een × 1 0



Fine compound nerve trunk with a bran m n fi composed of fibres which pursue opposite courses h n the submucous u o the pa'a × 0

PHILOSOPHICAL

TRANSACTIONS

OF THE

ROYAL SOCIETY

OF

LONDON.

FOR THE YEAR MDCCCLXV.

VOL. 155.—PART II.

LONDON:
PRINTED BY TAYLOR AND FRANCIS, RED LION COURT, FLEET STREET
MDCCCLXV

CONTENTS.

IX. *On the Embryogeny of* Antedon rosaceus, *Linck* (Comatula rosacea *of Lamarck*). *By Professor* WYVILLE THOMSON, *LL.D., F.R.S.E., M.R.I.A., F.G.S., &c. Communicated by* THOMAS HENRY HUXLEY, *F.R.S.* . . . . page 513

X. *On the Sextactic Points of a Plane Curve. By* A. CAYLEY, *F.R.S.* 545

XI. *A Description of some Fossil Plants, showing Structure, found in the Lower Coal-seams of Lancashire and Yorkshire. By* E. W. BINNEY, *F.R.S.* 579

XII. THE BAKERIAN LECTURE.—*On a Method of Meteorological Registration of the Chemical Action of Total Daylight. By* HENRY ENFIELD ROSCOE, *B.A., F.R.S., Professor of Chemistry in Owens College, Manchester* 605

XIII. *On the Commissures of the Cerebral Hemispheres of the Marsupialia and Monotremata as compared with those of the Placental Mammals. By* WILLIAM HENRY FLOWER, *F.R.S., F.R.C.S., Conservator of the Museum of the Royal College of Surgeons of England* 633

XIV. *On the Sextactic Points of a Plane Curve. By* WILLIAM SPOTTISWOODE, *M.A., F.R.S., &c.* 653

XV. *On the Marsupial Pouches, Mammary Glands, and Mammary Fœtus of the* Echidna Hystrix. *By Professor* OWEN, *F.R.S., &c.* 671

XVI. *On the Influence of Physical and Chemical Agents upon Blood; with special reference to the mutual action of the Blood and the Respiratory Gases. By* GEORGE HARLEY, *M.D., Fellow of the Royal College of Physicians, Professor of Medical Jurisprudence in University College, London. Communicated by Professor* SHARPEY, *M.D., Sec. R.S.* 687

XVII. *On a New Geometry of Space. By* J. PLÜCKER, *of Bonn, For. Memb. R.S.* 725

Index 793

APPENDIX.

Presents . . . . [1]

LIST OF ILLUSTRATIONS.

Plates XXIII. to XXVII.—Professor W. Thomson on the Embryogeny of *Antedon rosaceus*, Linck (*Comatula rosacea* of Lamarck).

Plates XXVIII. & XXIX.—Professor Roscoe on a Method of Meteorological Registration of the Chemical Action of Total Daylight.

Plates XXX. to XXXV.—Mr. E. W. Binney on some Lower-coal-seam Fossil Plants.

Plates XXXVI. to XXXVIII.—Mr. W. H. Flower on the Cerebral Commissures of the Marsupialia and Monotremata.

Plates XXXIX. to XLI.—Professor Owen on the Marsupial Pouches, Mammary Glands, and Mammary Fœtus of the *Echidna Hystrix*.

ADJUDICATION of the MEDALS of the ROYAL SOCIETY for the year 1865 by the PRESIDENT and COUNCIL.

The COPLEY MEDAL to Mons. MICHEL CHASLES, For. Memb. R.S., for his Historical and Original researches in Pure Geometry.

A ROYAL MEDAL to JOSEPH PRESTWICH, Esq., F.R.S., for his numerous and valuable Contributions to Geological Science, and more especially for his Papers published in the Philosophical Transactions, on the general question of the Excavation of River-valleys, and on the Superficial Deposits in France and England in which the Works of Man are associated with the remains of Extinct Animals.

A ROYAL MEDAL to ARCHIBALD SMITH, Esq., F.R.S., for his Papers in the Philosophical Transactions, and elsewhere, on the Magnetism of Ships.

Professor H. E. ROSCOE's Paper, entitled "On a Method of Meteorological Registration of the Chemical Action of Total Daylight," was appointed as the BAKERIAN LECTURE.

The CROONIAN LECTURE was delivered by Professor LIONEL SMITH BEALE, F.R.S.: it was entitled "On the Ultimate Nerve-fibres distributed to Muscle and some other tissues, with Observations upon the Structure and probable Mode of Action of a Nervous Mechanism."

IX. *On the Embryogeny of* Antedon rosaceus, *Linck* (Comatula rosacea *of Lamarck*). *By* Professor Wyville Thomson, *LL.D., F.R.S.E., M.R.I.A., F.G.S., &c. Communicated by* Thomas Henry Huxley, *F.R.S.*

Received December 29, 1862,—Read February 5, 1863*.

In the year 1827 Mr. J. V. Thomson, Deputy Inspector-General of Military Hospitals, described and figured what he believed to be a new recent Crinoid, under the name of *Pentacrinus Europæus*; and in June 1835 communicated to this Society a "Memoir on the Star-fish of the genus *Comatula*, demonstrative of the *Pentacrinus Europæus* being the young of our indigenous species." In this memoir the author describes and figures a series of *Pentacrinus Europæus* from its earliest stage, in which it is represented as "an attached ovum in the form of a flattened oval disk, by which it is permanently fixed to the point selected, giving exit to an obscurely jointed stem ending in a club-shaped head"; to its most perfect attached condition, in which the head is compared with, and found closely to resemble the youngest free *Antedon* taken with the dredge.

The period of the disappearance of the pentacrinoid larvæ on the oar-weed exactly corresponds with that of the appearance of the most minute free *Antedons* in the water. Mr. Thomson's observations were conclusive. I am not aware that they have hitherto been repeated in detail on the European species, but the "pentacrinoid" stage of *Antedon* has ever since been the frequent and familiar prize of the dredger, the wonderful beauty and gracefulness of its form and movements, and its singular relations to the Echinoderm inhabitants of modern and of primæval seas, rendering it an object of ever recurring admiration and interest.

The remarkable discoveries of Professors Sars and Johannes Muller on the metamorphoses of the embryo and its appendages in other Echinoderm orders rendered it probable that the germ of *Antedon* might pass through some earlier transitional stage before assuming the fixed pentacrinoid form.

Dr. W. Busch undertook this investigation, and for this purpose he visited Orkney in July 1849, and procured a supply of specimens in Kirkwall Bay. As those of Dr. Busch are the only recorded observations on the early stages in the embryology of the Crinoids, I shall briefly abstract his results published in Müller's 'Archiv,' 1849, and more fully in his own 'Beobachtungen über Anatomie und Entwickelung einiger wirbellosen Seethiere' (Berlin, 1851).

---

* Subsequently to the reading of this paper it was arranged that the author should take up a somewhat later stage in the development, which he had at first intended to leave to Dr. Carpenter. The paper was accordingly returned to him that it might receive the necessary additions; but no alteration of importance has been made in the description of the earlier developmental stages, which formed the subject of the memoir presented to the Royal Society.

The author alludes to the position of the ovary in *Antedon*, and to the peculiar way in which the impregnated ova remain hanging in bunches from the ovarian aperture. He describes the formation from the segmented yelk-mass of a uniformly ciliated club-shaped embryo, which escapes from the vitelline membrane and swims freely in the water (Beobachtungen, &c., pl. 13. fig. 13). During the next four-and-twenty hours a bunch of long cilia appears on the narrower anterior extremity, and near it, on the side of the embryo which is turned downwards in a state of rest, a small round opening which he regards as the provisional larval mouth. Three slightly elevated ridges now gird the body transversely at equal distances (*op. cit.* pl. 13. fig. 14), and gradually become clothed with long cilia, the smaller cilia disappearing from the intervening spaces. The integument between the first and third ciliated ring becomes inverted into a large oval depression, a fourth ciliated band appears near the posterior extremity of the embryo, and a few delicate areolated calcareous plates are developed within the integument. The embryo now becomes slightly curved, the large oval opening which the author regards as the excretory orifice becomes more distinct in the centre of the ventral surface, and the embryo attains its most perfect larval form (pl. 14. figs. 1 & 2). The form of the larva now rapidly alters; on the ninth day (pl. 14. fig. 3) the posterior extremity has become much enlarged and invested with a thick gelatinous integument. This distended extremity becomes slightly lobed, the anterior bunch of cilia and the posterior ciliated bands disappear, the mouth and anus become indistinct (pl. 14. fig. 5), and at length (pl. 14. fig. 6) a row of four delicate tubes bearing pinnules appears along either side of the larva, the rudiments of the arms of the Crinoid. Dr. BUSCH was unable to pursue his researches further. In many points his observations are inconsistent with those which I have repeated during the last three years with great care, and I believe that he has misconceived the nature and relations of the organs of the larval embryo. Dr. BUSCH'S account of the first appearance of the pentacrinoid form is certainly contrary to my experience; I have been led, however, by inconsistencies in my own observations upon different broods in different seasons, to believe that the mode of development may to a certain extent vary with circumstances. I find, for instance, that when the ova are liberally supplied with fresh sea-water and placed in a warm temperature, the later stages of larval growth are, as it were, hurried over; so that the free larva scarcely attains its perfect form before being distorted by the growing crinoid. In other instances, in colder seasons and in a less favourable medium, the larva reaches a much higher degree of independent development, and retains for a longer period the larval form.

In 1859 I communicated to this Society a short notice (Proc. Royal Society, vol. ix. p. 600) of the earlier stages in the development of *Antedon*. My observations were made upon one or two broods of *Antedon* in a single season. I had an opportunity at that time of tracing carefully the earliest phases in the development of the pseudembryo, but subsequent observations have led me to believe that in some of the later stages the young of *Antedon* were confounded with those of a Turbellarian, which resembled them closely, and which during that season accompanied them in great numbers. These earlier observations were imperfect and hurried in consequence of the difficulty which I then experienced in rearing the young, of their extreme delicacy, and of the rapidity with which they passed through their developmental steps. These difficulties have since been to a certain extent overcome by the frequent repetition of the observations, and by due regulation of the temperature of the tanks and of the supply of food and water.

M. Dujardin has figured* with great accuracy, but without any description, an early stage in the development of the pentacrinoid young of *Antedon Mediterraneus*, Lam., which he observed at Toulon in May 1835. The figure represents the oral valves partially open, with a group of tubular tentacles protruded from the cup. It is highly characteristic.

On the 16th of February, 1863, Professor Allman communicated to the Royal Society of Edinburgh† a paper "On a Prebrachial stage in the development of *Comatula*." The author procured a single specimen of the stage represented by Dujardin, and in Plate XXVI. of the present memoir, among the refuse of a dredging boat on the coast of South Devon. Dr. Allman describes this minute Crinoid as consisting of a body and a stem; the body formed of a calyx covered by a pyramidal roof. The calyx is composed of five large separate plates. Between the lower edges of these plates and the summit of the stem, there is a narrow zone, in which "no distinct indications of a composition out of separate plates can be detected." Between the upper angles of every two contiguous plates there may, with some care, be made out a minute intercalated plate. The pyramidal roof which closes the cup is composed of five large triangular plates, each supported by its base upon the upper edge of one of the large plates of the calyx, and with the small intercalated plates encroaching upon its basal angles. Long flexible appendages or cirri rise out of the calyx, and in the expanded state of the animal, are thrown out between the edges of the five diverging plates of the roof. Dr. Allman counted fourteen of these appendages, but could not determine their exact number. "They appear to be cylindrical with a canal occupying their axis; as far as they can be traced backwards they are seen to be furnished with two opposite rows of rigid setæ or fine blunt spines. Between every two opposite setæ a transverse line may be seen stretching across the cirrus, and indicating its division into transverse segments." The author never succeeded in tracing these appendages to their origin. Besides these long extensile cirri, there is also an inner circle of short apparently non-extensile appendages. It was only occasionally that the author succeeded in getting a glimpse of these. "They appear to constitute a circle of slightly curved rods or narrow plates probably five in number, which arch over the centre and are provided along their length with two opposite rows of little tooth-like spines. They seem to be articulated to the upper or

---

* Suites à Buffon. Zoophytes Echinodermes, par M. F. Dujardin et par M. F. Hupé. Paris, 1862.

† Transactions of the Royal Society of Edinburgh, vol. xxiii.

ventral side of the calyx by their base, and may be seen in a constant motion, which consists in a sudden inclination upon their base towards the centre, followed immediately by a resumption of their more erect attitude." The interior of the calyx is occupied by a reddish-brown visceral mass, obscurely visible through the walls. The author did not succeed in getting a view of the mouth, and detected no anal aperture. Dr. ALLMAN accurately describes the general structure of the stem (*loc. cit.* p. 243); he conceives, however, that "the multiplication of the segments of the stem seems to take place by the division of the pre-existing ones, and this division seems indicated by the transverse ridges, which in several of the segments may be seen running round the centre."

A detailed description of the developmental stage which forms the subject of Dr. ALLMAN'S communication will be found at pp. 525 & 526 of the present memoir. It is unfortunate that so able an observer had not an opportunity of making himself fully acquainted with this interesting form by the study of a sufficient number of specimens.

In 1856 Professor SARS communicated to the Seventh Meeting of the Scandinavian Association a most interesting paper on the Pentacrinoid stage of *Antedon Sarsii* (DUBEN and KOREN). The only specimen observed was dredged on the 14th of March with *Halichondria ventilabrum*, from a depth of 50 fathoms near Bergen. It was in every respect a fully developed *Antedon*, from the centre of whose centro-dorsal plate proceeded a long thin cylindrical articulated stem attached inferiorly to the sponge. The disk with its central mouth, the long, cylindrical, excentric anal tube, the radial grooves, the ten arms with their characteristic articulations and syzygies, the pinnules with their tentacles, the rows of red-brown spots on the margins of the grooves on the arms and pinnules, and the dorsal cirri, were completely developed as in the adult form. All the arms were unfortunately broken, the portions left bore nine to ten pairs of pinnules. Six of these were of the ordinary form; the three or four proximal pairs, which alternated less regularly, were setaceous, destitute of tentacles and pigment spots, the innermost pair longer than the others, as in the adult; all the pinnules were attenuated, the generative element being as yet undeveloped. The dorsal cirri, twenty to thirty in number, were thickly set round the circumference of the centro-dorsal plate. They were fully formed, and the joints and terminal claws had the form characteristic of *A. Sarsii*. The stem was 20 millimetres in length, and consisted of thirty-one joints; but as it was broken from its place of attachment, some of the inferior joints may have been lost. The two or three lowermost joints preserved became shorter towards the base, and the upper joints towards the attachment of the stem to the centro-dorsal plate decreased likewise in length; the second joint was about half the length of the third, and the first only half that of the second; but the first joint was dilated upwards to its insertion. The middle joints of the stem are three to three and a half times longer than wide, and are all dice-box shaped like the joints of the dorsal cirri of the species.

From this observation it would appear that the development of *A. Sarsii* is continued to a much later period in the pedunculated condition than that of *A. rosaceus*; the disengagement of the latter species from its stem constantly occurs between the middle of August and the middle of September. The capture of the specimen described by SARS in March would seem to indicate that the development of the Pentacrinoid of *A. Sarsii* extends over nearly a year.

The early portion of the history of the development of *Antedon* described in the following pages divides itself naturally into two stages.

The Echinoderms present in the most marked degree a peculiarity which seems to be only imperfectly indicated in the other invertebrate subkingdoms. This peculiarity consists in the successive development from a single egg, of two organisms, each apparently presenting all the essential characters of a perfect animal. These two beings seem to differ from one another entirely in plan of structure. The first, derived directly from the germ-mass, would appear at first sight to homologate with some of the lower forms of the Annulosa; the second, subsequently produced within or in close organic connexion with the first, is the true Echinoderm. The extreme form of this singular cycle, in which the development of a provisional zooid as a separate, independent, living organism, is carried to its full extent, is by no means constant throughout the whole subkingdom, although its existence has been established for all the recent orders. In each order it appears to be exceptional, and in certain cases it is known to be carried to its most abnormal degree in one species, while in a closely allied species of the same genus the mode of reproduction differs but slightly from the ordinary invertebrate type.

To avoid ambiguity in the discussion of such singular relations, I believe it is necessary to introduce certain new terms. For an organism which possesses all the apparent characters of a distinct animal, which is developed from the germ-mass, and which maintains a separate existence before the appearance of the embryo, I would propose the term *pseudembryo*; and for all the appendages which homologate with the whole or with parts of such a pseudembryo, even although they do not assume fully the characters of a distinct animal form, I would propose the term *pseudembryonic appendages*. The same prefix may distinguish the organs of the temporary zooid, where such exist, *pseudostome*, *pseudocele*, *pseudoproct*, &c. The reason for the retention of this series of terms, and for the rejection as applied to the provisional organism of the ordinary terms "embryo" and "larva," will be fully discussed hereafter.

The first stage includes the development, structure, and life-history of the pseudembryo.

While the special external form of the pseudembryo is still perfectly retained, and while its special functions are still in full activity, the form of the pentacrinoid embryo is gradually mapped out within the provisional zooid, and the permanent organs of the embryo are differentiated within its sarcode-substance. The pseudembryo then becomes gradually distorted by the embryo developing within it, its special assimilative and locomotive organs disappear, and the external layer of its sarcode-substance subsides into the general integument of the embryo, still retaining sufficiently the histological characters of the pseudembryonic integument to leave no doubt that it is simply produced by its modification and extension.

From the appearance of the first traces of the permanent embryonic structures within the pseudembryo, the development of the pentacrinoid larva advances steadily; and there is no natural separation into stages of its subsequent progress until the young *Antedon* drops from the larval stem. At one period, however, during the development of the pentacrinoid there is a marked change in the external form and in the anatomical relations of the larva, owing to the sudden widening out of the radial portion of the disk, and the breaking through of the anal opening. Division of labour has been found expedient in the present investigation, and my portion of the task ends just before the development of the Pentacrinoid has reached this point. I think it only right, however, to mention that Dr. CARPENTER, who has been at the same time working out the later stages in the development of the Pentacrinoid and the structure of the mature *Antedon*, has most freely communicated to me all his results. My description of the development of the pentacrinoid larva has had therefore all the advantage of the light thrown upon the earlier stages by Dr. CARPENTER'S researches on the later.

The observations whose combined results have been condensed into the present communication have extended over the last four years. I have had an opportunity each season of watching the more or less favourable development of one or two sets of embryos. As stated above, these observations have not in all cases thoroughly tallied; their inconsistencies depending, I believe, in some instances upon error of observation, and in others upon actual discrepancies in the process of development under different circumstances. In Arran, in June 1860, I had a most favourable opportunity of tracing a single brood from the segmentation of the yelk almost to the maturity of the pentacrinoid young. I took the opportunity to revise and check previous special observations; and each stage of the development of this group was described and figured with great care, and with the advantage of previous familiarity with the successive modifications in form. To avoid all possibility of confusion, I have incorporated in the following detailed description those results only which were confirmed by these later observations; and all the figures of the free pseudembryos, and of the origin of the pentacrinoid form, refer to the successive stages in the development of this single brood. On this occasion the pseudembryos remained for perhaps a somewhat shorter time than usual in their free condition, and their growth was early arrested by the development of the permanent calcareous plates. The pseudembryos, however, during their brief independent existence, attained their perfect and usual external form; and the subsequent transitions, though rapid, were normal.

The ovaries of *Antedon* have been frequently described. During the latter part of summer, autumn, and early winter they can only be traced as delicate lines of whitish stroma, beneath the integument of the upper (oral) surface of the pinnules, and immediately beneath the tentacular canals which in the ordinary condition of the pinnules lie in the groove of the calcareous joints. About the end of February or the beginning of March, the integument of the pinnules becomes slightly turgid; and this turgescence increases till towards the end of May or the beginning of June, when the eggs are fully formed.

The mature ovaries are short, entire, fusiform glands distending widely the integument of the pinnules, and provided with a special aperture which perforates the distended skin on that side of the pinnule which is turned towards the end of the arm. The aperture is bounded by a somewhat thickened ring of apparently elastic tissue, which acts as an imperfect sphincter. Examining the ovary by compression shortly after it has begun to enlarge, the meshes of the stroma (Plate XXIII. fig. 1) are found to contain a clear mucilaginous protoplasm with minute ova in various early stages of development. Tracing the development of the ova, the formative fluid first becomes slightly opalescent, and a minute, highly refractive, lenticular body makes its appearance, which subsequently declares itself as the germinal spot. This body remains some time slowly enlarging without much further change. A delicate film now rises from one side of it, and this film gradually extends till the germinal spot appears to be attached to the inner wall of a spherical cell with perfectly transparent fluid contents, the germinal vesicle (Plate XXIII. fig. 2, *a–c*). The blastema in the neighbourhood of the germinal vesicle becomes slightly granular, and the granules accumulate so as to form a distinct granular layer round the cell. This layer, the nascent yelk, is shortly found to be invested by a delicate vitelline membrane; but this membrane does not appear to originate from the germinal vesicle as a nucleus, as in the case of the latter from the germinal spot. The impression rather is that the surrounding fluid is influenced to a certain distance by the chemical forces acting in the germinal vesicle, and that a membrane is produced at the point of junction between the blastema so influenced and the general fluid contents of the ovary. The egg now increases in size without much further change in structure. The vitelline membrane rapidly expands (Plate XXIII. fig. 2, *d*-*o*), and its contents become more dense, till at length it has attained a diameter of about ·5 millimetre, and is entirely filled with a yelk-mass composed of oil-cells of the usual form.

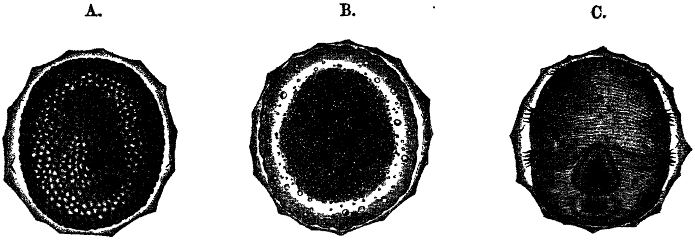
The ripe eggs are now discharged from the ovary; they remain, however, for some time (in some cases three or four days) entangled in the loose stroma of the ovary, and hanging from the ovarian aperture like a bunch of grapes.

The testis resembles the ovary in form and situation. A transparent mucus distends the integument of the pinnule. The fluid becomes opalescent, then granular, and finally the cavity becomes filled with a mass of fusiform parent cells (Plate XXIII. fig. 4). The contents of these cells are at first perfectly transparent; soon, however, they lose their transparency and become granular, and at length the cells are found to contain a progeny of ten or twelve minute spherical "vesicles of evolution." Bright refractive spots, the heads of the spermatozoa, three or four in number, appear in each of these secondary cells; and finally, the walls of the parent cells and vesicles give way, and the cavity of the pinnule is filled with a mucilaginous liquid charged with myriads of mature spermatozoa (Plate XXIII figs. 5 & 6).

The form of the spermatozoon is intermediate between that of a club on cards and a spade (Plate XXIII. fig. 7), with a vibratile filament of great length attached to the obtuse end. There is no special opening to the testis, so that the female may be at once distinguished by the ovarian aperture. The seminal fluid seems to be discharged by the thinning away and dehiscence of the integument. The spermatozoa are dispersed in the water. Impregnation appears to take place after the discharge of the ova, but while they are still hanging from the ovarian aperture.

An hour or two after impregnation the germinal vesicle disappears, or at all events leaves its former superficial position. The yelk-mass contracts and becomes more opaque and dense, leaving a clear space immediately within the vitelline membrane, which is thus more clearly defined, perfectly transparent and structureless, with the surface slightly and irregularly echinated (Plate XXIII. fig. 8). Consequently on the contraction of the yelk, a number of minute spherical pale yellow oil-globules are apparently pressed out into the space within the vitelline membrane (Plate XXIII. fig. 11). The appearance of the "richtungs-bläschen" may be very readily traced in the egg of *Antedon*. At a point on the circumference of the yelk a very distinct globule, about half the diameter of the germinal vesicle, with an obscure nucleus, passes out of the yelk-mass into the surrounding space. In all the cases in which I have observed it, this globule has been accompanied by two or three minute rounded granular masses. Plate XXIII. fig. 14, *a–c*, are careful representations of three groups of these globules. They remain perfectly distinct from the divisions of the yelk during the earlier stages of segmentation; at the close of this process, however, it becomes difficult to distinguish them from the ultimate divisions of the mulberry mass. In *Antedon*, yelk-segmentation is complete (Plate XXIII. figs. 9–13). Its first appearance is a slight groove passing inwards from the circumference of the yelk, immediately at the point where the so-called "richtungs-bläschen" have been extruded. If the egg be now subjected to slight pressure, a transparent nucleus may be observed in the centre; and at each stage of segmentation the nucleus may be readily detected in the centre of each segment. A few hours after segmentation has been completed, the surface of the germ-mass becomes slightly more transparent. The ultimate yelk-spherules are still sufficiently evident, giving the surface a distinctly mammillated appearance (woodcut A).

This gradually disappears, the spherules seem to coalesce upon the outer surface, remaining distinct a little longer towards the inner surface of this rudimentary germinal membrane, and a few hours later they have become entirely fused into a continuous structureless sarcode-layer (woodcut B). While these changes are taking place in the outer layer, the central portion of the germ-mass becomes resolved into a mucilaginous protoplasm sufficiently fluid towards the centre to allow of an active circulation of granules and oil-globules, but apparently continuous with, and graduating into, the lower surface of the more consistent peripheral layer.



A. Usual condition of the mulberry mass immediately after segmentation has been completed. B. Appearance of the nascent pseudembryo after the coalescence of the ultimate spherules of the germ-mass. C. Pseudembryo shortly before the rupture of the vitelline sac.

In this case the development of the pseudembryo from the germ-mass resembles in every way the development of the embryo in most of the invertebrate groups; on three occasions, however, during the examination of a series of eight or ten broods, a whole brood of embryos were evolved under somewhat different circumstances. The surface of the mulberry mass became somewhat looser and more transparent, and under slight pressure a large, somewhat darker and more consistent central nucleus was observed (Plate XXIV. fig. 1). This nucleus increased in size from hour to hour, the peripheral portion of the contents of the vitelline membrane gradually liquefying and becoming absorbed into the nucleus. At length the oval outline of the pseudembryo might be traced through the flocculent mass of semitransparent semifluid yelk. The remainder of the yelk now became completely transparent and liquid, the embryo increased rapidly in size, and its form was more clearly defined through the wall of the vitelline sac (Plate XXIV. figs. 1–4). I believe, however, that this latter is an abnormal mode of development, depending probably upon imperfect aëration.

Observed during the process of development within the vitelline membrane, the embryo is at first nearly regularly oval, and the surface appears to be uniformly ciliated. I have never met with an instance in which the embryo escaped in this condition. In all the cases which I have observed, the ciliated bands so characteristic of the pseudembryonic form have made their appearance before the rupture of the vitelline sac (woodcut, C); and frequently the pseudembryo has become somewhat reniform, a depressed ciliated patch indicating the position of the pseudostome. The pseudembryo frequently, but not constantly, rotates slowly and irregularly within the vitelline sac, the rotation depending evidently upon the action of the cilia on the surface of the pseudembryo. Immediately after escaping from the vitelline membrane, the pseudembryo is about ·8 millim. in length, oval, slightly enlarged towards one extremity, and girded by four nearly equidistant transverse ciliated bands. It consists throughout of very delicately vacuolated sarcode, which becomes more and more consistent towards the periphery, where it forms a smooth firm surface, which is not, however, bounded by any definite membrane. Towards the centre the substance becomes more fluid, and is turbid with oil-cells and granules. At this stage distinct molecular motion may be observed in the central portion, and a granular semifluid mass escapes if the larva be ruptured by pressure. The surface is dotted over with the wider ends of large pyriform lemon-coloured oil-cells immersed perpendicularly in the sarcode. Between these oil-cells the sarcode is nearly transparent, containing merely a few scattered granules. The ciliated bands project slightly above the general surface. They are greyish and granular, and appear to be rather more consistent than the surface of the sarcode, which rises up to them, sinking somewhat in the interspaces. The cilia are very long; they do not vibrate with the regular rhythmical lash of ordinary cilia, but seem to move independently, their motion regulating the rapidity and direction of the movements of the animal in the water. There is a large tuft of still longer cilia in perpetual vibratile motion at the narrower (posterior) extremity of the body. At first the pseudembryo is simply barrel-shaped, and regularly hooped by the four parallel transverse ciliated bands. Sometimes, while yet within the vitelline sac, but at all events within a few hours after its rupture, the body becomes slightly curved, somewhat like a kidney bean; and on the concave surface, the third band from the anterior extremity arches forwards towards the second band; and in the wider space thus left at this point between the third and fourth bands, a large pyriform inversion of the superficial sarcode-layer takes place (Plate XXIV. fig. 7).

This inversion is narrower anteriorly, becoming wider and deeper towards the posterior extremity. Its margins are richly ciliated. Simultaneously with the appearance of this depression, a small round aperture may be observed immediately behind it, separated from it by the fourth ciliated band, and close to the posterior tuft of cilia. This aperture is surrounded by a ring of darker granular tissue, and the outline of a short arched canal may be detected passing under the fourth ciliated band and uniting the deep posterior extremity of the larger aperture, which thus becomes irregularly funnel-shaped, with the smaller circular opening.

The large ciliated key-hole-like inversion of the sarcode is undoubtedly the pseudostome; and resembles closely in form and position the same organ in other echinoderm pseudembryos. The loop-like canal beneath the posterior ciliated band is the extremely rudimentary pseudocele, and the round aperture is the pseudoproct. The pseudembryo swims with either extremity in advance indifferently; the anterior and posterior extremities are therefore only defined at this stage by the relative positions of the mouth and anus. It swims rapidly with a peculiar swinging semi-rotatory motion. The oral surface is turned downwards in a state of rest. The pseudembryo sometimes remains for several days, increasing in size till it becomes from 1·5 millim. to 2 millims. in length, without undergoing any further change. In other cases indications of the areolated calcareous plates of the Echinoderm appear within a few hours of the rupture of the vitelline sac.

Usually not until the pseudembryo has assumed its mature and perfect form, but sometimes much earlier, several minute calcareous spicula make their appearance beneath the external layer of sarcode. The spicula are at first blunt irregular cylinders; but shortly they fork at either end, and at length, by repeatedly dichotomizing and anastomosing, they form delicate plates of calcareous network. When definitely developed, these plates are ten in number, and they arrange themselves in two transverse rings of five each, within the wider anterior portion of the pseudembryo, the posterior row being slightly in advance of the pseudostome. These plates are at first round and expand regularly; the plates of the anterior row being arranged symmetrically above those of the posterior series (Plate XXIV. fig. 7.). They are imbedded in the substance of the sarcode, which for some time remains transparent within and without; gradually, however, the space within the plates becomes turbid and opaque, and at length a rounded brownish granular mass fills up the lower portion of the cup formed by the calcareous trellis. A series, varying in number, of delicate calcareous rings may now be detected, forming a curved line passing backwards from beneath the centre of the lower ring of plates, behind and slightly to the left of the mouth of the pseudembryo; and a large cribriform plate is rapidly developed close to the posterior extremity behind the anus (Plate XXIV. fig. 6). The rings are regular in their inner contour, but externally they are rough with minute branching spicula and excrescences.

About twenty-four hours later the pseudembryo still retains its original form, and its rapidity of movement in the water is unimpaired. The anterior wider portion has become still more bulbous and enlarged, and a thick layer of firm transparent sarcode, thickly studded with columnar oil-cells, forms a dome-shaped arch over the anterior extremity. The sarcode external to the calcareous framework is extremely transparent, and the dark granular hemispherical brownish mass within the lower tier of plates is more clearly defined; while above it and within the upper part of the space included within the plates, the outline of a second more transparent delicately granular hemisphere has become apparent. The two rows of plates are now irregularly square in outline, the plates of the lower series slightly contracted beneath, and those of the upper tier above; so that the ten plates forming the two rows, and now placed in close juxtaposition, form a delicate calcareous basket pentagonal in transverse section and slightly contracted above and below. A hollow sheaf of parallel calcareous rods, united together by short anastomosing lateral branches, is formed within each of the calcareous rings of the series passing backwards from the base of the calcareous cup. These sheaves are, as it were, *bound* in the centre by the calcareous rings, and the rods remain irregular and constantly increasing in length at either end of the sheaf, the irregular growing ends of the rods of one sheaf meeting and mixing with those of the sheaves next it. Thus we have formed what at first appears to be a continuous curved calcareous rod; a slight amount of pressure, however, is sufficient to separate the joints from one another, and to show its true structure. The base of the sheaf of rods passing through the last ring of the series abuts against the centre of the upper surface of the circular cribriform plate, now rapidly increasing in size, and becoming more defined in contour, immediately behind the anus (Plate XXIV. figs. 8, 9, & 10).

We have thus the rudiments of the "pentacrinoid stage" of the *Antedon* clearly defined and rapidly advancing in development within the body of the pseudembryo, while the latter still retains in perfection its independent form and its special organs of locomotion and of assimilation.

I have found it utterly impossible at this stage to trace the formation of the viscera of the young pentacrinoid, on account of the close calcareous network in which the nascent organs are enveloped. From its colour and position, however, there can be no doubt that the mass occupying the base of the cup represents the origin of the stomach with its granular hepatic folds, while the upper more transparent sarcode-hemisphere indicates the nascent tissues of the vault, and at a subsequent stage originates the ambulacral ring with its radial branches and the tissues of the young arms. The two rows of plates, enclosing the viscera and forming the cup at this early period, represent the *basal* and the *oral* series of plates, which are remarkably suppressed and modified during the subsequent development of the crinoid. The jointed calcareous rod is the stem of the Pentacrinoid, and the circular calcareous plate afterwards supports the round fleshy disk by which the base of the stem adheres to its point of attachment. From six to twenty-four hours later the pseudembryo becomes more sluggish in its movements, and begins to lose its characteristic contour. The anterior extremity becomes somewhat flattened, and then slightly depressed in the centre. The stem of the included crinoid lengthens, and the sarcode of the body of the pseudembryo contracts towards it. The pseudostome and pseudoproct become obscure and are shortly obliterated, the sarcode forming a thick, smooth, uniform layer over the stem and over its terminal disk. The two posterior ciliated bands disappear, the anterior bands remaining entire a little longer, and still subserving the locomotion of the pseudembryo. The anterior bands then likewise gradually disappear, the pseudembryo sinking in the water and resting upon a sea-weed or a stone, to which it becomes finally adherent.

At this stage the pseudembryo is irregularly oval and in form slightly contracted posteriorly, expanded and gibbous anteriorly, the anterior extremity flattened or slightly cupped. The posterior extremity expands into a small rounded disk (Plate XXV. fig. 1). Slightly compressed and examined by transmitted light, the Pentacrinoid larva has but little altered from the description given above; the joints of the stem are somewhat lengthened, and the cup is rather more open by the growth and slight separation of the upper portions of the plates of the upper tier. The whole of the pentacrinoid is entirely invested by a thick layer of transparent sarcode, which is merely the substance of the body of the larva which has contracted uniformly over the body and stem of the crinoid, its surface retaining, with the exception of the absence of the bands of cilia, the same character as the surface of the pseudembryo, with the same pyriform oil-cells arranged in the same way, and leaving the same interstices of nearly transparent delicately vacuolated sarcode. The head of the crinoid now becomes more regularly pyriform, and the stem rapidly lengthens. The posterior disk becomes firmly and permanently fixed to its point of attachment. The wide anterior extremity now shows a distinct central depression, and the raised external rim indicates a division into five crescentic lobes.

The whole cup gradually expands and increases in size. The five basal plates enlarge and become more definite in form. Their upper edges are still irregular in outline, somewhat crescentic, arching upwards towards the bases of the orals; but the lateral edges are now bounded by smooth straight calcareous bands, the sides of each plate applied with the intervention of a narrow band of sarcode to the similar edges of the two contiguous plates. The narrow lower edges of the basals are rough and irregular, resting on the upper surface of the irregular ring-like rudiment of the centro-dorsal plate. The oral plates likewise undergo a change in form. They become wider inferiorly, and the sides of the plates towards the lower margin curve outwards, the lower borders thus becoming concave, the convexity turned inwards towards the centre of the body. At the same time the upper edges, which remain narrow and rounded, curve slightly forwards and inwards towards the opening of the cup. If the animal remain undisturbed in well aërated water, when the development of the skeleton has reached this stage, the five lobes (the "oral lobes") forming the edge of the calyx gradually expand, till the cup assumes the form of an open bell (Plate XXVI. fig. 1). Immediately on opening, at least five, and more usually fifteen, delicate, extremely extensile tentacles are protruded from the cup. The mouth, with the organs immediately surrounding it, is formed even before the separation of the oral lobes. It may be seen occupying the centre of the cup (Plate XXVI. fig. 3) immediately after its expansion, as a large patent aperture. When the cup is fully expanded, the transparent tissue continuous with the five oral lobes, and forming the margin of the disk, seems to curve over uniformly into the wide funnel-shaped central opening. The mouth, however, frequently contracts, though it never appears to close completely; and when contracted it is bordered by a slightly thickened very contractile rim, which projects over the cavity of the œsophagus and forms an imperfect sphincter. When this sphincter is relaxed and the mouth fully open, it is easy to see down to the very bottom of the digestive cavity, a sac-like space apparently simply hollowed out in the general sarcode-body (Plate XXVI. fig. 3).

Commencing immediately within the mouth, a series of irregularly-lobed glandular masses, of a pale yellowish-brown colour, project into the cavity of the stomach, curving in an irregular spiral down to the bottom of the cup. These glandular folds are richly clothed with long vibratile cilia. The merest film of sarcode separates their secretion from the stomach-cavity. The slightest touch, even of a hair, ruptures them and causes the effusion of a multitude of minute granules, some colourless and transparent, and others of a yellow or brownish hue. There can be little doubt from their position and colour that these lobes form a rudimentary liver. They appear very early in the pentacrinoid, colouring the lower portions of its body in the earlier stages of its growth within the pseudembryo. They increase steadily in bulk during its later stages, and with but little change of character make up a large portion of the visceral mass in the adult *Antedon*.

A wide vascular ring surrounds the mouth, occupying nearly the whole of the space between the lip and the base of the oral lobes. This ring seems to be simply hollowed out in the uniform sarcode. Its walls are not contractile, it maintains a constant diameter of about 0·08 millim. It is filled with a transparent liquid, which passes likewise into all its tubular appendages; and as granules move rapidly in this fluid, the walls of the ring would seem to be ciliated, though hitherto no cilia have been detected, even in sections and under high powers. The upper and outer margin of the ring gives origin to two classes of tubular tentacles. In a very few cases in which I had an opportunity of looking into the cup immediately after its expansion, the total number of these appendages has been fifteen, five extensile, and ten non-extensile. I have never seen fewer; and I feel convinced that these, with the vascular ring from which they spring, are developed towards the close of the pseudembryonic stage and within the closed cup; they are protruded so immediately after its first expansion.

Radially, the ring gives off five highly mobile, irritable, and extensile tubular tentacles, one opposite each of the intervals between the oral lobes. The cavity of these tentacles is continuous throughout, and immediately continuous with the cavity of the oral ring. Their wall seems to consist of a simple contractile sarcode-layer, studded with oval yellowish endoplasts. There is no definite differentiation of a contractile fibrous tissue. Under a high power, however, the sarcode appears to have a longitudinal arrangement; this may possibly be due to motion among the particles producing a play of light. The walls of these tentacles are produced into numerous delicate tubular processes (Plate XXVI. fig. 3 *e*), their cavities continuous with those of the tentacles. These processes are arranged in three or four irregular longitudinal rows. They are extensile, their walls when extended are extremely delicate, transparent, and apparently structureless. When contracted two or three delicate ring-like rugæ appear on the walls of each (Plate XXV. fig. 3). Each process is terminated by a minute three-lobed slightly granular head. At the base of each of these processes there is a delicate crescentic leaf-like fold, slightly granular, and most distinctly marked when the tentacle is retracted. When one of the extensile tentacles is wholly or partially retracted, it is thrown into obscure transverse wrinkles, which give it at first sight the appearance of being divided by a series of dissepiments. When the tentacle is fully extended these folds totally disappear. At the base of each of these five "azygous tentacles" there is a conical thickening and enlargement of the sarcode-tissue, contracting outwards towards the tentacle which is continuous with its apex, and whose cavity passes through it to unite at its base with the oral vascular ring. This conical projection is the commencement of the young arm. The azygous tentacle terminates it, and leads it out, as it were, up to the point of bifurcation. The tentacle remains persistent for some time in the angle between the two first brachial joints (Plate XXVII. figs. 1 & 3), and finally becomes absorbed and disappears. These five azygous tentacles are the first of a system of "extensile tentacles" which are subsequently developed in very extended series as appendages of the radial and brachial tentacular canals. In almost all cases, as soon as the interior of the cup can be examined after its expansion, the number of extensile tentacles has reached fifteen; but from the one or two instances in which the ten additional tentacles have been absent, there can be no doubt that they are developed somewhat later than the five already described. They arise in five pairs, one tentacle on either side of and slightly within the base of each of the azygous tentacles, which they resemble closely in character. They commence as minute cæcal diverticula from the canal which passes through the enlarged base of the azygous tentacle, and become rapidly developed into tubular prolongations. At this stage (Plate XXVI. fig. 1), when the cup is open, the fifteen tentacles are usually fully extended, curving over the edge of the cup in the angles between the oral lobes, in threes, the azygous tentacle somewhat longer in the centre, and one of the paired tentacles on either side.

Interradially, opposite each of the oral lobes, there is a pair of short tubular tentacles, their cavities likewise continuous with that of the oral vascular ring. These tentacles appear simultaneously with the five azygous extensile tentacles, immediately on the expansion of the cup. They are flexible, but not extensile, slightly club-shaped towards the distal extremity, which is fringed on either side by a single row of short conical tubercles. The base of these tentacles is involved in the contractile sarcode ring surrounding the mouth. When the disk is fully expanded they lie in pairs up against the inner surface of the oral lobes. They are frequently, however, gathered inwards together, or singly curving over the mouth. They form part of a very characteristic system of "non-extensile tentacles," which afterwards fringe the radial and brachial grooves. At this stage, then, the oral ring usually gives off twenty-five tentacular appendages, of which fifteen are radial and extensile, and ten are interradial and non-extensile.

Imbedded in the sarcode at the base of each of the azygous tentacles, a peculiar glandular body is very early developed. At first it consists of a minute vesicle containing a transparent fluid. The vesicle gradually increases in size till it attains a diameter of about 0·08 millim. in diameter. Its contents become granular, and at length it has the appearance of a large cell with a special wall, included in a capsule formed of a firm sarcode-layer, from which the cell can be turned out unbroken.

The cell contains a number of large, irregularly-formed, transparent, slightly granular masses, which are set free by the rupture of the cell-wall. These masses are quite colourless. They are coloured by carmine more deeply than the general substance of the body, and after death they become immediately strongly coloured by the red pigment set free from the perisom. I have been utterly unable to determine the function of these bodies. They are produced in great numbers, during the growth of the pentacrinoid, along the edges of the radial and brachial grooves, and are permanent in the mature *Antedon*. The only speculation which seems to me at all feasible, a speculation which derives some support from their peculiar affinity for colouring matter, is that they are glands connected with the secretion of calcareous solution for the development and nutrition of the skeleton, analogous to the calcareous glands so constantly met with in the pseudembryos and young of some of the other Echinoderm orders. At this early period no general body-cavity can be detected separating the wall of the stomach from the body. The stomach seems to be simply excavated in the structureless body-substance, and the organism corresponds generally with the Cœlenterate type. The external sarcode-layer still retains much the same character which it possessed in the pseudembryonic stage. Its basis is transparent and structureless, with imbedded pyriform oil-cells, endoplasts, and granules.

The stem now gradually lengthens, by additions to either end of the sheaf-like calcareous cylinders which form the axes of the stem joints, and by the addition of new rings which rapidly become filled up by the vertical tissue, at the top of the stem, immediately beneath the rudiment of the centro-dorsal plate (Plate XXVI. fig. 2). The disk of attachment becomes opaque by the addition of calcareous matter, and is firmly fixed. The centro-dorsal ring (Plate XXVI. fig. 2) is more definite in form, though it is still simply perforated in the centre, and in connexion with the sarcode-axis of the stem, and bears no traces of dorsal cirri. The basals expand and form a wide, nearly continuous cup. By the rapid expansion of the body, five diamond-shaped spaces are left at the points where the upturned angles of two oral plates are opposed to the bevelled-off upper angles of two adjacent basals. In these spaces cylindrical spicula appear, which soon become club-shaped, dichotomize, branch, and anastomose into delicate net-like superficial plates, irregularly oval, slightly produced superiorly, their upper, narrower portions resting beneath, and supporting, the gradually extending sarcode projections which are terminated by the azygous tentacles (Plate XXVII. fig. 1). The equatorial portion of the body, the band between the upper edges of the basals and the lower edges of the orals, now rapidly expands. The five young arms extend outwards, their bases carrying out with them a zone of sarcode which gives the central portion of the body a great additional width. The oral plates maintain their original position, so that they are now completely separated from the basals by this intervening equatorial band; and are left, a circle of five separate plates, each enclosed in its sarcode-lobe, on the centre of the upper surface surrounding the mouth, and enclosing the ten non-extensile tentacles only. The first radial plates begin to thicken, especially towards the upper margin, and this thickening is produced by the growth, beneath the cribriform superficial calcareous film, of a longitudinal mass of tissue of the same character as that which forms the cylindrical axis of the stem joints. On the lower surface of each arm, in linear series, immediately above the first radials, two spicula, horseshoe-shaped, with the opening above, appear almost simultaneously, and become quickly filled up with elongating sheaves of longitudinal trellis-work. These extend along beneath the extending arms, and indicate the second radials and the radial axillaries.

The upper surface of the arms now becomes grooved by the development, on either side of the central vessel, of a series of delicate crescentic leaves. These leaves are hollow, communicating by special apertures with the radial vessel, and filled with fluid from it. At the base of each of the leaves there is a pair of tentacles forming a group with the leaf, and along with it communicating with the vessel. One of these tentacles (the distal one) is somewhat larger than the proximal; they are both slightly club-shaped, the club-shaped extremity fringed on either side with conical papillæ. They are non-extensile, and resemble in every particular the ten non-extensile tentacles early developed from the oral ring. A group consisting of a crescentic leaf and two non-extensile tentacles lies immediately at the base of each extensile tentacle, and a little lower down the arm (Plate XXVII. fig. 3 *d*). Minute spicules, some of them simple or key-shaped, and others expanding into a cribriform film, appear in the superficial sarcode-layer along the back or edges of the arms; and, usually at the base of each of the tentacles, irregularly imbedded in the sarcode-substance, there is one of the calcareous glands.

Immediately on the expansion of the equatorial portion of the cup, the wall of the stomach becomes separated by a distinct body-cavity filled with fluid, from the body-wall. The stomach seems to hang in this cavity as a separate sac, attached to the body-wall here and there by sarcodic bands and threads. As the disk expands, the radial canal may be distinctly seen rising from the oral ring, crossing the narrow disk and running along the upper surface of the arm, communicating on either side with the various tentacles and respiratory leaves, and ending at the extremity of the arm in the azygous tentacle. Beneath the radial canal a tubular extension of the perivisceral space passes along the radial grooves. This series of vessels, for which Dr. CARPENTER proposes the term "cœliac canals," afterwards extends throughout the whole length of the arms. In the mature *Antedon* Dr. CARPENTER has observed a third vessel intermediate between the cœliac and tentacular canals; but no trace of this vessel can be detected in the earlier stages in the development of the pentacrinoid.

A little later, the end of the arm shows a tendency to bifurcate, and two half rings, with their enclosed sheaves of calcified tissue, give the first indication of the first two brachials. At the stage which I have described the arm is free, from the base of the second radial; at a later stage the visceral sac extends to the bifurcation, and the whole of the radial portion of the arm becomes included in the cup and disk. The azygous tentacles go no further than the bifurcation. They remain for some time in the centre, between the two divisions of the arm, while secondary branches from the radial canal run on in the brachial grooves. About the period of the development of the second radials, a forked spicule makes its appearance in one of the interradial spaces between the upper portions of two of the first radial plates. This gradually extends in the usual way till it becomes developed into a round cribriform superficial plate.

Simultaneously with the appearance of this "anal" plate, a cæcal process like the finger of a glove rises from one side of the stomach and curves towards the plate. The plate increases in size, becomes enclosed in a little flattened tubercle of sarcode, and maintaining its upright position it passes slightly outwards, leaving a space on the edge of the disk between itself and the base of the oral plate immediately within it.

Towards this space the cæcal intestinal process directs itself. It rises up through it in the form of an elongated tubular closed papilla. The summit of the papilla is finally absorbed, and a patent anal opening is formed. The details of these later changes belong, however, more properly to a subsequent stage.

Having thus described generally the development of the Pentacrinoid stage of *Antedon* up to a point when a marked change takes place in its structure and economy, I shall now discuss, in somewhat fuller detail, certain general considerations arising from the successive steps of the developmental process.

The relations of the Pseudembryo.—In *Antedon* the germ-mass is resolved, at all events to a great extent, into sarcode having the peculiar delicately vacuolated structure so characteristic of this zoological element. The sarcode contains multitudes of "endoplasts;" and of oil-cells and granules scattered through its substance, but these latter I must regard merely as stores of various organic compounds elaborated as secretions and excretions during the development of the organism. In the centre of the sarcode zooid there is usually a darker nucleus, indicating a special accumulation of granular matter. I have satisfied myself, however, that this condition is not essential, as in some cases in which the young were developed in clear water, with a scanty supply of nourishment, the pseudembryo became transparent throughout. Still it is conceivable that a germ of the original substance of the mulberry-mass may be retained to originate the Crinoidal embryo. At all events, the temporary organism which I have termed the Pseudembryo is entirely dependent for its form and structure upon the sarcode into which the whole or the greater portion of the germ-mass is resolved. This sarcode zooid possesses all the peculiarities of the sarcode organisms among the Protozoa and the lower forms of the Cœlenterata. Its external surface is richly ciliated, and if lightly touched with a bristle it moves off rapidly, by means of these cilia, in a direction opposite to the touch, giving evidence of a high degree of irritability and power of automatic motion, without the slightest trace of a special nervous system. During the early stages of its development, and before the differentiation of a special assimilative tract, the body increases rapidly in size; the sarcode is therefore capable, as in the case of the astomatous Protozoa, of absorption over the whole external surface, and of assimilation throughout the entire internal substance.

Whatever at this stage may be the relations of the granular nucleus of the pseudembryo, I believe the external ciliated absorbent and irritable sheet of sarcode must be regarded as a special provisional organ for the nutrition and aëration of the nascent embryo. Dr. CARPENTER * has already suggested a correspondence between the zooid pseudembryo in the Urchins and Starfishes, and the temporary embryonic structures in the higher animals; and I have developed the analogy* still further, in tracing the continuity of the cavity of the pseudembryonic appendages in Asteracanthion with the vascular system of the young Starfish. The sarcode cylinder preceding and afterwards investing the embryo of *Antedon* must undoubtedly be referred to the same category of structures.

---

* "We here find the yolk-mass converted into a structure, which is destined only to possess a transient existence, and which disappears entirely by the time that the development of the offset from it has advanced so far that it begins to assume the characters of the permanent organism. This, however, is what takes place in the higher vertebrata; for the structures first developed in the egg of the bird hold nearly the same relation to the rudimentary chick, that the 'Pluteus' bears to the incipient Echinus or Ophiura, or the 'Bipinnaria' to the incipient Starfish."—Principles of Comparative Physiology, 4th edit. p. 568.

As the development of the pseudembryo proceeds, a large funnel-shaped ciliated pseudostome with an obscure intestine and a minute pseudoproct are formed; and the zooid, which at first resembled a *Plagiophrys* or *Difflugia* in simplicity of structure, may now be compared to a *Vorticella* or *Bursaria.*

The alimentary system is, however, extremely simple. The digestive tract is rudimentary, and the function of the large funnel-shaped œsophagus, with its loop-like pseudocele, seems to be to produce a rapid and special current of fresh water to the general mass of absorbent sarcode rather than to localize the assimilative function. The functional activity of the pseudembryo appears to reside essentially in the peripheral layer. During the earlier stages of its development the central portion consists of a dusky granular semifluid substance, increasing gradually in opacity, and exhibiting active molecular motion; afterwards the centre is devoted to the building up of the viscera of the embryo at the expense of this previously secreted pabulum; but during the earlier stages of the growth of the embryo, its increasing bulk does not appear to interfere in any way with the functions of its nurse. Absorption, as indicated by increase in size and weight, is at no period more rapid than when the pseudembryo is losing its special organs of locomotion and assimilation, and becoming torpid and distorted by the growth of the included organism.

The hollow cylinder of sarcode forming the independent living body of the pseudembryo, at a certain stage loses its cilia, its special organs of assimilation are obliterated, it appears to merge its distinct life in a second harmonized combination of organs which has grown up within it, and the whole layer, without the slightest change in structure, subsides into the perisom of the Pentacrinus.

Histologically the ectosarc of the pseudembryo must be regarded as having been the integument of the Crinoid throughout, its functions highly modified and exalted for a special purpose. The hard structures of the perisom, the two rows of cup-plates and the stem, are accordingly developed in the substance of this integument; and the outline of the Crinoid is thus frequently mapped out in calcareous trellis-work before there is the least trace of the differentiation of internal organs. The stem has clearly no connexion with the viscera whatever, it is a temporary appendage to the radial skeleton.

Until we have accurate details of the embryogeny of a more extended series from the various Echinoderm orders, I believe it would be premature to discuss at length the morphology of the pseudembryo of *Antedon.* At present we are acquainted with many species belonging to widely differing genera, scattered apparently irregularly through the four orders of the subkingdom, which produce independently organized pseudembryonic nurses, presenting a distinct bilateral symmetry in the arrangement of their alimentary system and natatory apparatus. A certain community of plan appears to run through the swimming group described by Professor MÜLLER; but subsequent observations would seem to indicate that so high a development of the pseudembryo is exceptional.

---

* "On the Embryology of *Asteracanthion violaceus* (M. & T.)," Quarterly Journal of the Microscopic Society, 1861, p. 99.

In genera closely approximated to those in which the pseudembryo is most highly organized, or even in allied species of the same genus, the pseudembryonic appendage is reduced to a mere rudimentary vascular tuft, or to a simple investment of sarcode. My own observations would lead me to suspect that the independent development of the pseudembryo may be greatly modified, even in the same species, under different circumstances of light, warmth, aëration, and nourishment.

The pseudembryo of *Antedon* resembles very closely what Professor MÜLLER has described as the "pupa stage" in certain Holothuridea. The young Holothuria, however, has in these instances, according to MÜLLER'S observations, passed through the phase of a pseudembryonic zooid (Auricularia), with a special mouth and alimentary canal, special natatory lobes, and a regular bilateral symmetry, before assuming the pupa form of a closed sarcode-cylinder girded with ciliated bands and devoid of special organs. In *Antedon* the "Auricularia" and the "pupa" stages are, as it were, fused into one. The "pupa" form is at once developed from the germ-mass, but it is provided with the assimilative organs of the Auricularia, though in a very rudimentary degree. Further metamorphosis proceeds very similarly in both cases. In both the organs of the young are gradually differentiated within a sarcode-cylinder, the branchial tentacles finally protruding through an anterior sarcode dome. The close analogy is highly marked in the Synaptidæ, the group whose metamorphoses have been observed by MÜLLER, in which, as in the Crinoids, the oral tentacles are highly developed at the expense of the vessels of the ambulacral region. One or two remarkable differences, however, exist. In *Antedon* no part whatever of the alimentary canal is adopted by the nascent Crinoid. In *Antedon* the development of the organs of the embryo is confined to the anterior region of the pseudembryo, the posterior portion containing the stalk, a temporary appendage. In the Holothuridea the whole pupa passes by simple metamorphosis into the body of the perfect form, the apical pole being occupied by the excretory orifice of the alimentary canal. In the Holothuridea the madreporic tubercle and the sand canal, though frequently extremely rudimentary in the mature form, seem uniformly conspicuous during the development of the young. In the pseudembryonic stage of *Antedon* no trace of this organ has been observed.

I believe that, in zoological language, the term "embryo" has hitherto been understood to indicate a young animal during the early stages of its development; an organism which is produced by the differentiation of the whole or of part of the segmented yelk, and which is a stage in progress towards the mature form of its species. Any accessory or deciduous parts have usually been termed embryonic appendages; but these embryonic appendages have always been regarded as parts of the embryo, although temporary, yet partaking during their life, of the life of the embryo, and as affording no evidence of possessing independent vitality. I imagine that as the term Embryo has not been applied to the yelk, or to the germ-mass before the separation of the organs of the young, it would be a like misapplication of the term to apply it to any stage in the development from that germ-mass of a being whose organs do not homologate with, and never by any subsequent metamorphosis become converted into, the analogous organs of the perfect form. Again, according to the ordinary conception of a " larva," it is a stage in the development of an animal during which its external form differs to a greater or less degree from that of the "imago" or mature form, and its organs are greatly modified for the performance of certain functions at the expense of others; but the organs of the larva are essentially the organs of the imago; and the individual which is formed of the sum of these organs, and which manifests vital phenomena, is the same individual which subsequently lives as the imago. It is utterly inconceivable that the larva and the imago should exist as separate individuals at the same time. The relations of the pseudembryo are entirely different. It is developed from the germ-mass as a distinct animal form, manifesting a combination of vital phenomena, through a sum of organs which attain a distinct maturity of their own, and which never pass in combination into the sum of the organs of the perfect being. So complete is this independence, that in cases where this type of the reproductive process is carried out most fully, as in Bipinnaria, the embryo is at a certain period cast off from the pseudembryo, and both beings continue for some time to manifest independent life. I would therefore define a "pseudembryo" or a "pseudembryonic appendage" as any provisional appendage produced from the germ-mass, which manifests the functions of organic and animal life through the medium of a combination of organs which precede and do not homologate with the organs of the true embryo. This appendage may be reduced to a condition of extreme simplicity. It may exist merely as a layer of structureless sarcode, ciliated, and manifesting the form of life characteristic of the simpler Protozoa; within which the organs of the embryo are gradually built up.

In most, however, if not in all the invertebrate groups, the so-called embryo differs greatly in external form from the mature organism.

It usually commences in aquatic animals as a "ciliated germ"; and in this condition, whether within the vitelline sac or free after the rupture of the sac, it increases in size by absorption through the general surface. Very usually various lobes and fringes are produced, frequently richly ciliated, extensions of a transparent sarcodic investing layer, within which—but bearing to it only obscure relations in form—the nascent organs of the true embryo are slowly differentiated. During this period the permanent organs, so far as their special functions are concerned, are utterly inert. They are merely growing. The rudiments of the alimentary canal are being laid down, but probably the mouth has not yet "broken through." The entire zooid, however, is by no means inactive. It moves rapidly through the water, its movements beautifully characteristic, and apparently guided by an obstruction-perceiving and light-perceiving instinct.

The perfect organic and relative life of this being, closely comparable to the life of the most highly gifted members of the protozoic subkingdom, does not certainly exist in the sum of the permanent organs; it resides, I believe, simply in a pseudembryonic sarcodic layer, endowed with the same properties which this zoological element possesses when isolated, as in the Protozoa. Gradually the sarcode eliminates from the products of its own assimilation the constituents, and elaborates the tissues, of the permanent special organs; and when these are sufficiently developed, it loses its own individuality, its vital activity passing into the organs which it has produced, and performing through their medium more effectively and condensedly, functions, which, as a transient nurse-layer, it performed in a manner perfect as to its simple object of temporary nutrition, though somewhat feeble and diffuse. In respect to the essentials of this process, some of the Holothuridea among the Echinodermata seem to conform almost exactly to the ordinary Invertebrate type. The pseudembryonic sarcode-layer is here little more special or independent than it is in the embryos of the Annelids and Mollusks, and infinitely less so than in some Turbellarians; and the transition from this condition, through the Crinoids, in which a short alimentary canal is formed in the sarcode layer,—and the "Plutei" in which the "Echinoderm disk" with its accompanying permanent organs is developed within the pseudembryo and covered by its general integument, the whole substance of the pseudembryo being finally absorbed into the embryo,—to the "Bipinnaria," in which the independent life of the pseudembryonic zooid is apparently carried to its limit, is so perfectly gradual as to leave no doubt whatever of the uniformity of the embryogenic plan.

This being the case, that is to say, a vast number of invertebrate embryos combining in their earlier stages pseudembryonic appendages possessing independent vitality with the nascent organs, no special divergence from the ordinary mode of development is to be anticipated in cases in which the pseudembryo attains unusual individual independence. We find accordingly the earlier stages in the development of the pseudembryo in the Echinoderms conforming closely to the general mode of development of the "embryo" of aquatic invertebrates.

The earlier stages in the development of the Tissues of the Pentacrinoid.

The general connective tissue.—As stated above, the general transparent investment which during the earlier stages of its development makes up the greater portion of the substance of the pentacrinoid, is produced by the gradual extension and modification of the sarcode substance of the pseudembryo. The pseudembryo is moulded from the germ-mass, and at first its surface retains the mammillated structure, the result of the ultimate segmentation of the yelk. At first each spherule retains a trace of the original enclosed endoplast; this, however, shortly disappears. No cell-membrane can be detected investing these spherules at any period. An hour or two after the rupture of the vitelline sac, the mammillated structure entirely disappears, the ultimate spherules being fused into a structureless layer. The external layer is firm and consistent. If the pseudembryo die at this stage, shortly after its death, a delicate film is sometimes separated from the surface of portions of the body, similar to the film which is observed under similar circumstances on the surface of Infusoria. I do not believe, however, that this film previously existed as a special membrane; but am rather inclined to think that it is produced after death by the coagulation of a layer of mucous excretion. Pyriform capsules of considerable size, about 0·03 millim. in diameter, are imbedded here and there in the superficial layer. These cells are of a pale yellow colour, full of a yellow fluid, which when the cell is crushed escapes as a round refractive globule. The wide end of the capsule is superficial, the narrower extremity passes inwards and ends in a delicate thread-like process, which is lost in the substance of the sarcode.

I have been able to detect no special wall to these capsules, the fluid of which seems simply to be enclosed in a pyriform space in the continuous sarcode: I regard these as reservoirs of oil.

The peripheric layer is nearly free from granules; but passing from without inwards, minute granules, compound granular masses, and endoplasts become more numerous; the sarcode at the same time apparently losing in consistency, till at length, towards the inner surface of the consistent perisomatic layer, it becomes densely granular, and no distinct line of demarcation can be detected between the sarcode which still retains a certain consistency, and the central semifluid protoplasm, in which the granules exhibit active molecular motion. The outer layer, when compressed and examined with a high power, exhibits between the endoplasts and oil-cells a very finely vacuolated structure. Minute spaces, somewhat like the lacunæ of bone, filled with a clear liquid, are scattered through the sarcode; and uniting these there is a system of exceedingly delicate tubules which may be compared to the canaliculi; they are much less numerous, however, only about six or eight apparently radiating from each lacunar space. Even while under observation, the size of these spaces appears to vary, one or two which were prominent in one part of the field gradually contracting and becoming indistinct, while others previously scarcely visible seem to expand into view. I believe that this appearance is caused by the circulation of fluid through the system of vacuoles and vessels by movements depending upon the general contractility of the body-substance. Near the close of the free stage, when the embryo is beginning by its growth to distort the form of the pseudembryo, the integument of the wider anterior extremity of the pseudembryo immediately above the mouth of the embryo seems to become columnar in structure and opaque with closely packed long oil-cells, arranged vertically, and forming a kind of dome. In the earliest fixed stage this dome gradually splits up into the five oral lobes, each with its enclosed oral plate.

The devlopment of the Skeleton.—To make the description of the development and relations of the parts of the calcareous skeleton of the pentacrinoid stage of *Antedon* intelligible, I shall in the first place describe very briefly the arrangement of the hard parts in the mature *Antedon* and in some nearly allied forms. I shall touch on this part of the subject lightly, as Dr. CARPENTER is preparing an elaborate memoir on the skeleton of *Antedon.* I adopt, in concert with Dr. CARPENTER, a nomenclature differing very slightly from that proposed by M. DE KONINCK in his valuable work on the fossil Crinoids of the Carboniferous System of Belgium. I accept for convenience of description the division of the body of a Crinoid into three parts, the stem, the head, and the arms. The head consists of two hemispheres, a dorsal or apical, and an oral hemisphere. The former I shall term the cup of the Crinoid, and the latter the disk. It must be remembered, however, that all the radial portions of the head belong morphologically and physiologically to the arms. In the earlier stages of development the radial plates of the cup, and the radial vessels of the disk, *form* the budding arms; and it is only at a later period that a distinction is produced between radial and brachial portions, by the development of the visceral mass and the extension of the space for its accommodation.

The mature Antedon has no true stem. The cup is closed beneath by a large circular plate hollowed out above into a small rounded chamber. The inferior convex surface of this plate in *Antedon rosaceus* is pitted with a series of small rounded depressions perforated in the centre with minute channels communicating with the cavity of the plate. Into these depressions are inserted a number of jointed calcareous cirri. I shall term the circular plate the "centro-dorsal plate," and the appendages the "dorsal cirri." The centro-dorsal plate in *Antedon* does not belong to the cup. It represents a coalesced series of the nodal stem-joints in the stalked Crinoids.

In *Pentacrinus* (*Neocrinus*) *asterias* (L.), the stem grows by additions immediately beneath the row of basal plates of the cup. These plates are five in number, interradial, wedge-shaped, their outer wider ends knob-like, heading and corresponding with the salient angles of the pentagonal stem. Their inner narrower ends nearly meet in the centre, each being only slightly truncated and emarginated, so that the five grooved ends may unite in forming the walls of a canal, which is continuous with the central canal of the stem, and through which the central sarcode-cylinder of the stem passes to branch to special perforations in the first radials. The lower surface of each basal plate is hollowed by a longitudinal groove crenated on the edges, and the five grooves are so arranged that when the basals are in position, they form together a star-like mould, in which the joints of the stem are formed. This cavity holds from three to four stem-joints at a time; one extremely small at the bottom of the mould, the others gradually increasing in size and gradually forced out and added to the lengthening stem, by the growth of those behind them.

The joints developed in this position are all nodal, that is to say, they subsequently bear whorls of cirri. The internodal joints, varying in number in different species, are developed afterwards between these, each new internodal joint originating apparently immediately beneath the nodal joint.

The dorsal cirri represent a varying number of compressed whorls of the stem-cirri of stalked species which possess such appendages.

The centro-dorsal plate with its dorsal cirri in *Antedon* is therefore the homologue of the stem with its cirri in the stalked Crinoids.

The true cup in the mature *Antedon* consists inferiorly of a delicate rosette of more or less fully coalesced small cribriform calcareous plates; which have been shown by Dr. CARPENTER, in a series of beautiful observations, to be the remains of the row of five basal plates which occupy so prominent a place in the cup of the Pentacrinoid.

This rosette is completely concealed in the cavity of the ring formed by the first five radials. Around the basal rosette, and alternating with its segments, five elongated calcareous blocks, triangular in transverse section, the first radial plates, form a column within the base of the cup. In *A. rosaceus* these plates are entirely concealed by the centro-dorsal plate and by the series of second radials. In some species of the genus *Antedon*, they project beyond the centro-dorsal plate, forming above its upper edge a closed ring which supports the series of second radials. The centro-dorsal plate, the basals, and the first radials are immoveably cemented together; they do not, however, coalesce, and may be easily separated after boiling in weak caustic potash. A ring of five second radial plates placed in close contact, form, externally, the base of the cup in *Antedon rosaceus*, resting within upon the upper surfaces of the first radials, and externally upon the edge of the centro-dorsal plate.

Resting upon the second radials, we have next a row of five triangular axillary radial plates, each bevelled above into two diverging surfaces for the articulation of the first brachial joints. The axillary radials are not in immediate contact laterally, they are separated by minute wedge-shaped prolongations downwards of the perisom of the disk. In *Antedon rosaceus*, the basals, and the first, second, and axillary radials form the whole of the skeleton of the cup.

In certain species of *Antedon*, as in *A. Milleri* (Müller, sp.), a series of five minute interradial plates are intercalated between the angles of the axillary radials, and in other forms, as in *A. Solaris* (Lam., sp.), and *A. tessellatus* (Müller, sp.), the whole of the perisom of the disk is covered with a pavement of irregular flat plates. We are unacquainted with the development of *Pentacrinus* (*Neocrinus*) *asterias* (L.), but in the mature form the perisom of the disk is continuously tessellated, and some of the plates pass irregularly downwards between the axillary radials. In *Pentacrinus* (*Neocrinus*) *decorus* (nob.), the surface of the disk is rough with irregularly scattered blocks, like fragments of perforated bricks; and these descend into the spaces between the axillary radials, though without any regular arrangement.

The basal and oral plates.—The first portions of the skeleton which appear are the two rings of five plates each, the plates of the upper ring directly superposed on those of the lower, which form the trellised basket, completely enclosing the viscera of the Pentacrinoid during the early stages of its growth within the pseudembryo. The plates of the upper tier subsequently extend into the five oral lobes, and remain as five valve-like interradial oral plates during the greater part of the pentacrinoid stage.

The lower series are the basals. These are permanent, with some remarkable modifications in form, in the mature *Antedon*. These ten plates appear simultaneously as delicate spicula imbedded within the firm peripheric layer of the pseudembryo, usually only a few hours after its escape from the vitelline sac, and before there is any trace of the permanent organs of the embryo.

The spicula are hollow throughout. They are at first simple and cylindrical; shortly they become club-shaped at each end; each thickened end then divides into two diverging branches, equal in length to the original rod; these fork in their turn, till on their second bifurcation their branches meet and coalesce with the corresponding branches from the opposite end of the original spiculum. By thus constantly branching and anastomosing on one plane, the spiculum extends into a delicate net-like plate, the meshes of which are at first irregularly hexagonal, but afterwards become rounded. The extending calcareous tubes are constantly closed, and constantly hollow to the end. They appear to grow by the molecular removal of calcareous matter from the back of the growing point, and its deposition in advance. At first all the ten plates are round; but as they expand they become irregularly square, their edges during the free condition of the embryo remaining rough with sprouting spicules.

About the time of the fixing of the pentacrinoid, the basals, which have now assumed a somewhat definite form, narrower beneath and expanding above, have their lateral edges bounded by straight lines, so that the edges of two adjacent plates are closely applied to one another. Even after their edges have become thus defined, the plates go on steadily increasing in size, apparently by interstitial growth. The upper edges of the basals still remain rounded and rough. Their lower edges are likewise irregular, but these soon become obscured by the growth of the centro-dorsal ring. The oral plates extend principally upwards into the oral lobes, where they become lengthened and somewhat contracted, their edges fringed with diverging pointed spicules (Plate XXVI. fig. 1). As development proceeds they change somewhat in form. The upper angle is slightly depressed, and the sides at the inferior angles are raised, the raised edges at that stage lying up against the sides of the second radials. Absorption of the inferior portion of the oral plates commences about the time of the appearance of the first brachial joints and of the anal plate (Plate XXVII. fig. 1). Both basal and oral plates consist at first of a delicate cribriform calcified film, formed by the lateral extension of a single layer of calcareous tubing only. As they increase in size, however, they gradually thicken, and this thickening is effected by the network sending in from its inner surface irregular processes which branch and unite to form a second layer not quite so regular as the first, but resembling it in general character. This process is repeated till the plates have attained the required thickness. In the oral plates the thickening is very slight, and is confined to the lower portion of the plates.

The stem.—As described above, shortly after the appearance of the spicula indicating the basal and oral plates, a chain of six or seven calcareous rings may be observed curving from the centre of the space between the bases of the basal plates; behind, and usually somewhat to the left of the pseudostome and pseudocele, and abutting against a round cribriform plate which makes its appearance at the same time close to the posterior extremity of the pseudembryo, behind and below the pseudoproct. Immediately beneath the basal plates an irregular calcareous ring is early formed, considerably wider and broader than the ordinary rings of the stem. This ring, which is subsequently developed into the permanent centro-dorsal plate, gradually thickens and becomes more regular in form, maintaining its position at the top of the stem, the lower edges of the basal plates resting on its upper surface. During the earlier stages of the growth of the pentacrinoid it is simply a circular band of the ordinary calcified areolar tissue, enclosing a sheaf of the peculiar fasciculated tissue of the stem, gradually enlarging, with a central aperture continuous with the bore of the tube-like stem-joints. It is not till some time after the latest stage described in the present memoir, that the rudiments of the first dorsal cirri appear round its lower contour. The rings which originate the ordinary stem-joints commence as small curved hollow spicules. At first they may often be seen open and imperfect; afterwards they completely close (Plate XXIV. fig. 6). The inner surfaces of the rings are smooth, the outer roughened with projecting branches. I have only once or twice seen the rings of the stem in this early simple stage. Very soon after their appearance, usually before the pseudembryo has attained its full size, a hollow sheaf of calcareous rods united by minute calcareous trabeculæ arises within each ring. The stem-joint increases in length by additions to each end of these cylinders. The centre of the cylinder is occupied by a consistent sarcodic thread running through the whole length of the stem. At this stage no fibrous tissue can be detected, either mixed with the calcified tissue or in the outer perisom. Additions are made to the length of the stem by the formation of new rings immediately beneath the centro-dorsal plate, the new rings becoming, as in the former case, gradually filled up by cylinders of linear calcified tissue. As the calcareous axis of the stem increases in width, the original rings girding the centre of the joints expand. They remain permanent during the whole of the fixed stage, and give the stem of the Pentacrinoid its characteristic beaded appearance. The terminal plate of the stem is formed on the same plan as the basals and orals. It is developed as a simple round cribriform plate within the posterior extremity of the pseudembryo; and when this extremity becomes expanded into a disk of attachment, it supports and forms the skeleton of the terminal sucker. Afterwards it becomes thickened by irregularly deposited calcareous matter. The layer of soft tissue between the calcareous disk and the point of attachment seems to be at length absorbed, and the stem is permanently fixed by amorphous cement.

The first and second radial joints and the axillary radials.—Shortly after the fixing of the Pentacrinoid and the opening of the cup, a third series of five plates make their appearance as minute branching spiculæ occupying the spaces left by the bevelling off of the upper angles of the basal plates and the lower angles of the orals, thus forming an intermediate series between the basals and orals, and alternating with them. The spicula indicating the origin of these plates, the first radials, branch and extend in the manner already described, till at length they form diamond-shaped films consisting of a single layer of cribriform calcified tissue. The plates shortly begin to thicken; but their mode of growth at once distinguishes them as fundamentally different in structure from the basals and orals. Processes are sent inwards from the inner surface of the superficial film as before; but the added tissue is longitudinal and fasciculated, resembling precisely in structure and mode of growth the inner cylinder of the joints of the stem; and, as in the case of the stem, tubular perforations are formed in it for the passage of the sarcode-cords, which subsequently extend in like channels through the joints of the arms and pinnules. The second radial joints and the radial axillaries rapidly succeed the first radials, and are developed nearly in the same way. They first appear as horseshoe-shaped spicula, or imperfect rings, which have the same relation to the joints which the stem-rings have to their included cylinders. The spicula soon become filled up with lengthening fasciculated tissue; the joints at this period are slightly grooved longitudinally on their upper surfaces to accommodate the radial vessels.

The anal plate, the interradial plates, and the plates and spicula of the perisom.—Upon the appearance of the second and third radial joints, the perisom between and somewhat above two of the first radials rises into a rounded papilla, towards which a cæcal process of the digestive cavity is directed. On the outer side of this papilla a branching spicule appears which rapidly extends into a round plate. This, the anal plate, grows, and afterwards thickens precisely on the model of the basal and oral plates; it contains none of the fasciculated tissue proper to the radial system. The basal and oral plates, the first and second radials, the radial axillaries, and the anal plate seem to complete the series of essential parts entering into the cup of the pentacrinoid. In one or two cases however, I have observed about the time of the first appearance of the anal plate, a series of five minute rounded plates developed interradially between the lower edges of the oral plates and the upper edges of the basals. These interradial plates sometimes remain permanent in the mature *Antedon rosaceus*, and they appear to be constantly present in some species, as for instance in another and a rarer British form, *Antedon Milleri* (Müller). They usually occur, finally, in groups of three or five. They are irregular in form, and they resemble the anal plate in structure and mode of growth. Simple and key-like spicula and small round cribriform plates are imbedded irregularly in the perisom of the arms, often almost covering the second and third radial joints with a dermal calcified layer, but never overlying the basal or oral plates of the body.

General remarks on the Skeleton.—The skeleton of the pentacrinoid is composed of two systems of plates, which I shall term respectively the *radial* and the *perisomatic* system, thoroughly distinct in their structure and mode of growth. The radial system consists of the joints of the stem, the centro-dorsal plate, the radial plates, and the joints of the arms (and subsequently of the pinnules). The perisomatic system includes the basal and oral plates, the anal plate, the interradial plates, and any other plates or spicula which may be developed in the perisom of the cup or disk. In the recent *Pentacrini*, and in certain species of *Antedon*, the disk is paved or studded with plates belonging to the perisomatic system, and a double series of like plates fringe the radial and brachial grooves. The joints or plates of the radial system may be at once distinguished by their being chiefly made up of the peculiar fasciculated (or radial) tissue of parallel rods which I have already described, and by their being perforated for the lodgment of a sarcodic axis. At first each radial element appears to consist of two parts. A stem-joint always commences with an annular spicule, within which the cylinder of "radial" tissue seems to arise. An arm-joint begins with a crescentic spicule, and a radial plate with an expanded single cribriform film. From the strong contrast which these superficial portions present to the tissue which is afterwards developed beneath them, I am inclined to refer the outer rings and films, even of the brachial joints and radial plates, to the perisomatic system, and to regard the radial system of plates as composed essentially of the "radial" tissue alone. The plates and joints of the radial system are singularly uniform in their structure and arrangement throughout the whole of the crinoidal series.

They seem to form, as it were, an essential skeleton whose constant general arrangement stamps the order with its most important and prominent character. In the Pentacrinoid the radial system of radial- and arm-joints supports the extensions of the radial vessels, and the radial vessels with their œsophageal vascular ring clearly arise in connexion with the disk, on the oral aspect of the animal. The radial plates arise at the opposite or apical pole. The first portion of the radial system which appears is the stem. When the sarcode-axis of the stem enters the cup, passing through the centro-dorsal plate and between the lower edges of the basals, it splits into five threads which enter the first radial plates, and after a somewhat singular distribution in the walls of the cup, which is not apparent till a later stage, they follow out the growing arms, the arm-joints being moulded round them as they extend. The perivisceral sac lies in the cleft formed by the five radial branches of the stem. The plates of the perisomatic system commence as simple cribriform films imbedded in the outer layer of the perisom, and thicken by a repetition inwards of the same diffuse areolar tissue. They are essentially variable in number and in arrangement; most of the minor structural modifications throughout the group depend upon the multiplication or suppression of plates of this series. Even in the same species they are by no means constant. In *Antedon rosaceus* the perisom of the disk is usually naked, but specimens from certain localities have well-defined groups of perisomatic interradial plates developed in the angles between the radial axillaries, and in some individuals rows of similar plates are imbedded along the margins of the radial grooves in the perisom of the disk. The entire body of the Pentacrinoid is, at first, while yet included within the pseudembryo and during its earliest fixed stage, surrounded and enclosed by plates of the perisomatic system alone, and it is quite conceiveable that plates belonging to this system may expand and multiply so as to form a tessellated external skeleton to the mature animal, the radial system being entirely absent, or represented only in the most rudimentary form. I believe that all the modifications of the skeleton which characterize the principal divisions of the Echinoderm subkingdom will be found to depend mainly upon the relative development or suppression of the radial and perisomatic systems of plates.

With reference to the form and position of the oral plates, Professor ALLMAN has suggested some interesting analogies between this transition stage of *Antedon* and the permanent condition of the fossil genera *Haplocrinus*, *Coccocrinus*, *Stephanocrinus*, and *Lageniocrinus*. I thoroughly agree with Dr. ALLMAN, that the oral plates of the Pentacrinoid are in all probability homologous with valve-like plates surrounding the mouth only in all crinoidal genera in which such plates occur. In *Antedon rosaceus* they disappear during the later stages in the growth of the Pentacrinoid young, and in all known species of the genus *Antedon*, even in those with a tessellated disk, they are wanting in the mature form. In *Pentacrinus* (*Neocrinus*) *asterias*, (L.), the mature form to which the fixed stage of *Antedon* is evidently most analogous, they are said to remain permanent. The evidence on this point is as yet extremely defective. It rests entirely upon the descriptions and sketches of M. DUCHASSAING*, which are sufficiently graphic, but by no means technically exact. In two nearly allied species, *Pentacrinus* (*Neocrinus*) *Mülleri* (Oersted) and *P.* (*N.*) *decorus* (nob.), in both of which I have had an opportunity of examining the perisom of the disk, the oral plates are totally absent.

Almost all Dr. ALLMAN's illustrations are necessarily taken from a small aberrant family of Crinoids, the Haplocrinidæ, of whose structure we know as yet very little. With the exception of *Stephanocrinus*, which only doubtfully belongs to the group, all the genera are Devonian, preceded by the peculiar Cystideans of the Upper Silurians, and ushering in the carboniferous Blastoids.

Notwithstanding Professor MÜLLER's discovery of rudimentary free arms, I cannot help still leaning to the view that the triangular interradial valves in the Haplocrinidæ may, like the pointed upper tier of interradial plates in the Pentremites, surround not only the mouth, but ovarian and anal openings; a discussion of the homologies of the fossil Crinoids is however foreign to the object of the present memoir.

The development of the assimilative and vascular systems, so far as it has been possible to observe it at this early stage, has already been described in detail.

EXPLANATION OF THE PLATES.

PLATE XXIII.

Fig. 1. Portion of the ovary under slight pressure, showing ova in various stages of development, × 40 linear.

Fig. 2, *a–o*. Ova in various stages, from the first appearance of the germinal spot 2, *a* to the maturity of the egg 2, *o*, × 40 linear.

Fig. 3. Yelk-granules, × 120 linear.

---

* Quoted by M. DE KONINCK, "Recherches sur les Crinoïdes du terrain Carbonifère de Belgique," p. 53. Brussels, 1854.

Fig. 4. A group of parent cells containing vesicles of evolution, and forming a portion of the tissue of the testis, × 40 linear.

Fig. 5, *a–e*. Parent cells with vesicles of evolution in various stages of development, × 40 linear.

Fig. 6, *a–c*. Mature vesicles of evolution containing spermatozoa, × 80 linear.

Fig. 7. Spermatozoa, × 120 linear.

Fig. 8. Egg shortly after impregnation, × 40 linear.

Figs. 9–13. The process of yelk segmentation, × 40 linear.

Fig. 14, *a–c*. Further enlarged views of the earlier stages of yelk segmentation, showing three groups of the "direction vesicles," × 80 linear.

PLATE XXIV.

Figs. 1–4. The development of the pseudembryo within the vitelline membrane, × 40 linear. In this case the development is somewhat abnormal.

Fig. 5. Dorsal aspect of the pseudembryo shortly after the rupture of the vitelline sac, × 40 linear.

Fig. 6. Dorsal view of the pseudembryo a little more advanced, × 40 linear.

Fig. 7. Ventral aspect of the pseudembryo a little later, showing the pseudostome and pseudoproct, and the rudiments of the cup plates of the embryo, × 40 linear.

Figs. 8, 9, 10. Ventral, dorsal, and lateral aspects of the pseudembryo shortly before the disappearance of the ciliated bands, × 40 linear.

PLATE XXV.

Figs. 1–3. The pseudembryo losing its special organs of assimilation and locomotion and passing into the "pentacrinoid stage," × 40 linear.

PLATE XXVI.

Fig. 1. Pentacrinoid larva immediately after the complete separation of the oral valves, expanded, × 40 linear.

Fig. 2. Pentacrinoid in the same stage, the cup closed, × 40 linear, but afterwards slightly reduced to suit the size of the plate.

Fig. 3. A portion of the oral disk of the same stage seen from above, in a state of complete expansion: *a*, patent oral aperture bounded by a ring of contractile tissue, and showing yellow richly ciliated granular folds, arranged somewhat spirally on the walls of the digestive cavity; *b*, central ring of the radial vascular system; *c*, non-extensile tentacles in immediate connexion with the vascular

ring, ten in number, and laid up in a state of complete expansion in pairs against the inner surfaces of the oral valves *f*; *d*, first pair of extensile radial tentacles; *e*, azygous radial extensile tentacle leading out the growing arm to its bifurcation, and giving off pairs of tentacles of the same series from its base. ×40 linear.

PLATE XXVII.

Fig. 1. Pentacrinoid larva immediately before the expansion of the ventral disk: *a*, centro-dorsal plate; *b*, series of basal plates; *c*, first radial plates; *d*, second radial joint; *e*, third radial; *f*, first brachial joint; *g*, anal plate; *h*, stem-joint; *k*, cribriform plate supporting the disk of attachment; *l*, granular visceral mass; *m*, cæcal process passing from the stomach towards the papilla which indicates the position subsequently occupied by the anal tube; *n*, oral valve and plate. ×40 linear, slightly reduced.

Fig. 2. An example in a somewhat earlier stage, expanded, and showing the arrangement of the non-extensile tentacles in connexion with the oral vascular ring, ×40 linear, considerably reduced.

Fig. 3. End of an extending arm further enlarged: *a*, *b*, and *c*, first, second, and third radial joints; *d*, superficial spicules and small cribriform plates of the perisomatic system; *e*, lenticular "gland"? ; *f*, radial vessel passing out on the arm to terminate in the azygous extensile tentacle *h*, after giving off the second pair of extensile tentacles *k*, *k*; *g*, leaf and pair of tentacles of the non-extensile tentacular system. ×40 linear.

Fig. 4. Pseudembryo uncompressed and observed by reflected light: *a*, pseudostome; *b*, pseudoproct; *c*, *c*, *c*, *c*, ciliated bands. ×40 linear.

All the figures, except Plate XXVII. fig. 4, have been drawn from specimens under slight pressure, and with a special view to the details of internal structure. The contour has been thus in some cases to a certain extent lost, and the figures, especially those of the pseudembryo, must be understood to represent individuals slightly flattened.

X. *On the Sextactic Points of a Plane Curve.* *By* A. CAYLEY, *F.R.S.*

Received November 5,—Read December 22, 1864.

IT is, in my memoir "On the Conic of Five-pointic Contact at any point of a Plane Curve"*, remarked that as in a plane curve there are certain singular points, viz. the points of inflexion, where three consecutive points lie in a line, so there are singular points where six consecutive points of the curve lie in a conic; and such a singular point is there termed a "sextactic point." The memoir in question (here cited as "former memoir") contains the theory of the sextactic points of a cubic curve; but it is only recently that I have succeeded in establishing the theory for a curve of the order m. The result arrived at is that the number of sextactic points is $=m(12m-27)$, the points in question being the intersections of the curve m with a curve of the order $12m-27$, the equation of which is

$$\begin{aligned}&(12m^2-54m+57)\mathrm{H}\ \mathrm{Jac.}\,(\mathrm{U},\ \mathrm{H},\ \Omega_{\overline{\mathrm{H}}})\\ +&(m-2)(12m-27)\mathrm{H}\ \mathrm{Jac.}\,(\mathrm{U},\ \mathrm{H},\ \Omega_{\overline{\mathrm{U}}})\\ +&40(m-2)^2\qquad\qquad \mathrm{Jac.}\,(\mathrm{U},\ \mathrm{H},\ \Psi\ )=0,\end{aligned}$$

where $\mathrm{U}=0$ is the equation of the given curve of the order m, H is the Hessian or determinant formed with the second differential coefficients $(a,\ b,\ c,\ f,\ g,\ h)$ of U, and, $(\mathfrak{A},\ \mathfrak{B},\ \mathfrak{C},\ \mathfrak{F},\ \mathfrak{G},\ \mathfrak{H})$ being the inverse coefficients ($\mathfrak{A}=bc-f^2$, &c.), then

$$\Omega=(\mathfrak{A},\ \mathfrak{B},\ \mathfrak{C},\ \mathfrak{F},\ \mathfrak{G},\ \mathfrak{H})(\partial_x,\ \partial_y,\ \partial_z)^2\mathrm{H},$$
$$\Psi=(\mathfrak{A},\ \mathfrak{B},\ \mathfrak{C},\ \mathfrak{F},\ \mathfrak{G},\ \mathfrak{H})(\partial_x\mathrm{H},\ \partial_y\mathrm{H},\ \partial_z\mathrm{H})^2;$$

and Jac. denotes the Jacobian or functional determinant, viz.

$$\mathrm{Jac.}\,(\mathrm{U},\ \mathrm{H},\ \Psi)=\begin{vmatrix}\partial_x\mathrm{U}, & \partial_y\mathrm{U}, & \partial_z\mathrm{U}\\ \partial_x\mathrm{H}, & \partial_y\mathrm{H}, & \partial_z\mathrm{H}\\ \partial_x\Psi, & \partial_y\Psi, & \partial_z\Psi\end{vmatrix},$$

and Jac. $(\mathrm{U},\ \mathrm{H},\ \Omega)$ would of course denote the like derivative of $(\mathrm{U},\ \mathrm{H},\ \Omega)$; the subscripts $(_{\overline{\mathrm{H}}},\ _{\overline{\mathrm{U}}})$ of Ω denote restrictions in regard to the differentiation of this function, viz. treating Ω as a function of U and H,

$$\Omega=(\mathfrak{A},\ \mathfrak{B},\ \mathfrak{C},\ \mathfrak{F},\ \mathfrak{G},\ \mathfrak{H})(a',\ b',\ c',\ f',\ 2f',\ 2g',\ 2h'),$$

if $(a',\ b',\ c',\ f',\ g',\ h')$ are the second differential coefficients of H, then we have

$$\begin{aligned}\partial_x\Omega=&(\partial_x\mathfrak{A},\ ..)(\ a',\ ..)\qquad(=\partial_x\Omega_{\overline{\mathrm{H}}})\\ +&(\ \mathfrak{A},\ ..)(\partial_x a',\ ..)\qquad(=\partial_x\Omega_{\overline{\mathrm{U}}});\end{aligned}$$

---

* Philosophical Transactions, vol. cxlix. (1859) pp. 371—400.

viz. in $\partial_x\Omega_{\overline{\mathrm{H}}}$ we consider as exempt from differentiation (a', b', c', f', g', h') which depend upon H, and in $\partial_x\Omega_{\overline{\mathrm{U}}}$ we consider as exempt from differentiation $(\mathfrak{A}, \mathfrak{B}, \mathfrak{C}, \mathfrak{F}, \mathfrak{G}, \mathfrak{H})$ which depend upon U. We have similarly

$$\partial_y\Omega=\partial_y\Omega_{\overline{\mathrm{H}}}+\partial_y\Omega_{\overline{\mathrm{U}}}, \text{ and } \partial_z\Omega=\partial_z\Omega_{\overline{\mathrm{H}}}+\partial_z\Omega_{\overline{\mathrm{U}}};$$

and in like manner

$$\text{Jac.}(\mathrm{U}, \mathrm{H}, \Omega)=\text{Jac.}(\mathrm{U}, \mathrm{H}, \Omega_{\overline{\mathrm{H}}})+\text{Jac.}(\mathrm{U}, \mathrm{H}, \Omega_{\overline{\mathrm{U}}}),$$

which explains the signification of the notations Jac. $(\mathrm{U}, \mathrm{H}, \Omega_{\overline{\mathrm{H}}})$, Jac. $(\mathrm{U}, \mathrm{H}, \Omega_{\overline{\mathrm{U}}})$.

The condition for a sextactic point is in the first instance obtained in a form involving the arbitrary coefficients (λ, μ, ν); viz. we have an equation of the order 5 in (λ, μ, ν) and of the order $12m-22$ in the coordinates (x, y, z). But writing $\vartheta=\lambda x+\mu y+\nu z$, by successive transformations we throw out the factors ϑ^2, ϑ, ϑ, ϑ, thus arriving at a result independent of (λ, μ, ν); viz. this is the before-mentioned equation of the order $12m-27$. The difficulty of the investigation consists in obtaining the transformations by means of which the equation in its original form is thus divested of these irrelevant factors.

Article Nos. 1 to 6.—*Investigation of the Condition for a Sextactic Point.*

1. Following the course of investigation in my former memoir, I take (X, Y, Z) as current coordinates, and I write

$$\Upsilon=(*\between \mathrm{X}, \mathrm{Y}, \mathrm{Z})^m=0$$

for the equation of the given curve; (x, y, z) are the coordinates of a particular point on the given curve, viz. the sextactic point; and U, $=(*\between x, y, z)^m$, is what Υ becomes when (x, y, z) are written in place of (X, Y, Z): we have thus $\mathrm{U}=0$ as a condition satisfied by the coordinates of the point in question.

2. Writing for shortness

$$\mathrm{DU}=(\mathrm{X}\partial_x+\mathrm{Y}\partial_y+\mathrm{Z}\partial_z)\,\mathrm{U},$$
$$\mathrm{D}^2\mathrm{U}=(\mathrm{X}\partial_x+\mathrm{Y}\partial_y+\mathrm{Z}\partial_z)^2\mathrm{U},$$

and taking $\Pi=a\mathrm{X}+b\mathrm{Y}+c\mathrm{Z}=0$ for the equation of an arbitrary line, the equation

$$\mathrm{D}^2\mathrm{U}-\Pi\mathrm{DU}=0$$

is that of a conic having an ordinary (two-pointic) contact with the curve at the point (x, y, z); and the coefficients of Π are in the former memoir determined so that the contact may be a five-pointic one; the value obtained for Π is

$$\Pi=\tfrac{2}{3}\,\frac{1}{\mathrm{H}}\mathrm{DH}+\Lambda\mathrm{DU},$$

where

$$\Lambda=\frac{1}{9\mathrm{H}^3}(-3\Omega\mathrm{H}+4\Psi).$$

3. This result was obtained by considering the coordinates of a point of the curve as functions of a single arbitrary parameter, and taking

$$x+dx+\tfrac{1}{2}d^2x+\tfrac{1}{6}d^3x+\tfrac{1}{24}d^4x,\ y+\ \&\text{c.},\ z+\ \&\text{c.}$$

for the coordinates of a point consecutive to (x, y, z); for the present purpose we must go a step further, and write for the coordinates

$$\begin{aligned}&x+dx+\tfrac{1}{2}d^2x+\tfrac{1}{6}d^3x+\tfrac{1}{24}d^4x+\tfrac{1}{120}d^5x,\\&y+dy+\tfrac{1}{2}d^2y+\tfrac{1}{6}d^3y+\tfrac{1}{24}d^4y+\tfrac{1}{120}d^5y,\\&z+dz+\tfrac{1}{2}d^2z+\tfrac{1}{6}d^3z+\tfrac{1}{24}d^4z+\tfrac{1}{120}d^5z.\end{aligned}$$

4. Hence if

$$\partial_1=dx\partial_x+dy\partial_y+dz\partial_z,\ \partial_2=d^2x\partial_x+d^2y\partial_y+d^2z\partial_z,\ \&\text{c.},$$

we have, in addition to the equations

$$\begin{aligned}&\mathrm{U}=0,\\&\partial_1\mathrm{U}=0,\\&(\partial_1^2+2\partial_2)\mathrm{U}=0,\\&(\partial_1^3+3\partial_1\partial_2+\partial_3)\mathrm{U}=0,\\&(\partial_1^4+6\partial_1^2\partial_2+4\partial_1\partial_3+3\partial_2^2+\partial_4)\mathrm{U}=0,\end{aligned}$$

of my former memoir, the new equation

$$(\partial_1^5+10\partial_1^3\partial_2+10\partial_1^2\partial_3+15\partial_1\partial_2^2+5\partial_1\partial_4+10\partial_2\partial_3+\partial_5)\mathrm{U}=0,$$

and in addition to the equations, $(\mathrm{P}=ax+by+cz)$,

$$\begin{aligned}&-\quad(m-2)\partial_1^2\mathrm{U}+\mathrm{P}.\tfrac{1}{2}\partial_1^2\mathrm{U}=0\\&-\ \tfrac{1}{3}[(m-1)\partial_1^3+3(m-2)\partial_1\partial_2]\mathrm{U}+\mathrm{P}.\tfrac{1}{6}(\partial_1^3+3\partial_1\partial_2)\mathrm{U}+\partial_1\mathrm{P}.\tfrac{1}{2}\partial_1^2\mathrm{U}=0,\\&-\tfrac{1}{12}[(m-1)(\partial_1^4+6\partial_1^2\partial_2)+(m-2)(4\partial_1\partial_3+3\partial_2^2)]\mathrm{U}\\&\qquad+\mathrm{P}.\tfrac{1}{24}(\partial_1^4+6\partial_1^2\partial_2+4\partial_1\partial_3+3\partial_2^2)\mathrm{U}+\partial_1\mathrm{P}.\tfrac{1}{6}(\partial_1^3+3\partial_1\partial_2)\mathrm{U}+\tfrac{1}{2}\partial_2\mathrm{P}.\tfrac{1}{2}\partial_1^2\mathrm{U}=0,\end{aligned}$$

giving in the first instance

$$\begin{aligned}&\mathrm{P}=2(m-2),\\&\partial_1\mathrm{P}=\tfrac{2}{3}\,\frac{\partial_1^3\mathrm{U}}{\partial_1^2\mathrm{U}},\\&\partial_2\mathrm{P}=\tfrac{1}{2}\,\frac{(\partial_1^4+6\partial_1^2\partial_2)\mathrm{U}}{\partial_1^2\mathrm{U}}-\tfrac{4}{9}\,\frac{\partial_1^3\mathrm{U}}{\partial_1^2\mathrm{U}}\,\frac{(\partial_1^3+3\partial_1\partial_2)\mathrm{U}}{\partial_1^2\mathrm{U}},\end{aligned}$$

and leading ultimately to the before-mentioned value of Π, we have the new equation

$$\begin{aligned}&-\tfrac{1}{60}\ [(m-1)(\partial_1^5+10\partial_1^3\partial_2+10\partial_1^2\partial_3+15\partial_1\partial_2^2)+(m-2)(5\partial_1\partial_4+10\partial_2\partial_3)]\mathrm{U}\\&+\quad\mathrm{P}.\tfrac{1}{120}(\partial_1^5+10\partial_1^3\partial_2+10\partial_1^2\partial_3+15\partial_1\partial_2^2\qquad+5\partial_1\partial_4+10\partial_2\partial_3)\ \mathrm{U}\\&+\ \partial_1\mathrm{P}.\tfrac{1}{24}\ (\partial_1^4+\ 6\partial_1^2\partial_2+\ 4\partial_1\partial_3+\quad3\partial_2^2)\mathrm{U}\\&+\tfrac{1}{2}\partial_2\mathrm{P}.\ \tfrac{1}{6}\ (\partial_1^3+\ 3\partial_1\partial_2)\mathrm{U}\\&+\tfrac{1}{6}\partial_3\mathrm{P}.\ \tfrac{1}{2}\ \partial_1^2\mathrm{U}=0.\end{aligned}$$

5. This may be written in the form

$$\begin{aligned}
&-2[(m-1)(\partial_1^5+10\partial_1^3\partial_2+10\partial_1^2\partial_3+15\partial_1\partial_2^2)+(m-2)(5\partial_1\partial_4+10\partial_2\partial_3)]\mathrm{U}\\
&+\quad \mathrm{P}(\quad \partial_1^5+10\partial_1^3\partial_2+10\partial_1^2\partial_3+15\partial_1\partial_2^2 \quad +5\partial_1\partial_4+10\partial_2\partial_3)\ \mathrm{U}\\
&+\ 5\partial_1\mathrm{P}(\quad \partial_1^4+\ 6\partial_1^2\partial_2+\ 4\partial_1\partial_3+\ 3\partial_2^2)\mathrm{U}\\
&+10\partial_2\mathrm{P}(\quad \partial_1^3+\ 3\partial_1\partial_2)\mathrm{U}\\
&+10\partial_3\mathrm{P}(\quad \partial_1^2\mathrm{U})=0;
\end{aligned}$$

or putting for P its value, $=2(m-2)$, the equation becomes

$$\begin{aligned}
&-\quad 2(\partial_1^5+10\partial_1^3\partial_2+10\partial_1^2\partial_3+15\partial_1\partial_2^2)\mathrm{U}\\
&+\ 5\partial_1\mathrm{P}(\partial_1^4+\ 6\partial_1^2\partial_2+\ 4\partial_1\partial_3+\ 3\partial_2^2\ )\mathrm{U}\\
&+10\partial_2\mathrm{P}(\partial_1^3+\ 3\partial_1\partial_2)\mathrm{U}\\
&+10\partial_3\mathrm{P}\,.\,\partial_2^2\mathrm{U}=0;
\end{aligned}$$

or, as this may also be written,

$$\begin{aligned}
&2(\partial_1^5+10\partial_1^3\partial_2+10\partial_1^2\partial_3+15\partial_1\partial_2^2)\mathrm{U}\\
&\qquad+5\partial_1\mathrm{P}.\partial_4\mathrm{U}+10\partial_2\mathrm{P}.\partial_3\mathrm{U}+10\partial_3\mathrm{P}.\partial_2\mathrm{U}=0.
\end{aligned}$$

6. But the equation

$$\Pi=\tfrac{2}{3}\,\frac{1}{\mathrm{H}}\mathrm{DH}+\Lambda\mathrm{DU},$$

which is an identity in regard to (X, Y, Z), gives

$$\partial_1\mathrm{P}=\tfrac{2}{3}\,\frac{1}{\mathrm{H}}\partial_1\mathrm{H},$$

$$\partial_2\mathrm{P}=\tfrac{2}{3}\,\frac{1}{\mathrm{H}}\partial_2\mathrm{H}+\Lambda\partial_2\mathrm{U},$$

$$\partial_3\mathrm{P}=\tfrac{2}{3}\,\frac{1}{\mathrm{H}}\partial_3\mathrm{H}+\Lambda\partial_3\mathrm{U};$$

and substituting these values, the foregoing equation becomes

$$\begin{aligned}
&2(\partial_1^5+10\partial_1^3\partial_2+10\partial_1^2\partial_3+15\partial_1\partial_2^2)\mathrm{U}\\
&\qquad+(5\partial_4\mathrm{U}\partial_1\mathrm{H}+10\partial_3\mathrm{U}\partial_2\mathrm{H}+10\partial_2\mathrm{U}\partial_3\mathrm{H})\tfrac{2}{3}\,\frac{1}{\mathrm{H}}+\Lambda.20\partial_2\mathrm{U}\partial_3\mathrm{U}=0;
\end{aligned}$$

or putting for Λ its value, $=\frac{1}{9\mathrm{H}^3}(-3\Omega\mathrm{H}+4\Psi)$, and multiplying by $\tfrac{9}{2}\mathrm{H}^2$ this is

$$\begin{aligned}
&9\mathrm{H}^2(\partial_1^5+10\partial_1^3\partial_2+10\partial_1^2\partial_3+15\partial_1\partial_2^2)\mathrm{U}\\
&+15\mathrm{H}\,(\partial_4\mathrm{U}\partial_1\mathrm{H}+2\partial_3\mathrm{U}\partial_2\mathrm{H}+2\partial_2\mathrm{U}\partial_3\mathrm{H})\\
&+\quad \frac{1}{\mathrm{H}}\,(-3\Omega\mathrm{H}+4\Psi).10\partial_2\mathrm{U}\partial_3\mathrm{U}=0,
\end{aligned}$$

which is, in its original or unreduced form, the condition for a sextactic point.

Article Nos. 7 & 8.—*Notations and Remarks.*

7. Writing, as in my former memoir, A, B, C for the first differential coefficients of U, we have $B\nu-C\mu$, $C\lambda-A\nu$, $A\mu-B\lambda$ for the values of dx, dy, dz, and instead of the symbol $\mathbf{D}$ used in my former memoir, I use indifferently the original symbol ∂_1, or write instead thereof ∂, to denote the resulting value

$$\partial_1(=\partial)=(B\nu-C\mu)\partial_x+(C\lambda-A\nu)\partial_y+(A\mu-B\lambda)\partial_z,$$

and I remark here that for any function whatever Ω, we have

$$\partial\Omega=\begin{vmatrix} A\,, & B\,, & C \\ \lambda\,, & \mu\,, & \nu \\ \partial_x\Omega, & \partial_y\Omega, & \partial_z\Omega \end{vmatrix}=\text{Jac. }(U,\vartheta,\Omega),$$

where $\vartheta=\lambda x+\mu y+\nu z$. I write, as in the former memoir,

$$\Phi=(\mathfrak{A},\mathfrak{B},\mathfrak{C},\mathfrak{F},\mathfrak{G},\mathfrak{H}\between\lambda,\mu,\nu)^2;$$

and also

$$\nabla=(\mathfrak{A},\mathfrak{B},\mathfrak{C},\mathfrak{F},\mathfrak{G},\mathfrak{H}\between\lambda,\mu,\nu\between\partial_x,\partial_y,\partial_z),$$

which new symbol ∇ serves to express the functions Π, $\square$, occurring in the former memoir; viz. we have $\Pi=2\nabla\Phi$, $\square=2\nabla H$, so that the symbols Π, $\square$ are not any longer required.

8. I remark that the symbols ∂, ∇ are each of them a linear function of $(\partial_x,\partial_y,\partial_z)$, with coefficients which are functions of the variables (x, y, z); and this being so, that for any function Π whatever, we have

$$\partial(\nabla\Pi)=(\partial\,.\,\nabla)\Pi+\partial\nabla\Pi,$$

viz. in $\partial(\nabla\Pi)$ we operate with ∇ on Π, thereby obtaining $\nabla\Pi$, and then with ∂ on $\nabla\Pi$; in $(\partial\,.\,\nabla)\Pi$ we operate with ∂ upon ∇ in so far as ∇ is a function of (x, y, z), thus obtaining a new operating symbol $\partial\,.\,\nabla$, a linear function of $(\partial_x,\partial_y,\partial_z)$, and then operate with $\partial\,.\,\nabla$ upon Π; and lastly, in $\partial\nabla\Pi$, we simply multiply together ∂ and ∇, thus obtaining a new operating symbol $\partial\nabla$ of the form $(\partial_x,\partial_y,\partial_z)^2$, and then operate therewith on Π; it is clear that, as regards the last-mentioned mode of combination, the symbols ∂ and ∇ are convertible, or $\partial\nabla=\nabla\partial$, that is, $\partial\nabla\Pi=\nabla\partial\Pi$.

It is to be observed throughout the memoir that the point (.) is used (as above in $\partial\,.\,\nabla$) when an operation is performed upon a symbol of operation as operand; the mere apposition of two or more symbols of operation (as above in $\partial\nabla$) denotes that the symbols of operation are simply multiplied together; and when $\partial\nabla$ is followed by a letter Π denoting not a symbol of operation, but a mere function of the coordinates, that is in an expression such as $\partial\nabla\Pi$, the resulting operation $\partial\nabla$ is performed upon Π as operand; if instead of the single letter Π we have a compound symbol such as HU or $H\nabla\vartheta$, so that the expression is ∂HU, $\partial H\nabla\vartheta$, $\partial\nabla HU$ or $\partial\nabla H\nabla\vartheta$, then it is to be understood that it is merely the immediately following function H which is operated upon by ∂ or $\partial\nabla$; in the few instances where any ambiguity might arise a special explanation is given.

Article Nos. 9 to 11.—*First Transformation.*

9. We have, assuming always $U=0$, the following formulæ (*see post*, Article Nos. 31 to 33):—

$$(\partial_1^5+10^3_1\partial_2+10\partial_1^2\partial_3+15\partial_1\partial_2^2)U$$

$$=\frac{\vartheta^2}{(m-1)^4}\{(27m^2-96m+81)H\partial\Phi+(17m^2-56m+51)\Phi\partial H\}$$

$$+\frac{\vartheta^3}{(m-1)^4}\{(-14m-22)(\partial\,.\,\nabla)H\ \ -(10m-18)\partial\nabla H\}$$

$$+\frac{\vartheta^4}{(m-1)^4}\{\partial\Omega\},$$

$$\partial_4U\partial_1H+2\partial_3U\partial_2H+2\partial_2U\partial_3H$$

$$=\frac{\vartheta^2}{(m-1)^4}\{(-6m^2+18m-12)H^2\partial\Phi+(-17m^2+60m-55)H\Phi\partial\Phi\}$$

$$+\frac{\vartheta^3}{(m-1)^4}\{(2m-2)H(\partial\,.\,\nabla)H\qquad+(8m-16)\partial H\nabla H\}$$

$$+\frac{\vartheta^4}{(m-1)^4}\{-\Omega\partial H\},$$

$$\partial_2U\partial_3U=\frac{\vartheta^4}{(m-1)^4}H\partial H.$$

10. And by means of these the condition becomes

$$0=\frac{\vartheta^2H^2}{(m-1)^4}\{(153m^2-594m+549)H\partial\Phi+(-102m^2+396m+366)\Phi\partial H\}$$

$$+\frac{\vartheta^3H}{(m-1)^4}\{(-96m+168)H(\partial\,.\,\nabla)H+(-90m+162)H\partial\nabla H+(120m-240)\partial H\nabla H\}$$

$$+\frac{\vartheta^4}{(m-1)^4}\{9H^2\partial\Omega-45H\Omega\partial H+40\Psi\partial H\},$$

being, as already remarked, of the degree 5 in the arbitrary coefficients (λ,μ,ν), and of the order $12m-22$ in the coordinates (x, y, z).

11. But throwing out the factor ϑ^2, and observing that in the first line the quadric functions of m are each a numerical multiple of $51m^2-198m+183$, the condition becomes

$$\begin{aligned}0=&\ \ (51m^2-198m+183)H^2(3H\partial\Phi-2\Phi\partial H)\\ &+\vartheta\ \{(-96m+168)H^2(\partial\,.\,\nabla)H+(-90m+162)H^2\partial\nabla H+(120m-240)\partial H\nabla H\}\\ &+\vartheta^2\{9H^2\partial\Omega-45H\Omega\partial H+40\Psi\partial H\}.\end{aligned}$$

Article Nos 12 & 13.—*Second transformation.*

12. We effect this by means of the formula

$$(m-2)(3H\partial\Phi-2\Phi\partial H)=-\vartheta\,\text{Jac.}(U,\Phi,H),\quad .\ .\ .\ .\quad (J)^*$$

---

* (J) here and elsewhere refers to the Jacobian Formula, *see post*, Article Nos. 34 & 35.

for substituting this value of $(3H\partial\Phi-2\Phi\partial H)$ the equation becomes divisible by ϑ; and dividing out accordingly, the condition becomes

$$\begin{aligned}&-\frac{51m^2-198m+183}{m-2}H^2\,\text{Jac.}(U,\Phi,H)\\&+(-96m+168)H^2(\partial\,.\,\nabla)H+(-90m+162)H^2\partial\nabla H+(120m-240)H\partial H\nabla H\\&+\vartheta(9H^2\partial\Omega-45H\Omega\partial H+40\Psi\partial H)=0.\end{aligned}$$

13. We have (*see post*, Article Nos. 36 to 40)

$$\text{Jac.}(U,\Phi,H)=-(\partial\,.\,\nabla)H\,;$$

and introducing also $\partial\,.\,\nabla H$ in place of $\partial\nabla H$ by means of the formula

$$\partial\nabla H=\partial(\nabla H)-(\partial\,.\,\nabla)H,$$

the condition becomes

$$\begin{aligned}&\left\{\frac{51m^2-198m+183}{m-2}-(6m-6)\right\}H^2(\partial\,.\,\nabla)H\\&+(-90m+162)H^2\partial(\nabla H)\quad+120(m-2)H\partial H\nabla H\\&+\vartheta(9H^2\partial\Omega-45H\Omega\partial H+40\Psi\partial H)=0,\end{aligned}$$

or, as this may be written,

$$\begin{aligned}&(45m^2-180m+171)H^2(\partial\,.\,\nabla)H\\&\quad+(-90m+162)(m-2)H^2\partial(\nabla H)+120(m-2)^2H\partial H\nabla H\\&\quad+(m-2)\vartheta(9H^2\partial\Omega-45H\Omega\partial H+40\Psi\partial H)=0.\end{aligned}$$

Article Nos. 14 to 17.—*Third transformation.*

14. We have the following formulæ,

$$\vartheta\,\text{Jac.}(U,\nabla H,H)-(5m-11)\partial H\nabla H+(3m-6)H\partial(\nabla H)=0,\quad\ldots\ldots\quad(\text{J})$$

$$\vartheta\,\text{Jac.}(U,\nabla,H)H-(2m-4\ )\partial H\nabla H+(3m-6)H(\partial\,.\,\nabla)H=0,\quad\ldots\ldots\quad(\text{J})$$

in the latter of which, treating ∇ as a function of the coordinates, we first form the symbol Jac. (U,∇,H), and then operating therewith on H, we have Jac. $(U,\nabla,H)H$; these give

$$H\partial(\nabla H)=\frac{5m-11}{3(m-2)}\partial H\nabla H-\frac{\vartheta}{3(m-2)}\text{Jac.}(U,\nabla H,H),$$

$$H(\partial\,.\,\nabla)H=\qquad\tfrac{2}{3}\partial H\nabla H-\frac{\vartheta}{3(m-2)}\text{Jac.}(U,\nabla\ ,H)H;$$

and substituting these values, the resulting coefficient of $H\partial H\nabla H$ is

$$\begin{aligned}&(\ 45m^2-180m+171)\tfrac{2}{3}\\&+(-90m+162)\frac{5m-11}{3}\\&+120(\ \ m-2)^2,\end{aligned}$$

which is $=0$.

15. Hence the condition will contain the factor ϑ, and throwing out this, and also the constant factor $\frac{1}{m-2}$, it becomes

$$\begin{aligned}&(-15m^2+60m-57)\mathrm{H}\,\mathrm{Jac.}\,(\mathrm{U},\nabla\ \ ,\ \mathrm{H})\mathrm{H}\\&+(30m-54)(m-2)\ \ \mathrm{H}\,\mathrm{Jac.}\,(\mathrm{U},\nabla\mathrm{H},\ \mathrm{H})\\&+(m-2)^2(9\mathrm{H}^2\partial\Omega-45\mathrm{H}\Omega\partial\mathrm{H}+40\Psi\partial\mathrm{H})=0.\end{aligned}$$

16. We have

$$\partial_x(\nabla\mathrm{H})=(\partial_x\,.\,\nabla)\mathrm{H}+\partial_x\nabla\mathrm{H},$$

viz. in $(\partial_x\,.\,\nabla)\mathrm{H}$, treating ∇ as a function of (x, y, z) we operate upon it with ∂_x to obtain the new symbol $\partial_x\,.\,\nabla$, and with this we operate on H; in $\partial_x\nabla$ we simply multiply together the symbols ∂_x and ∇, giving a new symbol of the form $(\partial_x^2, \partial_x\partial_y, \partial_x\partial_z)$ which then operates on H. We have the like values of $\partial_y(\nabla\mathrm{H})$ and $\partial_z(\nabla\mathrm{H})$; and thence also

$$\mathrm{Jac.}\,(\mathrm{U},\nabla\mathrm{H},\mathrm{H})=\mathrm{Jac.}\,(\mathrm{U},\nabla,\mathrm{H})\mathrm{H}+\mathrm{Jac.}\,(\mathrm{U},\bar{\nabla}\mathrm{H},\mathrm{H}),$$

viz. in the determinant Jac. (U, ∇, H) the second line corresponding to ∇ is $\partial_x\,.\,\nabla$, $\partial_y\,.\,\nabla$, $\partial_z\,.\,\nabla$ (∇ being the operand); and the Jacobian thus obtained is a symbol which operates on H giving Jac. (U, ∇, H)H; and in the determinant Jac. (U, $\bar{\nabla}$H, H) the second line is $\partial_x\nabla\mathrm{H}$, $\partial_y\,\nabla\mathrm{H}$, $\partial_z\nabla\mathrm{H}$ (∇ being simply multiplied by ∂_x, ∂_y, ∂_z respectively).

17. Substituting, the condition becomes

$$\begin{aligned}&(-15m^2+60m-57\,)\ \mathrm{H}\,\mathrm{Jac.}\,(\mathrm{U},\nabla,\mathrm{H})\mathrm{H}\\&+(30m-54)(m-2)\{\mathrm{H}\,\mathrm{Jac.}\,(\mathrm{U},\nabla,\mathrm{H})\mathrm{H}+\mathrm{Jac.}\,(\mathrm{U},\bar{\nabla}\mathrm{H},\mathrm{H})\}\\&+\ \ (m-2)^2\qquad\{9\mathrm{H}^2\partial\Omega-54\mathrm{H}\Omega\partial\mathrm{H}+40\Psi\partial\mathrm{H}\}=0,\end{aligned}$$

or, what is the same thing,

$$\begin{aligned}&(15m^2-54m+51)\mathrm{H}\,\mathrm{Jac.}\,(\mathrm{U},\nabla\ \ ,\ \mathrm{H})\mathrm{H}\\&+(30m-54)(m-2)\mathrm{H}\,\mathrm{Jac.}\,(\mathrm{U},\bar{\nabla}\mathrm{H},\mathrm{H})\\&+(m-2)^2\{9\mathrm{H}^2\partial\Omega-45\mathrm{H}\Omega\partial\mathrm{H}+40\Psi\partial\mathrm{H}\}=0.\end{aligned}$$

Article Nos. 18 to 27.—*Fourth transformation, and final form of the condition for a Sextactic Point.*

18. I write

$$\begin{aligned}(5m-12)\Omega\partial\mathrm{H}-(3m-6)\mathrm{H}\partial\Omega&=\vartheta\,\mathrm{Jac.}\,(\mathrm{U},\Omega,\mathrm{H})\quad.\ .\ .\ .\ .\quad(\mathrm{J})\\\Omega\partial\mathrm{H}+\qquad\mathrm{H}\partial\Omega&=\qquad\partial(\Omega\mathrm{H}),\end{aligned}$$

and, introducing for convenience the new symbol W,

$$-5\Omega\partial\mathrm{H}+\qquad\mathrm{H}\partial\Omega=\mathrm{W},$$

so that

$$\begin{vmatrix}5m-12, & -(3m-6), & \vartheta\,\mathrm{Jac.}\,(\mathrm{U},\Omega,\mathrm{H})\\1\ , & 1\ , & \partial\,.\,\Omega\mathrm{H}\\-5\ , & 1\ , & \mathrm{W}\end{vmatrix}=0,$$

or what is the same thing,

$$(8m-18)\mathrm{W}+6\vartheta\,\mathrm{Jac.}\,(\mathrm{U},\ \Omega,\ \mathrm{H})+(10m-18)\partial(\Omega\mathrm{H})=0,$$

we have

$$\mathrm{W}=\mathrm{H}\partial\Omega-5\Omega\partial\mathrm{H}=\frac{-3}{4m-9}\vartheta\,\mathrm{Jac.}\,(\mathrm{U},\ \Omega,\ \mathrm{H})-\frac{5m-9}{4m-9}\partial(\Omega\mathrm{H}).$$

19. We have also

$$(8m-18)\Psi\partial\mathrm{H}-(3m-6)\mathrm{H}\partial\Psi-\vartheta\,\mathrm{Jac.}\,(\mathrm{U},\ \Psi,\ \mathrm{H})=0,\quad .\ .\ .\ .\ .\quad (\mathrm{J})$$

that is

$$\Psi\partial\mathrm{H}\qquad=\frac{\frac{1}{2}}{4m-9}\vartheta\,\mathrm{Jac.}\,(\mathrm{U},\ \Psi,\ \mathrm{H})+\frac{\frac{3}{2}(m-2)}{4m-9}\mathrm{H}\partial\Psi,$$

and thence

$$\begin{aligned}&9\mathrm{HW}+40\Psi\partial\mathrm{H}\\ &\quad=9\mathrm{H}^2\partial\Omega-45\mathrm{H}\Omega\partial\mathrm{H}+40\Psi\partial\mathrm{H}\\ &\quad=-\frac{9(5m-9)}{4m-9}\mathrm{H}\partial(\Omega\mathrm{H})+\frac{60(m-2)}{4m-9}\mathrm{H}\partial\Psi\\ &\qquad+\frac{\vartheta}{4m-9}\{-27\mathrm{H}\,\mathrm{Jac.}\,(\mathrm{U},\ \Omega,\ \mathrm{H})+40\,\mathrm{Jac.}\,(\mathrm{U},\ \Psi,\ \mathrm{H})\}.\end{aligned}$$

20. The condition thus becomes

$$\begin{aligned}&(15m^2-54m+51)\ (4m-9)\mathrm{H}\,\mathrm{Jac.}\,(\mathrm{U},\ \nabla\quad,\ \mathrm{H})\mathrm{H}\\ &+6(5m-9)(m-2)(4m-9)\mathrm{H}\,\mathrm{Jac.}\,(\mathrm{U},\ \bar\nabla\mathrm{H},\ \mathrm{H})\\ &+3(m-2)\{-3(5m-9)(m-2)\mathrm{H}\partial(\Omega\mathrm{H})+20(m-2)^2\mathrm{H}\partial\Psi\}\\ &+(m-2)^2\vartheta\{-27\mathrm{H}\,\mathrm{Jac.}\,(\mathrm{U},\ \Omega,\ \mathrm{H})+40\,\mathrm{Jac.}\,(\mathrm{U},\ \Psi,\ \mathrm{H})\}=0,\end{aligned}$$

which for shortness I represent by

$$3\mathrm{H}\text{Ш}+(m-2)^2\vartheta\{-27\mathrm{H}\,\mathrm{Jac.}\,(\mathrm{U},\ \Omega,\ \mathrm{H})+40\,\mathrm{Jac.}\,(\mathrm{U},\ \Psi,\ \mathrm{H})\}=0,$$

so that we have

$$\begin{aligned}\text{Ш}=\ &(5m^2-18m+17)(4m-9)\,\mathrm{Jac.}\,(\mathrm{U},\ \nabla\quad,\ \mathrm{H})\mathrm{H}\\ &+2(5m-9)(m-2)(4m-9)\,\mathrm{Jac.}\,(\mathrm{U},\ \bar\nabla\mathrm{H},\ \mathrm{H})\\ &+(m-2)\{-3(5m-9)(m-2)\partial(\Omega\mathrm{H})+20(m-2)^2\partial\Psi\}.\end{aligned}$$

21. Write

$$\Psi_1=(\mathfrak{A}',\ \mathfrak{B}',\ \mathfrak{C}',\ \mathfrak{F}',\ \mathfrak{G}',\ \mathfrak{H}'\between \mathrm{A},\ \mathrm{B},\ \mathrm{C})^2,$$

where (A, B, C) are as before the first differential coefficients of U, and $(a',\ b',\ c',\ f',\ g',\ h')$ being the second differential coefficients of H, $(\mathfrak{A}',\ \mathfrak{B}',\ \mathfrak{C}',\ \mathfrak{F}',\ \mathfrak{G}',\ \mathfrak{H}')$ are the inverse coefficients, viz., $\mathfrak{A}'=b'c'-f'^2$, &c. We have

$$-(m-1)^2\partial\Psi_1=(3m-6)(3m-7)\partial(\Omega\mathrm{H})-(3m-7)^2\partial\Psi\ (\textit{see post},\ \text{Nos. 41 to 46}),$$

that is

$$(3m-6)\partial(\Omega\mathrm{H})=(3m-7)\partial\Psi-\frac{(m-1)^2}{3m-7}\partial\Psi_1,$$

and thence

$$\begin{aligned}\Pi= &\ (5m^2-18m+17)(4m-9)\,\text{Jac.}\,(\mathrm{U},\nabla\ \ ,\mathrm{H})\mathrm{H}\\ &+2(5m-9)(m-2)(4m-9)\,\text{Jac.}\,(\mathrm{U},\bar{\nabla}\mathrm{H},\mathrm{H})\\ &+(m-2)\left\{(5m^2-18m+17)\partial\Psi+\frac{(m-1)^2(5m-9)}{3m-7}\partial\Psi_1\right\}=0.\end{aligned}$$

22. Now

$$\Psi=(\mathfrak{A},\mathfrak{B},\mathfrak{C},\mathfrak{F},\mathfrak{G},\mathfrak{H})(\mathrm{A}',\mathrm{B}',\mathrm{C}')^2,\quad \Psi_1=(\mathfrak{A}',\mathfrak{B}',\mathfrak{C}',\mathfrak{F}',\mathfrak{G}',\mathfrak{H}')(\mathrm{A},\mathrm{B},\mathrm{C})^2,$$

and writing for shortness

$$\begin{aligned}\mathrm{E}\Psi\ &=(\partial\mathfrak{A},\ldots)(\mathrm{A}',\mathrm{B}',\mathrm{C}')^2, &\mathrm{F}\Psi\ &=(\mathfrak{A},\ldots)(\mathrm{A}',\mathrm{B}',\mathrm{C}')(\partial\mathfrak{A}',\partial\mathfrak{B}',\partial\mathfrak{C}'),\\ \mathrm{E}\Psi_1&=(\partial\mathfrak{A}',\ldots)(\mathrm{A},\mathrm{B},\mathrm{C})^2, &\mathrm{F}\Psi_1&=(\mathfrak{A}',\ldots)(\mathrm{A},\mathrm{B},\mathrm{C})(\partial\mathfrak{A},\partial\mathfrak{B},\partial\mathfrak{C}),\end{aligned}$$

(we might, in a notation above explained, write $\mathrm{E}\Psi=\partial\Psi_{\bar{\mathrm{H}}}$, $\mathrm{F}\Psi=\frac{1}{2}\partial\Psi_{\bar{\mathrm{U}}}$, and in like manner $\mathrm{E}\Psi_1=\partial\Psi_{1\bar{\mathrm{U}}}$, $\mathrm{F}\Psi_1=\frac{1}{2}\partial\Psi_{1\bar{\mathrm{H}}}$), then we have

$$\partial\Psi=\mathrm{E}\Psi+2\mathrm{F}\Psi,\quad \partial\Psi_1=\mathrm{E}\Psi_1+2\mathrm{F}\Psi_1.$$

We have moreover

$$\left.\begin{aligned}\text{Jac.}\,(\mathrm{U},\bar{\nabla}\mathrm{H},\mathrm{H})\ \ &=-\frac{m-1}{3m-7}\mathrm{E}\Psi_1,\\ \text{Jac.}\,(\mathrm{U},\nabla\ \ ,\mathrm{H})\mathrm{H}&=-\qquad\ \ \mathrm{E}\Psi\ ,\end{aligned}\right\}\begin{aligned}&\textit{post}, \text{ Nos. 47 to 50.}\\ &\textit{post}, \text{ Nos. 51 to 53.}\end{aligned}$$

23. The just-mentioned formulæ give

$$\begin{aligned}\Pi=&-(5m^2-18m+17)(4m-9)\mathrm{E}\Psi\\ &-2(5m-9)(m-2)(4m-9)\frac{m-1}{3m-7}\mathrm{F}\Psi_1\\ &+(m-2)(5m^2-18m+17)(\mathrm{E}\Psi\ +2\mathrm{F}\Psi\ )\\ &+\frac{(5m-9)(m-1)^2(m-2)}{3m-7}\quad(\mathrm{E}\Psi_1+2\mathrm{F}\Psi_1),\end{aligned}$$

that is

$$\begin{aligned}\Pi=&-(3m-7)(5m^2-18m+17)\quad\mathrm{E}\Psi\\ &+2(m-2)(5m^2-18m+17)\quad\mathrm{F}\Psi\\ &+\frac{(5m-9)(m-1)^2(m-2)}{3m-7}\qquad\mathrm{E}\Psi_1\\ &-\frac{2(m-1)(m-2)(3m-8)(5m-9)}{3m-7}\mathrm{F}\Psi_1,\end{aligned}$$

or, as this may also be written,

$$\begin{aligned}(3m-7)\Pi=&-(5m^2-18m+17)\{-2(m-1)(\ m-2)\mathrm{F}\Psi_1 &&+(3m-7)^2\mathrm{E}\Psi\ \}\\ &-(5m-9)(m-2)\ \ \{\quad(m-1)(3m-8)\mathrm{F}\Psi_1+(3m-7)(3m-8)\mathrm{F}\Psi &&-(\ m-1)^2\mathrm{E}\Psi_1\}\\ &+(25m^2-103m+106)(m-2)\{\qquad-(\ m-1)\mathrm{F}\Psi_1+\qquad(3m-7)\mathrm{F}\Psi &&\qquad\qquad\}.\end{aligned}$$

24. But recollecting that

$$\Omega=(\mathfrak{A}, \mathfrak{B}, \mathfrak{C}, \mathfrak{F}, \mathfrak{G}, \mathfrak{H}\between\partial_x, \partial_y, \partial_z)^2\mathrm{H}$$
$$=(\mathfrak{A}, \mathfrak{B}, \mathfrak{C}, \mathfrak{F}, \mathfrak{G}, \mathfrak{H}\between a', b', c', 2f', 2g', 2h'),$$

and putting

$$\mathrm{E}\Omega=(\partial\mathfrak{A}, \ldots\between\ a', \ldots) \qquad (=\partial\Omega_{\overline{\mathrm{H}}}),$$
$$\mathrm{F}\Omega=(\ \mathfrak{A}, \ldots\between\partial a', \ldots) \qquad (=\partial\Omega_{\overline{\mathrm{U}}}),$$

we have, *post*, Nos. 41 to 46,

$$-2(m-1)(m-2)\ \mathrm{F}\Psi_1 \qquad\qquad +(3m-7)^2\mathrm{E}\Psi=(3m-6)(3m-7)\mathrm{HE}\Omega$$
$$(m-1)(3m-8)\mathrm{F}\Psi_1+(3m-7)(3m-8)\mathrm{F}\Psi-(\ m-1)^2\mathrm{E}\Psi_1=(3m-6)(3m-7)\mathrm{HF}\Omega$$
$$-(\ m-1)\mathrm{F}\Psi_1+ \qquad (3m-7)\mathrm{F}\Psi \qquad\qquad = \qquad (3m-7)\Omega\partial\mathrm{H},$$

and the foregoing equation becomes

$$(3m-7)\Pi=-(5m^2-18m+17)(3m-6) \qquad (3m-7)\mathrm{HE}\Omega$$
$$-(5m-\ 9)(m-2)(3m-6) \qquad (3m-7)\mathrm{HF}\Omega$$
$$+(\ m-\ 2)(25m^2-103m-106)(3m-7)\Omega\partial\mathrm{H}.$$

25. But we have

$$\vartheta\,\mathrm{Jac.}(\mathrm{U}, \mathrm{H}, \Omega_{\overline{\mathrm{H}}})-(3m-6)\mathrm{HE}\Omega+(2m-4)\Omega\partial\mathrm{H}=0, \quad .\ .\ .\ .\ . \quad (\mathrm{J})$$
$$\vartheta\,\mathrm{Jac.}(\mathrm{U}, \mathrm{H}, \Omega_{\overline{\mathrm{U}}})-(3m-6)\mathrm{HF}\Omega+(3m-6)\Omega\partial\mathrm{H}=0, \quad .\ .\ .\ .\ . \quad (\mathrm{J})$$

that is

$$3(m-2)\mathrm{HE}\Omega=2(m-2)\Omega\partial\mathrm{H}+\vartheta\,\mathrm{Jac.}(\mathrm{U}, \mathrm{H}, \Omega_{\overline{\mathrm{H}}}),$$
$$3(m-2)\mathrm{HF}\Omega=(3m-8)\Omega\partial\mathrm{H}+\vartheta\,\mathrm{Jac.}(\mathrm{U}, \mathrm{H}, \Omega_{\overline{\mathrm{U}}}),$$

and we thus obtain

$$\Pi=-(5m^2-18m+17)\{2(m-2)\Omega\partial\mathrm{H}+\vartheta\,\mathrm{Jac.}(\mathrm{U}, \mathrm{H}, \Omega_{\overline{\mathrm{H}}})\}$$
$$-(5m-9)(m-2)\ \ \{(3m-8)\Omega\partial\mathrm{H}+\vartheta\,\mathrm{Jac.}(\mathrm{U}, \mathrm{H}, \Omega_{\overline{\mathrm{U}}})\}$$
$$+(25m^2-103m+106)(m-2)\Omega\partial\mathrm{H},$$

where the coefficient of $(m-2)\Omega\partial\mathrm{H}$ is

$$-(10m^2-36m+34)$$
$$-(5m-9)(3m-8)$$
$$+(25m^2-103m+106),$$

which is $=0$. Hence

$$\Pi=-(5m^2-18m+17)\vartheta\,\mathrm{Jac.}(\mathrm{U}, \mathrm{H}, \Omega_{\overline{\mathrm{H}}})$$
$$-(5m-9)(m-2)\ \ \vartheta\,\mathrm{Jac.}(\mathrm{U}, \mathrm{H}, \Omega_{\overline{\mathrm{U}}}).$$

26. Substituting this in the equation

$$3\mathrm{H}\Pi+(m-2)^2\{-27\mathrm{H}\,\mathrm{Jac.}(\mathrm{U}, \Omega, \mathrm{H})+40\,\mathrm{Jac.}(\mathrm{U}, \Psi, \mathrm{H})\}=0,$$

the result contains the factor ϑ, and, throwing this out, the condition is

$$3\mathrm{H}\{-(5m^2\ \ 18m+17)\,\mathrm{Jac.}(\mathrm{U}, \mathrm{H}, \Omega_{\overline{\mathrm{H}}})-(5m-9)(m-2)\,\mathrm{Jac.}(\mathrm{U}, \mathrm{H}, \Omega_{\overline{\mathrm{U}}})\}$$
$$+(m-2)^2\{27\mathrm{H}\,\mathrm{Jac.}(\mathrm{U}, \mathrm{H}, \Omega)-40\,\mathrm{Jac.}(\mathrm{U}, \mathrm{H}, \Psi)\}=0,$$

or, as this may also be written,

$$\begin{aligned}&-(15m^2-54m+51)\mathrm{H\,Jac.}(\mathrm{U},\mathrm{H},\Omega_{\bar{\mathrm{H}}})-3(5m-9)(m-2)\mathrm{H\,Jac.}(\mathrm{U},\mathrm{H},\Omega_{\bar{\mathrm{U}}})\\&+27(m-2)^2\quad\{\mathrm{H\,Jac.}(\mathrm{U},\mathrm{H},\Omega_{\bar{\mathrm{H}}})+\quad\mathrm{H\,Jac.}(\mathrm{U},\mathrm{H},\Omega_{\bar{\mathrm{U}}})\}\\&-40(m-2)^2\quad\mathrm{Jac.}(\mathrm{U},\mathrm{H},\Psi)=0.\end{aligned}$$

27. Hence the condition finally is

$$\begin{aligned}&(12m^2-54m+57)\,\mathrm{H\,Jac.}(\mathrm{U},\mathrm{H},\Omega_{\bar{\mathrm{H}}})+(m-2)(12m-27)\mathrm{H\,Jac.}(\mathrm{U},\mathrm{H},\Omega_{\bar{\mathrm{U}}})\\&\qquad-40(m-2)^2\,\mathrm{Jac.}(\mathrm{U},\mathrm{H},\Psi)=0,\end{aligned}$$

or, as this may also be written,

$$\begin{aligned}&-3(m-1)\mathrm{H\,Jac.}(\mathrm{U},\mathrm{H},\Omega_{\bar{\mathrm{H}}})+(m-2)(12m-27)\mathrm{H\,Jac.}(\mathrm{U},\mathrm{H},\Omega)\\&\qquad-40\,(m-2)^2\,\mathrm{Jac.}(\mathrm{U},\mathrm{H},\Psi)=0,\end{aligned}$$

viz. the sextactic points are the intersections of the curve m with the curve represented by this equation; and observing that U, H, HΩ and Ψ are of the orders m, $3m-6$, $8m-18$ respectively, the order of the curve is as above mentioned $=12m-27$.

Article Nos. 28 to 30.—*Application to a Cubic.*

28. I have in my former memoir, No. 30, shown that for a cubic curve

$$\Omega=(\mathfrak{A},\mathfrak{B},\mathfrak{C},\mathfrak{F},\mathfrak{G},\mathfrak{H})(\partial_x,\partial_y,\partial_z)^2\mathrm{H}=-2\mathrm{S}\,.\,\mathrm{U}=0,$$

this implies Jac. (U, H, Ω)=0, and hence if one of the two Jacobians, Jac. $(\mathrm{U},\mathrm{H},\Omega_{\bar{\mathrm{U}}})$, Jac. $(\mathrm{U},\mathrm{H},\Omega_{\bar{\mathrm{H}}})$ vanish, the other will also vanish. Now, using the canonical form

$$\mathrm{U}=x^3+y^3+z^3+6lxyz,$$

we have

$$\begin{aligned}\Omega&=(\mathfrak{A},\,.\;.\,)(a',\ldots)\\&=(yz-l^2x^2,\ zx-l^2y^2,\ xy-l^2z^2,\ l^2yz-lx^2,\ l^2zx-ly^2,\ l^2xy-lz^2)(\\&\quad)(\ -3l^2x,\quad -3l^2y,\quad -3l^2z,\ (1+2l^3)x,\ (1+2l^3)y,\ (1+2l^3)z\,),\end{aligned}$$

the development of which in fact gives the last-mentioned result. But applying this formula to the calculation of Jac. $(\mathrm{U},\mathrm{H},\Omega_{\bar{\mathrm{U}}})$, then disregarding numerical factors, we have

$$\begin{aligned}\partial_x\Omega_{\bar{\mathrm{U}}}&=(yz-l^2x^2,\,.\,,\,.\;l^2yz-lx^2,\,.\,,\,.\,)(-3l^2,\,0,\,0,\,(1+2l^3),\,0,\,0)\\&=-3l^2\quad(yz-l^2x^2\,)\\&\quad+(1+2l^3\,)(l^2yz-lx^2)\\&=(-l+l^4)(x^2+2lyz)=\mathrm{S}\partial_x\mathrm{U};\end{aligned}$$

and in like manner

$$\partial_y\Omega_{\bar{\mathrm{U}}}=\mathrm{S}\partial_y\mathrm{U},\quad\partial_z\Omega_{\bar{\mathrm{U}}}=\mathrm{S}\partial_z\mathrm{U},$$

and therefore

$$\mathrm{Jac.}(\mathrm{U},\mathrm{H},\Omega_{\bar{\mathrm{U}}})=\mathrm{S\,Jac.}(\mathrm{U},\mathrm{H},\mathrm{U})=0,$$

whence also

$$\text{Jac.}\,(\mathrm{U},\ \mathrm{H},\ \Omega_{\mathrm{H}})=0\,;$$

and the condition for a sextactic point assumes the more simple form,

$$\text{Jac.}\,(\mathrm{U},\ \mathrm{H},\ \Psi)=0.$$

29. Now (former memoir, No. 32) we have

$$\begin{aligned}\Psi&=(\mathfrak{A},\ \mathfrak{B},\ \mathfrak{C},\ \mathfrak{F},\ \mathfrak{G},\ \mathfrak{H}\between\partial_x\mathrm{H},\ \partial_y\mathrm{H},\ \partial_z\mathrm{H})^2\\ &=\ \ (1+8l^3)^2 \qquad\qquad (y^3z^3+z^3x^3+x^3y^3)\\ &\quad+(-9l^6) \qquad\qquad\quad (x^3+y^3+z^3)^2\\ &\quad+(-2l-5l^4-20l^7)\ \ (x^3+y^3+z^3)xyz\\ &\quad+(-15l^2-78l^5+12l^8)x^2y^2z^2,\end{aligned}$$

or observing that $x^3+y^3+z^3$ and xyz, and therefore the last three lines of the expression of Ψ are functions of $\mathrm{U}(=x^3+y^3+z^3+6lxyz)$ and $\mathrm{H}(=-l^2(x^3+y^3+z^3)+(1+2l^3)xyz)$, and consequently give rise to the term$=0$ in Jac. (U, H, Ψ), we may write

$$\Psi=(1+8l^3)^2(y^3z^3+z^3x^3+x^3y^3).$$

30. We have then, disregarding a constant factor,

$$\begin{aligned}\text{Jac.}\,(\mathrm{U},\ \mathrm{H},\ \Psi)&=\text{Jac.}\,(x^3+y^3+z^3,\ xyz,\ y^3z^3+z^3x^3+x^3y^3)\\ &=\begin{vmatrix} x^2, & y^2, & z^2\\ yz, & zx, & xy\\ x^2(y^3+z^3), & y^2(z^3+x^3), & z^2(x^3+y^3)\end{vmatrix}\\ &=x^3(y^6-z^6)+y^3(z^6-x^6)+z^3(x^6-y^6)\\ &=(y^3-z^3)(z^3-x^3)(x^3-y^3),\end{aligned}$$

so that the sextactic points are the intersections of the curve

$$\mathrm{U}=x^3+y^3+z^3+6lxyz=0,$$

with the curve

$$(y^3-z^3)(z^3-x^3)(x^3-y^3)=0.$$

Article Nos. 31 to 33.—*Proof of identities for the first transformation.*

31. Calculation of $(\partial_1^5+10\partial_1^3\partial_2+10\partial_1^2\partial_3+15\partial_1\partial_2^2)\mathrm{U}$.

Writing ∂ in place of $\mathbf{D}$, we have (former memoir, No. 20)

$$(\partial_1^4+6\partial_1^2\partial_2)\mathrm{U}=\frac{\vartheta^2}{(m-1)^2}\Big(-2\partial_2\mathrm{H}-\partial^2\mathrm{H}+\frac{3m-6}{m-1}\mathrm{H}\Phi-\frac{2\vartheta}{m-1}\nabla\mathrm{H}\Big).$$

But

$$\left.\begin{aligned}-2\partial_2\mathrm{H}&=\frac{6m-12}{m-1}\mathrm{H}\Phi-\frac{2\vartheta}{m-1}\nabla\mathrm{H},\\ -\partial^2\mathrm{H}&=\frac{(3m-6)(3m-7)}{(m-1)^2}\mathrm{H}\Phi-\frac{6m-14}{(m-1)^2}\vartheta\nabla\mathrm{H}+\frac{\vartheta^2}{(m-1)}\Omega,\end{aligned}\right\}\ \begin{matrix}\text{former memoir,}\\ \text{Nos. 21 \& 22;}\end{matrix}$$

and thence

$$
\begin{aligned}
(\partial_1^4+6\partial_1^2\partial_2)\mathrm{U}= {}& \frac{\vartheta^2}{(m-1)^4}(18m^2-66m+60)\mathrm{H}\Phi \\
&+\frac{\vartheta^3}{(m-1)^4}(-10m+18)\nabla\mathrm{H} \\
&+\frac{\vartheta^4}{(m-1)^4}(\Omega);
\end{aligned}
$$

whence operating on each side with ∂_1, $=\partial$, we have

$$
\begin{aligned}
(\partial_1^5+10\partial_1^3\partial_2+6\partial_1^2\partial_3+12\partial_1\partial_2^2)\mathrm{U}= {}& \frac{\vartheta^2}{(m-1)^4}(18m^2-66m+60)(\mathrm{H}\partial\Phi+\Phi\partial\mathrm{H}) \\
&+\frac{\vartheta^3}{(m-1)^4}(-10m+18)\{(\partial.\nabla)\mathrm{H}+\partial\nabla\mathrm{H}\} \\
&+\frac{\vartheta^4}{(m-1)^4}\partial\Omega.
\end{aligned}
$$

We have besides (*see Appendix*, Nos. 69 to 74),

$$
\begin{aligned}
\partial_1^2\partial_3\mathrm{U}= {}& \frac{\vartheta^2}{(m-1)^3}\{(3m-6)\mathrm{H}\partial\Phi+(-m+3)\Phi\partial\mathrm{H}\} \\
&+\frac{\vartheta^3}{(m-1)^3}\{-(\partial.\nabla)\mathrm{H}\}, \\
\partial_1\partial_2^2\mathrm{U}= {}& \frac{\vartheta^2}{(m-1)^2}(-\mathrm{H}\partial\Phi+\Phi\partial\mathrm{H});
\end{aligned}
$$

and thence

$$
\begin{aligned}
(4\partial_1^2\partial_3+3\partial_1\partial_2^2)\mathrm{U}= {}& \frac{\vartheta^2}{(m-1)^3}\{(90m-21)\mathrm{H}\partial\Phi+(-m+9)\Phi\partial\mathrm{H}\} \\
&+\frac{\vartheta^3}{(m-1)^3}\{-4(\partial\,.\,\nabla)\mathrm{H}\};
\end{aligned}
$$

and adding this to the foregoing expression for

$$(\partial_1^5+10\partial_1^3\partial_2+6\partial_1^2\partial_3+12\partial_1\partial_2^2)\mathrm{U},$$

we have

$$
\begin{aligned}
&(\partial_1^5+10\partial_1^3\partial_2+10\partial_1^2\partial_3+15\partial_1\partial_2^2)\mathrm{U}= \\
&\qquad \frac{\vartheta^2}{(m-1)^4}\{(27m^2-96m+81)\mathrm{H}\partial\Phi+(17m^2-56m+51)\Phi\partial\mathrm{H}\} \\
&\qquad +\frac{\vartheta^3}{(m-1)^4}\{(-14m+22)(\partial\,.\,\nabla)\mathrm{H}+(-10m+18)\partial\nabla\,.\,\mathrm{H}\} \\
&\qquad +\frac{\vartheta^4}{(m-1)^4}\partial\Omega.
\end{aligned}
$$

32. Calculation of

$$\partial_4\mathrm{U}\partial_1\mathrm{H}+2\partial_3\mathrm{U}\partial_2\mathrm{H}+2\partial_2\mathrm{U}\partial_3\mathrm{H}.$$

We have

$$\begin{array}{l|l} \partial_4\mathrm{U}=\frac{\vartheta^2}{(m-1)^2}\,\tfrac{2}{3}\partial_2\mathrm{H}+\partial^2\mathrm{H}-\frac{1}{m-1}\mathrm{H}\Phi-\frac{\frac{2}{3}\vartheta}{m-1}\nabla\mathrm{H}, & \partial_1\mathrm{H}=\partial\mathrm{H}, \\ \partial_3\mathrm{U}=\frac{\vartheta^2}{(m-1)^2}\,\partial\mathrm{H}, & \partial_2\mathrm{H}=\partial_2\mathrm{H}, \\ \partial_2\mathrm{U}=\frac{\vartheta^2}{(m-1)^2}\,\mathrm{H}, & \partial_3\mathrm{H}=\frac{1}{m-1}(-3m+6)\partial\Phi-\Phi\partial\mathrm{H}+\frac{\vartheta}{m-1}(\partial\,.\,\nabla)\mathrm{H}, \end{array}$$

for which values *see Appendix*, No. 58. And hence the expression sought for is

$$\begin{aligned} =\frac{\vartheta^2}{(m-1)^3}\Big\{&\Big((m-1)(\tfrac{2}{3}\partial_2\mathrm{H}+\partial^2\mathrm{H})-\mathrm{H}\Phi-\tfrac{2}{3}\vartheta\nabla\mathrm{H}\Big)\partial\mathrm{H} \\ &+2(m-1)\partial\mathrm{H}\partial_2\mathrm{H} \\ &+2\mathrm{H}\Big((-3m+6)\mathrm{H}\partial\Phi-\Phi\partial\mathrm{H}+\vartheta(\partial\,.\,\nabla)\mathrm{H}\Big)\Big\}, \end{aligned}$$

which is

$$\begin{aligned} =\frac{\vartheta^2}{(m-1)^3}\{\quad &\tfrac{8}{3}(m-1)\partial\mathrm{H}\partial_2\mathrm{H} \\ +\;&(m-1)\partial\mathrm{H}\partial^2\mathrm{H} \\ +\;&(-6m+12)\mathrm{H}^2\partial\Phi-3\mathrm{H}\Phi\partial\mathrm{H}\} \\ +\frac{\vartheta^3}{(m-1)^3}\{&2\mathrm{H}(\partial\,.\,\nabla)\mathrm{H}-\tfrac{2}{3}\partial\mathrm{H}\nabla\mathrm{H}\}. \end{aligned}$$

But we have, former memoir, Nos. 21 & 25,

$$\partial_2\mathrm{H}=-\frac{(3m-6)}{m-1}\,\mathrm{H}\Phi-\frac{\vartheta}{m-1}\,\nabla\mathrm{H},$$

$$\partial^2\mathrm{H}=-\frac{(3m-6)(3m-7)}{(m-1)^2}\,\mathrm{H}\Phi+\frac{6m-14}{(m-1)^2}\vartheta\nabla\mathrm{H}-\frac{\vartheta^2}{(m-1)^2}\,\Omega,$$

so that the foregoing expression becomes

$$\begin{aligned} =\frac{\vartheta^2}{(m-1)^3}\{&-(8m-16)\mathrm{H}\Phi\partial\mathrm{H}+\tfrac{8}{3}\vartheta\partial\mathrm{H}\nabla\mathrm{H} \\ &-\frac{(3m-6)(3m-7)}{m-1}\,\mathrm{H}\Phi\partial\mathrm{H}+\frac{6m-14}{m-1}\,\vartheta\partial\mathrm{H}\nabla\mathrm{H}-\frac{\vartheta^2}{m-1}\,\Omega\partial\mathrm{H} \\ &-\qquad 3\mathrm{H}\Phi\partial\mathrm{H}-(6m-12)\mathrm{H}^2\partial\Phi\} \\ +\frac{\vartheta^3}{(m-1)^3}\{&2\mathrm{H}(\partial.\nabla)\mathrm{H}-\tfrac{2}{3}\partial\mathrm{H}\nabla\mathrm{H}\}\;; \end{aligned}$$

or finally

$$\begin{aligned} &\partial_4\mathrm{U}\partial_1\mathrm{H}+2\partial_3\mathrm{U}\partial_2\mathrm{H}+2\partial_2\mathrm{U}\partial_3\mathrm{H} \\ &=\frac{\vartheta^2}{(m-1)^4}\{(-6m^2+18m-12)\mathrm{H}^2\partial\Phi+(-17m^2+60m-55)\mathrm{H}\Phi\partial\mathrm{H}\} \\ &+\frac{\vartheta^3}{(m-1)^4}\{(2m-2)\mathrm{H}(\partial.\nabla)\mathrm{H}+(8m-16)\partial\mathrm{H}\nabla\mathrm{H}\} \\ &+\frac{\vartheta^4}{(m-1)^4}\{-\Omega\partial\mathrm{H}\}. \end{aligned}$$

33. Calculation of $\partial_2 U \partial_3 U$.

This is

$$=\frac{\vartheta^4}{(m-1)^4} H\partial H.$$

Article Nos. 34 & 35.—*The Jacobian Formula.*

34. In general, if P, Q, R, S be functions of the degrees p, q, r, s respectively, we have identically

$$\left|\begin{array}{llll} pP, & qQ, & rR, & sS \\ \partial_x P, & \partial_x Q, & \partial_x R, & \partial_x S \\ \partial_y P, & \partial_y Q, & \partial_y R, & \partial_y S \\ \partial_z P, & \partial_z Q, & \partial_z R, & \partial_z S \end{array}\right| = 0,$$

or, what is the same thing,

$$pP \,\text{Jac.}\,(Q, R, S) - qQ \,\text{Jac.}\,(R, S, P) + rR \,\text{Jac.}\,(S, P, Q) - sS \,\text{Jac.}\,(P, Q, R) = 0.$$

Hence in particular if $P=U$, and assuming $U=0$, we have

$$-qQ \,\text{Jac.}\,(R, S, U) + rR \,\text{Jac.}\,(S, U, Q) - sS \,\text{Jac.}\,(U, Q, R) = 0.$$

If moreover $Q=\vartheta$, and therefore $q=1$, we have

$$-\vartheta \,\text{Jac.}\,(R, S, U) + rR \,\text{Jac.}\,(S, U, \vartheta) - sS \,\text{Jac.}\,(U, \vartheta, R) = 0\,;$$

or, as this may also be written,

$$-\vartheta \,\text{Jac.}\,(U, R, S) + rR \,\text{Jac.}\,(U, \vartheta, S) - sS \,\text{Jac.}\,(U, \vartheta, R) = 0\,;$$

that is

$$-\vartheta \,\text{Jac.}\,(U, R, S) + rR\partial S - sS\partial R = 0.$$

35. Particular cases are

$$\begin{array}{llll}
(2m-\ 4)\ \ \Phi\partial H-(3m-6)H\partial\Phi & =\vartheta \,\text{Jac.}\,(U, \Phi\ \ , H), & \textit{ante}, \text{No.} & 12, \\
(5m-11)\nabla H\partial H-(3m-6)H\partial(\nabla H) & =\vartheta \,\text{Jac.}\,(U, \nabla H, H), & \quad ,, & 14, \\
(2m-\ 4)\,\dot{\nabla} : \partial H-(3m-6)H\partial\,.\nabla & =\vartheta \,\text{Jac.}\,(U, \nabla\ \ , H), & \quad ,, & ,, \\
(5m-12)\ \ \Omega\partial H-(3m-6)H\partial\Omega & =\vartheta \,\text{Jac.}\,(U, \Omega\ \ , H), & \quad ,, & 18, \\
(8m-18)\ \ \Psi\partial H-(3m-6)H\partial\Psi & =\vartheta \,\text{Jac.}\,(U, \Psi\ \ , H), & \quad ,, & 19, \\
(2m-\ 4)\ \ \Omega\partial H-(3m-6)HE\Omega & =\vartheta \,\text{Jac.}\,(U, \Omega_{\overline{H}}\ , H), & \quad ,, & 25, \\
(3m-\ 8)\ \ \Omega\partial H-(3m-6)HF\Omega & =\vartheta \,\text{Jac.}\,(U, \Omega_{\overline{U}}\ , H), & \quad ,, & ,,
\end{array}$$

where it is to be observed that in the third of these formulæ I have, in accordance with the notation before employed, written $\partial\,.\nabla$ to denote the result of the operation ∂ performed on ∇ as operand. I have also written $\nabla : \partial H$ to show that the operation ∇ is not to be performed on the following ∂H as an operand, but that it remains as an unperformed operation. As regards the last two equations, it is to be remarked that the demonstration in the last preceding number depends merely on the homogeneity of the functions, and the orders of these functions: in the former of the two formulæ, the differentiation of Ω is performed upon Ω in regard to the coordinates (x, y, z) in so far only as they enter through U, and Ω is therefore to be regarded as a function of the order $2m-4$; in the latter of the two formulæ the differentiation is to be performed in regard to the coordinates in so far only as they enter through H, and Ω is therefore to be regarded as a function of the order $3m-8$. The two formulæ might also be written

$$(2m-4)\Omega\partial\mathrm{H}-(3m-6)\mathrm{H}\partial\Omega_{\overline{\mathrm{H}}}=\vartheta\,\mathrm{Jac.}\,(\mathrm{U},\Omega_{\overline{\mathrm{H}}},\mathrm{H}),$$
$$(3m-8)\Omega\partial\mathrm{H}-(3m-6)\mathrm{H}\partial\Omega_{\overline{\mathrm{U}}}=\vartheta\,\mathrm{Jac.}\,(\mathrm{U},\Omega_{\overline{\mathrm{U}}},\mathrm{H});$$

and it may be noticed that, adding these together, we obtain the foregoing formula,

$$(5m-12)\Omega\partial\mathrm{H}-(3m-6)\mathrm{H}\partial\Omega=\vartheta\,\mathrm{Jac.}\,(\mathrm{U},\Omega,\mathrm{H}).$$

Article Nos. 36 to 40.—*Proof of equation* $(\partial.\nabla)\mathrm{H}=\mathrm{Jac.}(\mathrm{U},\mathrm{H},\Phi)$, *used in the second transformation.*

36. We have

$$\begin{aligned}\nabla&=(\mathfrak{A},\ldots\between\lambda,\mu,\nu\between\partial_x,\partial_y,\partial_z)\\&=(\mathfrak{A}\partial_x+\mathfrak{H}\partial_y+\mathfrak{G}\partial_x,\ \mathfrak{H}\partial_x+\mathfrak{B}\partial_y+\mathfrak{F}\partial_z,\ \mathfrak{G}\partial_x+\mathfrak{F}\partial_y+\mathfrak{C}\partial_z\between\lambda,\mu,\nu).\end{aligned}$$

Also

$$\begin{aligned}\partial&=(\mathrm{B}\nu-\mathrm{C}\mu)\partial_x+(\mathrm{C}\lambda-\mathrm{A}\nu)\partial_y+(\mathrm{A}\mu-\mathrm{B}\lambda)\partial_z\\&=\lambda\mathrm{P}+\mu\mathrm{Q}+\nu\mathrm{R},\end{aligned}$$

if for a moment

$$\mathrm{P},\mathrm{Q},\mathrm{R}=\mathrm{C}\partial_y-\mathrm{B}\partial_z,\ \mathrm{A}\partial_z-\mathrm{C}\partial_x,\ \mathrm{B}\partial_x-\mathrm{A}\partial_y.$$

Hence

$$\partial.\nabla=(\mathrm{P}\lambda+\mathrm{Q}\mu+\mathrm{R}\nu).(\mathfrak{A}\partial_x+\mathfrak{H}\partial_y+\mathfrak{G}\partial_z,\ \mathfrak{H}\partial_x+\mathfrak{B}\partial_y+\mathfrak{F}\partial_z,\ \mathfrak{G}\partial_x+\mathfrak{F}\partial_y+\mathfrak{C}\partial_z\between\lambda,\mu,\nu),$$

viz. coefficient of λ^2

$$=\mathrm{P}\mathfrak{A}\partial_x+\mathrm{P}\mathfrak{H}\partial_y+\mathrm{P}\mathfrak{G}\partial_z,$$

and so for the other terms; whence also in $(\partial.\nabla)\mathrm{H}$ the coefficients of λ^2, &c. are

$$(\mathrm{P}\mathfrak{A}\partial_x+\mathrm{P}\mathfrak{H}\partial_y+\mathrm{P}\mathfrak{G}\partial_z)\mathrm{H},\ \text{\&c.}$$

37. Again, in Jac. $(\mathrm{U},\mathrm{H},\Phi)$, where $\Phi=(\mathfrak{A},\mathfrak{B},\mathfrak{C},\mathfrak{F},\mathfrak{G},\mathfrak{H}\between\lambda,\mu,\nu)^2$, the coefficients of λ^2, &c. are Jac. $(\mathrm{U},\mathrm{H},\mathfrak{A})$, &c.; and hence the assumed equation

$$(\partial.\nabla)\mathrm{H}=\mathrm{Jac.}\,(\mathrm{U},\mathrm{H},\Phi),$$

in regard to the term in λ^2, is

$$(\mathrm{P}\mathfrak{A}\partial_x+\mathrm{P}\mathfrak{H}\partial_y+\mathrm{P}\mathfrak{G}\partial_z)\mathrm{H}=\mathrm{Jac.}\,(\mathrm{U},\mathrm{H},\mathfrak{A}),$$

and we have

$$\begin{aligned}\mathrm{Jac.}\,(\mathrm{U},\mathrm{H},\mathfrak{A})&=\begin{vmatrix}\mathrm{A}, & \mathrm{B}, & \mathrm{C}\\ \partial_x\mathrm{H}, & \partial_y\mathrm{H}, & \partial_z\mathrm{H}\\ \partial_x, & \partial_y, & \partial_z\end{vmatrix}\mathfrak{A}\\&=[\partial_x\mathrm{H}(\mathrm{C}\partial_y-\mathrm{B}\partial_z)+\partial_y\mathrm{H}(\mathrm{A}\partial_z-\mathrm{C}\partial_x)+\partial_z\mathrm{H}(\mathrm{B}\partial_x-\mathrm{A}\partial_y)]\mathfrak{A}\\&=(\partial_x\mathrm{H}.\mathrm{P}+\partial_y\mathrm{H}.\mathrm{Q}+\partial_z\mathrm{H}.\mathrm{R})\mathfrak{A};\end{aligned}$$

so that the equation is

$$\begin{aligned}&\mathrm{P}\mathfrak{A}\partial_x\mathrm{H}+\mathrm{P}\mathfrak{H}\partial_y\mathrm{H}+\mathrm{P}\mathfrak{G}\partial_z\mathrm{H}\\ &=\mathrm{P}\mathfrak{A}\partial_x\mathrm{H}+\mathrm{Q}\mathfrak{A}\partial_y\mathrm{H}+\mathrm{R}\mathfrak{A}\partial_z\mathrm{H},\end{aligned}$$

or, as this may be written,

$$\begin{aligned}&[(\mathrm{B}\partial_z-\mathrm{C}\partial_y)\mathfrak{H}-(\mathrm{C}\partial_x-\mathrm{A}\partial_z)\mathfrak{A}]\partial_y\mathrm{H}\\ &+[(\mathrm{B}\partial_z-\mathrm{C}\partial_y)\mathfrak{G}-(\mathrm{A}\partial_y-\mathrm{B}\partial_x)\mathfrak{A}]\partial_z\mathrm{H}=0.\end{aligned}$$

38. The coefficient of $\partial_y\mathrm{H}$ is

$$=\mathrm{A}\partial_z\mathfrak{A}+\mathrm{B}\partial_z\mathfrak{H}-\mathrm{C}(\partial_x\mathfrak{A}+\partial_y\mathfrak{H}),$$

which, in virtue of the identity, *post*, No. 40,

$$\partial_x\mathfrak{A}+\partial_y\mathfrak{H}+\partial_z\mathfrak{G}=0,$$

is

$$=\mathrm{A}\partial_z\mathfrak{A}+\mathrm{B}\partial_z\mathfrak{H}+\mathrm{C}\partial_z\mathfrak{G}.$$

And in like manner the coefficient of $\partial_z\mathrm{H}$

$$=-(\mathrm{A}\partial_y\mathfrak{A}+\mathrm{B}\partial_y\mathfrak{H}+\mathrm{C}\partial_y\mathfrak{G}),$$

so that the equation is

$$(\mathrm{A}\partial_z\mathfrak{A}+\mathrm{B}\partial_z\mathfrak{H}+\mathrm{C}\partial_z\mathfrak{G})\partial_y\mathrm{H}-(\mathrm{A}\partial_y\mathfrak{A}+\mathrm{B}\partial_y\mathfrak{H}+\mathrm{C}\partial_z\mathfrak{G})\partial_z\mathrm{H}=0.$$

39. But we have

$$\begin{aligned}&\mathfrak{A}a+\mathfrak{H}h+\mathfrak{G}g=\mathrm{H},\\ &\mathfrak{A}h+\mathfrak{H}b+\mathfrak{G}f=0,\\ &\mathfrak{A}g+\mathfrak{H}f+\mathfrak{G}c=0,\end{aligned}$$

or multiplying by x, y, z and adding,

$$(m-1)(\mathfrak{A}\mathrm{A}+\mathfrak{H}\mathrm{B}+\mathfrak{G}\mathrm{C})=x\mathrm{H}\,;$$

whence also

$$(m-1)(\mathfrak{A}h+\mathfrak{H}b+\mathfrak{G}c+\mathrm{A}\partial_y\mathfrak{A}+\mathrm{B}\partial_y\mathfrak{H}+\mathrm{C}\partial_y\mathfrak{G})=x\partial_y\mathrm{H},$$

that is

$$(m-1)(\mathrm{A}\partial_y\mathfrak{A}+\mathrm{B}\partial_y\mathfrak{H}+\mathrm{C}\partial_y\mathfrak{G})=x\partial_y\mathrm{H}\,;$$

and in like manner

$$(m-1)(\mathrm{A}\partial_z\mathfrak{A}+\mathrm{B}\partial_z\mathfrak{H}+\mathrm{C}\partial_z\mathfrak{G})=x\partial_z\mathrm{H},$$

whence the equation in question. The terms in λ^2 are thus shown to be equal, and it might in a similar manner be shown that the terms in $\mu\nu$ are equal; the other terms will then be equal, and we have therefore

$$(\partial\,.\,\nabla)\mathrm{H}=\text{Jac.}\,(\mathrm{U},\ \mathrm{H},\ \Phi).$$

40. The identity

$$\partial_x\mathfrak{A}+\partial_y\mathfrak{H}+\partial_z\mathfrak{G}=0$$

assumed in the course of the foregoing proof is easily proved. We have in fact

$$\begin{aligned}\partial_x\mathfrak{A}+\partial_y\mathfrak{H}+\partial_z\mathfrak{G}&=\partial_x(bc-f^2)+\partial_y(fg-ch)+\partial_z(fh-bg)\\ &=b(\partial_x c-\partial_z g)+c(\partial_x b-\partial_y h)\\ &\quad+f(-2\partial_x f+\partial_y g+\partial_z h)+g(\partial_y f-\partial_z b)+h(-\partial_y c+\partial_z f),\end{aligned}$$

where the coefficients of b, c, f, g, h separately vanish: we have of course the system

$$\partial_x\mathfrak{A}+\partial_y\mathfrak{H}+\partial_z\mathfrak{G}=0,$$
$$\partial_x\mathfrak{H}+\partial_y\mathfrak{B}+\partial_z\mathfrak{F}=0,$$
$$\partial_x\mathfrak{G}+\partial_y\mathfrak{F}+\partial_z\mathfrak{C}=0.$$

Article Nos. 41 to 46.—*Proof of identities for the fourth transformation.*

41. Consider the coefficients (a, b, c, f, g, h) and the inverse set $(\mathfrak{A}, \mathfrak{B}, \mathfrak{C}, \mathfrak{F}, \mathfrak{G}, \mathfrak{H})$, and the coefficients (a', b', c', f', g', h'), and the inverse set $(\mathfrak{A}', \mathfrak{B}', \mathfrak{C}', \mathfrak{F}', \mathfrak{G}', \mathfrak{H}')$; then we have identically

$$\begin{aligned}&(a, \ldots \between x, y, z)^2(\mathfrak{A}', \ldots \between a, \ldots)-(\mathfrak{A}', \ldots \between ax+hy+gz, \ldots)^2\\ =&(a', \ldots \between x, y, z)^2(\mathfrak{A}, \ldots \between a', \ldots)-(\mathfrak{A}, \ldots \between a'x+h'y+g'z, \ldots)^2,\end{aligned}$$

where $(\mathfrak{A}', \ldots \between a, \ldots)$ and $(\mathfrak{A}, \ldots \between a', \ldots)$ stand for

$$(\mathfrak{A}', \mathfrak{B}', \mathfrak{C}', \mathfrak{F}', \mathfrak{G}', \mathfrak{H}' \between a, b, c, 2f, 2g, 2h)$$

and

$$(\mathfrak{A}, \mathfrak{B}, \mathfrak{C}, \mathfrak{F}, \mathfrak{G}, \mathfrak{H} \between a', b', c', 2f', 2g', 2h')$$

respectively.

42. Taking (a, b, c, f, g, h), the second differential coefficients of a function U of the order m, and in like manner (a', b', c', f', g', h'), the second differential coefficients of a function U$'$ of the order m', we have

$$\begin{aligned}&m(m-1)\mathrm{U}\,.(\mathfrak{A}', \ldots \between \partial_x, \partial_y, \partial_z)^2\mathrm{U}'-(m-1)^2(\mathfrak{A}', \ldots \between \partial_x\mathrm{U}, \partial_y\mathrm{U}, \partial_z\mathrm{U})^2\\ =&m'(m'-1)\mathrm{U}'.(\mathfrak{A}, \ldots \between \partial_x, \partial_y, \partial_z)^2\mathrm{U}-(m'-1)^2(\mathfrak{A}, \ldots \between \partial_x\mathrm{U}', \partial_y\mathrm{U}', \partial_z\mathrm{U}')^2;\end{aligned}$$

and in particular if U$'$ be the Hessian of U, then $m'=3m-6$.

43. Hence writing

$$\Omega=(\mathfrak{A}, \ldots \between \partial_x, \partial_y, \partial_z)^2\mathrm{H},\ \ \Psi=(\mathfrak{A}, \ldots \between \partial_x\mathrm{H}, \partial_y\mathrm{H}, \partial_z\mathrm{H})^2,$$
$$\Omega_1=(\mathfrak{A}', \ldots \between \partial_x, \partial_y, \partial_z)^2\mathrm{U},\ \ \Psi_1=(\mathfrak{A}', \ldots \between \partial_x\mathrm{U}, \partial_y\mathrm{U}, \partial_z\mathrm{U})^2,$$

we have

$$m(m-1)\mathrm{U}\Omega_1-(m-1)^2\Psi_1=(3m-6)(3m-7)\mathrm{H}\Omega-(3m-7)^2\Psi;$$

or if $\mathrm{U}=0$, then

$$-(m-1)^2\Psi_1=(3m-6)(3m-7)\mathrm{H}\Omega-(3m-7)^2\Psi;$$

whence also

$$-(m-1)^2\partial\Psi_1=(3m-6)(3m-7)(\mathrm{H}\partial\Omega+\Omega\partial\mathrm{H})-(3m-7)^2\partial\Psi,$$

which is the formula, *ante* No. 21.

44. Recurring to the original formula, since this is an actual identity, we may operate on it with the differential symbol ∂ on the three assumptions,—

1. (a, b, c, f, g, h), $(\mathfrak{A}, \mathfrak{B}, \mathfrak{C}, \mathfrak{F}, \mathfrak{G}, \mathfrak{H})$ are alone variable.
2. (a', b', c', f', g', h'), $(\mathfrak{A}', \mathfrak{B}', \mathfrak{C}', \mathfrak{F}', \mathfrak{G}', \mathfrak{H}')$ are alone variable.
3. (x, y, z) are alone variable.

We thus obtain

$$\begin{aligned}
&(\partial a, \ldots\between x, y, z)^2(\mathfrak{A}', \ldots\between a, \ldots) && =(a', \ldots\between x, y, z)^2(\partial\mathfrak{A}, \ldots\between a', \ldots)\\
&+(a, \ldots\between x, y, z)^2(\mathfrak{A}', \ldots\between\partial a, \ldots) && \quad -(\partial\mathfrak{A}, \ldots\between a'x+h'y+g'z, \ldots)^2,\\
&-2(\mathfrak{A}', \ldots\between ax+hy+gz, \ldots\between x\partial a+y\partial b+z\partial c, \ldots) &&
\end{aligned}$$

$$\begin{aligned}
&(a, \ldots\between x, y, z)^2(\partial\mathfrak{A}', \ldots\between a, \ldots) && =(\partial a', \ldots\between x, y, z)^2(\mathfrak{A}, \ldots\between a', \ldots)\\
&-(\partial\mathfrak{A}', \ldots\between ax+hy+gz, \ldots)^2 && \quad +(a', \ldots\between x, y, z)^2(\mathfrak{A}, \ldots\between\partial a', \ldots)\\
& && \quad -2(\mathfrak{A}, \ldots\between a'x+h'y+g'z, \ldots\between x\partial a'+y\partial h'+z\partial g', \ldots),
\end{aligned}$$

$$\begin{aligned}
&2(a, \ldots\between x, y, z\between\partial x, \partial y, \partial z)(\mathfrak{A}', \ldots\between a, \ldots) && =2(a', \ldots\between x, y, z\between\partial x, \partial y, \partial z)(\mathfrak{A}, \ldots\between a', \ldots)\\
&-2(\mathfrak{A}', .\between ax+hy+gz, .\between a\partial x+h\partial y+g\partial z, .) && \quad -2(\mathfrak{A}, .\between a'x+h'y+g'z, \ldots\between a'\partial x+h'\partial y+g'\partial z, \ldots).
\end{aligned}$$

45. If in these equations respectively we suppose as before that (a, b, c, f, g, h) are the second differential coefficients of a function U of the order m, and (a', b', c', f', g', h') the second differential coefficients of a function U′ of the order m'; and that (A, B, C), (A′, B′, C′) are the first differential coefficients of these functions respectively, then after some easy reductions we have

$$\begin{aligned}
&(m-1)(m-2)\partial\mathrm{U}(\mathfrak{A}', \ldots\between a, \ldots) && = \quad m'(m'-1)\mathrm{U}'(\partial\mathfrak{A}, \ldots\between a', \ldots)\\
&\quad +m(m-1)\mathrm{U}(\mathfrak{A}', \ldots\between\partial a, \ldots) && \quad -(m'-1)^2(\partial\mathfrak{A}, \ldots\between \mathrm{A}', \mathrm{B}', \mathrm{C}')^2,\\
&\quad -2(m-1)(m-2)(\mathfrak{A}', \ldots\between \mathrm{A}, \mathrm{B}, \mathrm{C}\between\partial\mathrm{A}, \partial\mathrm{B}, \partial\mathrm{C}) &&
\end{aligned}$$

$$\begin{aligned}
&m(m-1)\mathrm{U}(\partial\mathfrak{A}', \ldots\between a', \ldots) && = \quad (m'-1)(m'-2)\partial\mathrm{U}'(\mathfrak{A}, \ldots\between a', \ldots)\\
&\quad -(m-1)^2(\partial\mathfrak{A}', \ldots\between \mathrm{A}, \mathrm{B}, \mathrm{C})^2 && \quad +m'(m'-1)\mathrm{U}'(\mathfrak{A}, \ldots\between\partial a', \ldots)\\
& && \quad -2(m'-1)(m'-2)(\mathfrak{A}, \ldots\between \mathrm{A}', \mathrm{B}', \mathrm{C}'\between\partial\mathrm{A}', \partial\mathrm{B}', \partial\mathrm{C}')
\end{aligned}$$

$$\begin{aligned}
&\quad 2(m-1)\partial\mathrm{U}(\mathfrak{A}', \ldots\between a, \ldots) && = \quad 2(m'-1)\partial\mathrm{U}'(\mathfrak{A}, \ldots\between a', \ldots)\\
&-2(m-1)(\mathfrak{A}', \ldots\between \mathrm{A}, \mathrm{B}, \mathrm{C}\between\partial\mathrm{A}, \partial\mathrm{B}, \partial\mathrm{C}) && \quad -2(m'-1)(\mathfrak{A}, \ldots\between \mathrm{A}', \mathrm{B}', \mathrm{C}'\between\partial\mathrm{A}', \partial\mathrm{B}, \partial\mathfrak{C}'),
\end{aligned}$$

equations which may be verified by remarking that their sum is

$$\begin{aligned}
&m(m-1)\{\partial\mathrm{U}(\mathfrak{A}', \ldots\between a, \ldots)+\mathrm{U}[(\mathfrak{A}', \ldots\between\partial a, \ldots)+(\partial\mathfrak{A}', \ldots\between a, \ldots)]\}\\
&-(m-1)^2\{\partial\mathfrak{A}', \ldots\between \mathrm{A}, \mathrm{B}, \mathrm{C})^2+(\mathfrak{A}', \ldots\between \mathrm{A}, \mathrm{B}, \mathrm{C}\between\partial\mathrm{A}, \partial\mathrm{B}, \partial\mathrm{C})\}=m'(m'-1)\ \&\text{c.},
\end{aligned}$$

viz., this is the derivative with ∂ of the equation

$$m(m-1)\mathrm{U}(\mathfrak{A}', \ldots\between a, \ldots)-(m-1)^2(\mathfrak{A}', \ldots\between \mathrm{A}, \mathrm{B}, \mathrm{C})^2=m'(m'-1)\ \&\text{c.}$$

46. Taking now $\mathrm{U}'=\mathrm{H}$, and therefore $m'=3m-6$; putting also $\mathrm{U}=0$, $\partial\mathrm{U}=0$, and writing as before

$$\begin{aligned}
\mathrm{E}\Psi\ &=(\partial\mathfrak{A}, \ldots\between \mathrm{A}', \mathrm{B}', \mathrm{C}')^2,\\
\mathrm{F}\Psi\ &=(\mathfrak{A}, \ldots\between \mathrm{A}', \mathrm{B}', \mathrm{C}'\between\partial\mathrm{A}', \partial\mathrm{B}', \partial\mathrm{C}'),\\
\mathrm{E}\Psi_1&=(\partial\mathfrak{A}', \ldots\between \mathrm{A}, \mathrm{B}, \mathrm{C})^2,\\
\mathrm{F}\Psi_1&=(\mathfrak{A}', \ldots\between \mathrm{A}, \mathrm{B}, \mathrm{C}\between\partial\mathrm{A}, \partial\mathrm{B}, \partial\mathrm{C}),\\
\mathrm{E}\Omega\ &=(\partial\mathfrak{A}, \ldots\between a', \ldots),\\
\mathrm{F}\Omega\ &=(\mathfrak{A}, \ldots\between\partial a', \ldots),
\end{aligned}$$

then the three equations are

$$
\begin{aligned}
-2(m-1)(m-2)\mathrm{F}\Psi_1 &= (3m-6)(3m-7)\mathrm{HE}\Omega-(3m-7)^2\mathrm{E}\Psi,\\
-(m-1)^2\mathrm{E}\Psi &= (3m-7)(3m-8)\Omega\partial\mathrm{H}\\
&\quad +(3m-6)(3m-7)\mathrm{HF}\Omega-2(3m-7)(3m-8)\mathrm{F}\Psi,\\
-2(m-1)\mathrm{F}\Psi_1 &= 2(3m-7)\Omega\partial\mathrm{H}-2(3m-7)\mathrm{F}\Psi,
\end{aligned}
$$

whence, adding, we have

$$
\begin{aligned}
-(m-1)^2(\mathrm{E}\Psi_1+2\mathrm{F}\Psi_1) &= -(3m-7)^2(\mathrm{E}\Psi+2\mathrm{F}\Psi)\\
&\quad +(3m-6)(3m-7)\{\Omega\partial\mathrm{H}+\mathrm{H}(\mathrm{E}\Omega+\mathrm{F}\Omega)\}
\end{aligned}
$$

(that is

$$
-(m-1)^2\partial\Psi_1 = -(3m-7)^2\partial\Psi+(3m-6)(3m-7)\partial\,.\,\Omega\mathrm{H},
$$

which is right).

And by linearly combining the three equations, we deduce

$$
\begin{aligned}
(3m-6)(3m-7)\mathrm{HE}\Omega &= -2(m-1)(m-2)\ \mathrm{F}\Psi_1 \qquad\qquad +(3m-7)^2\mathrm{E}\Psi,\\
(3m-7)\Omega\partial\mathrm{H} &= \ -(m-1)\qquad \mathrm{F}\Psi_1+(3m-7)\qquad \mathrm{F}\Psi,\\
(3m-6)(3m-7)\mathrm{HF}\Omega &= \quad (m-1)(3m-8)\mathrm{F}\Psi_1+(3m-7)(3m-8)\mathrm{F}\Psi-(m-1)^2\mathrm{E}\Psi_1,
\end{aligned}
$$

which are the formulæ, *ante*, No. 24.

Article Nos. 47 to 50.—*Proof of an identity used in the fourth transformation*, viz.,

$$
\mathrm{Jac.}\,(\mathrm{U},\overline{\nabla}\mathrm{H},\mathrm{H})=-\frac{m-1}{3m-7}\,\mathrm{F}\Psi_1,
$$

or say

$$
\mathrm{Jac.}\,(\mathrm{U},\mathrm{H},\overline{\nabla}\mathrm{H})=\ \frac{m-1}{3m-7}\,(\mathfrak{A}',\,..\,)(\mathrm{A},\mathrm{B},\mathrm{C})(\partial\mathrm{A},\partial\mathrm{B},\partial\mathrm{C}).
$$

47. We have

$$
\begin{aligned}
\nabla &= (\mathfrak{A},\,..\,)(\lambda,\mu,\nu)(\partial_x,\partial_y,\partial_z)\\
&= \big((\mathfrak{A},\mathfrak{H},\mathfrak{G})(\lambda,\mu,\nu),\ (\mathfrak{H},\mathfrak{B},\mathfrak{F})(\lambda,\mu,\nu),\ (\mathfrak{G},\mathfrak{F},\mathfrak{C})(\lambda,\mu,\nu)\big)(\partial_x,\partial_y,\partial_z\big);
\end{aligned}
$$

or, attending to the effect of the bar as denoting the exemption of the $(\mathfrak{A},\,..)$ from differentiation,

$$
\begin{aligned}
\mathrm{Jac.}\,(\mathrm{U},\mathrm{H},\overline{\nabla}\mathrm{H}) &= \ (\mathfrak{A},\mathfrak{H},\mathfrak{G})(\lambda,\mu,\nu)\,\mathrm{Jac.}\,(\mathrm{U},\mathrm{H},\partial_x\mathrm{H})\\
&\quad +(\mathfrak{H},\mathfrak{B},\mathfrak{F})(\lambda,\mu,\nu)\,\mathrm{Jac.}\,(\mathrm{U},\mathrm{H},\partial_y\mathrm{H})\\
&\quad +(\mathfrak{G},\mathfrak{F},\mathfrak{C})(\lambda,\mu,\nu)\,\mathrm{Jac.}\,(\mathrm{U},\mathrm{H},\partial_z\mathrm{H}).
\end{aligned}
$$

48. Now

$$
\mathrm{Jac.}(\mathrm{U},\mathrm{H},\partial_x\mathrm{H})=\frac{1}{3m-6}\,\mathrm{Jac.}\,(\mathrm{U},x\partial_x\mathrm{H}+y\partial_y\mathrm{H}+z\partial_z\mathrm{H},\partial_x\mathrm{H}),
$$

and the last-mentioned Jacobian is

$$
\begin{aligned}
&=\partial_x\mathrm{H}\,\mathrm{Jac.}\,(\mathrm{U},x,\partial_x\mathrm{H})+\partial_y\mathrm{H}\,\mathrm{Jac.}\,(\mathrm{U},y,\partial_x\mathrm{H})+\partial_z\mathrm{H}\,\mathrm{Jac.}\,(\mathrm{U},z,\partial_x\mathrm{H})\\
&\quad +y\,\mathrm{Jac.}\,(\mathrm{U},\partial_y\mathrm{H},\partial_x\mathrm{H})+z\,\mathrm{Jac.}\,(\mathrm{U},\partial_z\mathrm{H},\partial_x\mathrm{H}),
\end{aligned}
$$

where the second line is

$$=-y\,\text{Jac.}\,(U,\ \partial_x H,\ \partial_y H)+z\,\text{Jac.}\,(U,\ \partial_z H,\ \partial_x H),$$

or writing (A', B', C') for the first differential coefficients and (a', b', c', f', g', h') for the second differential coefficients of H, this is

$$=-y\begin{vmatrix} A, & B, & C \\ a', & h', & g' \\ h', & b', & f' \end{vmatrix}+z\begin{vmatrix} A, & B, & C \\ g', & f', & c' \\ a', & h', & g' \end{vmatrix}$$

$$=-y(\mathfrak{G}', \mathfrak{F}', \mathfrak{C}')(A, B, C)+z(\mathfrak{H}', \mathfrak{B}', \mathfrak{F}')(A, B, C).$$

The first line is

$$=\begin{vmatrix} A, & B, & C \\ A', & B', & C' \\ a', & h', & g' \end{vmatrix}$$

$$=A(B'g'-C'h')+B(C'a'-A'g')+C(A'h'-B'a'),$$

or reducing by the formulæ,

$$(3m-7)(A', B', C')=(a'x+h'y+g'z,\ h'x+b'y+f'z,\ g'x+f'y+c'z),$$

this is

$$=\frac{1}{3m-7}\{A(-\mathfrak{G}'y+\mathfrak{H}'z)+B(-\mathfrak{F}'y+\mathfrak{B}'z)+C(-\mathfrak{C}'y+\mathfrak{F}'z)\}$$

$$=\frac{1}{3m-7}\{-y(\mathfrak{G}', \mathfrak{F}', \mathfrak{C}')(A, B, C)+z(\mathfrak{H}', \mathfrak{B}', \mathfrak{F}')(A, B, C)\}.$$

Hence we have

$$\text{Jac.}(U,\ H,\ \partial_x H)=\frac{1}{3m-6}\left(1+\frac{1}{3m-7}\right)\{-y(\mathfrak{G}', \mathfrak{F}', \mathfrak{C}')(A, B, C)+z(\mathfrak{H}', \mathfrak{B}', \mathfrak{F}')(A, B, C)\}$$

$$=\frac{1}{3m-7}\qquad\{-y(\mathfrak{G}', \mathfrak{F}', \mathfrak{C}')(A, B, C)+z(\mathfrak{H}', \mathfrak{B}', \mathfrak{F}')(A, B, C)\};$$

and in like manner

$$\text{Jac.}(U,\ H,\ \partial_y H)=\frac{1}{3m-7}\qquad\{-z(\mathfrak{A}', \mathfrak{H}', \mathfrak{G}')(A, B, C)+x(\mathfrak{G}', \mathfrak{F}', \mathfrak{C}')(A, B, C)\},$$

$$\text{Jac.}(U,\ H,\ \partial_z H)=\frac{1}{3m-7}\qquad\{-x(\mathfrak{H}', \mathfrak{B}', \mathfrak{F}')(A, B, C)+y(\mathfrak{A}', \mathfrak{H}', \mathfrak{G}')(A, B, C)\}.$$

49. And we thence have

$$\text{Jac.}\,(U,\ H,\ \overline{\nabla}H)=\frac{1}{3m-7}\begin{vmatrix} (\mathfrak{A}, \mathfrak{H}, \mathfrak{G})(\lambda, \mu, \nu), & (\mathfrak{H}, \mathfrak{B}, \mathfrak{F})(\lambda, \mu, \nu), & (\mathfrak{G}, \mathfrak{F}, \mathfrak{C})(\lambda, \mu, \nu) \\ (\mathfrak{A}', \mathfrak{H}', \mathfrak{G}')(A, B, C), & (\mathfrak{H}', \mathfrak{B}', \mathfrak{F}')(A, B, C), & (\mathfrak{G}', \mathfrak{F}', \mathfrak{C}')(A, B, C) \\ x, & y, & z, \end{vmatrix},$$

or multiplying the two sides by

$$H,=\begin{vmatrix} a, & h, & g \\ h, & b, & f \\ g, & f, & c \end{vmatrix},$$

the right hand side is

$$=\frac{1}{3m-7}\begin{vmatrix} \mathrm{H}\lambda\ , & \mathrm{H}\mu\ , & \mathrm{H}\nu \\ \mathrm{X}\ , & \mathrm{Y}\ , & \mathrm{Z} \\ (m-1)\mathrm{A}, & (m-1)\mathrm{B}, & (m-1)\mathrm{C}, \end{vmatrix}$$

which is

$$=\mathrm{H}\frac{m-1}{3m-7}\begin{vmatrix} \lambda\ , & \mu, & \nu \\ \mathrm{X}, & \mathrm{Y}, & \mathrm{Z} \\ \mathrm{A}, & \mathrm{B}, & \mathrm{C}, \end{vmatrix},$$

if for a moment

$$\begin{aligned} \mathrm{X}&=(\mathfrak{A}', \ldots \between \mathrm{A}, \mathrm{B}, \mathrm{C} \between a, h, g), \\ \mathrm{Y}&=(\mathfrak{A}', \ldots \between \mathrm{A}, \mathrm{B}, \mathrm{C} \between h, b, f), \\ \mathrm{Z}&=(\mathfrak{A}', \ldots \between \mathrm{A}, \mathrm{B}, \mathrm{C} \between g, f, c). \end{aligned}$$

50. Hence observing that these equations may be written

$$\begin{aligned} \mathrm{X}&=(\mathfrak{A}', \ldots \between \mathrm{A}, \mathrm{B}, \mathrm{C} \between \partial_x\mathrm{A}, \partial_x\mathrm{B}, \partial_x\mathrm{C}), \\ \mathrm{Y}&=(\mathfrak{A}', \ldots \between \mathrm{A}, \mathrm{B}, \mathrm{C} \between \partial_y\mathrm{A}, \partial_y\mathrm{B}, \partial_y\mathrm{C}), \\ \mathrm{Z}&=(\mathfrak{A}', \ldots \between \mathrm{A}, \mathrm{B}, \mathrm{C} \between \partial_z\mathrm{A}, \partial_z\mathrm{B}, \partial_z\mathrm{C}), \end{aligned}$$

and that we have

$$\partial=\begin{vmatrix} \lambda\ , & \mu\ , & \nu \\ \partial_x, & \partial_y, & \partial_z \\ \mathrm{A}, & \mathrm{B}, & \mathrm{C}, \end{vmatrix}$$

we obtain for H Jac. (U, H, $\overline{\overline{\nabla}}$, H) the value

$$=\mathrm{H}\frac{m-1}{3m-7}(\mathfrak{A}', \ldots \between \mathrm{A}, \mathrm{B}, \mathrm{C} \between \partial\mathrm{A}, \partial\mathrm{B}, \partial\mathrm{C}),$$

or throwing out the factor H, we have the required result.

Article Nos. 51 to 53.—*Proof of identity used in the fourth transformation*, viz.,

$$\text{Jac.}\,(\mathrm{U}, \nabla, \mathrm{H})\mathrm{H}=-\mathrm{E}\Psi,$$

or say

$$\text{Jac.}\,(\mathrm{U}, \mathrm{H}, \nabla)\mathrm{H}=(\partial\mathfrak{A}, \ldots \between \mathrm{A}', \mathrm{B}', \mathrm{C}')^2.$$

51. We have

$$\nabla=\big((\mathfrak{A}, \mathfrak{H}, \mathfrak{G}, \between \lambda, \mu, \nu), (\mathfrak{H}, \mathfrak{B}, \mathfrak{F} \between \lambda, \mu, \nu), (\mathfrak{G}, \mathfrak{F}, \mathfrak{C} \between \lambda, \mu, \nu) \between \partial_x, \partial_y, \partial_z\big),$$

and thence

$$\partial\,.\,\nabla\quad=\big((\partial_x\mathfrak{A}, \partial_x\mathfrak{H}, \partial_x\mathfrak{G} \between \lambda, \mu, \nu), (\partial_x\mathfrak{H}, \partial_x\mathfrak{B}, \partial_x\mathfrak{F} \between \lambda, \mu, \nu), (\partial_x\mathfrak{G}, \partial_x\mathfrak{F}, \partial_x\mathfrak{C} \between \lambda, \mu, \nu) \between \partial_x, \partial_y, \partial_z\big),$$

and

$$(\partial_x\,.\,\nabla)\mathrm{H}=\big((\partial_x\mathfrak{A}, \partial_x\mathfrak{H}, \partial_x\mathfrak{G} \between \lambda, \mu, \nu), (\partial_x\mathfrak{H}, \partial_x\mathfrak{B}, \partial_x\mathfrak{F} \between \lambda, \mu, \nu), (\partial_x\mathfrak{G}, \partial_x\mathfrak{F}, \partial_x\mathfrak{C} \between \lambda, \mu, \nu) \between \mathrm{A}', \mathrm{B}', \mathrm{C}'\big),$$

with the like values for $(\partial_y . \nabla)\mathrm{H}$ and $(\partial_z . \nabla)\mathrm{H}$. And then

$$\text{Jac.}\,(\mathrm{U}, \mathrm{H}, \nabla)\mathrm{H} = \begin{vmatrix} \mathrm{A} , & \mathrm{B} , & \mathrm{C} \\ \mathrm{A}' , & \mathrm{B}' , & \mathrm{C}' \\ (\partial_x . \nabla)\mathrm{H}, & (\partial_y . \nabla)\mathrm{H}, & (\partial_z . \nabla)\mathrm{H}, \end{vmatrix}$$

in which the coefficient of A'^2 is

$$=(\mathrm{C}\partial_y - \mathrm{B}\partial_z)(\mathfrak{A}, \mathfrak{H}, \mathfrak{G})(\lambda, \mu, \nu);$$

or putting for shortness

$$(\mathrm{C}\partial_y - \mathrm{B}\partial_z, \mathrm{A}\partial_z - \mathrm{C}\partial_x, \mathrm{B}\partial_x - \mathrm{A}\partial_y) = (\mathrm{P}, \mathrm{Q}, \mathrm{R});$$

the coefficient is

$$(\mathrm{P}\mathfrak{A}, \mathrm{P}\mathfrak{H}, \mathrm{P}\mathfrak{G})(\lambda, \mu, \nu).$$

52. We have

$$\partial = (\mathrm{P}\lambda + \mathrm{Q}\mu + \mathrm{R}\nu),$$

and thence

$$\text{coefficient } \mathrm{A}'^2 - \partial\mathfrak{A} = (\mathrm{P}\mathfrak{A}, \mathrm{P}\mathfrak{H}, \mathrm{P}\mathfrak{G})(\lambda, \mu, \nu) - (\mathrm{P}\mathfrak{A}, \mathrm{Q}\mathfrak{A}, \mathrm{R}\mathfrak{A})(\lambda, \mu, \nu),$$

which is

$$\begin{aligned} = \ & \mu\{(\mathrm{C}\partial_y - \mathrm{B}\partial_z)\mathfrak{H} - (\mathfrak{A}\partial_z - \mathrm{C}\partial_x)\mathfrak{A}\} \\ & + \nu\{(\mathrm{C}\partial_y - \mathrm{B}\partial_z)\mathfrak{G} - (\mathrm{B}\partial_x - \mathfrak{A}\partial_y)\mathfrak{A}\}, \end{aligned}$$

where coefficient of μ is

$$\begin{aligned} & = - \mathrm{A}\partial_z\mathfrak{A} - \mathrm{B}\partial_z\mathfrak{H} + \mathrm{C}(\partial_x\mathfrak{A} + \partial_y\mathfrak{H}) \\ & = -(\mathrm{A}\partial_z\mathfrak{A} + \mathrm{B}\partial_z\mathfrak{H} + \mathrm{C}\partial_z\mathfrak{G}) = -\frac{1}{m-1}x\partial_z\mathrm{H}, \end{aligned}$$

and coefficient of ν is

$$= +(\mathrm{A}\partial_y\mathfrak{A} + \mathrm{B}\partial_y\mathfrak{H} + \mathrm{C}\partial_y\mathfrak{G}) = \frac{1}{m-1}x\partial_y\mathrm{H},$$

so that

$$\text{coefficient } \mathrm{A}'^2 - \partial\mathfrak{A} = -\frac{1}{m-1}x(\mu\partial_z\mathrm{H} - \nu\partial_y\mathrm{H}).$$

53. And by forming in a similar manner the coefficients of the other terms, it appears that

$$\begin{aligned} & \text{Jac.}\,(\mathrm{U}, \mathrm{H}, \nabla)\mathrm{H} - (\partial\mathfrak{A}, \ldots)(\mathrm{A}', \mathrm{B}', \mathrm{C}')^2 \\ & = -\frac{1}{m-1}(\mathrm{A}'x + \mathrm{B}'y + \mathrm{C}'z) \begin{vmatrix} \mathrm{A}' , & \mathrm{B}' , & \mathrm{C}' \\ \lambda , & \mu , & \nu \\ \partial_x\mathrm{H}, & \partial_y\mathrm{H}, & \partial_z\mathrm{H}. \end{vmatrix}, \end{aligned}$$

or since the determinant is

$$\begin{vmatrix} \mathrm{A}', & \mathrm{B}', & \mathrm{C}' \\ \lambda , & \mu , & \nu \\ \mathrm{A}', & \mathrm{B}', & \mathrm{C}' \end{vmatrix}, =0,$$

we have the required equation,

$$\text{Jac.}\,(\mathrm{U}, \mathrm{H}, \nabla)\mathrm{H} = (\partial\mathfrak{A}, \ldots)(\mathrm{A}', \mathrm{B}', \mathrm{C}')^2.$$

This completes the series of formulæ used in the transformations of the condition for the sextactic point.

APPENDIX, Nos. 54 to 74.

For the sake of exhibiting in their proper connexion some of the formulæ employed in the foregoing first transformation of the condition for a sextactic point, I have investigated them in the present Appendix, which however is numbered continuously with the memoir.

54. The investigations of my former memoir and the present memoir have reference to the operations

$$\begin{aligned}
&\partial_1 = dx\,\partial_x + dy\,\partial_y + dz\,\partial_z,\\
&\partial_2 = d^2x\partial_x + d^2y\partial_y + d^2z\partial_z,\\
&\partial_3 = d^3x\partial_x + d^3y\partial_y + d^3z\partial_z,\\
&\&\text{c.},
\end{aligned}$$

where if (A, B, C) are the first differential coefficients of a function $U = (*\between x, y, z)^m$, and λ, μ, ν are arbitrary constants, then we have

$$dx = B\nu - C\mu, \quad dy = C\lambda - A\nu, \quad dz = A\mu - B\lambda;$$

so that putting

$$\begin{aligned}
\partial &= (B\nu - C\mu)\partial_x + (C\lambda - A\nu)\partial_y + (A\mu - B\lambda)\partial_z\\
&= \begin{vmatrix} A, & B, & C \\ \lambda\,, & \mu\,, & \nu \\ \partial_x, & \partial_y, & \partial_z, \end{vmatrix}\,,
\end{aligned}$$

we have $\partial_1 = \partial$. The foregoing expressions of (dx, dy, dz) determine of course the values of (d^2x, d^2y, d^2z), (d^3x, d^3y, d^3z), &c., and it is throughout assumed that these values are substituted in the symbols ∂_2, ∂_3, &c., so that ∂_1, $=\partial$, and ∂_2, ∂_3, &c. denote each of them an operator such as $X\partial_x + Y\partial_y + Z\partial_z$, where (X, Y, Z) are functions of the coordinates; such operator, in so far as it is a function of the coordinates, may therefore be made an operand, and be operated upon by itself or any other like operator.

55. Taking (a, b, c, f, g, h) for the second differential coefficients of U, $(\mathfrak{A}, \mathfrak{B}, \mathfrak{C}, \mathfrak{F}, \mathfrak{G}, \mathfrak{H})$ for the inverse coefficients, and H for the Hessian, I write also

$$\begin{aligned}
&\Phi = (\mathfrak{A}, \ldots \between \lambda, \mu, \nu)^2,\\
&\nabla = (\mathfrak{A}, \ldots \between \lambda, \mu, \nu \between \partial_x, \partial_y, \partial_z),\\
&\square = (\mathfrak{A}, \ldots \between \partial_x, \partial_y, \partial_z)^2,\\
&\vartheta = \lambda x + \mu y + \nu z,\\
&\Omega = (\mathfrak{A}, \ldots \between \partial_x, \partial_y, \partial_z)^2 H, = \square H,\\
&\Psi = (\mathfrak{A}, \ldots \between \partial_x H, \partial_y H, \partial_z H)^2,\\
&\Gamma = (a, \ldots \between \mu\partial_z - \nu\partial_y, \nu\partial_x - \lambda\partial_z, \lambda\partial_y - \mu\partial_x)^2,
\end{aligned}$$

and I notice that we have

$$\Gamma\mathrm{U}=2\Phi,\quad \nabla\mathrm{U}=\frac{\vartheta}{m-1}\mathrm{H},\quad \square\mathrm{U}=3\mathrm{H},$$

$$\nabla\vartheta=\Phi,\quad \nabla^2\mathrm{U}=\mathrm{H}\Phi\quad,\quad \nabla.\partial=0\quad,$$

the last of which is proved, *post* No. 65; the others are found without any difficulty.

56. I form the Table

$$\partial_1\mathrm{U}=0,$$

$$\partial_1^2\mathrm{U}=\frac{m\mathrm{U}}{m-1}\Phi \qquad +\frac{\vartheta^2}{(m-1)^2}(-\mathrm{H}),$$

$$\partial_2\mathrm{U}=\frac{m\mathrm{U}}{m-1}(-\Phi) \qquad +\frac{\vartheta^2}{(m-1)^2}(\mathrm{H}),$$

$$\partial_1^3\mathrm{U}=\frac{m\mathrm{U}}{m-1}\partial\Phi \qquad +\frac{\vartheta^2}{(m-1)^2}(-\partial\mathrm{H}),$$

$$\partial_1\partial_2\mathrm{U}=0,$$

$$\partial_3\mathrm{U}=\frac{m\mathrm{U}}{m-1}(-\partial\Phi) \qquad +\frac{\vartheta^2}{(m-1)^2}(\partial\mathrm{H}),$$

$$\partial_1^4\mathrm{U}=\frac{m\mathrm{U}}{m-1}\left(\partial^2\Phi-\frac{2m\vartheta}{m-1}\nabla\Phi\right) \qquad +\frac{\vartheta^2}{(m-1)^2}\left(-\ \partial^2\mathrm{H}-\frac{3m-6}{m-1}\mathrm{H}\Phi+\frac{2\vartheta}{m-1}\nabla\mathrm{H}\right),$$

$$\partial_1^2\partial_2\mathrm{U}=\frac{m\mathrm{U}}{m-1}\left(\tfrac{1}{3}\partial_2\Phi+\frac{\frac{2}{3}\vartheta}{m-1}\nabla\Phi\right) \qquad +\frac{\vartheta^2}{(m-1)^2}\left(-\tfrac{1}{3}\partial_2\mathrm{H}+\frac{m-2}{m-1}\ \mathrm{H}\Phi-\frac{\frac{2}{3}\vartheta}{m-1}\nabla\mathrm{H}\right),$$

$$\partial_1\partial_3\mathrm{U}=\frac{m\mathrm{U}}{m-1}\left(-\tfrac{1}{3}\partial_2\Phi-\Phi^2-\frac{\frac{2}{3}\vartheta}{m-1}\nabla\Phi\right) \qquad +\frac{\vartheta^2}{(m-1)^2}\left(\ \tfrac{1}{3}\partial_2\mathrm{H}+\frac{1}{m-1}\ \mathrm{H}\Phi+\frac{\frac{2}{3}\vartheta}{m-1}\nabla\mathrm{H}\right),$$

$$\partial_2^2\mathrm{U}=\frac{m\mathrm{U}}{m-1}(\Phi^2) \qquad +\frac{\vartheta^2}{(m-1)^2}(-\mathrm{H}\Phi),$$

$$\partial_4\mathrm{U}=\frac{m\mathrm{U}}{m-1}\left(-\tfrac{2}{3}\partial_2\Phi-\partial^2\Phi+\Phi^2+\frac{\frac{2}{3}\vartheta}{m-1}\nabla\Phi\right)+\frac{\vartheta^2}{(m-1)^2}\left(\tfrac{2}{3}\partial_2\mathrm{H}+\partial^2\mathrm{H}-\frac{1}{m-1}\mathrm{H}\Phi-\frac{\frac{2}{3}\vartheta}{m-1}\nabla\mathrm{H}\right),$$

$$\partial_2\mathrm{H}=-\frac{3m-6}{m-1}\mathrm{H}\Phi+\frac{\vartheta}{m-1}\nabla\mathrm{H},$$

and assuming U=0,

$$\partial_1^2\mathrm{H}=\partial^2\mathrm{H}=-\frac{(3m-6)(3m-7)}{(m-1)^2}\mathrm{H}\Phi+\frac{6m-14}{(m-1)^2}\vartheta\nabla\mathrm{H}-\frac{\vartheta^2}{(m-1)^2}\Omega,$$

$$(\partial_1\mathrm{H})^2=(\partial\mathrm{H})^2=-\frac{(3m-6)^2}{(m-1)^2}\mathrm{H}^2\Phi+\frac{6m-12}{(m-1)^2}\vartheta\mathrm{H}\nabla\mathrm{H}-\frac{\vartheta^2}{(m-1)^2}\Psi,$$

which are for the most part given in my former memoir; the expressions for $\partial_2\mathrm{U}$, $\partial_3\mathrm{U}$, which are not explicitly given, follow at once from the equations

$$(\partial_1^2+\partial_2)\mathrm{U}=0,\quad (\partial_1^3+2\partial_1\partial_2+\partial_3)\mathrm{U}=0;$$

those for $\partial_1\partial_3\mathrm{U}$, $\partial_2^2\mathrm{U}$, and $\partial_4\mathrm{U}$ are new, but when the expressions for $\partial_1\partial_3\mathrm{U}$ and $\partial_2^2\mathrm{U}$ are known, that for $\partial_4 U$ is at once found from the equation

$$(\partial_1^4+6\partial_1^2\partial_2+4\partial_1\partial_3+3\partial_2^2+\partial_4)U=0.$$

57. Before going further, I remark that we have identically

$$\begin{aligned}
&(a, \ldots \between x, y, z)^2(a, \ldots \between \mu\gamma-\nu\beta, \nu\alpha-\lambda\gamma, \lambda\beta-\mu\alpha)^2 \\
&- \begin{vmatrix} ax+hy+gz, & hx+by+fz, & gx+fy+cz \\ \lambda \quad , & \mu \quad , & \nu \\ \alpha \quad , & \beta \quad , & \gamma \end{vmatrix}^2 \\
&=(\mathfrak{A}, \ldots \between \lambda p-\alpha\vartheta, \mu p-\beta\vartheta, \nu p-\gamma\vartheta)^2,
\end{aligned}$$

(if for shortness $p=\alpha x+\beta y+\gamma z$, $\vartheta=\lambda x+\mu y+\nu z$)

$$\begin{aligned}
= \quad & p^2(\mathfrak{A}, \ldots \between \lambda, \mu, \nu)^2 \\
& -2p\vartheta(\mathfrak{A}, \ldots \between \lambda, \mu, \nu \between \alpha, \beta, \gamma) \\
& +\vartheta^2(\mathfrak{A}, \ldots \between \alpha, \beta, \gamma)^2.
\end{aligned}$$

58. If in this equation we take (a, b, c, f, g, h) to be the second differential coefficients of U, and write also $(\alpha, \beta, \gamma)=(\partial_x, \partial_y, \partial_z)$, the equation becomes

$$\begin{aligned}
m(m-1)\mathrm{U}\Gamma-(m-1)^2\partial^2= \quad & \Phi(x\partial_x+y\partial_y+z\partial_z)^2 \\
& -2\vartheta(x\partial_x+y\partial_y+z\partial_z)\nabla \\
& +\vartheta^2\square,
\end{aligned}$$

which is a general equation for the transformation of $\partial^2(=\partial_1^2)$.

59. If with the two sides of this equation we operate on U, we obtain

$$\begin{aligned}
m(m-1)\mathrm{U}\Gamma\mathrm{U}-(m-1)^2\partial^2\mathrm{U}= \quad & m(m-1)\Phi\mathrm{U} \\
& -2(m-1)\vartheta\nabla\mathrm{U} \\
& +\vartheta^2\square\mathrm{U};
\end{aligned}$$

and substituting the values

$$\Gamma\mathrm{U}=2\Phi,\ \nabla\mathrm{U}=\frac{\vartheta}{m-1}\mathrm{H},\ \square\mathrm{U}=3\mathrm{H},$$

we find the before-mentioned expression of $\partial_1^2\mathrm{U}$.

60. Operating with the two sides of the same equation on a function H of the order m', we find

$$\begin{aligned}
m(m-1)\mathrm{U}\Gamma\mathrm{H}-(m-1)^2\partial^2\mathrm{H}= \quad & m'(m'-1)\Phi\mathrm{H} \\
& -2(m'-1)\vartheta\nabla\mathrm{H} \\
& + \qquad \vartheta^2\square\mathrm{H};
\end{aligned}$$

and in particular if H is the Hessian, then writing $m'=3m-6$, and putting $\mathrm{U}=0$, we find the before-mentioned expression for $\partial^2\mathrm{H}$.

61. But we may also from the general identical equation deduce the expression for $(\partial\mathrm{H})^2$. In fact taking H a function of the degree m' and writing

$$(\alpha, \beta, \gamma)=(\partial_x\mathrm{H}, \partial_y\mathrm{H}, \partial_z\mathrm{H}),$$

we have

$$m(m-1)\mathrm{U}(a,\ldots)(\mu\partial_z\mathrm{H}-\nu\partial_y\mathrm{H},\ \nu\partial_x\mathrm{H}-\lambda\partial_z\mathrm{H},\ \lambda\partial_y\mathrm{H}-\mu\partial_x\mathrm{H})^2-(m-1)^2(\partial\mathrm{H})^2$$
$$=m'^2\Phi\mathrm{H}^2-2m'\vartheta\mathrm{H}\nabla\mathrm{H}+\vartheta^2(\mathfrak{A},\ldots)(\partial_x\mathrm{H},\ \partial_y\mathrm{H},\ \partial_z\mathrm{H})^2;$$

and if H be the Hessian, then writing $m'=3m-6$ and putting also $\mathrm{U}=0$, we find the before-mentioned expression for $(\partial\mathrm{H})^2$.

62. Proof of equation

$$\partial_2=-\frac{1}{m-1}(x\partial_x+y\partial_y+z\partial_z)+\frac{\vartheta}{m-1}\nabla.$$

We have

$$\partial_2=\partial.\partial=\{(\mathrm{B}\nu-\mathrm{C}\mu)\partial_x+(\mathrm{C}\lambda-\mathrm{A}\nu)\partial_y+(\mathrm{A}\mu-\mathrm{B}\lambda)\partial_z\}.$$
$$\big(\lambda(\mathrm{C}\partial_y-\mathrm{B}\partial_z)+\mu(\mathrm{A}\partial_z-\mathrm{C}\partial_x)+\nu(\mathrm{B}\partial_x-\mathrm{A}\partial_y)\big),$$

which is

$$=\lambda(\mathrm{C}'\partial_y-\mathrm{B}'\partial_z)+\mu(\mathrm{A}'\partial_z-\mathrm{C}'\partial_x)+\nu(\mathrm{B}'\partial_x-\mathrm{A}'\partial_y),$$

where

$$\mathrm{A}'=\partial\mathrm{A}=a(\mathrm{B}\nu-\mathrm{C}\mu)+h(\mathrm{C}\lambda-\mathrm{A}\nu)+g(\mathrm{A}\mu-\mathrm{B}\lambda)$$
$$=\lambda(h\mathrm{C}-g\mathrm{B})+\mu(g\mathrm{A}-a\mathrm{C})+\nu(a\mathrm{B}-h\mathrm{A}),$$

with the like values for B′ and C′. Substituting the values

$$(m-1)(\mathrm{A},\ \mathrm{B},\ \mathrm{C})=(ax+hy+gz,\ hx+by+fz,\ gx+fy+cz),$$

we have

$$(m-1)\mathrm{A}'=\lambda(\mathfrak{G}y-\mathfrak{H}z)+\mu(\mathfrak{F}y-\mathfrak{B}z)+\nu(\mathfrak{C}y-\mathfrak{F}z);$$

and similarly

$$(m-1)\mathrm{B}'=\lambda(\mathfrak{A}z-\mathfrak{G}x)+\mu(\mathfrak{H}z-\mathfrak{F}x)+\nu(\mathfrak{G}z-\mathfrak{C}x),$$
$$(m-1)\mathrm{C}'=\lambda(\mathfrak{H}x-\mathfrak{A}y)+\mu(\mathfrak{B}x-\mathfrak{H}y)+\nu(\mathfrak{F}x-\mathfrak{G}y),$$

and then

$$\begin{aligned}(m-1)(\mathrm{C}'\partial_y-\mathrm{B}'\partial_z)=\ &\lambda[(\mathfrak{H}x-\mathfrak{A}y)\partial_y-(\mathfrak{A}z-\mathfrak{G}x)\partial_z]\\&+\mu[(\mathfrak{B}x-\mathfrak{H}y)\partial_y-(\mathfrak{H}z-\mathfrak{F}x)\partial_z]\\&+\nu[(\mathfrak{F}x-\mathfrak{G}y)\partial_y-(\mathfrak{G}z-\mathfrak{C}x)\partial_z]\end{aligned}$$

$$\begin{aligned}=\ &\lambda[x(\mathfrak{A},\ \mathfrak{H},\ \mathfrak{G})(\partial_x,\ \partial_y,\ \partial_z)-\mathfrak{A}(x\partial_x+y\partial_y+z\partial_z)]\\&+\mu[x(\mathfrak{H},\ \mathfrak{B},\ \mathfrak{F})(\partial_x,\ \partial_y,\ \partial_z)-\mathfrak{H}(x\partial_x+y\partial_y+z\partial_z)]\\&+\nu[x(\mathfrak{G},\ \mathfrak{F},\ \mathfrak{C})(\partial_x,\ \partial_y,\ \partial_z)-\mathfrak{G}(x\partial_x+y\partial_y+z\partial_z)]\end{aligned}$$

$$=\ x(\mathfrak{A},\ldots)(\lambda,\ \mu,\ \nu)(\partial_x,\ \partial_y,\ \partial_z)-(\mathfrak{A},\ \mathfrak{H},\ \mathfrak{G})(\lambda,\ \mu,\ \nu)(x\partial_x+y\partial_y+z\partial_z);$$

that is

$$(m-1)(\mathrm{C}'\partial_y-\mathrm{B}'\partial_z)=x\nabla-(\mathfrak{A},\ \mathfrak{H},\ \mathfrak{G})(\lambda,\ \mu,\ \nu)(x\partial_x+y\partial_y+z\partial_z),\text{ and so}$$
$$(m-1)(\mathrm{A}'\partial_z-\mathrm{C}'\partial_x)=y\nabla-(\mathfrak{H},\ \mathfrak{B},\ \mathfrak{F})(\lambda,\ \mu,\ \nu)(x\partial_x+y\partial_y+z\partial_z),$$
$$(m-1)(\mathrm{B}'\partial_x-\mathrm{A}'\partial_y)=z\nabla-(\mathfrak{G},\ \mathfrak{F},\ \mathfrak{C})(\lambda,\ \mu,\ \nu)(x\partial_x+y\partial_y+z\partial_z);$$

whence

$$(m-1)\partial_2=(\lambda x+\mu y+\nu z)\nabla-(\mathfrak{A},\ldots)(\lambda,\ \mu,\ \nu)^2(x\partial_x+y\partial_y+z\partial_z)$$
$$=\vartheta\nabla-\Phi(x\partial_x+y\partial_y+z\partial_z);$$

or finally

$$\partial_2=-\frac{1}{m-1}\Phi(x\partial_x+y\partial_y+z\partial_z)+\frac{\vartheta}{m-1}\nabla.$$

63. This leads to the expression for $\partial_2^2\mathrm{U}$; we have

$$\begin{aligned}\partial_2^2= &\ \frac{1}{(m-1)^2}\Phi^2(x\partial_x+y\partial_y+z\partial_z)^2\\ &-\frac{2\vartheta}{(m-1)^2}\Phi\nabla(x\partial_x+y\partial_y+z\partial_z)\\ &+\frac{\vartheta^2}{(m-1)^2}\nabla^2;\end{aligned}$$

and operating herewith on U, we find

$$\begin{aligned}\partial_2^2\mathrm{U}= &\ \frac{m(m-1)}{(m-1)^2}\Phi^2\mathrm{U}\\ &-\frac{2(m-1)\vartheta}{(m-1)^2}\Phi\nabla\mathrm{U}\\ &+\frac{\vartheta^2}{(m-1)^2}\nabla^2\mathrm{U};\end{aligned}$$

or since

$$\nabla\mathrm{U}=\frac{\vartheta}{m-1}\mathrm{H},\ \nabla^2\mathrm{U}=\mathrm{H}\Phi,$$

this is

$$\partial_2^2\mathrm{U}=\frac{m\mathrm{U}}{(m-1)^2}\Phi^2+\frac{\vartheta^2}{(m-1)^2}\mathrm{H}\Phi.$$

64. We have $\partial_1\partial_2\mathrm{U}=0$, and thence

$$(\partial_1^2\partial_2+\partial_1\partial_3+\partial_2^2)\mathrm{U}=0,$$

that is

$$\partial_1\partial_3\mathrm{U}=-\partial_1^2\partial_2\mathrm{U}-\partial_2^2\mathrm{U};$$

or substituting the values of $\partial_1^2\partial_2\mathrm{U}$ and $\partial_2^2\mathrm{U}$, we find the value of $\partial_1\partial_3\mathrm{U}$ as given in the Table. And then from the equation

$$(\partial_1^4+6\partial_1^2\partial_2+4\partial_1\partial_3+3\partial_2^2+\partial_4)\mathrm{U},$$

or

$$\partial_4\mathrm{U}=-(\partial_1^4+6\partial_1^2\partial_2+4\partial_1\partial_3+3\partial_2^2)\mathrm{U},$$

we find the value of $\partial_4\mathrm{U}$, and the proof of the expressions in the Table is thus completed.

65. Proof of equation $\nabla.\partial=0$.

We have

$$\begin{aligned}\nabla.\partial&=\nabla.\big((\mathrm{B}\nu-\mathrm{C}\mu)\partial_x+(\mathrm{C}\lambda-\mathrm{A}\nu)\partial_y\ \ +(\mathrm{A}\mu-\mathrm{B}\lambda)\partial_z\big)\\ &=\nabla.\big(\mathrm{A}(\mu\partial_z-\nu\partial_y)+\mathrm{B}(\nu\partial_x-\lambda\partial_z)\ \ +\mathrm{C}(\lambda\partial_y-\mu\partial_x)\big)\\ &=\nabla\mathrm{A}(\mu\partial_z-\nu\partial_y)\ \ +\nabla\mathrm{B}(\nu\partial_x-\lambda\partial_z)+\nabla\mathrm{C}(\lambda\partial_y-\mu\partial_x);\end{aligned}$$

and then

$$\begin{aligned}\nabla\mathrm{A}&=(\mathfrak{A},\ldots)(\lambda,\mu,\nu)(a,h,g)=\mathrm{H}\lambda,\\ \nabla\mathrm{B}&=(\mathfrak{A},\ldots)(\lambda,\mu,\nu)(h,b,f)=\mathrm{H}\mu,\\ \nabla\mathrm{C}&=(\mathfrak{A},\ldots)(\lambda,\mu,\nu)(g,f,c)=\mathrm{H}\nu;\end{aligned}$$

or substituting these values, we have the equation in question.

66. Proof of expression for ∂_3.

We have

$$\partial_2 = -\frac{1}{m-1}\Phi(x\partial_x+y\partial_y+z\partial_z)+\frac{\vartheta}{m-1}\nabla;$$

and thence operating on the two sides respectively with ∂_1, $=\partial$, we have

$$\partial_3 = -\frac{1}{m-1}\{\partial\Phi(x\partial_x+y\partial_y+z\partial_z)+\Phi\partial\,.(x\partial_x+y\partial_y+z\partial_z)\}$$
$$+\frac{1}{m-1}\{\partial\vartheta\nabla+\vartheta\partial\,.\nabla\};$$

or since

$$\partial\,.(x\partial_x+y\partial_y+z\partial_z)=\partial,\ \partial\vartheta=0,$$

this is

$$\partial_3 = -\frac{1}{m-1}\partial\Phi(x\partial_x+y\partial_y+z\partial_z)-\frac{1}{m-1}\Phi\partial+\frac{\vartheta}{m-1}\partial\,.\nabla.$$

67. Proof of expression for ∂_3H.

Operating with ∂_3 upon H, we have at once

$$\partial_3\mathrm{H} = -\frac{3m-6}{m-1}\mathrm{H}\partial\Phi-\frac{1}{m-1}\Phi\partial\mathrm{H}+\frac{\vartheta}{m-1}(\partial.\nabla)\mathrm{H}.$$

The remainder of the present Appendix is preliminary, or relating to the investigation of the expressions for $\partial_1\partial_2^2\mathrm{U}$ and $\partial_1^2\partial_3\mathrm{U}$, used *ante*, No. 31.

68. Proof of equation $\nabla^2\partial\mathrm{U}=\Phi\partial\mathrm{H}-\mathrm{H}\partial\Phi$.

We have identically

$$(\mathfrak{A}, \ldots \between \lambda, \mu, \nu)^2(\mathfrak{A}, \ldots \between \partial_x, \partial_y, \partial_z)^2-[(\mathfrak{A}, \ldots \between \lambda, \mu, \nu \between \partial_x, \partial_y, \partial_z)]^2$$
$$=(abc-\&c.)(a, \ldots \between \nu\partial_y-\mu\partial_z,\ \lambda\partial_z-\nu\partial_x,\ \mu\partial_x-\lambda\partial_y)^2;$$

that is

$$\Phi\Box-\nabla^2=\mathrm{H}\Gamma;$$

and then multiplying by ∂, and with the result operating on U, we find

$$\Phi\Box\partial\mathrm{U}-\nabla^2\partial\mathrm{U}=\mathrm{H}\Gamma\partial\mathrm{U}.$$

Now

$$\Box\mathrm{U}=(\mathfrak{A}, \ldots \between \partial_x, \partial_y, \partial_z)^2\mathrm{U}$$
$$=(\mathfrak{A}, \ldots \between a, b, c, 2f, 2g, 2h);$$

and thence

$$\Box\partial\mathrm{U}=(\mathfrak{A}, \ldots \between \partial a, \partial b, \partial c, 2\partial f, 2\partial g, 2\partial h);$$

and observing that

$$\mathrm{H}=\begin{vmatrix} a, & h, & g \\ h, & b, & f \\ g, & f, & c \end{vmatrix},$$

and thence that

$$\begin{aligned}\partial\mathrm{H}=&\begin{vmatrix}\partial a, & \partial h, & \partial g\\ h, & b, & f\\ g, & f, & c\end{vmatrix}+\begin{vmatrix}a, & h, & g\\ \partial h, & \partial b, & \partial f\\ g, & f, & c\end{vmatrix}+\begin{vmatrix}a, & h, & g\\ h, & b, & f\\ \partial g, & \partial f, & \partial c\end{vmatrix}\\ &=(\mathfrak{A},\ \mathfrak{H},\ \mathfrak{G}\between\partial a,\ \partial h,\ \partial g)+(\mathfrak{H},\ \mathfrak{B},\ \mathfrak{F}\between\partial h,\ \partial b,\ \partial f)+(\mathfrak{G},\ \mathfrak{F},\ \mathfrak{C}\between\partial g,\ \partial f,\ \partial c)\\ &=(\mathfrak{A},\ \ldots\between\partial a,\ \partial b,\ \partial c,\ 2\partial f,\ 2\partial g,\ 2\partial h,\end{aligned}$$

we see that

$$\square\partial\mathrm{U}=\partial\mathrm{H}.$$

Moreover

$$\begin{aligned}\Gamma\mathrm{U}=&\ (a,\ \ldots\between\nu\partial_y-\mu\partial_z,\ \ldots)^2\mathrm{U}\\ =&\ a(b\nu^2+c\mu^2-2f\mu\nu)\\ &+b(c\lambda^2+a\nu^2-2g\nu\lambda)\\ &+c(a\mu^2+b\lambda^2-2h\lambda\mu)\\ &+2f(-f\lambda^2+g\lambda\mu+h\lambda\nu-a\mu\nu)\\ &+2g(\ f\lambda\mu-g\mu^2+h\mu\nu-b\nu\lambda)\\ &+2h(\ f\nu\lambda+g\nu\mu-h\nu^2-c\lambda\mu);\end{aligned}$$

and thence

$$\begin{aligned}\Gamma\partial\mathrm{U}=&\ (a,\ \ldots\between\nu\partial_y-\mu\partial_z,\ \ldots)^2\partial\mathrm{U}\\ =&\ a(\nu^2\partial b+\mu^2\partial c-2\mu\nu\partial f)\\ &+\&c.\\ =&\ \lambda^2(b\partial c+c\partial b-2f\partial f)\\ &+\&c.\\ =&(\partial\mathfrak{A},\ \partial\mathfrak{B},\ \partial\mathfrak{C},\ \partial\mathfrak{F},\ \partial\mathfrak{G},\ \partial\mathfrak{H}\between\lambda,\ \mu,\ \nu)^2,\end{aligned}$$

that is

$$\Gamma\partial\mathrm{U}=\partial\Phi.$$

Hence the equation

$$\Phi\square\partial\mathrm{U}-\nabla^2\partial\mathrm{U}=\mathrm{H}\Gamma\partial\mathrm{U}$$

becomes

$$\Phi\partial\mathrm{H}-\nabla^2\partial\mathrm{U}=\mathrm{H}\partial\Phi,$$

that is,

$$\nabla^2\partial\mathrm{U}=\Phi\partial\mathrm{H}-\mathrm{H}\partial\Phi.$$

69. Proof of equation $\partial_1\partial_2^2\mathrm{U}=\frac{\vartheta^2}{(m-1)^2}(\Phi\partial\mathrm{H}-\mathrm{H}\partial\Phi)$.

We have

$$\begin{aligned}\partial_2^2=&\ \frac{1}{(m-1)^2}\Phi^2(x\partial_x+y\partial_y+z\partial_z)^2\\ &-\frac{2\vartheta}{(m-1)^2}\Phi(x\partial_x+y\partial_y+z\partial_z)\nabla\\ &+\frac{\vartheta^2}{(m-1)^2}\nabla^2;\end{aligned}$$

and thence multiplying by $\partial_1, =\partial$, and with the result operating upon U, we find

$$\partial_1\partial_2^2\mathrm{U}=\frac{(m-1)(m-2)}{(m-1)^2}\Phi^2\partial\mathrm{U}-\frac{2(m-2)}{(m-1)^2}\vartheta\Phi\partial\nabla\mathrm{U}+\frac{\vartheta^2}{(m-1)^2}\partial\nabla^2\mathrm{U}.$$

But $\partial\mathrm{U}=0$, and thence also $\nabla(\partial\mathrm{U})=0$, that is $(\nabla\,.\,\partial)\mathrm{U}+\nabla\partial\mathrm{U}=0$; moreover $\nabla\,.\,\partial=0$, and therefore $(\nabla\,.\,\partial)\mathrm{U}=0$, whence also $\nabla\partial\mathrm{U}=0$. Therefore

$$\partial_1\partial_2^2\mathrm{U}=\frac{\vartheta^2}{(m-1)^2}\partial\nabla^2\mathrm{U}\,;$$

or substituting for $\partial\nabla^2\mathrm{U}$ its value $=\Phi\partial\mathrm{H}-\mathrm{H}\partial\Phi$, we have the required expression for $\partial_1\partial_2^2\mathrm{U}$.

70. Proof of equation $\partial_1^2\partial_3\mathrm{U}=\frac{\vartheta^2}{(m-1)^3}\big((3m-6)\mathrm{H}\partial\Phi+(-m+3)\Phi\partial\mathrm{H}\big)+\frac{\vartheta^3}{(m-1)^3}\{-(\partial\,.\,\nabla)\mathrm{H}\}$. We have

$$\partial_3=-\frac{1}{m-1}\partial\Phi(x\partial_x+y\partial_y+z\partial_z)-\frac{1}{m-1}\Phi\partial+\frac{\vartheta}{m-1}\partial\,.\,\nabla,$$

and thence multiplying by $\partial_1^2=\partial^2$, and operating on U,

$$\partial_1^2\partial_3\mathrm{U}=-\frac{m-2}{m-1}\partial\Phi\partial^2\mathrm{U}-\frac{1}{m-1}\Phi\partial^3\mathrm{U}+\frac{\vartheta}{m-1}(\partial\,.\,\nabla)\partial^2\mathrm{U}.$$

To reduce $(\partial\,.\,\nabla)\partial^2\mathrm{U}$, we have

$$\begin{aligned}\partial(\nabla\partial^2\mathrm{U})&=\nabla\partial^3\mathrm{U}+(\partial\,.\nabla\partial^2)\mathrm{U}\\&=\nabla\partial^3\mathrm{U}+[(\partial\,.\,\nabla)\partial^2+\nabla(\partial\,.\,\partial^2)]\mathrm{U}\\&=\nabla\partial^3\mathrm{U}+(\partial\,.\,\nabla)\partial^2\mathrm{U}+2\nabla\partial\partial_2\mathrm{U},\end{aligned}$$

and since

$$\partial_2=-\frac{1}{m-1}\Phi(x\partial_x+y\partial_y+z\partial_z)+\frac{\vartheta}{m-1}\nabla\,;$$

multiplying by $\nabla\partial$, and with the result operating on U, we obtain

$$\nabla\partial\partial_2\mathrm{U}=-\frac{m-2}{m-1}\Phi\nabla\partial\mathrm{U}+\frac{\vartheta}{m-1}\nabla^2\partial\mathrm{U}\,;$$

or since $\nabla\partial\mathrm{U}=0$, this is

$$\nabla\partial\partial_2\mathrm{U}=\frac{\vartheta}{m-1}\nabla^2\partial\mathrm{U}.$$

Hence

$$\partial(\nabla\partial^2\mathrm{U})=\nabla\partial^3\mathrm{U}+(\partial\,.\,\nabla)\partial^2\mathrm{U}+\frac{2\vartheta}{m-1}\nabla^2\partial\mathrm{U},$$

that is

$$(\partial\,.\,\nabla)\partial^2\mathrm{U}=\partial(\nabla\partial^2\mathrm{U})-\nabla\partial^3\mathrm{U}-\frac{2\vartheta}{m-1}\nabla^2\partial\mathrm{U}.$$

Substituting this value of $(\partial\,.\,\nabla)\partial^2\mathrm{U}$, we find

$$\begin{aligned}\partial_1^2\partial_3\mathrm{U}=&-\frac{m-2}{m-1}\partial\Phi\partial^2\mathrm{U}-\frac{1}{m-1}\Phi\partial^3\mathrm{U}\\&+\frac{\vartheta}{m-1}\big(\partial(\nabla\partial^2\mathrm{U})-\nabla\partial^3\mathrm{U}\big)\\&+\frac{\vartheta^2}{(m-1)^2}(-2\nabla^2\partial\mathrm{U}),\end{aligned}$$

the three lines whereof are to be separately further reduced.

71. For the first line we have

$$\partial^2 U = -\frac{\vartheta^2}{(m-1)^2}H,\ \partial^3 U = -\frac{\vartheta^2}{(m-1)^2}\partial H,$$

and hence

$$\text{first line of } \partial_1^2\partial_3 U = \frac{\vartheta^2}{(m-1)^3}\Big((m-2)H\partial\Phi + \Phi\partial H\Big).$$

72. For the second line, we have

$$\nabla(\partial^2 U) = \nabla\partial^2 U + 2(\nabla . \partial)\partial U$$

$$= \nabla\partial^2 U, \text{ since } \nabla . \partial = 0, \text{ and therefore } (\nabla . \partial)\partial U = 0;$$

that is

$$\nabla\partial^2 U = \nabla(\partial^2 U) = \nabla\left(\frac{mU}{m-1}\Phi - \frac{\vartheta^2}{(m-1)^2}H\right)$$

$$= \frac{m}{m-1}(U\nabla\Phi + \Phi\nabla U) - \frac{1}{(m-1)^2}(\vartheta^2\nabla H + 2\vartheta H\nabla\vartheta);$$

or writing

$$U=0,\ \nabla U = \frac{\vartheta}{m-1}H,\ \nabla\vartheta = \Phi,$$

this is

$$\nabla\partial^2 U = \frac{(m-2)\vartheta}{(m-1)^2}H\Phi - \frac{\vartheta^2}{(m-1)^2}\nabla H,$$

whence also

$$\partial(\nabla\partial^2 U) = \frac{(m-2)\vartheta}{(m-1)^2}(H\partial\Phi + \Phi\partial H) - \frac{\vartheta^2}{(m-1)^2}\partial(\nabla H).$$

Similarly

$$\nabla\partial^3 U = \nabla(\partial^3 U)$$

$$= \nabla\left(\frac{mU}{m-1}\partial\Phi - \frac{\vartheta^2}{(m-1)^2}\partial H\right)$$

$$= \frac{m}{m-1}\Big(\nabla U\partial\Phi + U\nabla(\partial\Phi)\Big) - \frac{1}{(m-1)^2}\Big(\vartheta^2\nabla(\partial H) + 2\vartheta\nabla\vartheta\partial H\Big);$$

or putting

$$U=0,\ \nabla U = \frac{\vartheta}{m-1}H,\ \nabla\vartheta = \Phi,$$

and observing also that $\nabla(\partial H)$, $=\nabla\partial H + (\nabla . \partial)H$ is equal to $\nabla\partial H$, that is to $\partial\nabla H$, we obtain

$$\nabla\partial^3 U = \frac{\vartheta}{(m-1)^2}(mH\partial\Phi - 2\Phi\partial H) - \frac{\vartheta^2}{(m-1)^2}\partial\nabla H;$$

and then from the above value of $\partial(\nabla\partial^2 U)$, we find

$$\partial(\nabla\partial^2 U) - \nabla\partial^3 U = \frac{\vartheta}{(m-1)^2}(-2H\partial\Phi + m\Phi\partial H) + \frac{\vartheta^2}{(m-1)^2}\Big(-\partial(\nabla H) + \partial\nabla H\Big);$$

or observing that the term multiplied by $\frac{\vartheta^2}{(m-1)^2}$ is $= -(\partial . \nabla)H$, we find

$$\text{second line of } \partial_1^2\partial^3 U = \frac{\vartheta^2}{(m-1)^3}(-2H\partial\Phi + m\Phi\partial H) + \frac{\vartheta^3}{(m-1)^3}\Big(-(\partial . \nabla)H\Big).$$

73. For the third line, substituting for $\nabla^2\partial\mathrm{U}$ its value $=\Phi\partial\mathrm{H}-\mathrm{H}\partial\Phi$, we have

$$\text{third line of } \partial_1^2\partial_3\mathrm{U}=-\frac{2\vartheta^2}{(m-1)^2}(\Phi\partial\mathrm{H}-\mathrm{H}\partial\Phi).$$

74. Hence, uniting the three lines, we have

$$\begin{aligned}\partial_1^2\partial_3\mathrm{U}= {} & \frac{\vartheta^2}{(m-1)^3}\big((m\ \ -2)\mathrm{H}\partial\Phi+\qquad\qquad \Phi\partial\mathrm{H}\big)\\ & +\frac{\vartheta^2}{(m-1)^3}(\qquad -2\ \mathrm{H}\partial\Phi+\qquad\qquad m\Phi\partial\mathrm{H})+\frac{\vartheta^3}{(m-1)^3}\big(-(\partial\,.\,\nabla)\mathrm{H}\big)\\ & +\frac{\vartheta^2}{(m-1)^3}\big((2m-2)\mathrm{H}\partial\Phi+(-2m+2)\Phi\partial\mathrm{H}\big),\end{aligned}$$

and reducing, we have the above-mentioned value of $\partial_1^2\partial_3\mathrm{U}$.

XI. *A Description of some Fossil Plants, showing Structure, found in the Lower Coal-seams of Lancashire and Yorkshire.* By E. W. Binney, *F.R.S.*

Received May 12,—Read June 15, 1865.

Introductory Remarks.

Although great attention has been devoted to the collection of the fossil remains of plants with which our coal-fields abound, the specimens are generally in very fragmentary and distorted conditions as they occur imbedded in the rocks in which they are entombed; but when they have been removed, cut into shape, and trimmed, and are seen in cabinets, they are in a far worse condition. This is as to their external forms and characters. When we come to examine their internal structure, and ascertain their true nature, we find still greater difficulties, from the rarity of specimens at the same time displaying both the external form and the internal structure of the original plant. It is often very difficult to decide which is the outside, different parts of the stem dividing and exposing varied surfaces which have been described as distinct genera of plants.

The specimens were collected by myself, and taken out of the seams of coal just as they occurred in the matrix in which they were found imbedded, by my own hands. This enables me to speak with certainty as to the condition and locality in which they were met with.

By the ingenuity of the late Mr. Nicol of Edinburgh, we were furnished with a beautiful method of slicing specimens of fossil-wood so as to examine their internal structure. The late Mr. Witham, assisted by Mr. Nicol, first applied this successfully, and his work on the internal structure of fossil vegetables was published in 1833. In describing his specimens, he notices one which he designated *Anabathra pulcherrima*. This did not do much more than afford evidence of the internal vascular cylinder arranged in radiating series, somewhat similar to that which had been found and described by Messrs. Lindley and Hutton as occurring in *Stigmaria ficoides*, in their third volume of the 'Fossil Flora.'

In 1839 M. Adolphe Brongniart published his truly valuable memoir, "Observations sur la structure intérieure du *Sigillaria elegans* comparée à celle des *Lepidodendron* et des *Stigmaria* et à celle des végétaux vivants." His specimen of *Sigillaria elegans* was in very perfect preservation, and showed its external characters and internal structure in every portion except the pith and a broad part of the plant intervening betwixt the internal and external radiating cylinders. Up to this time nothing had been seen at all to be compared to Brongniart's specimen, and no savant could have been better selected to describe and illustrate it. His memoir will always be considered as one of the most valuable ever contributed on the fossil flora of the Carboniferous period.

In 1849, AUGUST JOSEPH CORDA published his 'Beiträge zur Flora der Vorwelt,' a work of great labour and research. Amongst his numerous specimens, he describes and illustrates one of *Diploxylon cycadoideum*, which, although not to be compared to BRONGNIART'S specimen, still affords us valuable information, confirming some of that author's views rather than affording much more original information. All these last three specimens BRONGNIART, in his 'Tableau de végétaux fossiles considérés sous le point de vue de leur classification botanique et de leur distribution géologique,' published in 1849, classes as *Dicotyledones gymnospermes* under the family of *Sigillarées*; amongst other plants his *Sigillaria elegans*, WITHAM'S *Anabathra*, and CORDA'S *Diploxylon*.

In 1862 the writer published an account of specimens in the 'Quarterly Journal of the Geological Society' of that year, which confirmed the views of the three learned authors above named as to *Sigillaria* and *Diploxylon* being allied plants; he also showed that their supposed pith or central axis was not composed of cellular tissue, but of different sized vessels arranged without order, having their sides barred by transverse striæ like the internal vascular cylinders of *Sigillaria* and *Lepidodendron*. These specimens were in very perfect preservation, and showed the external as well as the internal characters of the plants.

All the above specimens were of comparatively small size, with the exception of that described by CORDA, which, although it showed the external characters in a decorticated state, did not exhibit any outward cylinder of a plant allied to *Sigillaria* with large ribs and deep furrows so commonly met with in our coal-fields, but rather to plants allied to *Sigillaria elegans* and *Lepidodendron*.

In the present communication it is intended to describe some specimens of larger size than those previously alluded to, and to endeavour to show that the *Sigillaria vascularis* gradually passes as it grows older into ribbed and furrowed *Sigillaria*, and that this singular plant not only possessed two woody cylinders, an internal one and an external one, both increasing on their outsides at the same time, but likewise had a central axis composed of hexagonal vessels, arranged without order, having all their sides marked with transverse striæ. Evidence will also be adduced to show that *Sigillaria* dichotomized in its branches something like *Lepidodendron*, and that, as in the latter plant, a *Lepidostrobus* was its fructification. The outer cylinder in large *Sigillaria* was composed of thick-walled quadrangular tubes or utricles arranged in radiating series, and exhibiting every appearance of having been as hard-wooded a tree as *Pinites*, but as yet no disks or striæ have been observed on the walls of the tubes. *Stigmaria* is now so generally considered to be the root of *Sigillaria*, that it is scarcely necessary to bring any further proof of this proposition; but specimens will be described which will prove by similarity of structure that the former is the root of the latter.

The chief specimens described in this memoir are eight in number, and were found by me in the lower divisions of the Lancashire and Yorkshire coal-measures imbedded in calcareous nodules occurring in seams of coal.

Specimen No. 1, from the first-named district, is from the same locality as the *Trigonocarpon*, described by Dr. J. D. Hooker, F.R.S., and myself, in a memoir "On the Structure of certain Limestone Nodules enclosed in seams of Bituminous Coal, with a Description of some *Trigonocarpons* contained therein"*, and the other seven specimens are from the same seam of coal in the lowercoal-measures as that in which the specimens described in a paper entitled "On some Fossil Plants, showing Structure, from the Lower Coal-measures of Lancashire"†, were met with, but from a different locality.

The position of the seams of coal in which the fossil-woods were found in the carboniferous series will be shown by the following sections of the lower coal-measures.

In Lancashire.	yds.	ft.	in.
Arley or Royley seam	1	1	0
Strata	69	0	0
Seam	0	0	3
Strata	57	0	0
Seam	0	0	6
Strata	45	0	0
Upper flagstone (Upholland)	50	0	0
Strata	20	0	0
Seam (90 yards)	0	0	5
Strata	20	0	0
Seam (40 yards)	0	1	6
Strata	64	0	0
***Upper Foot seam (Dog Hill)	0	1	2
Strata	15	0	0
**Gannister seam	1	0	0
Strata	13	0	0
Lower Foot seam (Quarlton)	0	2	0
Strata	17	0	0
Bassy seam (New Mills)	0	2	6
Strata	40	0	0
Seam	0	0	10
Strata	10	0	0
Sand or Featheredge seam	0	2	0
Rough Rock of Lancashire (Upper Millstone of Geological Survey)	20	0	0
Strata (Rochdale or Lower Flags)	120	0	0
*Seam	0	0	6
Strata	2	0	0
Seam	0	0	10
Strata	14	0	0
Seam	0	1	3
Upper Millstone of Lancashire.			

In Yorkshire.	yds.	ft.	in.
Beeston or Silkstone seam	2	0	0
Strata	77	0	0
Royds or Black seam	0	2	10
Strata	38	0	0
Better Bed seam	0	1	4
Strata	51	0	0
Upper Flagstone (Elland)	40	0	0
Strata	40	0	0
Seam (90 yards)	0	0	6
Strata	56	0	0
Seam (40 yards)	0	1	0
Strata	39	0	0
**Halifax Hard seam	0	2	3
Strata	14	0	0
Middle seam	0	0	11
Strata	24	0	0
Soft seam	0	1	6
Strata	56	0	0
Sand seam	0	0	4
Upper Millstone of Phillips (Halifax)	36	0	0
Strata (Lower Flagstone)	72	0	0
Little seam	0	0	3

In the Lancashire coal-field all the seams of coal, from the forty yards downwards, have at places afforded the *Aviculopecten* and other marine shells in their roofs of black shale, and these latter strata generally contain calcareous nodules. The nodules in the seams of coal commonly known by the name of Bullions have chiefly been found in the beds marked *, **, and *** in Lancashire, whilst in Yorkshire they have as yet been only observed in the Halifax Hard seam marked **.

---

* Philosophical Transactions, 1855, p. 149.

† Quarterly Journal of the Geological Society of London for May 1862, p. 106.

Description of No. 1 *Specimen.*

The first specimen intended to be described in this communication is from the thin seam of coal marked * in the lower coal-measures of Lancashire arranged in the vertical section previously given, and is from the same mine from which the specimens described by Dr. HOOKER and myself were obtained. It was found associated with *Calamodendron*, *Halonia*, *Sigillaria*, *Lepidodendron*, *Stigmaria*, *Trigonocarpon*, *Lycopodites*, *Lepidostrobus*, *Medullosa*, and other genera of plants not yet determined in the order of relative abundance in which they have been just named.

A portion of a similar specimen of fossil-wood obtained by me from the same locality, on analysis* gave

Carbonate of lime	76·66
Carbonate of magnesia	12·87
Sesquioxide of iron	4·95
Sulphate of iron	0·73
Carbonaceous matter	4·95

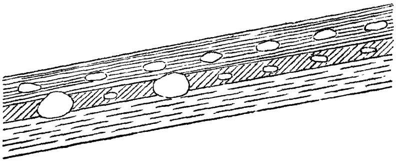
The stratum lying immediately above the seam of coal in which the specimen occurred, generally termed the roof, was composed of black shale containing large calcareous nodules, and for a distance of about 2 feet 6 inches upwards was one entire mass of fossil shells of the genera *Goniatites*, *Orthoceratites*, *Aviculopecten*, and *Posidonia*.

The beds in the vicinity of the coal occurred in the following order, namely,

	yds.	ft.	in.
1. Black shale with nodules containing fossil shells	0	2	6
2. Upper seam of coal enclosing the nodules full of fossil-wood	0	0	6
3. Fire-clay floor full of *Stigmaria*	0	2	0
4. Clay and rock	2	0	0
5. Lower seam of coal	0	0	10
6. Fire-clay full of *Stigmaria*.			

The fossil-wood occurred in circular, lenticular, and elongated and flattened oval-shaped nodules, varying from an inch to a foot in diameter, the round and uncompressed specimens being in general small, whilst the flattened ones were nearly always of a large size. No fossil shells were met with in the nodules found in the coal itself, although, as previously stated, they were very abundant in the nodules found in the roof of the seam, which there rarely contained any remains of plants. The large nodules of 10 to 12 inches in diameter, when they occurred, swelled out the seam of coal both above and below as in the annexed woodcut, fig. 1.

Fig. 1.



---

* For this analysis I am indebted to the kindness of Mr. HERMANN.

Specimen No. 1 was originally, when first found, 6 inches in length by 7 in breadth, and of an oval form. Its exterior surface was not very well preserved, the outer bark coming off with the matrix of coal in which it was imbedded, but the inner bark showed an irregularly fluted surface marked with fine longitudinal striæ.

In Plate XXX. fig. 1, one half of the specimen is represented. The middle portion of the specimen in transverse section shows a central axis, marked *a*, having its inner portion, *a'*, somewhat compressed, and composed of hexagonal-shaped vessels showing all their sides marked with transverse striæ, arranged without order. Around this axis is a cylinder of hexagonal vessels, *b*, arranged in radiating series of considerably less size than those of the central axis, but having all their sides similarly marked with transverse striæ, and increasing in size as they extend from the centre to the circumference. On the outside of this radiating cylinder is a part of the specimen not showing much structure, but apparently having been once composed of coarse cellular tissue. Beyond this is another zone, for the most part now consisting of mineral matter, chiefly crystallized carbonate of lime, sometimes affording evidence of structure in the form of tubes or elongated utricles arranged in radiating series, and forming an outer cylinder in the plant.

Figs. 2 & 3 show longitudinal and tangential sections of the natural size, taken from the lower and upper portions of fig. 1.

Fig. 4 shows a part of the transverse section, magnified five diameters, where the commencement of the wedge-shaped masses are seen with convex ends adjoining the central axis, and parted by medullary rays or bundles extending from the centre to the circumference, and probably communicating with the leaves on the outside of the plant.

Figs. 5 & 6 show longitudinal and tangential sections of a little more than one half of the specimen, magnified five diameters, the latter displaying the oval-shaped bundles of vessels traversing the internal cylinder of the plant from the centre to the circumference.

This specimen is evidently of the same genus as that described by WITHAM, and obtained by him from Allenbank in Berwickshire, from the mountain-limestone series, and named *Anabathra pulcherrima*, although in a much more perfect state of preservation [*]. My specimen, however, does not show a pith of cellular tissue, it being rather imperfect in that part; but it distinctly confirms WITHAM's opinion as to the occurrence of medullary rays or bundles dividing the woody cylinder; and it appears to be nearly identical in structure with *Diploxylon cycadoideum* of CORDA[†], with which it will be classed.

This specimen is not in so perfect a state of preservation as those fossil-woods intended to be hereinafter described in this communication, especially as regards its central and external parts; but it certainly differs from them in having a larger mass of scalariform tissue composing the central axis, and having the inner portions of the wedge-shaped bundles forming the internal radiating cylinder of a convex shape as they approach the central axis, somewhat like those represented by BRONGNIART in his *Sigillaria elegans*, and still more resembling those described by CORDA in *Diploxylon cycadoideum**; but my specimen shows within those convex bundles a broad zone of scalariform tissue arranged without order and marked with transverse striæ.

---

[*] On the Internal Structure of Fossil Vegetables found in the Carboniferous and Oolitic Districts of Great Britain, by H. T. M. WITHAM, F.G.S. &c. Edinburgh, 1833.

[†] Beiträge zur Flora der Vorwelt, Taf x.

It has been assumed, both by CORDA and BRONGNIART, that *Diploxylon* had a pith composed of cellular tissue, surrounded by a medullary sheath of hexagonal vessels arranged without order, barred on all their sides with transverse striæ. My specimen is evidently more complete in structure than those of the last-named authors, or even that which WITHAM himself described; but although it shows the so-called medullary sheath in a very perfect state, there is nothing to indicate the former existence of a pith of cellular tissue. All the specimens examined by WITHAM, CORDA, and BRONGNIART appear to have had their central axes removed altogether and replaced by mineral matter, or else only showing slight traces of their structure; and these authors appear to have inferred the former existence of a pith of cellular tissue, rather than to have had any direct evidence of it in the specimens of *Anabathra*, *Diploxylon*, and *Sigillaria* respectively figured by them. Every collector of coal-plants is well aware of the blank space so generally left in the above fossil plants as well as in the roots *Stigmariæ*. It is quite true that a little disarrangement of the scalariform vessels (a') in the specimen is seen; but the part which remains undisturbed shows that the whole of the central axis was formerly composed of hexagonal vessels arranged without order, having all their sides marked with transverse striæ and not of cellular tissue. This view is confirmed by another and more perfect specimen of *Anabathra* in my cabinet, and enables me to speak with positive certainty, and to show that these three plants had a similar structure in the central axes to the specimens of *Sigillaria vascularis* described by me in my paper published in the Quarterly Journal of the Geological Society.

My specimen clearly proves the existence of medullary rays or bundles traversing the internal woody cylinder, which originate on the outside of the central axis; and it appears to me pretty certain that CORDA's specimen of *Diploxylon cycadoideum*, if tangential sections had been made and carefully examined, would have done the same.

The exterior of the specimen is not in a very complete state of preservation, but it seems to have been covered by irregular ribs and furrows, with slight indications of remains of the cicatrices of leaf-scars. Its marked character, as previously alluded to, is the great space occupied by the central axis. This is of much larger size than in either the *Sigillaria vascularis* or the specimens intended to be next described.

The lunette-shaped ends of the wedge-like bundles of the inner woody cylinder bear some resemblance to the form of the same parts of the *Sigillaria elegans* of BRONGNIART, but much more to those of CORDA's *Diploxylon cycadoideum*, with which it appears to be identical.

---

* See M. BRONGNIART's paper on *Sigillaria*, previously quoted.

The lunette-shaped extremities of the inner radiating cylinder of *Diploxylon cycadoideum*, as well as those in my specimen, remind us of a similar arrangement shown to occur in *Stigmaria* by Dr. Hooker, in plate 2. fig. 14*; and they appear to differ from those found in *Sigillaria vascularis* in not being divided from the central axis by a distinct line of demarcation, just as the same author's *Stigmaria* fig. 5 differs from fig. 14. The exterior of the inner radiating cylinder of the former plant is more free and open, and not so sharp and compact as that of the latter plant. Indeed, from structure alone, it would appear probable that the first-named *Stigmaria* was the root of *Diploxylon*, whilst the last one was the root of *Sigillaria vascularis*.

As Brongniart has preferred Corda's name of *Diploxylon* to *Anabathra*, and as the former is a more expressive generic term in my opinion, probably it is better to adopt it, and accordingly the specimen has been denominated *Diploxylon cycadoideum*.

Description of Specimens Nos. 2, 3, 4, 5, 6, 7, & 8.

The second specimen intended to be described in this memoir is from a small seam of coal about 2 feet in thickness in the lower coal-measures, marked ⁎⁎ in the vertical section previously given, and from the same seam that the specimens of *Sigillaria vascularis*, described by me in the paper published in the Quarterly Journal of the Geological Society previously quoted, came from, although from a different locality. This specimen, as well as those numbered respectively 3, 4, 5, 6, & 7, all came from the Halifax Hard seam, the Gannister coal, at South Owram near Halifax. It was found associated with *Sigillaria*, *Stigmaria*, *Lepidodendron*, *Calamodendron*, *Halonia*, *Diploxylon*, *Lepidostrobus*, and *Trigonocarpon*, and other fossil plants not well determined in the order of relative abundance in which they have been just named.

A portion of one of the specimens, a large *Sigillaria*, gave, on analysis†,

Sulphates of potash and soda	1·62
Carbonate of lime	45·61
Carbonate of magnesia	26·91
Bisulphide of iron	11·65
Oxides of iron	13·578
Silica	0·23
Moisture	0·402

The stratum found lying immediately above the seam of coal in which the nodules occurred was composed of black shale containing large calcareous concretions, and for about 18 inches was one entire mass of fossil shells of the genera *Aviculopecten*, *Goniatites*, *Orthoceratites*, and *Posidonia*.

---

* Memoirs of the Geological Survey of Great Britain, vol. ii. part 1.

† For this analysis I am indebted to the kindness of Dr. R. Angus Smith, F.R.S., who had it done in his laboratory by Mr. Browning.

The beds occurred in the following (descending) order, namely,

	ft.	in.
1. Black shale full of fossil shells and containing calcareous concretions	1	6
2. Halifax Hard seam with the nodules containing the fossil plants .	2	0
3. Floor of fire-clay and Gannister, full of *Stigmaria ficoides.*		

The fossil-wood is found in nodules dispersed throughout the coal, some being spherical and others elongated and flattened ovals, varying in size from the bulk of a common pea to 8 and 10 inches in diameter. In some portions of the seam of coal the nodules are so numerous as to render it utterly useless, and they will occur over a space of several acres, and then for the most part disappear and again occur as numerous as ever. For a distance of from twenty-five to thirty miles the nodules occur in this seam of coal in more or less abundance, but always containing the same plants. Fossil shells are rarely met with in the nodules found in the coal, but they occur abundantly in the large calcareous concretions found in the roof of the mine, and are there associated with *Dadoxylon* containing *Sternbergia*-piths, which plant has not yet been noticed in the coal, and *Lepidostrobus.* So far as my experience extends, the nodules in the coal are always found associated with the occurrence of fossil shells in the roof, and may probably be owing to the presence of mineral matter held in solution in water, and precipitated upon or aggregated around certain centres in the mass of the vegetable matter now forming coal before the bituminization of such vegetables took place. No doubt such nodules contain a fair sample of the plants of which the seams of coal in which they are found was formed, and their calcification was most probably chiefly due to the abundance of shells afterwards accumulated in the soft mud now forming the shale overlying the coal.

The specimen illustrated in Plate XXXI. fig. 1, is of an irregular oval shape, 1 foot 3 inches in circumference, 7 inches across its major, and 3½ inches across its minor axis. When first discovered it was 8 inches in length, and only a fragment of a much larger stem. The light-coloured disk in the middle, about an inch in diameter, shows the central axis and the internal radiating cylinder of woody tissue, while the indistinct radiating lines towards the circumference indicate the outer cylinder, formed of thick-walled tubes or utricles of quadrangular form arranged in wedge-shaped masses divided by coarse muriform tissue, increasing in the opposite direction as to their size that the wedge-shaped masses do: all of the natural size.

Fig. 2 shows the outside appearance of the specimen marked with fine longitudinal striæ, irregular ribs and furrows, and some cicatrices of leaf-scars, which would induce most collectors of coal-plants to class it with a decorticated specimen of *Sigillaria.* It most resembles *Sigillaria organum.* The bark of a portion of the specimen remains attached to it in the form of coal, that is united to the matrix of the seam in which the fossil was found imbedded. The reverse side of the specimen does not show the character so distinctly.

Here we have a *Stigmaria*-like woody cylinder, with a central axis composed of barred vessels arranged without order, found in the inside of a stem of *Sigillaria* in such a position as it existed in the living plant. It is not a solitary instance, but one of more than fifty specimens exhibiting similar characters which have come under my observation.

In Plate XXXII. fig. 1, is represented the light-coloured disk previously alluded to, and shown in Plate XXXI. of the natural size, but here magnified 5 diameters, exhibiting the central axis composed of hexagonal vessels arranged without order, of several sizes, those in the middle being smaller and becoming larger towards the outside, where they come in contact with the internal radiating cylinder *b*, and then again diminishing in size. This latter was no doubt cylindrical, like the stem of the plant, but both parts in the process of petrification have been altered by pressure to their present forms. It consists of a broad cylinder (*b*) of about an inch in diameter, composed of parallel elongated tetragonal or hexagonal tubes of equal diameter throughout for the greater part of their length, obtuse and rounded at either extremity, and everywhere marked with crowded parallel lines which are free or anastomosing all over the surface. The tubes towards the axis are of the smallest diameter; they gradually enlarge towards the circumference, where the largest are situated, though bundles of smaller tubes occasionally occur among the larger. This cylinder, which for convenience may be called the internal woody system of the plant, is divided into elongated wedge-shaped masses, pointed at their posterior or inner extremity, and parted by fine medullary rays of various breadths, some much narrower than the diameter of the tubes, others considerably broader, but none are conspicuous to the naked eye, except towards the outer circumference in some rare instances.

Fig. 2 represents a transverse section of the central axis and the commencement of the internal radiating cylinder, magnified 12 diameters. The hexagonal vessels in the centre and at the circumference, where they come in contact with the internal radiating cylinder, are smaller in size than those seen in the other parts of the axis. The dark line across the axis, as well as the dark space in the centre, both seem to be the result of a disarrangement of the tubes during the process of mineralization, as similar appearances have not been observed in many other specimens examined by me, which in those parts are in a more perfect state of preservation. The dark and sharp line separating the vessels of the central axis from those of the internal radiating cylinder does not permit us to clearly see the origin of the medullary rays or bundles which undoubtedly traverse the latter.

Fig. 3 represents a longitudinal section taken on the right-hand side of the specimen, and extending across the whole of the internal radiating cylinder through the central axis, the intermediate space between the internal radiating cylinder and the outer cylinder, and the external radiating cylinder to the outside of the stem, magnified 4 diameters: *a a* showing the smaller barred vessels of the central axis, having some (*a′ a′*) which appear to have been disarranged; *b b* the internal radiating cylinder of larger barred vessels; *c* the space occupied by lax cellular tissue traversed by bundles of vessels; and *d* the external radiating cylinder, consisting of elongated tubes or utricles arranged in radiating series diverging from certain circular openings, and divided by masses of muriform tissue which contain the medullary rays or bundles.

Fig. 4 is a tangential section of the same parts of the specimen as lastly described, magnified 4 diameters; *b' b'* showing the medullary rays or bundles traversing the inner radiating cylinder, and *d' d'* those traversing the outer radiating cylinder.

Plate XXXIII. fig. 1 is a longitudinal section of a portion of the same specimen, exhibiting the central axis* and the inner radiating cylinder, magnified 15 diameters.

Fig. 2 shows several of the vessels of the central axis as they would be if they were not ground away in the operations of slicing and polishing, magnified 45 times.

Fig. 3 is a tangential section of the inner radiating cylinder, *b* showing the barred vessels, and *b''* the medullary rays or bundles, magnified 15 diameters.

Figs. 4 & 5, longitudinal and tangential sections of the same specimens, showing the structure of the outer radiating cylinder, *d* denoting the tubes or elongated utricles of which it is composed, and *d'* the medullary rays or bundles which traverse it, magnified 10 diameters.

Plate XXXIV. fig. 1 represents a transverse section of a ribbed and furrowed stem (No. 3), displaying similar cicatrices to that of No. 2 given in Plate XXXI., and having a like central axis, as well as like internal and external radiating cylinders and other parts, magnified 2 diameters. It is given for the purpose of more distinctly showing the tubes or elongated utricles, *d*, and the fusiform openings formed of very open muriform tissue, *d'* enclosing the medullary rays or bundles which traverse the external radiating cylinder. This it does in a very marked manner: magnified 20 diameters.

In Plate XXXV. figs. 1, 2 & 3 (Nos. 4, 5 & 6), are shown the exteriors of three central axes separated from large ribbed and furrowed stems, in every respect similar to those described in Plate XXXI. and Plate XXXIV., and such as might easily be taken for small *Calamites*, magnified 2½ diameters. Fig. 4 (No. 7) shows the outside of the internal woody cylinder of a *Stigmaria* with ribbed and furrowed characters, resembling those shown on the outsides of the central axes lastly described, also magnified 2½ diameters.

The first three specimens, Nos. 4, 5 & 6, are from the Halifax Hard seam of coal at South Owram, but No. 7 is from the Wigan Five Feet Mine, a seam in the middle coal-measures.

The tangential sections which show the medullary rays or bundles that traverse the inner and outer radiating cylinders, afford clear evidence of the different appearance of the bundles marked *b''* in Plate XXXIII. fig. 3, from those in Plate XXXIV. fig. 2 marked *d'*.

Specimens Nos. 2 & 3 bear considerable resemblance to the *Sigillaria elegans* of BRONGNIART, with respect to their internal radiating cylinder and the medullary rays or bundles which traverse it, assuming that such vessels come from the outside of the central axis, and not from the exterior of the internal radiating cylinder, as that distinguished savant supposed. Certainly there is no evidence in my specimens to support the latter view. A great many specimens have been broken up and destroyed for the purpose of examining the inner radiating cylinder, and in every case medullary rays or bundles were found traversing it, just as you find in the same part of *Stigmaria*. On the outside of the inner cylinder, at the extreme part of the zone of coarse and lax cellular tissue which bounds it, are some circular openings, from which spring the wedge-shaped masses of quadrangular, tubular, or elongated utricles which form the outer radiating cylinder. The lax cellular tissue has nearly always been displaced and disarranged in the process of mineralization, and sometimes the outer radiating cylinder and the circular orifices connected with it have been pushed towards the inner cylinder. This may have been the case in BRONGNIART'S specimen, and caused him to suppose that the medullary rays or bundles originated only on the outside, and were not joined to those which traversed the inner cylinder. So far as my large specimens show, there were medullary rays which had their origin next the central axis, passed through the inner cylinder, and after traversing the zone of lax cellular tissue outside the latter, apparently communicated with similar rays or bundles of vessels of much larger size, which are always found traversing the outer radiating cylinder, and then went on to the leaves on the outside of the stem.

---

* In the Plate the small tubes *a' a''* appear to be divided by septæ. This is certainly the case in one slice, but in another of the same specimen these septæ are not seen, but small barred vessels appear in their places, so the former may probably be due to the direction of the slice being cut along the dark line which traverses the central axis, as shown in Plate XXXII. figs. 1 & 2.

In BRONGNIART'S specimen the tubes or elongated utricles composing the outer radiating cylinder appear to have been far more delicate in structure than the thick-walled tubes in specimens Nos. 2 & 3[*], but probably not more so than might be expected from the difference in size of the plants, my specimens being about twelve times as large as his, and in all probability so much older individuals. The tubes in mine might easily be mistaken for similar tubes in *Pinites* if their size and the thickness of their walls were merely considered, and no notice were taken of the discigerous characters of that genus. In my specimens no disks have as yet been observed on the walls of the tubes, nor have they afforded any evidence of the transverse striæ which characterize the tubes of the central axis and internal radiating cylinder. It is possible that these markings may have once existed on the walls of the tubes, and been afterwards obliterated during the process of mineralization. The thick walls of the tubes in my specimens often exhibit circular dots of a yellow colour, bearing some resemblance to coloured disks. The absence of the disks is the only reason for distinguishing the outer tissue in my specimens from the woody portion of *Pinites*, and this absence of disks is sometimes found to prevail on the walls of the tubes of small specimens of *Dadoxylon*, which are found with piths of *Sternbergia* inside them.

The late Mr. J. E. BOWMAN, F.G.S., in his paper on the Fossil Trees discovered on the line of the Bolton Railway, near Manchester[†], and which were in all probability old *Sigillariæ*, at considerable length endeavoured to prove that they were hard-wooded solid timber trees, in opposition to the then common opinion that they were soft or hollow stems. In my company that author first saw the trees, and he then observed to me that the roots of those fossil trees clearly indicated by their great size and strength that the trees when living had heavy tops.

---

[*] In the longitudinal section represented in the Plates these tubes are made more delicate than they appear in the specimens.

[†] Transactions of the Manchester Geological Society, vol. i. p. 112.

In all the numerous specimens of large *Sigillaria* which have come under my observation, the outer radiating cylinder shows more or less evidence of lines of growth, and is generally divided into rectangular masses parted by straight lines of sparry matter, just as a piece of oak taken out of a peat bog and dried does at the present day. This similarity in divisional structure strongly supports the view of the late Mr. BOWMAN as to *Sigillaria* being a hard-wooded tree, which has lately been revived by Dr. DAWSON, F.R.S., in his paper "On the Vegetable Structures of Coal," who says, "I am even inclined to suspect that some of the described specimens of Conifers of the coal may be the woody axes of large *Sigillariæ*, or at least approaching quite as nearly to those plants as to modern Conifers"*. All the large specimens of fossil trees found in seams of coal give evidence of having been subject to considerable pressure when in a soft state, and this might also cause the divisional lines above alluded to, without resorting to a process like that which takes place in drying bog oak.

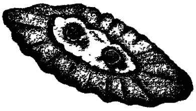
In the specimens Nos. 2 & 3 the outer radiating cylinders are nearly an inch and a half in breadth of thick-walled tubes, or elongated utricles arranged in radiating series, and diverging from a circular opening, while in BRONGNIART'S *Sigillaria elegans* the outer radiating cylinder was not more than $\frac{1}{12}$th of that breadth. Probably my specimens may not prove to be of the same species as that of the celebrated Autun specimen, still they may be of the same genus, although of considerably greater age. But they have the greatest resemblance to the *Sigillaria vascularis* described by me in a paper read before the Geological Society, and printed in its Journal†. All the specimens described in that communication, as well as those in the present one, were obtained by me from the same seam of coal, but at different places, still the two, namely, the large ribbed and furrowed specimens and the small rhomboidal scarred stems, are always found associated together, and they can be traced gradually passing from one into the other. These facts, when taken in connexion with the similarity of structure in the central axis, the internal radiating cylinder, the space filled with lax cellular tissue between the latter and the outer radiating cylinder diverging from circular openings, clearly prove that the smaller specimen is but the young branch of the older stem, No. 2. It is true that the earlier authors who have written on these plants, would scarcely have admitted a ribbed and furrowed *Sigillaria* to have been so intimately connected with a rhomboidal scarred plant, but it is now generally allowed that such differences in external characters would afford no grounds for ignoring the structural similarity of the specimens. Undoubtedly the small *Sigillaria vascularis* was part of a branching stem; for in my cabinet there is a specimen clearly showing two internal radiating cylinders just at the point where the branches dichotomized, as shown in woodcut (fig. 2), so often met with in *Lepidodendron*.

---

* Quarterly Journal of the Geological Society, vol. xv. p. 636.

† Quarterly Journal of the Geological Society for May 1862.

Fig. 2.



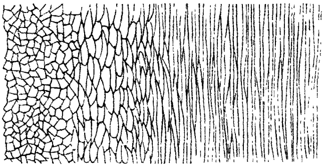
Whatever evidence Dr. DAWSON had for supposing a large *Sigillaria* to have been possessed of the obtuse top and the flat main roots, as shown in his restored specimen, figured in vol. xv. of the 'Quarterly Journal of the Geological Society,' it is impossible to say, but certainly in all the numerous specimens which have come under my observation nothing has occurred to warrant me in supposing *Sigillaria* to be such a plant. Everything has led me to believe that the leaves and branches, and probably the fructification of *Sigillaria*, would prove to be very analogous to those of *Lepidodendron*.

In order to show the identity in structure of specimens 2 & 3 with *Sigillaria vascularis*, previously described by me*, in Plate XXXV. fig. 5 is a specimen of *Sigillaria vascularis* from the same pit and seam of coal as the larger specimen No. 2, showing a transverse section, and fig. 6 exhibiting the external characters of the plant, part being covered with its bark, and part being decorticated, magnified 4 diameters.

On comparing this specimen with those figured in Plates XXXI., XXXII., XXXIII., and XXXIV., the greatest difference is seen in the external characters of the stems; but, as before stated, these can be traced from a regular rhomboidal scar, like that of the *Lepidodendron*, to the irregularly ribbed and furrowed *Sigillaria*. When we examine their internal structure it is found that their central axes are alike. The internal radiating cylinders are the same in both, making allowance for the greater age of the large specimen, each having been undoubtedly exogenous. The space on the outside of the inner radiating cylinder, filled with lax tissue and traversed by medullary bundles, is well marked and defined in the smaller specimen, much more so than in the larger one; but neither show the nature and position of these bundles, which will be noticed more at large in a specimen from a different locality hereinafter described. The outer boundary of this space in the small specimen is marked by a well-defined line of carbonaceous matter. The coarse cellular tissue on the outside of the latter, with the circular openings from which proceed the bundles of vessels traversing the outer zone of tubes or elongated utricles in radiating series, forming the outer cylinder, are the same in both.

The term tubes, or elongated utricles, has been previously employed to denote the structure of the outer cylinder. The inner portion of this zone is made up of what appears to be coarse cellular tissue. This gradually elongates as it proceeds outwards into utricles, which in their turn pass into tubes of a quadrangular form, of which the outer part of the cylinder is composed. The accompanying woodcut (fig. 3) represents a longitudinal section of No. 8, described in Plate XXXV. figs. 5 & 6. From this it is seen that the elongated utricles are more prominent and numerous in the small specimens, whilst in the large specimens, like those in Plates XXXIII. & XXXIV., the tubes are chiefly seen.

Fig. 3.



---

* Quarterly Journal of the Geological Society for May 1862, p. 106.

The outer cylinder seems to surround the band of lax cellular tissue enveloping the inner cylinder, and appears something in the nature of a pith to it. The inner cylinder no doubt increased on its outside by encroaching on the zone of lax cellular tissue, as may be proved by comparing a young with an old specimen, No. 8 with No. 2.

This outer zone of pseudo-wood increased externally like the inner cylinder, as is evident on comparing the younger with the older plant, the walls of the tubes of the latter being stronger, as might be expected to be the case; and in both we have the singular phenomenon of a tree increasing externally in two different zones at the same time.

As to the internal radiating cylinders described as occurring in the *Diploxylon* and *Sigillaria*, given in this communication, they are evidently like two different *Stigmaria*-cylinders, which afford no structure in their central axes, exactly resembling those figured by Dr. HOOKER in his paper on *Stigmaria ficoides* printed in the 'Memoirs of the Geological Survey of Great Britain'*, in plate 2. figs. 14 & 5. In the latter we have the wedge-form masses of wood of a lunette shape running into the central axis, whilst in the former we have them separated by a sharp and well-defined line from the central axis. The identity of structure between *Sigillaria* and *Diploxylon* and these two *Stigmariæ* is further proved by some specimens which have lately come under my notice.

After the researches of Dr. LINDLEY, Professor GOEPPERT, Mr. PRESTWICH, Dr. HOOKER and others, it really seemed that we had obtained almost a complete knowledge of the internal structure of *Stigmaria*. It is true that only GOEPPERT had seen the isolated bundles in the pith; all the specimens of the other observers having been imperfect in that portion of the plant, and not giving indication of structure there †. In my own researches it has rarely fallen to my lot to meet with a *Stigmaria* showing any structure in the central axis, even where the small stems of *Sigillaria vascularis*, affording all the structure in that part, are in great abundance.

Many years since, after an examination of a great number of specimens of *Stigmaria* in my collection, it occurred to me that an outer radiating cylinder would ultimately be discovered. In my remarks on *Stigmaria* ‡ is the following passage:—"That part of *Stigmaria* which intervened between the vascular axis and the bark appears to have consisted of two different kinds of cellular tissue. These, in most cases, have been unfortunately destroyed, so that we cannot positively know their true nature; but they appear to be of different characters, for there generally appears to be a well-marked division. This is often shown in specimens composed of clay ironstone which have not been flattened, and the boundary line is generally about a quarter of an inch from the outside of the specimen. Most probably the outer part of the zone has been composed of stronger tissue than the inner one, as is the case with well-preserved specimens of

---

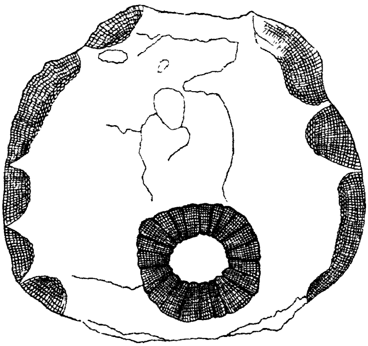
* Memoirs of the Geological Survey of Great Britain, vol. ii. part 1.

† I have written to Professor GOEPPERT for the purpose of obtaining further information as to the pith of this specimen, but I have not been successful in my endeavour.

‡ Quarterly Journal of the Geological Society, vol. iv. part 1. p. 20.

Lepidodendron." It is singular that Drs. LINDLEY and HOOKER, as well as such acute observers as BRONGNIART and GOEPPERT, had not noticed this line of division, but it was no doubt owing to the imperfect specimens which they had examined. After the discovery of the outer radiating cylinder by WITHAM in *Lepidodendron*, and the same arrangement in *Sigillaria* by BRONGNIART, it was to be expected that such outer radiating cylinder would be found to occur in *Stigmaria*, if it were the root of *Sigillaria.* After an inspection of a great number of specimens, the cabinet of Mr. RUSSELL, of Chapel Hall, Airdrie, has afforded me four or five distinct specimens which give clear evidence of the existence of this outer radiating cylinder in *Stigmaria.* They are all in clay ironstone, and have not been much compressed. He has kindly allowed me to slice two of the specimens, which afford decisive evidence of the former existence of both an inner and an outer radiating cylinder. The space on the outside of the inner cylinder does not distinctly show the bundles of vessels communicating with the rootlets, although there is some evidence of their former occurrence. The bell-shaped orifices from which the rootlets spring are well displayed, and the space between them is occupied by wedge-shaped masses of tubes or elongated utricles arranged in radiating series, and not to be distinguished in any way from those shown in Plate XXXV. fig. 5. Indeed the transverse section of the specimen there figured would almost do for a representation of the *Stigmaria* if the latter had the central axis preserved, which it unfortunately has not. There is the same internal radiating cylinder, the same space occupied by lax cellular tissue, which gradually passes into tubes or elongated utricles arranged in radiating series, apparently diverging from circular openings, and parted by large bundles of muriform tissue containing vessels barred on all their sides, extending to the outer bark. The accompanying woodcut (fig. 4) will give a much better idea of its structure than any laboured description.

Fig. 4.

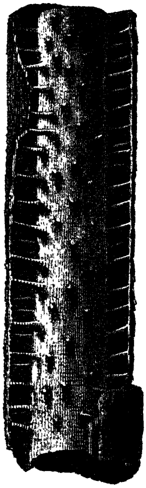


This specimen clearly proves, by the evidence of internal structure alone, that *Stigmaria* is the root of *Sigillaria*, each of them having an inner radiating cylinder composed of barred vessels, a space occupied by lax cellular tissue, and an outer radiating cylinder composed of tubes or elongated utricles.

The broad space intervening between the internal and external radiating cylinders, filled with lax cellular tissue and traversed by medullary bundles communicating with the leaves on the outside of the stem, as shown in the specimens described in this paper, is the only part on which information is required to complete our knowledge of the structure of the stem of *Sigillaria.* Fortunately a small specimen of *Sigillaria vascularis*, kindly presented to me by Mr. WARD, of Longton, a most indefatigable collector, has enabled me to obtain considerable information on this point. This specimen shows the rhomboidal scars on the outside of the stem, the two radiating cylinders and the space between occupied by lax cellular tissue, and traversed by medullary bundles.

The specimen in this woodcut (fig. 5, magnified twice) is of smaller size than any previously described by me, but it is, from both its internal structure and external characters, a small *Sigillaria vascularis* in its young state, when the two radiating cylinders, especially the outer one of the plant, were only slightly developed. The medullary bundles are seen on the outside of the inner radiating cylinder, and pass, inclining upwards at a small angle, from the inner cylinder to nearly the outside of the stem. No trace of the outer cylinder can be seen, so as to enable us to see whether the smaller-sized medullary bundles coming from the inner cylinder join the larger ones in the outer cylinder, described in Plate XXXIV. fig. 2, and there marked *d'*. All the tangential sections show the medullary bundles, both in large and small specimens, to be much greater and stronger in the outer than in the inner radiating cylinder; but no evidence has yet been foundof the junction of these medullary bundles to prove that the former run into the latter, or whether the two are distinct. They consist of hexagonal tubes, barred on all their sides, surrounded by muriform tissue, that on the outside of the specimen being of very coarse texture.

Fig. 5.



Up to this time we possess little information as to the organs of fructification belonging to *Sigillaria.* In a paper many years since printed by me *, some *Stigmariæ* were described which were found with their insides full of spores, resembling those which were found by Dr. HOOKER in *Lepidodendron.* Similar spores are met with in great abundance in all the seams of splint coal which have been examined by me, the floors of which, it is well known, are one mass of *Stigmariæ.* In the strata lying around the large *Sigillaria* found at Dixon Fold, described by the late Mr. J. E. BOWMAN†, that author says, "they (the trees) lie in a stratum of soft shale about four feet thick, among which great quantities of nodules containing cones of *Lepidostrobus*, with pieces of *Stigmariæ*, &c., were found."

---

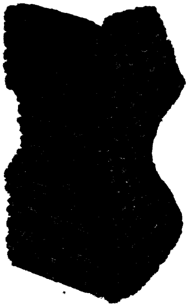
* Quarterly Journal of the Geological Society, vol. vi. p. 17.

† Transactions of the Manchester Geological Society, vol. i. p. 113.

GOLDENBERG gives a description and figures of a cone and spores which he considers to be the fructification of *Sigillaria* [*]. That author, however, does not give any further evidence of the connexion of the supposed organs of fructification with the stem of *Sigillaria* than had been known in England for years, as previously mentioned. The spores he figures as belonging to *Sigillaria* are exactly the same as those found by me in the inside of *Stigmaria*.

A specimen found in the roof of the same seam of coal in which Nos. 2, 3 & 8 were met with, but at a different place, was given to me by Mr. W. BUTTERWORTH, junior, of Moorside, near Oldham, and enables me to give evidence, equally strong with that adduced by Dr. HOOKER to prove that *Lepidostrobus* was the fruit of *Lepidodendron*, to show that a *Lepidostrobus* was the fruit of *Sigillaria*. Dr. HOOKER, in his excellent paper on this subject [†], says, "The doctrine of morphology teaches us that the cone is nothing more than the leafy apex of a branch whose leaves are modified in form, generally to the end that they shall perform the office of protecting organs to reproductive bodies; this is the case of the pine cone, that of the *Lycopodium*, or Club Moss, and many other plants." This specimen is shown in the annexed woodcut (fig. 6), of its natural size, and exhibits sporangia, like those described by Dr. HOOKER in his memoir previously quoted, arranged around the axis of the cone, which does not afford the rhomboidal scars characteristic of the *Lepidodendron*, but presents ribs and furrows, with scars, arranged in quincuncial order, like a small specimen of *Sigillaria organum*. Certainly, if the axis of Dr. HOOKER's cone is to be regarded as nothing more than the continuation of a branch of *Lepidodendron*, the axis of this cone is equally entitled to be classed as the branch of a *Sigillaria*.

Fig. 6.



The organs of fructification, which have been called by geologists fossil cones, and have been classed under the genus *Lepidostrobus*, may not only have belonged to *Lepidodendron* and *Sigillaria*, but it appears nearly certain in my mind that some of them also belonged to *Calamites*. In a paper published many years since, the apparent connexion of *Calamites* and *Sigillariæ* was discussed and noticed by the author [‡]. Since that time he has collected much further evidence on the structure of *Calamites*, which he proposes at some future time to communicate to the Society in a separate memoir.

In all the large specimens of *Sigillaria vascularis* the outer radiating cylinder has been considerably disarranged by pressure, the original cylindrical form of the plant having been changed into that of an elongated oval. This has been more especially the case with that part of the plant composed of lax and coarse cellular tissue, forming the inner portion of the outer cylinder, as well as the thick tubes or elongated utricles, arranged in radiating series, composing the outer part next the bark. Nevertheless in the former there is nearly always some evidence left of circular openings or eyes surrounded by coarse cellular tissue, which gradually assumes a radiating character, and from which the wedge-shaped bundles of tubes or elongated utricles proceed and extend to the outside of the stem. The character of these circular openings, and the wedge-shaped bundles proceeding from them, is well shown in the young specimen of *Sigillaria vascularis*, drawn in Plate XXXIII. fig. 5, and remind us much of what is seen in *Calamodendron*, except that in the latter plant the walls of the tubes exhibit oval openings, sometimes approaching the form of disks, characters which have not as yet, so far as my knowledge extends, been observed in the outer cylinder of *Sigillaria*. In larger and older specimens, as previously stated, the walls of these tubes or elongated utricles of a quadrangular form have become much thicker, and cannot be distinguished from those of *Pinites*, except by the absence of disks.

---

[*] Flora Saraepontana fossilis, Die flora der Vorwelt Saarbrückens, von FR. GOLDENBERG, 11tes Heft, Tafel x. figs. 1 & 2.

[†] Memoirs of the Geological Survey of Great Britain, vol. ii. part 2. p. 452.

[‡] Philosophical Magazine for November 1847, p. 259.

The outer cylinder, as before noticed, in large specimens always presents divisional lines of a rectangular form, filled by spathose matter, in shape very like those now seen in hard-wooded trees. These appear to me as if made by pressure, but they may have been formed in the process of drying, before the mineralization of the specimen, as previously stated; however, it is still my opinion that these lines originate from pressure rather than desiccation, as there is little evidence yet published of the subaërial decay of the vegetable matter now forming coal, while, on the contrary, nearly every seam of cannel-coal affords abundance of fish remains, and no doubt seams of soft bright coal, if equally favourable for their preservation, would yield them. My cabinet contains specimens from the Oldham coal-field of soft bright coal containing undoubted scales of *Rhizodus*, given to me by Mr. WILD, of Glodwick, and doubtless many more such specimens will be found if carefully looked for.

In the outer portion there is always some appearance of concentric rings, not unlike those seen in our present hard-wooded trees, and which my friend Mr. J. S. DAWES, F.G.S., first noticed in *Calamodendron*[*]. This observation of Mr. DAWES many specimens in my cabinet amply confirm, although they do not bear out that author's statement as to *Calamodendron* having had a pith composed of cellular tissue, as it undoubtedly possessed a central axis composed of large vessels apparently barred on all their sides by transverse striæ, and not to be distinguished from the same part of *S. vascularis*.

Concluding Remarks.

In this memoir the reader will no doubt have seen that it was intended to be more of a descriptive character than an attempt to trace the analogy of the plants whose remains have formed our beds of coal with living vegetables. The subject is surrounded with difficulties, and although the author has been singularly fortunate in meeting with specimens in a good state of preservation, when compared with most hitherto described,

[*] Quarterly Journal of the Geological Society, vol. vii. p. 198.

still his information is confined to two plants. These, no doubt, have contributed by their remains in a great measure to form the two seams of coal in which they were found, as is evident from the abundance of *Sigillaria*-roots now found in floors of the beds. In addition to this fact, the Halifax Hard or Gannister seam yields the *Sigillaria vascularis* as by far the most common plant found in it.

The large specimens Nos. 2 & 3, now described and figured, some persons may doubt as being the older forms of the *Sigillaria vascularis* described by me some years since in the Geological Society's Journal previously quoted, as well as the medium-sized specimen No. 8 given in Plate XXXV. fig. 5 of this memoir; but the one has been traced gradually passing into the other so as to leave no doubt on this point, and the internal structure is unquestionably the same both in the large and small plants, after making due allowance for the greater age of the former.

The general opinion of botanists and geologists, that *Sigillaria* was a hollow and succulent plant, no doubt arose from the flat specimens generally found compressed into thin plates in indurated clays or shales. The same view was taken with regard to *Calamites*, owing to their being nearly always found in a similar condition; but it is now well known that many specimens of *Calamites* are nothing more than the casts of the central axis of a hard-wooded tree with concentric rings, the whole of which has in most cases disappeared and left no trace of its former existence. Now, although till the discovery of my specimens few, if any, large *Sigillaria* had been found exhibiting structure, it has been shown that the late Mr. BOWMAN, an eminent botanist, many years since pronounced the Dixon Fold fossil trees to be large *Sigillariæ* and hard-wooded dicotyledonous trees with heavy tops, and this he inferred chiefly from the size and form of their roots. Long after the last-named author's death, Dr. DAWSON, in 1859, as previously quoted, was inclined "to suspect that some of the described species of conifers of the coal may be the woody axes of large *Sigillariæ*, or at least of trees approaching quite as nearly to those plants as to modern conifers." Although my specimens do not altogether support Dr. DAWSON'S views as to the woody axis he no doubt refers to, namely, the internal radiating cylinder and not the outward one, which he terms a very thick cellular inner bark, his opinion is entitled to considerable weight as to *Sigillariæ* being hard-wooded trees, he having paid great attention to the different structures found in the charcoal now met with in our coals, the floors of which so constantly testify to the presence of *Sigillaria* in the form of roots, and the great part it contributed to their formation. The size of the external cylinder of this plant, when compared with its internal one, is so much greater, that by far the larger portion of the coal must have been derived from the former. It is this part of the fossil tree that so generally divides into rectangular masses, and not the small internal cylinder evidently alluded to by Dr. DAWSON, as any person who has examined many large specimens will well know.

Specimen No. 2 probably may not be considered as so marked an example of the genus *Sigillaria*, owing to the small size and indistinctness of the cicatrices left by the leaves, which are not so well shown in the Plate as they are generally found on specimens of *Sigillaria organum*. No doubt it cannot be regarded as a good example of the species *organum*, but from the ribs, furrows, and scars on its outside no one will question its being a *Sigillaria*, even if its internal structure did not prove its relationship to *Sigillaria elegans*.

In all my investigations as to the origin of coal, the marine character of the water in which the plants that formed it by their decomposition grew, becomes to my mind more evident. It is now well known to all parties conversant with coal-mining, that in most deep mines where the surface water cannot get down the water found in the coal is quite salt, and contains iodine, bromine, and the usual constituents of sea-water. Any person carefully examining each of the seams of coal in which the fossil woods described in this memoir were found, placed as they are upon an under clay full of *Sigillaria*-roots with their radicles traversing it in every direction, will be convinced that the plants which formed the coal grew on the spot where it is now met with, and were not drifted there, while the presence of such a mass of marine shells as is found in the roof of each seam evidently where they lived and died, equally proves the salt nature of the water.

Little evidence is to be obtained of the character of the dry land of the Carboniferous epoch except what is afforded by a few sun cracks on some of the rocks, but from the shallow seas more resembling marine swamps than the oceans of the present day, it was probably little above the surface of the water. Shallow seas and low lands would of course greatly influence the climate of the period. The strata found in the vicinity of seams of coal, with some few exceptions, show that they were deposited from water during periods of great tranquillity, and the vast range over the old and new worlds of the genus *Sigillaria* found in all their true coal-fields, indicates a uniformity of conditions of which we have now no parallel, and areas of such immense extent as is only equalled by some of our present oceans.

In the Lancashire coal-field, probably one of the best developed in Great Britain, from the bottom to the top there are about 120 different seams of coal, great and small. These indicate 120 periods of rest or repose of the earth's crust, when a primeval forest reared its top above the waters until the vegetable matter now forming each bed of coal was grown and deposited*. Then such forest was submerged and buried under mud and sand now found as shale and sandstone rocks. The hollow caused by such subsidence was silted up until it was again covered by shallow water. Then, again, a fresh crop of vegetation flourished so as to form another bed of coal. For 120 different times did this successive growth of vegetable matter, submergence and silting up go on. In some instances whole forests of *Sigillaria*, standing upright in fine shale, on the top of the seams of coal are met with, thus clearly showing that they were submerged quietly and slowly, whilst at other times the prostrate stems now found lying in sandstone roofs show that the submergence was rapid, causing strong currents that tore up and drifted the trees. Every one of the floors of these coal-seams is full of the roots of *Sigillaria*; so with the stems of these trees in the roof, the vegetable matter in the seam of coal, and the roots in the floor, there can scarcely be a doubt as to the remains of the vegetables now composing coal having grown on the spots where it is now found, and that *Stigmaria* was the characteristic root of the plants which for the most part produced coal.

---

* Although upright *Sigillariæ* are generally found in the roof of a seam of coal, they are also met with in fine-grained shales, midway between seams, less frequently in coal floors, and more rarely still in the seams of coal themselves.—Transactions of the Manchester Literary and Philosophical Society, vol. viii. 2nd series, p. 176.

The above conditions of the growth of vegetables in shallow seas very different to any state of things now existing, would require a plant suited to them and very different from any now living. After a careful investigation of the structure of *Sigillaria elegans*, BRONGNIART came to this conclusion: "Tous ces motifs doivent nous porter à conclure que les *Sigillaria* et les *Stigmaria* constituaient une famille spéciale entièrement détruite, appartenant probablement à la grande division des Dicotylédones gymnospermes, mais dont nous ne connaissons encore ni les feuilles ni les fruits."

If we take particular parts of *Sigillaria vascularis*, as before described, we can trace resemblances to some living plants. The central axis when taken by itself might appear to connect the plant with ferns, as it certainly bears some resemblance to the root of *Aspidium exaltatum*, as figured by BRONGNIART in plate 8, figs. 10 & 11*. The internal radiating cylinder is somewhat like similar cylinders in *Echinocactus* and *Melocactus*, as figured by the same author.

The vessels with barred and dotted sides in some respects resemble those of *Zamia integrifolia*, also noticed by BRONGNIART, and the outer radiating cylinder in the thickness of the walls of its tubes, or elongated utricles, and their arrangement, points to conifers. Although *Sigillaria* has resemblance in some of its parts to such widely different living plants, there can scarcely be a doubt in the mind of any one who has had the advantage of examining the fossil plant with its far extending roots and long radicles, but that it had an aquatic *habitat*. It attained a large size, as upright specimens have been traced by me nearly 60 feet in height without showing much diminution in size, and the bases of others have come under my observation which have measured over 7 feet in diameter.

DESCRIPTION OF THE PLATES.

PLATE XXX.

Diploxylon cycadoideum.

Fig. 1. Specimen (No. 1) of one-half of a stem of *Diploxylon cycadoideum* in a calcified state, found in the lower coal-measures of Lancashire, in the middle of a seam of coal, showing a transverse section: natural size.

---

* Observations sur la structure intérieure du *Sigillaria elegans*, p. 447.

Fig. 2. A longitudinal section of the same specimen taken across the minor axis from *d* to *d* in fig. 1: natural size.

Fig. 3. A tangential section of the same specimen taken across the upper part: natural size.

Note.—The same letters indicate the same parts in this and the preceding figures, and also in the subsequent ones.

a a. The middle part, showing the central axis or pith composed of large hexagonal vessels, having all their sides barred by transverse striæ.

a′ a′. The smaller hexagonal vessels in the central axis or pith found sometimes interspersed amongst the larger ones, and divided by horizontal septæ.

a″ a″. Small vessels of very delicate elongated tissue found mixed with the other vessels in the axis or pith.

b b. The vascular internal cylinder, in wedge-shaped bundles and radiating series, composed of hexagonal vessels, barred on all their sides by transverse striæ, and divided by medullary rays or bundles, *b″ b″*.

b′ b′. Portions of the same cylinder disarranged or destroyed.

b″ b″. Medullary rays or bundles passing through the internal cylinder, and extending to the outside of the stem.

c c. Space on the outside of the internal cylinder, composed of lax cellular tissue, and traversed by vascular bundles frequently disarranged or destroyed, and replaced by mineral matter.

d d. Outer cylinder of tubes or elongated utricles in wedge-shaped bundles, and radiating series of quadrangular form, divided by wide openings filled with coarse muriform tissue, which enclose medullary rays or bundles of an oval or circular form leading to the leaves.

d′ d′. Medullary rays or bundles of barred vessels traversing the coarse muriform tissue.

d″ d″. Elongated tissue divided by horizontal septæ (muriform tissue) surrounding the medullary rays or bundles.

Fig. 4. A transverse section of a portion of the same specimen taken across the minor axis, showing the whole of the central axis or pith, one side of the inner radiating cylinder, and the space between the latter and the outside of the stem: magnified 5 diameters.

Fig. 5. A longitudinal section of the same specimen, showing the same parts of the stem as are named in the last figure, magnified 5 diameters.

Fig. 6. A tangential section of the same specimen (upper part), magnified 5 diameters.

PLATE XXXI.

Sigillaria vascularis.

Fig. 1 (No. 2). Specimen of a stem of *Sigillaria vascularis* in a calcified state, found in the lower coal-measures of the West Riding of the County of York, at North Owram near Halifax, in the middle of the Hard bed of coal, showing a front view of the upper part, containing the central axis, internal vascular cylinder, space on the outside of the latter composed of coarse cellular tissue, and external radiating cylinder: natural size.

Fig. 2. Side view of the same specimen, which not only shows the upper part of the specimen like fig. 1, with the central axis, internal radiating cylinder, intervening space of lax cellular tissue, and external radiating cylinder, but a side view of the decorticated portion of the stem with irregular ribs and furrows, on the former of which are traces of the cicatrices left by the leaves of the plant: natural size.

PLATE XXXII.

Sigillaria vascularis.

Fig. 1 shows a transverse section of the central axis and internal radiating cylinder of the same specimen, magnified 5 diameters.

Fig. 2. A part of the same specimen, *a* denoting the central axis, and *b* the internal radiating cylinder: magnified 12 diameters.

Fig. 3. A longitudinal section of the same specimen, commencing on the outside of the internal radiating cylinder passing through the central axis, the other portion of the internal radiating cylinder, the part composed of coarse cellular tissue generally disarranged adjoining to it, and the external radiating cylinder to the outside of the specimen: magnified 4 diameters.

a a. Parts of the central axis composed of hexagonal vessels arranged without order, having all their sides marked by transverse striæ.

b b Parts of the internal cylinder, composed of hexagonal vessels in wedge-shaped bundles, and radiating series marked on all their sides by transverse striæ parted by medullary rays or vascular bundles communicating from the outside of the central axis to the exterior of the cylinder, and probably extending on to the leaves.

c c. Parts of the coarse cellular tissue, generally a good deal disarranged, traversed by large vascular bundles, most probably connected with the medullary rays or vascular bundles of the internal cylinder, and communicating with the leaves.

d d. Parts of the external cylinder, composed of tubes or elongated utricles of a quadrangular form arranged in radiating series, and parted by large vascular bundles surrounded by coarse muriform tissue.

Fig. 4. A tangential section of a portion of the same specimen, magnified 4 diameters.

b. Parts of the internal cylinder, showing a section of the medullary rays or vascular bundles, *b''*.

c. Portions of the coarse cellular tissue, generally a good deal disarranged, traversed by large vascular bundles, most probably connected with the medullary rays or vascular bundles of the internal cylinder, and communicating with the leaves.

d d. Parts of the external cylinder, composed of tubes or elongated utricles of a quadrangular form arranged in radiating series, and parted by large vascular bundles surrounded by coarse muriform tissue.

Fig. 4. A tangential section of a portion of the same specimen, magnified 4 diameters.

b b. Parts of the internal cylinder, showing a section of the medullary rays or vascular bundles, *b''*.

c c. Parts of the coarse cellular tissue somewhat disarranged, but showing some structure, and traversed by vascular bundles.

d d. Parts of the external radiating cylinder, showing the large oval bundles of vascular tissue (*d'*) surrounded by coarse muriform tissue.

PLATE XXXIII.

Sigillaria vascularis.

Fig. 1 shows a longitudinal section of a portion of the same specimen, exhibiting the central axis composed of barred vessels, *a a*, parted by smaller vessels divided by horizontal septæ, *a'*, as well as portions of the internal cylinder composed of barred vessels, *b b*: magnified 15 diameters.

Fig. 2 represents two of the barred vessels of the central axis as they would appear if not ground away in the slicing and polishing, magnified 45 times.

Fig. 3. A tangential section of a portion of the same specimen across a part of the internal cylinder, showing the medullary rays or bundles (*b''*) traversing the cylinder *b*: magnified 15 diameters.

Fig. 4. A longitudinal section of a portion of the external cylinder *d*, composed of tubes or elongated utricles arranged in radiating series, magnified 10 diameters.

Fig. 5. A tangential section of a portion of the external cylinder, showing the large vascular bundles of an oval shape, *d'*, surrounded by coarse muriform tissue which traverse it: magnified 10 diameters.

PLATE XXXIV.

Sigillaria vascularis.

Fig. 1. Specimen (No. 3) of a stem of *Sigillaria vascularis* in a calcified state, found also in the lower coal-measures of North Owram in the middle of the Hard bed of coal, in company with the last specimen described, showing a portion of the central axis divided and partly disarranged, portions of the internal cylinder composed of hexagonal vessels having all their sides marked with transverse striæ, arranged in radiating series parted by medullary rays or vascular bundles; also a part of the space on the outside of the internal cylinder, composed of coarse cellular tissue, and parts of the external cylinder, composed of tubes or elongated utricles arranged in radiating series, and parted by large vascular bundles surrounded by coarse muriform tissue communicating with the leaves.

The outside of the specimen presented the same kind of ribs and furrows, with indistinct traces of cicatrices, as the specimen No. 2, described in Plates XXXI., XXXII., and XXXIII. It is given chiefly for the purpose of showing the tubes or elongated utricles of the external cylinder, traversed by the large vascular bundles of an oval form, surrounded by coarse muriform tissue which are much more distinctly represented than in the first-named specimen No. 2: magnified 2 diameters.

Fig. 2. A tangential section of the same specimen, showing a portion of the outer cylinder, composed of tubes or elongated utricles, *d d*, traversed by large vascular bundles of the shape of a double cone, composed of very large horizontally-divided tissue, *d'*, and more finely divided tissue, *d'' d''*, and having an oval-shaped vascular bundle in the middle, most probably communicating with the cicatrices to which the leaves were attached on the outside of the plant: magnified 20 diameters.

Fig. 3. A longitudinal section of the same specimen, showing a portion of the outer cylinder, composed of tubes or elongated utricles, *d*, arranged in radiating series, as well as a portion of a vascular bundle with the fine tissue divided by horizontal partitions, *d''*: magnified 20 diameters.

PLATE XXXV.

Sigillaria vascularis.

Figs. 1, 2, & 3 (Nos. 4, 5, & 6) represent the external appearance of the central axes of three different specimens of *Sigillaria vascularis*, found in the middle of the Hard seam of coal in company with the specimens Nos. 2 & 3 described in Plates XXXI., XXXII., XXXIII., and XXXIV. They were enclosed in three stems, exactly resembling those specimens in external characters and internal structure in every respect. The horizontal division in fig. 1 may probably owe its origin to a fissure in the stone rather than a division such as is usually seen in a *Calamites*, but the outside longitudinal striæ in all the specimens remind us of that fossil plant, while the vascular bundles of the central axis of these specimens bear considerable resemblance to some of the species of *Medullosa*: magnified 2½ diameters.

Fig. 4 (No. 7) represents the outside of the inner radiating cylinder of *Stigmaria ficoides* arranged in wedge-shaped bundles, showing the finely marked longitudinal striæ with which it was furnished, but not affording any evidence of structure in the central axis: magnified 2½ diameters. This specimen is from the Wigan Five Feet seam of coal of the Ince Hall Coal and Cannel Company, in the middle division of the Lancashire coal-measures, and is the only specimen which has come under my notice which shows the outside of the inner radiating cylinder: magnified 2½ diameters.

Fig. 5 (No. 8) represents a transverse section of a small specimen of *Sigillaria vascularis*, found also in the lower coal-measures of North Owram, in the middle of the Hard bed of coal. It is in a more perfect condition, as a whole, than any of the other specimens described in this paper, and appears to be a younger individual of the same genus and species as the larger and more imperfect ones, Nos. 2 & 3, figured in Plates XXXI., XXXII., XXXIII., and XXXIV., associated with which it was found. It shows the central axis, composed of hexagonal vessels arranged without order, and having all their sides marked with horizontal striæ, the internal cylinder of hexagonal vessels arranged in radiating series, and having all their sides marked with transverse striæ and parted by medullary rays or vascular bundles, the space outside that cylinder occupied by lax cellular tissue traversed by vascular bundles, sections of some of which are seen as circular openings, a dark line bounding it, the zone of coarse cellular tissue outside that last named containing circular and oval openings, and passing into tubes or elongated utricles arranged in radiating series, and divided by large medullary rays or vascular bundles, forming the external cylinder, and an outer bark enveloping the plant: magnified 4 diameters.

Fig. 6 (No. 8) represents the outside view of the same specimen partly covered by a thick carbonaceous coating, probably representing the outer bark and partly decorticated, displaying rhomboidal scars, having a rib running through their major axis, in the middle of which is a cicatrix of a circular form left by the leaf. The scars and cicatrices upon them were arranged in quincuncial order. The specimen appears to be older than those described by me in the Geological Journal previously alluded to, and younger than specimens 2 & 3 of this paper: magnified 2½ diameters.

XII. THE BAKERIAN LECTURE.—*On a Method of Meteorological Registration of the Chemical Action of Total Daylight**. *By* HENRY ENFIELD ROSCOE, *B.A.*, *F.R.S.*, *Professor of Chemistry in Owens College, Manchester.*

Received November 8,—Read December 22, 1864.

IN the last memoir on Photochemical Measurements, presented to the Royal Society[†], Professor BUNSEN and I described a method for determining, by simple observations, the varying amount of chemical action effected by the direct and diffuse sunlight on photographic paper, founded upon a law discovered by us, viz. that equal products of the intensity of the light into the times of insolation correspond within very wide limits to equal shades of tints produced on chloride-of-silver paper of uniform sensitiveness—so that light of the intensity 50, acting for the time 1, produces the same blackening effect as light of the intensity 1 acting for the time 50. For the purpose of exposing this paper to light for a known but very short length of time, a pendulum photometer was constructed; and by means of this instrument a strip of paper is so exposed that the different times of insolation for all points along the length of the strip can be calculated to within small fractions of a second, when the duration and amplitude of vibration of the pendulum are known. The strip of sensitive paper insolated during the oscillation of the pendulum exhibits throughout its length a regularly diminishing shade from dark to white; and by reference to a Table, the time needed to produce any one of these shades can be ascertained. The unit of photo-chemical intensity is assumed to be that of the light which produces upon the standard paper in the unit of time (one second) a given but arbitrary degree of shade termed the normal tint. The reciprocals of the times during which the points on the strip have to be exposed in order to attain the normal tint, give the intensities of the acting light expressed in terms of the above unit.

According to this method the chemical action of the total daylight (*i. e.* the direct sunlight and the reflected light from the whole heavens) has been determined, by means of observations made at frequent intervals throughout the day, and curves representing the variation of daily chemical intensity at Manchester have been drawn[‡]. The labour of obtaining a regular series of such daily measurements of the chemical action of daylight according to this method is, however, very considerable; the apparatus required

---

[*] It is to be carefully borne in mind that no absolute measurement of the more refrangible solar rays falling on the earth's surface is possible, except by the expression of their heat-producing effect; and that all methods of measuring the intensity of these rays depending upon the action which they produce on any single chemical compound, give results which are only true for the particular rays affecting the compound selected as the standard of comparison.

[†] Philosophical Transactions, 1863, p. 139.

[‡] Ibid. 1863, p. 160.

is bulky, the observations can only be made in calm weather, and the quantity of sensitive paper needed for a day's observations is large.

The aim of the following communication is to describe a very simple mode of determining at any moment the chemical action of the whole direct and diffuse sunlight (as measured by chloride-of-silver paper) adapted to the purpose of regular meteorological registration, and founded upon the principles laid down in the memoir above alluded to. According to this method a regular series of daily observations can without difficulty be kept up at frequent intervals. The whole apparatus needed for exposure can be packed into very small space; the observations can be carried on without regard to wind or weather; and no less than forty-five separate determinations can be made upon 36 square centimetres of sensitive paper.

Strips of the standard chloride-of-silver paper tinted in the pendulum photometer remain as the basis of the more simple mode of measurement now to be described. Two strips of this paper are exposed as usual in the pendulum photometer; one of these strips is fixed in hyposulphite-of-sodium solution, washed, dried, and pasted upon a board furnished with a millimetre-scale. This fixed strip is now graduated in terms of the unfixed pendulum strip by reading off, with the light of a soda-flame, the position of those points on each strip which possess equal degrees of tint, the position of the normal tint upon the unfixed strip being ascertained for the purpose of the graduation. The fixed strip thus becomes in every respect equivalent to the unfixed strip. Upon this comparison with the unfixed pendulum strip depends the subsequent use of the fixed strip. In order to understand how the chemical action of daylight can be measured by help of this fixed and graduated strip, let us suppose, in the first place, that we have ascertained the position of those points upon the fixed strip which possess an equal degree of tint to points on the unfixed strip situated at regular intervals, say 10 millims. from each other. By reference to Table I. of the above-mentioned memoir, given below, we then find the relation between the times of exposure necessary to effect the tints in question when the intensity of the light remains constant.

Let us suppose, in the second place, that the position on the unfixed strip of which the shade corresponds to that of the normal tint has been found; and that the time of exposure, placed opposite to this position in Table I., has been noticed. If, now, the various tints on the strip had been produced in one and the same time by lights of different intensities, instead of being effected by light of the same intensity acting for different times, the law above alluded to shows that the numbers found in the Table would represent the relation of these different intensities; so that in order to express this relation in terms of the unit of intensity employed, it is only necessary to multiply the numbers thus obtained by a constant, viz. the reciprocal of the number found in column II. of the Table, opposite to the position in column I., giving the point on the unfixed strip equal in shade to the normal tint. An example may serve to make this calculation plain: (1) The position on the unfixed strip equal in shade to the normal tint was found to be 112 millims.; (2) the positions on the fixed strip equal in

TABLE I.

I. Millims.	II. Seconds.	I. Millims.	II. Seconds.	I. Millims.	II. Seconds.	I. Millims.	II. Seconds.	I. Millims.	II. Seconds.	I. Millims.	II. Seconds.
0	1·200	32	1·003	64	0·846	96	0·700	128	0·549	160	0·369
1	1·193	33	0·998	65	0·841	97	0·695	129	0·544	161	0·363
2	1·186	34	0·993	66	0·837	98	0·691	130	0·539	162	0·357
3	1·179	35	0·988	67	0·832	99	0·686	131	0·534	163	0·350
4	1·172	36	0·983	68	0·828	100	0·682	132	0·528	164	0·343
5	1·165	37	0·977	69	0·823	101	0·677	133	0·523	165	0·336
6	1·158	38	0·972	70	0·819	102	0·672	134	0·518	166	0·329
7	1·151	39	0·967	71	0·814	103	0·668	135	0·513	167	0·321
8	1·144	40	0·962	72	0·809	104	0·663	136	0·508	168	0·314
9	1·137	41	0·957	73	0·805	105	0·659	137	0·502	169	0·309
10	1·131	42	0·952	74	0·800	106	0·654	138	0·497	170	0·300
11	1·125	43	0·947	75	0·796	107	0·650	139	0·492	171	0·291
12	1·119	44	0·942	76	0·791	108	0·645	140	0·487	172	0·283
13	1·113	45	0·937	77	0·786	109	0·640	141	0·482	173	0·274
14	1·106	46	0·932	78	0·782	110	0·635	142	0·476	174	0·266
15	1·100	47	0·927	79	0·777	111	0·631	143	0·470	175	0·257
16	1·094	48	0·922	80	0·773	112	0·626	144	0·465	176	0·249
17	1·087	49	0·917	81	0·768	113	0·621	145	0·459	177	0·240
18	1·081	50	0·912	82	0·764	114	0·617	146	0·453	178	0·229
19	1·076	51	0·907	83	0·759	115	0·612	147	0·448	179	0·219
20	1·070	52	0·903	84	0·755	116	0·607	148	0·442	180	0·208
21	1·064	53	0·898	85	0·750	117	0·603	149	0·436	181	0·198
22	1·058	54	0·893	86	0·745	118	0·598	150	0·431	182	0·187
23	1·053	55	0·888	87	0·741	119	0·593	151	0·425	183	0·176
24	1·047	56	0·884	88	0·736	120	0·588	152	0·419	184	0·161
25	1·041	57	0·879	89	0·732	121	0·583	153	0·413	185	0·146
26	1·036	58	0·874	90	0·727	122	0·578	154	0·407	186	0·131
27	1·030	59	0·870	91	0·723	123	0·573	155	0·401	187	0·116
28	1·025	60	0·865	92	0·718	124	0·568	156	0·394		
29	1·019	61	0·860	93	0·714	125	0·563	157	0·388		
30	1·014	62	0·856	94	0·709	126	0·558	158	0·382		
31	1·009	63	0·851	95	0·704	127	0·553	159	0·376		

tint to two points on the unfixed strip situated 10 millims. on each side of this, were found to be 100 millims. and 123 millims; (3) by reference to the Table, the relation between the intensities on these two positions is found to be as 0·672 to 0·578; (4) these numbers, multiplied by $\frac{1}{0\cdot626}$, the reciprocal of the intensity corresponding to 112 millims., give the intensities expressed in terms of the unit formerly employed, which acting for one second produce the tints in question.

The method of observation thus becomes very simple. To each of the fixed and graduated strips an Intensity Table is attached, giving the value of the tints upon each millimetre of its length in terms of the described unit; a piece of standard sensitive paper is exposed for a known number of seconds to the light which it is required to measure, until a tint is attained equal to some one of the tints upon the strip; the exact position upon the strip of equality of tint to the exposed paper is next read off by the light of the soda-flame; the number found in the Intensity Table opposite to this position, divided by the time of exposure in seconds, gives the intensity of the acting light in terms of the required unit.

A detailed description of the apparatus employed, and of the methods of preparing and graduating the strips, will be given under separate headings.

The following conditions must be fulfilled in order that this method can be adopted as a reliable measurement of the chemical action of light:—.

1st. The tint of the standard strips fixed in hyposulphite must remain perfectly unalterable during a considerable length of time.

2nd. The tints upon these fixed strips must shade regularly into each other, so as to render possible an accurate comparison with, and graduation in terms of, the unfixed pendulum strips.

3rd. Simultaneous measurements made with different strips thus graduated must show close agreement amongst themselves, and they must give the same results as determinations made by means of the pendulum photometer, according to the method described on pages 158, 159 of the last memoir.

I. *Preparation of the Standard fixed Strips.*

For the purpose of preparing the fixed strips, sheets of good white photographic paper are salted in a solution containing 3 per cent. of chloride of sodium, exactly according to the directions given in the last memoir (p. 155) for the preparation of the standard paper. The salted paper after drying is cut into pieces, 16 centimetres in length by 15 centimetres in breadth, and silvered on a bath containing 12 parts of nitrate of silver to 100 parts of water. After drying, one of these papers is fixed at the corners upon a board covered by a well-fitting lid of sheet zinc, so made that it does not touch the paper; the paper is then blackened by exposure to the action of light in the pendulum apparatus. For this purpose, the thin elastic sheet of the blackened mica usually employed, is replaced by a piece of thin sheet zinc 16 centimetres broad. The frame carrying the paper is clamped on to the horizontal plate of pendulum photometer, and the sheet of blackened zinc placed over it; the cover is then withdrawn, and the paper exposed by allowing the pendulum, with the sheet of zinc attached to it, to vibrate until the required tint has been attained. The cover is then replaced, the frame opened in the dark room, the paper washed to remove excess of nitrate of silver, fixed in a saturated solution of hyposulphite of sodium, and well washed for three days. As the tints of the foxy-red colour which the paper possesses after fixing can be accurately compared with the bluish-grey tint of the freshly-exposed paper by means of the monochromatic light of the soda-flame, the use of a toning-bath was specially avoided as likely to render the paper liable to fade. Each sheet thus prepared is cut into four strips, 160 millims. long and 30 millims. broad, which are then preserved for graduation.

In order to ascertain whether these fixed strips undergo any alteration in tint by exposure to light, or when preserved in the dark, two consecutive strips were cut off from several different sheets, and the point on each at which the shade was equal to that of the standard tint (see last memoir, p. 157) was determined by reading off with the light of the soda-flame, by means of the arrangements fully described on p. 143 of the above-cited memoir. One-half of these strips were carefully preserved in the dark, the other half exposed to direct and diffuse sunlight for periods varying from fourteen days to six months, and the position of equality of tint with the standard tint from time to time determined. It appears, from a large number of such comparisons, a few of which only are given below, that in almost all cases an irregular, and in some instances a rapid fading takes place immediately after the strips have been prepared, and that this fading continues for about six to eight weeks from the date of the preparation. It is, however, seen that, after this length of time has elapsed, neither exposure to sunlight nor preservation in the dark produces the slightest change of tint, and that, for many months from this time forward, the tint of the strips may be considered as perfectly unalterable.

(1) *Experiments showing the alteration of tint ensuing immediately after preparation.*

Each number given below represents the intensity (see Table II., p. 159 of the last memoir) corresponding to the mean of ten independent readings on each strip upon the under-mentioned days.

Sheet No. 1, prepared December 9, 1863.			
	Intensity. 1st Reading. Dec 16, 1863.	Intensity. 2nd Reading, Jan. 7, 1864.	Diminution in three weeks.
Strip A, exposed to sunlight...	2·49	2·05	0·44
Strip B, preserved in the dark	2·49	2·01	0·48
Sheet No. 2, prepared December 9, 1863.			
Strip A, exposed to sunlight...	2·21	1·86	0·35
Strip B, kept in the dark	2·21	2·03	0·18

From these numbers it is seen that the fading which occurs immediately after preparation is not dependent upon exposure, a change of the same kind being observed in those strips which were protected from the action of light.

(2) *Experiments showing the permanency of tint after lapse of some time from date of preparation.*

Sheet No. 3, prepared September 21, 1863.				
	Intensity. 1st Reading, Dec. 10, 1863.	Intensity. 2nd Reading, Dec. 18, 1863.	Intensity. 3rd Reading, Jan. 11, 1864.	Intensity. 4th Reading, Feb. 4, 1864.
Strip A, exposed to sunlight...	1·40	1·40	1·38	1·36
Strip B, kept in the dark	1·38	1·37	1·39	1·35
Sheet No. 4, prepared September 21, 1863.				
Strip A, exposed to sunlight...	1·45	1·39	1·39	1·38
Strip B, kept in the dark	1·43	1·43	1·45	1·46

(3) *Experiments showing alteration and subsequent permanency of Tint.*

	Intensity. 1st Reading, Mar. 12, 1864.	Intensity. 2nd Reading, Mar. 21, 1864.	Intensity. 3rd Reading, Apr. 27, 1864.	Intensity. 4th Reading, May 11, 1864.	Intensity. 5th Reading, June 3, 1864.	Intensity. 6th Reading, July 18, 1864.
Sheet No. 5, prepared March 10, 1864.						
Strip A, exposed to sunlight...	2·08	2·13	1·93	1·99	2·03	1·89
Strip B, in the dark	2·10	2·13	1·93	1·93	1·89	1·89
Sheet No. 6, prepared March 10, 1864.						
Strip A, exposed to sunlight...	2·23	2·23	2·13	2·15	2·15	2·10
Strip B, kept in the dark	2·23	2·23	1·99	2·01	2·08	1·97
Sheet No. 7, prepared March 10, 1864.						
Strip A, exposed to sunlight...	2·35	2·42	2·08	2·18	2·13	2·01
Strip B, kept in the dark	2·35	2·54	2·01	2·03	2·08	2·03

The above numbers show that, after the standard fixed strips have been prepared for about two months, the tints remain constant both when the paper is exposed to light and when it is kept in the dark. The small differences seen in some instances arise from unavoidable experimental errors of various kinds.

II. *Graduation of the fixed Strips in terms of the Standard Pendulum Strips.*

The value of the proposed method of measurement entirely depends upon the possibility of accurately determining the intensities of the various shades of the fixed strips in terms of the known intensities of the standard strips prepared in the pendulum photometer.

Two modes of effecting this graduation, and of comparing the accuracy of the graduation of one strip with that of another, were employed.

The first of these methods consists in determining by direct comparison the points on the fixed strip having equal intensities to points on the pendulum strip. For this purpose the position of the standard tint upon the pendulum strip was first observed; circular pieces of this strip, situated 20 millims. apart, were then stamped out with a punch 5 millims. in diameter, and half of each circle pasted on to the wooden reading block (fig. 4 of the last memoir), so that the centre of the paper circle came into the centre of the hole. The readings were conducted in the way described on p. 159 of the last memoir, every comparison being made independently ten times by each of two observers, and the mean reading taken as the result, whilst several pendulum strips were used for the graduation of one fixed strip. The following may serve as an example of the first method of graduation. Four pendulum strips were employed for the graduation of the fixed strip A.

Graduation of fixed strip A.

Position of standard tint upon pendulum strip No. 1=85 millims., from which the constant $\frac{1}{0{\cdot}750}$ is found in Table I. p. 607.

The position 20 mm. on pendulum strip=1·427 intensity, and corresponds to 67·4 mm. on fixed strip.
„ 40 „ 1·283 „ 79·8 „
„ 60 „ 1·154 „ 83·0 „
„ 80 „ 1·031 „ 91·6 „
„ 100 „ 0·910 „ 94·5 „
„ 120 „ 0·784 „ 119·8 „
„ 140 „ 0·650 „ 121·6 „

In like manner the constants for three other pendulum strips were determined.

$$\text{Constant for pendulum strip No. } 2=\frac{1}{0{\cdot}431}.$$

$$\text{Constant for pendulum strip No. } 3=\frac{1}{0{\cdot}607}.$$

$$\text{Constant for pendulum strip No. } 4=\frac{1}{0{\cdot}508}.$$

By comparison of each of these three pendulum strips with the fixed strip the following numbers were obtained. Column I. gives the readings on the millimetre-scale of the fixed strip; Column II. the corresponding intensities calculated as in the foregoing example.

No. 2.		No. 3.		No. 4.	
I.	II.	I.	II.	I.	II.
26·0	2·12	49·9	1·76	34·6	2·10
35·3	1·90	60·0	1·59	40·4	1·89
55·5	1·69	70·5	1·43	53·4	1·70
72·6	1·47	81·5	1·27	64·8	1·52
80·1	1·25	92·4	1·12	82·5	1·16
90·5	1·00	103·0	0·97	93·0	0·96
		121·4	0·80	123·6	0·72
		131·5	0·61		

In order to obtain the mean result of these numbers, the curve for each of the four graduations was drawn, the abscissæ giving the positions on the fixed strip in millimetres, and the ordinates the intensities corresponding to these positions. A curve was then interpolated, lying as nearly as possible between the points determining the single observations, and from this mean curve the intensity for each millimetre on the scale was calculated. The following are these tabular values for every 10 millims. Column I. gives the position in millims. on the fixed strip, Column II. the corresponding intensity, and Column III. the mean tabular error.

I.	II.	III.	I.	II.	III.
20	2·30	0·10	70	1·47	0·022
30	2·10	0·09	80	1·28	0·010
40	1·90	0·02	90	1·07	0·045
50	1·76	0·016	100	0·916	0·053
60	1·62	0·013	110	0·830	0·056
			120	0·755	0·050

A comparison of the several curves of the graduation of strip A found in Plate XXVIII. fig. 1 shows that the determinations agree as well as can be expected from such photometric experiments; the mean tabular error between the positions 40 and 80 millims. on the strip not exceeding one per cent. of the measured intensity.

For the second method of graduation sheets of paper tinted by lithography of a brownish colour and of different shades are employed, and a portion of each sheet is cut out, so that the several tints differ considerably from each other, and correspond to the tints taken at definite intervals along the fixed strip. These are then gummed over half the reading block, and the value of each read off on several pendulum strips, the intensity of which had previously been determined by the normal tint. Having thus obtained the intensity of each of the fixed tints, the fixed strip is graduated in terms of the pendulum strip by determining the points on the former equal in intensity to the fixed tints. This method possesses several advantages over that just described, and is to be preferred to it, although the comparison is an indirect one, as the intensity of the fixed tints can be found with a great degree of accuracy by repeated measurements; and when their intensities are once determined they can be preserved for a length of time, as they do not undergo any change of shade, and therefore can serve for the graduation of a large number of fixed strips; the preparation of which is accordingly not dependent, as is the case in the first method, upon the state of the weather.

The following numbers may serve as an example of this method:

(1) Determination of the intensity of fixed tints upon pendulum strips.

	No. 1.	No. 2.	No. 3.	No. 4.	No. 5.	No. 6.	No. 7.	No. 8.	No 9.	No. 10.	No. 11.	No. 12.
	mm.	mm.	mm.	mm.	mm.	mm.	mm.	mm.	mm.	mm.	mm.	mm.
Reading of normal tint on pendulum strip	153·2	82·0	131·1	136·6	105·7	17·5	121·6	19·2	51·9	131·6	119·4	98·0
Reading on pendulum strip of fixed tint.........No. I.	40·3	...	5·0	...	...	...	...	...	...	15·7	24·5	...
„ No. II.	90·4	...	50·2	67·2	25·4	...	...	...	...	50·1	52·3	22·6
„ No. III.	115·7	29·7	91·0	100·7	66·1	...	99·0	...	...	98·4	89·3	51·7
„ No. IV.	...	108·7	159·5	...	...	50·5	...	51·2	93·0	...	145·5	134·0
„ No. V.	...	...	...	...	...	125·8	...	120·6	150·0	...	...	...

The intensities for each determination of a fixed tint are obtained from the above numbers by dividing the numbers found in Column II. of Table I. (p. 607) opposite the millimetre readings of each fixed tint by those found in the same Table opposite to the readings of the normal tint.

Intensity of Fixed Tints.

Fixed Tint.	Expt. 1.	Expt. 2.	Expt. 3.	Expt. 4.	Expt. 5.	Expt. 6.	Expt. 7.	Expt. 8.	Expt. 9.	Expt. 10.	Expt. 11.	Expt. 12.
I.	2·336	......	2·185	......	......	......	......	......	......	2 067	1·768	......
II.	1·767	......	1·709	1·647	1·585	......	......	......	......	1·689	1·524	1·548
III.	1·480	1·328	1·356	1·346	1·276	......	1·182	......	......	1·289	1·235	1·317
IV.	......	0·840	0·698	......	......	0·891	......	0·838	0·794	......	0·773	0·755
V.	......	......	......	......	......	0·515	......	0·544	0·473			

Mean Intensity.

Fixed Tint No. I....... 2·089　　Fixed Tint No. IV....... 0·798
„ II....... 1·637　　„ V....... 0·512
„ III....... 1·312

(2) Graduation of fixed strips B and C, by means of the fixed tints. The graduation of the fixed strips by means of the fixed tints is now made in the way described in the first method.

	Readings on fixed strip B.	Readings on fixed strip C.	Corresponding intensity.
	millims.	millims.	millims.
Fixed tint I.......	20·2	27·7	2·089
„ II.......	3·88	42·8	1·637
„ III.......	67·3	71·7	1·312
„ IV.......	105·1	100·6	0·798
„ V.......	129·0	122·6	0·512
Standard tint	96·0	97·5	1·000

The Intensity Tables for these two strips are obtained by careful graphical interpolation from the above numbers; the curves are given (in black) on Plate XXVIII. fig. 2, the abscissæ representing the position on the millimetre-scale of the strips, and the ordinates the corresponding intensities. In every case the normal tint (intensity=1·00) is read off on the fixed strip, serving as a control of the accuracy of the graduation.

A second series of intensity determinations of the same fixed tints with pendulum strips is appended for the purpose of controlling the accuracy of the first series. The intensities of the fixed tints thus obtained are given in the 3rd column of the following Table. A new fixed tint, No. III. A, was introduced of a shade between Nos. III. and IV. This new tint was found to coincide with the positions 82·1 millims. and 82·3 millims. on the strips B and C respectively. The readings of the remaining tints are the same as in the first series.

(3) Second graduation of Strips B and C.

	I. Readings on strip B.	II. Readings on strip C.	III. Corresponding intensity.
Fixed tint I.......	20·2	27·7	1·955
„ II.......	38·8	42·8	1·597
„ III.......	67·3	71·7	1·291
„ III A....	82·1	82·3	1·123
„ IV.......	105·1	100·6	0·807
„ V.......	129·0	122·6	0·547
Standard tint	96·0	97·5	1·000

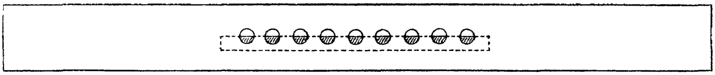
The Intensity Tables for strips B and C obtained by graphical interpolation from both the above determinations, are those used in most of the observations of daily chemical intensity about to be described. The curves of these two last graduations are given (dotted lines) on Plate XXVIII. fig. 2; and from these curves the close agreement of the graduations is seen.

The fixed strip graduated according to the above method is gummed upon the brass drum (M) of the reading-apparatus, fig. 6, care being taken to place a thick sheet of white paper between the metal and the fixed strip. In this position it is ready for use.

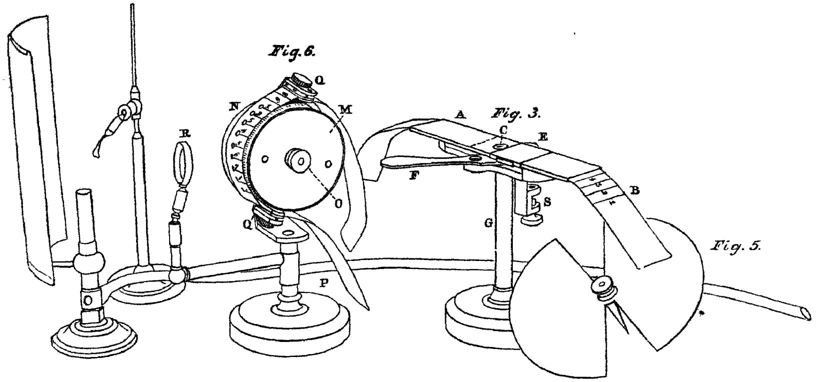
III. *Method of Exposure and Reading.*

For the purpose of making the observations, standard sensitive paper is prepared, according to the directions given on p. 155 of the last memoir, by salting photographic paper in a 3 per cent. solution of chloride of sodium, and subsequently silvering on a bath containing 12 parts of nitrate of silver to 100 of water. After drying in the dark, the paper is cut into pieces 100 millims. long by 10 millims. wide, and each piece gummed upon the back of an *insolation-band* (fig. 4) in the position denoted by the dotted lines, so that the lower half of each of the nine holes (5 millims. in diameter) stamped out of the paper 10 millims. apart, is filled up with the sensitive preparation. These insolation-bands may be easily cut out of white cartridge paper by means of an iron ruler 400 millims. long and 35 millims. broad, the holes in the paper being stamped out by a punch fitting into nine corresponding holes in the ruler. The holes in the paper are numbered, and the numbers are repeated upon the band at a distance of 87 millims. from each hole for the purpose of subsequent adjustment.

Fig. 4.



The insolation-apparatus (fig. 3) consists of a thin metal slide (A) 174 millims. in length and 40 millims. wide, with space enough between the sides to allow the paper band (B) to pass through easily. A circular opening (C) 10 millims. in diameter is cut in the middle of the upper side of the slide, and the marks on the bands are so arranged that the line marked No. 1 coincides with one end of the slide when the centre of the hole No. 1 in the band coincides with the centre of the opening (C) in the slide. A thin slip of brass (E) moves easily over the slide, and when brought into the position shown by the dotted lines, effectually protects the sensitive paper from the action of the light. If the slide (A) be used alone, the cover (E) can be moved by means of a button placed at the back of the slide; it is, however, more convenient to place the slide upon the stand (G), to which a lever handle (F) is attached, fitting into the button for the purpose of enabling the observer to cover and uncover the opening with greater ease and exactitude than is practicable when the hand alone is used. When the intensity of the light is such that the time of insolation does not exceed 2 or 3 seconds, the error introduced by this opening and closing may become considerable; for the purpose of diminishing this error by increasing the duration of exposure, the intensity of the acting light is decreased by



a known amount by allowing the circular disk of blackened metal (fig. 5), out of which two segments, each of $\frac{1}{12}$th of the whole area, have been cut, to revolve rapidly close above the upper surface of the slide (A); the spindle of the disk, for this purpose, fitting into the socket (S, fig. 3) on the stand. As the rate of rotation of the disk does not affect the accuracy of the result, it is made to revolve by turning the spindle with the hand. In order that the insolation-band carrying the sensitive paper may be made to press close against the lower edge of the opening (C), a piece of cartridge paper is placed underneath it, having several thicknesses of paper pasted at the part underlying the opening, whilst the ends of the same are made fast at the back of the slide. To enable the operator to observe when the paper has been sufficiently exposed, a small piece of photographically-tinted fixed paper of the requisite degree of shade is gummed upon the surface of the permanent paper band so as to lie directly under the opening (C).

When one observation has been made and the time and duration of the insolation noted, the remaining papers can be similarly exposed at any required time, by successively bringing them under the central opening (C), the right adjustment being ensured by making the corresponding mark coincide with the end of the slide. When all the nine papers upon the band have thus been exposed, it can be withdrawn and a second band prepared, as the first can be substituted without the necessity of bringing the apparatus into a dark room. This is done by means of a small black silk bag or sleeve, open at both ends; one end can be closed round the end of the brass slide by an elastic band, and the other is left open to admit the hand. When it is required to withdraw an insolation-band from the slide, the end of the paper is drawn out into the bag and the band rolled up into a small coil, and thus preserved until it can be read off, whilst the new band is introduced into the bag in the form of a coil, then unwound and pushed into the slide.

The reading-instrument is represented by fig. 6. It consists essentially of a metallic drum 80 millims. in diameter and 37 millims. broad, upon which a piece of thick white cartridge paper, and over it the graduated strip, is fastened. The edge of the drum is furnished with a millimetre-scale, and the dark end of the strip is made to coincide with the commencement of the scale. The drum turns upon a horizontal fixed axis against a vertical circular plate (N), being held in position by the screw (O). The drum and vertical plate are fixed upon a pillar and foot (P). The insolation-band is held against the graduated strip by means of two spring clamps (QQ′), placed apart at a distance of 130 millims. and fixed to the vertical plate (N). By moving the drum on its horizontal axis, the various shades of the fixed strip can be made to pass and repass each of the holes on the insolation-band, and the points of coincidence in tint on the strip and each of the insolated papers can be easily ascertained by reading off by the light of a soda-flame in a dark room. The lens (R) fixed upon the brass pillar of the instrument serves to concentrate the light from the flame upon the small surface under examination. If a coal-gas flame can be procured at the Observatory, the best mode of obtaining the monochromatic light is to place two beads of sodic carbonate upon fine platinum loops into the colourless flame of a Bunsen burner; if a coal-gas flame cannot be obtained, the flame of a lamp fed with spirit saturated with common salt can be used, and beads of the more volatile sodic chloride held into the flame. The reading of each observation is made ten times, and the mean of these readings taken as the result.

The following observations of the intensity of the chemical action of light on July 8, 1864, may serve as an example of the detail of the determinations.

Solar time. T.	Duration of exposure, n.	Mean reading, R.	Tabulated intensity of strip, I.	Calculated intensity, $\frac{I}{n}$.	Condition of solar disk.	Amount of cloud.	Barom.	Temperature.	
								Dry bulb.	Wet bulb.
h m	″	millims.							
7 10 A.M.	18	96	1·00	0·055	Clouded over	8			
7 50	15	93	1·03	0·068	Clouds	7			
8 25	12	90	1·06	0·089	„	9			
9 0	10	76	1·20	0·12	„	„			
9 30	10	75	1·21	0·12	„	„	millims.		
10 30	10	64	1·33	0·13	„	„	765·1	18°·6 C.	13°·9 C.
11 0	10	76	1·20	0·12	Clouded over	10			
11 30	10	67	1·30	0·13	„	„			
12 0	10	86	1·10	0·11	„	„		18·7	13·3
12 30 P.M.	6	107	0·78	0·13	Light clouds	9		19·3	13·5
1 10	8	73	1·24	0·15	„	7			
1 40	5	105	0·80	0·16	„	„		19·3	13·7
2 15	4	93	1·03	0·26	Unclouded ...	4		19·7	13·9
3 0	4	80	1·16	0·29	„	3		20·0	14·4
3 30	21 (with disk)	99	0·93	0·26	„	„			
4 0	5	86	1·10	0·22	„	„		21·1	14·4
4 30	8	76	1·20	0·15	„	1			
5 0	11	66	1·31	0·12	„	„			
6 10	60	116	0·66	0·011	„	„			

IV. *Concerning the accuracy and trustworthiness of the method.*

The most satisfactory mode of testing the reliability and accuracy of the method of measurement just described, is to compare the results of two series of independent determinations of the chemical action of daylight, made simultaneously at the same spot with the present arrangement and with the pendulum photometer, according to the method described in the last memoir, upon which the present mode of measurement is founded. For the purpose of making these comparisons, the strips of standard photographic paper placed in the pendulum apparatus (see fig. 1 of last memoir) and the pieces of the same material placed on the insolation-band in the exposing slide (fig. 3, A) were simultaneously insolated, each for a known length of time, both instruments being placed near one another in a position (on the roof of the laboratory of Owens College, Manchester) having a tolerably free horizon. If the varying daily intensities thus measured by the two methods are found to agree, we may conclude that the unavoidable experimental errors arising from graduation, exposure, and reading are not of sufficient magnitude materially to affect the accuracy of the measurement. The intensity with the pendulum photometer was determined exactly as described on pp. 158 & 159 of the above-cited memoir; the time of exposure and the number of vibrations were noted, the position at which the strip possessed a shade equal to that of the normal tint was then read off, and the corresponding intensity obtained by dividing the number found in Table II. of the above memoir by the number of the vibrations. The intensity, according to the new method, was obtained by insolating the standard paper in the exposing slide (fig. 3, A) for a known number of seconds, and then reading off, by means of the arrangement shown in fig. 6, the position in millimetres on the calibrated strip equal in shade to the exposed paper. The number found in the second column of the Intensity Table, of the strip opposite to this position, when divided by the time of exposure in seconds, gives the required intensity. In this way comparisons of the working of the two modes of measurement have been made during four different days. On each of these days a large number of simultaneous observations were made, and on some of them two or more determinations were made with each instrument immediately succeeding each other. An examination of the following Tables, giving the results of these observations, shows that the agreement between the intensities as obtained by the two methods is as close as can be expected.

Simultaneous Measurements with Pendulum Instrument and New Photometer.

April 29th, 1864.			
Time.	Intensity.		Difference.
	Pendulum instrument.	New photometer.	
h m			
9 30 A.M.	0·210	0·180	−0·03
10 0	0·160	0·160	0·00
11 0	0·073	0·083	+0·010
11 5	0·064	0·078	+0·014
12 30 P.M.	0·200	0·210	−0·01
12 32	0·210	0·220	+0·01
1 30	0·068	{0·072 0·056} 0·064	−0·04
2 0	0·105	0·105	0·00
2 30	0·124	{0·133 0·133}	+0·009
3 0	0·136	0·144	+0·008
3 0	0·117	0·114	−0·003
3 30	0·157	0·182	+0·025

May 10th, 1864.			
Time.	Intensity.		Difference.
	Pendulum instrument.	New photometer.	
h m			
9 0 A.M.	0·093	{0·079 0·085} 0·082	−0·011
10 0	0·100	0·110	+0·010
11 15	0·130	0·150	+0·020
12 30 P.M.	0·220	0·250	+0·030
1 0	0·100	{0·099 0·102} 0·100	0·000
2 30	0·105	{0·109 0·096} 0·102	−0·003
2 33	0·115	0·116	+0·001
4 30	0·0125	0·0106	−0·002

Simultaneous Measurements (continued).

June 8, 1864.			
Time.	Intensity.		Difference.
	Pendulum photometer.	New instrument.	
h m			
10 40 A.M.	0·229	0·203	−0·026
10 42	0·232	{0·226 0·238} 0·233	+0·001
11 25	0·218	0·207	−0·011
11 27	0·225	0·217	−0·008
1 33 P.M.	0·205	0·231	+0·026
2 15	0·218	0·230	+0·012
2 17	0·224	0·233	+0·009
3 20	0·072	0·064	−0·010
3 22	0·077	0·068	−0·009
4 0	0·039	0·048	+0·009
4 3	0·031	0·036	+0·005

July 16, 1864.			
Time.	Intensity.		Difference.
	Pendulum photometer.	New instrument.	
h m			
9 50 A.M.	…………	0·24	
10 25	…………	0·16	
10 40	{0·19 0·19} 0·19	{0·18 0·20 0·17 0·21} 0·19	0·00
11 45	{0·21 0·17} 0·19	0·17	−0·02
12 15 P.M.	{0·19 0·17} 0·18	{0·24 0·20 0·18 0·20} 0·205	+0·02
12 45	{0·14 0·13} 0·135	{0·17 0·14 0·11 0·17} 0·147	+0·012
1 30	{0·15 0·14} 0·145	{0·12 0·12 0·12} 0·12	−0·025
2 21	0·14	{0·17 0·15} 0·16	+0·02
2 46	{0·13 0·13 0·12 0·13} 0·127	{0·12 0·14 0·14 0·14} 0·135	+0·008

The curves on figs. 7, 8, & 9, Plate XXVIII. exhibit these results graphically for the first three days, and a glance at these curves show how closely the measurements made by the two methods agree. The black line represents the intensity as determined by the pendulum instrument, the dotted line that obtained by the new photometer, the abscissæ giving the times of observation, and the ordinates the chemical intensity in the terms of the unit above described. The mean chemical intensities, as observed on the above days by the two methods, are represented by the following numbers, for the definition of which the reader is referred to page 621.

Daily Mean Chemical Intensity.

Plate XXVIII.	1. Pendulum photometer.	2. New instrument.
Fig. 7, April 29, 1864 . . .	62·0	62·3
Fig. 8, May 10, 1864 . . .	41·3	43·3
Fig. 9, June 8, 1864 . . .	64·7	65·3

From these results the agreement of the two methods is well seen.

As a second test of the trustworthiness and availability of the method for actual measurement, I give the following results of determinations, made at the same time and on the same spot, by two observers with two of the new instruments. These determinations, made with the two graduated fixed strips B and C (page 613), were conducted in every way independently, so that the results serve as a fair sample of the accuracy with which the measurements can be practically carried out.

Simultaneous Determinations made independently with two Instruments by two observers.

July 11, 1864.			July 15, 1864.		
Time.	Chemical Intensity.		Time.	Chemical Intensity.	
	Instrument 1. Strip B.	Instrument 2. Strip C.		Instrument 1. Strip B.	Instrument 2. Strip C.
h m			h m		
10 30 A.M.	0·16	0·14	10 0 A.M.	0·16	0·17
”	0·14	0·14	10 1	0·19	0·19
10 31	0·14	0·15	11 0	0·049	0·046
”	0·12	0·13	11 1	0·049	0·046
10 32	0·13	0·11	11 35	0·12	0·12
”	0·15	0·12	”	0·12	0·12
10 33	0·14	0·12	11 36	0·12	0·13
11 0	0·13	0·12	”	0·11	0·11
12 0	0·31	0·27	12 30 P.M.	0·13	0·10
12 30 P.M.	0·31	0·29	”	0·13	0·12
12 31	0·38	0·37	”	0·14	0·13
12 32	0·33	0·31	”	0·14	0·12
12 33	0·35	0·32	1 0	0·17	0·17
1 5	0·13	0·13	”	0·18	0·18
2 0	0·27	0·25	2 30	0·057	0·060
”	0·27	0·25	”	0·068	0·070
3 10	0·24	0·23	3 30	0·059	0·057
3 11	0·21	0·24	”	0·067	0·062
3 12	0·18	0·23	3 31	0·063	0·045
3 13	0·17	0·18	”	0·054	0·045
3 40	0·24	0·23	4 20	0·028	0·025
3 41	0·14	0·15	”	0·028	0·025
4 0	0·21	0·20	”	0·032	0·028
4 30	0·11	0·13			
”	0·14	0·14			
4 31	0·14	0·15			
”	0·15	0·14			
4 32	0·16	0·14			

Figs. 10 & 11, Plate XXVIII. exhibit the daily curve of chemical intensity thus determined; the close agreement of the two curves for each day shows that the errors of graduation, exposure, and reading do not materially affect the accuracy of the measurements; whilst the values of the Daily Mean chemical intensities obtained from each curve, viz. 42·0 and 41·7 for fig. 11, July 15, 1864; and 74·3 and 70·0 for fig. 10, July 11, 1864, confirm this conclusion.

V. *Application of the Method to actual Registration.*

A series of determinations of the varying intensity of the chemical action of total daylight, made at Manchester on more than forty days, at the most widely differing seasons of the year, extending from August 1863 to September 1864, serves to show, in the first place, that the daily determination of the varying chemical intensity can without difficulty be carried on; whilst, secondly, they reveal a few of the many interesting results to which an extended series of such measurements must lead. The whole of the observations, with a few exceptions, were carried on in Manchester, upon the roof of the laboratory of Owens College. As a rule, one observation was made every half-hour; frequently, however, when the object was either to control the measurements, or to record the great changes which suddenly occur when the sun is obscured or appears from behind a cloud, the determinations were made at intervals of a few minutes or even seconds. Sometimes, when the sky was overclouded, or when no great changes in the light occurred, the observations were made once every hour. On most of the days employed for observation, the temperature, atmospheric moisture, barometric pressure, varying amount of cloud, and the condition of the sun's disk were noted.

The curves given on Plate XXIX. serve to exhibit these same results graphically, the abscissæ representing the hours of the day (solar time), and the ordinates giving corresponding chemical intensity expressed in terms of the unit above described.

Consecutive observations were carried on each day for nearly a month, from June 16 to July 9, 1864; the labour thus incurred was found to be comparatively light, so that, when all the preliminary arrangements are made, the daily measurements take up but a small portion of the attention and time of one observer. From the results of these measurements the great difference becomes perceptible which often exists between the chemical intensity of neighbouring days; examples of this variation are seen on PlateXXIX. figs. 12 & 13, for June 27th and 28th, and on figs. 14 & 15, for June 29th and 30th. The tabular results show that the amount of chemical action generally corresponds to the degree of cloud or sunshine, as noted in the observation. Irregular changes in the chemical action are, however, observed on some days (as on March 19, 1864, fig. 16), on which the sun shone continuously, and these are to be mainly attributed to the variation in the amount of cloud passing at the time of observation. In several cases, when no apparent change in the amount of light as affecting the eye could be noticed, a considerable and sudden alteration in the chemical intensity occurred. This was clearly seen on September 26, 1864, when the whole sky was apparently unclouded throughout the day; at $9^{h}\,25'$ A.M. the chemical intensity was found to be 0·13; at 10^{h}, without any visible change in the light, the chemical action sank to 0·07, and continued at this point for more than half an hour, rising again to 0·11 at 11 o'clock. That this diminution of the chemical activity arises from the presence of mist, or of suspended particles of water imperceptible to the eye, is rendered probable by the very powerful absorptive action which a light haze or mist exerts upon the chemical rays. Thus on March 18, 1864, the action at 8^{h} A.M., when a light mist obscured the sun, amounted to 0·0026, whereas the normal action for that day and hour, with an unclouded sky, is twenty-five times as large. It is scarcely necessary to remark that on this occasion the ratio of decrease of visible luminosity was not nearly so great. The same absorptive action of mist is well seen in the following measurements on September 27 and 28, 1864.

September 27, clear sun.			September 28, sun obscured by haze.		
Time.	Intensity.	Weather.	Time.	Intensity.	Weather.
h m			h m		
10 0 A.M.	0·13	Clear sky and direct sun.	10 0 A.M.	0·016	Hazy.
10 30	0·17	„	10 30	0·039	„
11 0	0·18	„	11 0	0·053	„
11 30	0·13	„	11 30	0·075	„
12 40 P.M.	0·16	„	12 0	0·042	Sunshine, haze gradually disappearing.
1 10	0·13	„	12 45 P.M.	0·056	„
1 40	0·17	„	1 0	0·053	„
2 10	0·14	„	1 30	0·10	Haze gone.
			2 15	0·12	„

For the purpose of expressing the relation of the sums of all these various hourly intensities, giving the *daily mean chemical intensity* of the place, a rough, but sufficiently accurate method of integration may be resorted to. This consists simply in cutting the curves out in strong homogeneous paper or cardboard, and in determining in each case the weights of the paper enclosed between the base-line and the curve. A portion of the paper of given size is cut out between every four or five curves, and the small variations in weight caused by irregularity in the thickness of the paper thus allowed for.

In the following Table the numbers are compared with the action, taken as 1000, which would be produced by light of the intensity 1 acting uniformly throughout the twenty-four hours.

Daily Mean Chemical Intensities at Manchester, 1863–64.

(Intensity 1·0 acting for 24 hours = 1000.)

Date.	Intensity.	Date.	Intensity.	Date.	Intensity.
1863.		1864.		1864.	
August 26	40·5	March 19	36·8	June 28	26·6
27	29·8	April 19	78·6	29	26·7
Sept. 4	41·8	20	85·3	30	64·4
16	30·8	June 16	100·7	July 1	61·5
23	12·4	17	47·2	2	19·1
24	18·7	18	118·7	4	51·2
25	18·1	20	50·9	5	76·2
28	29·1	21	99·0	6	78·9
Dec. 21	3·3	22	119·0	7	39·1
22	4·7	23	81·4	8	72·2
		25	83·0	9	83·6
		27	83·0	Sept. 26	48·8

The remarkable differences observed in the chemical intensity on two neighbouring days is shown on fig. 17, in which the curves for the 20th and 22nd June 1864 are represented. The integrals for these days are 50·9 and 119; or the total chemical action on the 20th and 22nd June is in the ratio of 1 to 2·34.

The chemical action of daylight at Manchester at the winter and summer solstice, and the vernal and autumnal equinoxes, is clearly seen by reference to the curves on fig. 18, in which the actions on September 28, 1863, December 22, 1863, March 19, 1864, and June 22, 1864, are represented graphically. These days were chosen out from amongst the observations made near the required periods, as being days upon which the sun shone most brightly, and as therefore giving the nearest approach to the maximum actions for the several periods in question. The integral for the winter solstice is 4·7, that of the vernal equinox 36·8, that of the summer solstice is 119, and that of the autumnal equinox 29·1. Hence if the total chemical action on the shortest day be taken as the unit, that upon the equinox will be represented by 7, and that upon the longest day by 25. From these numbers, as well as from the curves (fig. 18), it is seen that the increase of chemical action from December to March is not nearly so great as that from March to June. With the small amount of experimental data which we as yet possess upon this subject, it is useless to attempt to give an explanation of the probable cause of this difference; suffice it to say that it does not appear to be mainly produced by the absorptive action exerted by the direct sunlight in passing through the different lengths of the columns of air which the rays have to traverse on the days in question.

In carrying out a regular series of meteorological observations upon the variation of mean daily chemical intensity at any spot, a fair average result may be obtained by a much smaller number of observations than is necessary when the object is to indicate the rapid changes occurring in the intensity. Thus, for instance, if determinations had been made on the following days once every two hours, viz. at 8^{h}, 10^{h} A.M., 12^{h}, 2^{h}, 4^{h}, and 6^h P.M., instead of about every fifteen minutes, the numbers for mean chemical intensity would have been—

Date.	Mean Chemical Intensity.	
	From 26 observations.	From 6 observations.
1863, August 26 ……	40·5	43·0
„ Sept. 4 ……	41·8	42·7
1864, April 20 ……	85·3	96·3

As examples of simultaneous determinations made in different localities, I give the results of observations made by myself in Heidelberg, lat. 49° 24′ N., on July 4, 1864, and near Dingwall in Rossshire, lat. 57° 35′ N, on September 27, 1864, compared with the results of observations made in Manchester, 53° 20′ N. latitude, by my assistant. The curves for Heidelberg and Manchester are given in fig. 19, those of Dingwall and Manchester on fig. 20. The integral giving the mean action at Heidelberg on July 4 is 160, that at Manchester on the same day being 51·2; so that the chemical action at Manchester and Heidelberg was on July 4 in the ratio of 1 to 3·12. The integral for Dingwall on September 27 is 66·4, whilst that of Manchester is 49·5; or the ratio of chemical action at Manchester and Dingwall on the day in question was 1 to 1·34. From these observations it would appear that the chemical action at Manchester is smaller than accords with the latitude of the place. This is easily accounted for by the absorptive action exerted by the atmosphere of coal smoke in which the whole of South Lancashire is constantly immersed. Indeed, from the frequent occurrence in Manchester of dull or rainy days, and of fogs or mists, it would be difficult to choose a spot more unsuited to the prosecution of experiments on the chemical action of light.

From the integrals of daily intensity giving the mean chemical action for each day, the mean monthly or yearly chemical intensity of the place of observation can, in like manner, be ascertained; so that, should this method of measurement prove capable of general adoption, we may look forward to obtaining in this way a knowledge of the distribution of the chemically active rays over the surface of our planet analogous to that which we already possess respecting the heating rays.

TABLES giving the Results of the Measurement of Daily Chemical Intensity in 1863–64, at Manchester, Heidelberg, and Dingwall.

Daily Chemical Intensity, Manchester, 1863.

August 26, 1863.
Barom. = 746 millims.

Solar time.	Chemical intensity of light.	Sun's disk.
h m		
7 3 A.M.	0·060	Unclouded.
7 33	0·038	Cloudy.
7 45	0·092	Unclouded.
8 15	0·077	”
8 45	0·070	Unclouded, hazy.
9 15	0·086	Unclouded, haze.
9 45	0·97	”
10 30	0·133	”
10 50	0·187	”
11 10	0·148	”
11 13	0·191	”
11 30	0·229	”
11 50	0·203	Light clouds.
12 0	0·160	”
12 20 P.M.	0·210	Unclouded.
12 40	0·075	Cloudy.
1 0	0·062	”
1 22	0·062	”
1 40	0·094	Light clouds.
2 20	0·069	Clouds.
3 0	0·021	”
3 30	0·016	Clouded over.
4 0	0·016	”
4 30	0·018	”
5 0	0·009	”
5 30	0·004	”
6 0	0·010	”

September 4, 1863.
Barom. = 756 millims.

Solar time.	Chemical intensity of light.	Sun's disk.
h m		
7 45 A.M.	0·062	Unclouded.
8 15	0·075	”
8 45	0·083	Ditto, hazy.
9 20	0·098	Unclouded.
9 40	0·097	”
10 0	0·166	”
10 30	0·115	”
10 45	0·173	”
11 0	0·165	”
11 30	0·135	Cloud.
11 42	0·079	”
11 50	0·128	Unclouded.
11 57	0·137	”
12 10 P.M.	0·072	Clouded.
12 26	0·159	Unclouded.
12 29	0·143	”
12 45	0·165	”
1 20	0·099	Light clouds.
1 21	0·105	”
2 25	0·149	Unclouded.
2 45	0·038	Cloudy.
3 0	0·024	”
3 30	0·035	”
4 0	0·040	Cloudy, rain.
5 0	0·035	Clouds.
5 30	0·016	”

August 27, 1863.
Barom. = 745 millims.

Solar time.	Chemical intensity of light.	Sun's disk.
8 5 A.M.	0·026	Cloudy.
8 33	0·068	Clouds.
9 0	0·041	”
9 45	0·039	”
10 30	0·098	Light clouds.
11 0	0·146	”
11 4	0·132	Unclouded.
11 30	0·115	Light clouds.
12 0	0·059	Cloudy.
12 30 P.M.	0·122	Unclouded.
1 0	0·057	Clouds.
1 30	0·078	Clouded over.
2 0	0·159	Sunshine.
2 20	0·155	”
3 0	0·027	Clouded over.
3 20	0·051	Light clouds.
3 50	0·066	Unclouded.
4 10	0·004	Overclouded.
4 30	0·002	Thunder-storm.

September 16, 1863.
Barom. = 767 millims.

Solar time.	Chemical intensity of light.	Sun's disk.
9 0 A.M.	0·059	Cloudy.
9 35	0·120	Light clouds.
10 15	0·078	Overclouded.
10 45	0·077	”
11 15	0·041	”
11 45	0·104	”
12 0	0·103	”
12 35 P.M.	0·080	”
1 0	0·086	”
2 0	0·091	”
2 40	0·093	”
3 20	0·037	”
4 0	0·027	Rain.
4 45	0·034	Clouds.
6 0	0·007	”

Daily Chemical Intensity, Manchester, 1863 (continued).

September 23, 1863.

Barom. = 738 millims.

Solar time.	Chemical intensity of light.	Sun's disk.
h m		
9 0 A.M.	0·026	Overclouded.
9 30	0·054	Light clouds.
10 0	0·063	Overclouded.
10 30	0·042	„
11 0	0·065	Light clouds.
11 30	0·077	Sun, clouds.
12 0	0·013	Overclouded.
12 20 P.M.	0·031	„
12 45	0·041	„
1 0	0·056	„
1 50	0·062	„
2 10	0·038	„
2 30	……	Rain.
……	……	Heavy rain.
4 0	0·01	

September 24, 1863.

Barom. = 744 millims.

Solar time.	Chemical intensity of light.	Sun's disk.
9 0 A.M.	0·068	Light clouds.
9 30	0·069	„
10 10	0·105	Sunshine.
10 40	0·016	Overclouded.
11 20	0·038	Light clouds.
11 40	0·015	Overclouded.
12 0	0·033	Light clouds.
12 30 P.M.	0·046	„
12 45	0·087	Unclouded.
12 46	0·099	„
1 0	0·110	„
1 55	0·088	Light clouds.
2 10	0·068	Unclouded.
2 40	0·042	Overclouded.
3 0	0·021	„
3 30	0·014	„
……	……	Rain.
5 0	0·014	Overclouded.

September 25, 1863.

Barom. = 753 millims.

Solar time.	Chemical intensity of light.	Sun's disk.
9 0 A.M.	0·042	Overclouded.
9 40	0·077	Unclouded, hazy.
10 20	0·035	Light clouds.
11 0	0·042	„
11 30	0·037	„
1 50 P.M.	0·031	Overclouded.
2 20	0·055	Light clouds.
2 21	0·075	Unclouded.
2 22	0·081	„

September 25, 1863 (continued).

Barom. = 753 millims.

Solar time.	Chemical intensity of light.	Sun's disk.
h m		
2 50 P.M.	0·065	Unclouded.
3 15	0·050	Light clouds.
3 20	0·064	Unclouded.
3 50	0·063	„
3 50	0·063	„
5 0	0·012	Overclouded.

September 28, 1863.

Barom. = 755 millims.

Solar time.	Chemical intensity of light.	Sun's disk.
9 20 A.M.	0·045	Light clouds.
10 20	0·108	Unclouded.
10 21	0·108	„
10 55	0·101	„
10 56	0·106	„
11 20	0·125	„
11 48	0·133	„
12 20 P.M.	0·047	Overclouded.
1 0	0·052	„
1 40	0·055	Light clouds.
2 30	0·099	Unclouded.
2 31	0·094	„
3 0	0·080	„
3 1	0·079	„
3 40	0·072	„
3 50	0·059	„
4 0	0·044	„
4 10	0·043	„
4 30	0·037	Light clouds.
5 0	0·019	„
5 30	0·004	„

December 21, 1863.

Barom. = 760 millims.

Solar time.	Chemical intensity of light.	Sun's disk.
11 0 A.M.	0·013	Clouds.
11 10	0·011	„
11 20	0·012	Hazy.
11 30	0·014	„
11 43	0·019	Unclouded.
12 0	0·003	Rain.
12 15 P.M.	0·018	Clouds.
12 30	0·010	Overclouded.
1 0	0·017	Light clouds.
1 30	0·013	Overclouded.
2 0	0·066	„
2 30	0·0066	„
3 0	0·0084	„
3 30	0·0017	„

Daily Chemical Intensity, Manchester, 1863–64.

December 22, 1863.

Barom. = 761 millims.

Solar time.	Chemical intensity of light.	Sun's disk.
h m		
9 10 A.M.	0·0077	Hazy.
9 40	0·0057	Cloudy.
10 20	0·011	"
11 20	0·020	"
11 40	0·025	"
11 50	0·026	Unclouded.
11 55	0·028	"
12 0	0·023	Light clouds.
12 30 P.M.	0·020	Hazy.
12 35	0·032	Unclouded.
1 0	0·029	Hazy.
1 30	0·017	Unclouded.
2 0	0·017	"
2 30	0·0066	"

March 19, 1864.

Barom. = 753 millims.

Solar time.	Chemical intensity of light.	Sun's disk.
8 0 A.M.	0·0026	Misty.
9 0	0·070	Unclouded.
9 40	0·120	"
10 25	0·080	"
10 45	0·13	"
11 0	0·13	"
11 15	0·080	"
11 35	0·10	"
11 45	0·11	"
11 55	0·10	"
12 0	0·12	"
12 5 P.M.	0·12	"
12 10	0·12	"
12 33	0·14	"
1 0	0·12	"
1 35	0·045	"
2 20	0·11	"
3 30	0·069	"
4 40	0·039	Light clouds.
6 0	0·007	"

April 19, 1864.

Barom. = 758 millims.

Solar time.	Chemical intensity of light.	Sun's disk.
7 50 A.M.	0·10	Unclouded.
9 25	0·22	"

April 19, 1864 (continued).

Barom. = 758 millims.

Solar time.	Chemical intensity of light.	Sun's disk.
h m		
10 0 A.M.	0·29	Unclouded.
10 46	0·20	"
11 0	0·33	"
12 0	0·25	"
1 0 P.M.	0·26	"
2 14	0·15	"
2 45	0·20	"
3 15	0·13	"
3 45	0·11	"
4 20	0·10	"
4 50	0·081	"

April 20, 1864.

Barom. = 759 millims.

Solar time.	Chemical intensity of light.	Sun's disk.
6 50 A.M.	0·067	Hazy.
7 45	0·17	Unclouded.
8 15	0·22	Hazy.
8 45	0·22	"
9 20	0·35	"
10 0	0·26	Unclouded.
10 50	0·30	"
11 15	0·16	"
11 30	0·17	"
11 40	0·19	"
11 50	0·17	"
12 0	0·16	"
12 30 P.M.	0·16	"
12 45	0·14	"
1 1	0·18	"
1 30	0·14	"
2 5	0·23	"
2 46	0·12	Cloudy.
3 13	0·11	"
3 30	0·10	"
4 15	0·091	"
5 5	0·094	"
5 30	0·060	"
6 5	0·041	"
6 50	0·014	"
7 30	0·0037	"

Daily Chemical Intensity, Manchester 1864.

June 16th, 1864. Barom.=758·3 millims. Mean Temp. Dry bulb 17°·9. ,, Wet bulb 12°·9C.

Solar time.	Chemical intensity of light.	Amount of cloud.	Sun's disk.
h m			
6 25 A.M.	0·039	......	Clouded over.
7 0	0·019	......	,,
7 30	0·10	......	Clouds breaking.
8 0	0·13	......	,,
8 30	0·15	......	Light clouds.
9 0	0·13	......	Clouded over.
9 30	0·24	......	Unclouded.
10 0	0·38	......	,,
10 30	0·29	......	,,
11 0	0·38	......	,,
11 30	0·35	......	,,
12 0	0·22	......	Light cloud.
12 30 P.M.	0·37	......	Unclouded.
1 0	0·31	......	,,
1 30	0·26	......	,,
2 0	0·24 0·23	......	,,
2 30	0·17	......	Clouds.
3 0	0·13	......	Unclouded.
3 30	0·15	......	Light clouds.
4 0	0·10	......	,,
4 30	0·052	......	Clouded over.
5 0	0·045	......	,,
5 30	0·087	......	Light clouds.
7 15	0·030	......	,,
8 15	0·010	......	,,
8 40	0·0027	......	,,

June 17th. 1864. Barom.=760·9 millims. Mean Temp. Dry bulb 20°·5. ,, Wet bulb 17°·1 C.

Solar time.	Chemical intensity of light.	Amount of cloud.	Sun's disk.
6 40 A.M.	0·053	......	Clouded over.
7 10	0·086	......	,,
7 50	0·18	......	,,
8 30	0·11	......	Light clouds.
9 0	0·11	......	,,
9 30	0·28	......	,,
9 55	0·13	......	Clouded over.
10 25	0·045	......	,,
11 10	0·15	......	,,
11 40	0·12	......	,,
12 10 P.M.	0·14	......	,,
12 30	0·14	......	,,
1 0	0·35	......	,,
1 35	0·18	......	,,
2 0	0·12	......	,,
2 40	0·059	......	,,
3 10	0·062	......	,,
3 40	0·027	......	,,
4 20	......	......	Rain.

June 18th, 1864. Barom.=761 millims.

Solar time.	Chemical intensity of light.	Amount of cloud.	Sun's disk.
h m			
7 50 A.M.	0·19	......	Unclouded.
8 40	0·30	......	,,
9 10	0·19	......	Clouds.
9 55	0·19	......	,,
10 45	0·13	......	,,
11 30	0·19	......	,,
12 35 P.M.	0·33	......	Light clouds.
1 30	0·38	......	,,
3 0	0·21	......	,,
4 0	0·22	......	Unclouded.
6 30	0·033	......	Clouds.
8 0	0·0079	......	,,

June 20th, 1864. Barom.=763·8 millims. Mean Temp. Dry bulb 19°·5. ,, Wet bulb 15°·9 C.

Solar time.	Chemical intensity of light.	Amount of cloud.	Sun's disk.
8 0 A.M.	0·14	......	Light clouds.
8 45	0·14	......	Clouds.
9 15	0·099	......	,,
9 55	0·094	......	,,
10 30	0·16	......	,,
11 0	0·12	......	,,
11 30	0·15	......	,,
12 0	0·13	......	,,
12 15 P.M.	0·13	......	,,
12 45	0·16	......	,,
1 0	0·15	......	,,
1 30	0·11	......	,,
2 10	0·074	......	,,
2 45	0·075	......	,,
3 15	0·044	......	,,
3 50	0·053	......	Clouded over.
4 30	0·031		
5 30	0·030	......	Rain.
7 0	0·010	......	,,

June 21st, 1864. Mean Temp. Dry bulb 16°·1. ,, Wet bulb 11°·1 C.

Solar time.	Chemical intensity of light.	Amount of cloud.	Sun's disk.
6 40 A.M.	0·12		
7 15	0·13	......	Light clouds.
7 45	0·074	......	,,
8 30	0·080	......	,,
9 30	0·21	......	Unclouded.
10 0	0·27	......	Light clouds.
10 30	0·27		
11 10	0·33	5	
11 30	0·29	......	Sun shining.
12 0	0·072	8	Clouds.
12 30 P.M.	0·22	8	Unclouded.

Daily Chemical Intensity, Manchester, 1864 (continued).

June 21st, 1864 (continued). Mean Temp. Dry bulb 16°·1.
„ Wet bulb 11°·1.

Solar time.	Chemical intensity of light.	Amount of cloud.	Sun's disk.
h m			
1 0 P.M.	0·29	6	Unclouded.
1 35	0·28	6	Clouds.
2 45	0·21	4	Unclouded.
3 15	0·24	3	Hazy sunshine.
4 15	0·13	......	Unclouded.
5 30	0·038	......	Clouds.
6 10	0·031	......	„
7 40	0·012	......	„

June 22nd, 1864. Mean Temp. Dry bulb 17°·6.
Barom.=761 millims. „ Wet bulb 13°·5 C.

8 0 A.M.	0·15	......	Clouded over.
......	......	......	Rain.
8 45	0·017	10	Clouded over.
9 15	0·22	6	Clouds.
10 0	0·22	9	„
10 30	0·21	8	„
11 0	0·19	8	„
11 30	0·45	6	Unclouded.
12 15 P.M.	0·49	5	„
1 30	0·28	3	„
1 50	0·27	......	„
2 0	0·26	2	„
2 30	0·38	......	„
3 0	0·17	5	Light clouds.
3 30	0·17	2	„
4 0	0·16	3	Unclouded.
5 0	0·15	1	„
6 0	0·068	......	Clouds.

June 23rd, 1864. Mean Temp. Dry bulb 15°·1.
Barom.=757·6 millims. „ Wet bulb 11°·6 C.

7 0 A.M.	0·090	10	Heavy rain.
9 20	0·18	10	Clouded over.
10 10	0·18	9	„
11 30	0·18	9	Rain.
12 0	0·21	10	Clouds.
1 0 P.M.	0·22	7	Rain.
3 0	0·16		
3 40	0·17	6	Unclouded.
4 35	0·12	8	Clouded.
5 0	0·093	9	„

June 25th, 1864. Mean Temp. Dry bulb 16°·7.
Barom.=761·2 millims. „ Wet bulb 13°·4.

7 45 A.M.	0·055	10	Clouded over.
8 30	0·14	10	„
10 10	0·27	10	„
11 0	0·18	10	„
12 0	0·27	10	„
12 30 P.M.	0·22	......	„

June 25th, 1864 (continued). Mean Temp. Dry bulb 16°·7.
Barom.=761·2 millims. „ Wet bulb 13·4.

Solar time.	Chemical intensity of light.	Amount of cloud.	Sun's disk.
h m			
1 0 P.M.	0·16	......	Clouds.
1 45	0·33	......	Clouded over.
2 30	0·23	8	Unclouded.
3 10	0·13	10	Clouded over.
5 15	0·10	......	„
6 30	0·037	......	„

June 27th. 1864. Mean Temp. Dry bulb 16°·4.
Barom.=765·2 millims. „ Wet bulb 12°·0 C.

7 45 A.M.	0·15	4	Light clouds.
8 30	0·22	4	Unclouded.
9 10	0·11	8	Clouds.
9 30	0·25	7	Unclouded.
10 0	0·12	6	Clouds.
10 40	0·34	4	Unclouded.
11 30	0·11	9	Clouded over.
12 0	0·21	7	Unclouded.
1 15 P.M.	0·050	9	Clouded over.
4 30	0·17	1	Unclouded.
5 7	0·15	1	„
5 30	0·092	1	„
6 0	0·020	......	„

June 28th. 1864. Mean Temp Dry bulb 15°·0.
Barom.=763·2 millims. „ Wet bulb 13°·4 C.

7 30 A.M.	0·031	10	Clouded over.
8 40	0·043	10	„
9 30	0·15	10	„
10 20	0·060	10	„
11 0	0·037	10	Rain.
11 30	0·034	10	„
12 30 P.M.	......	10	„
2 30	0·095	10	„

June 29th, 1864. Mean Temp. Dry bulb 13°·0.
Barom.=759·2 millims. „ Wet bulb 11°·4 C.

7 40 A.M.	0·11	10	Clouds.
8 30	0·13	10	„
9 40	0·042	10	„
10 20	0·044	10	„
11 20	0·047	......	„
11 35	0·026	......	„
12 0	0·022	......	„
12 30 P.M.	0·040	......	„
1 15	0·018	......	„
2 20	0·013	......	„
3 0	......	......	Rain.
4 0	0·028	......	Clouds.
5 0	0·014	......	„

Daily Chemical Intensity, Manchester, 1864 (continued).

June 30th, 1864. Mean Temp. Dry bulb 12°·6.
Barom. = 758 millims. " Wet bulb 12°·1.

Solar time.	Chemical intensity of light.	Amount of cloud.	Sun's disk.
h m			
7 15 A.M.	0·021	10	Clouded over.
8 15	0·10	10	"
9 10	0·21	8	Sunshine cloud.
10 0	0·060	9	Clouds.
11 0	0·37	5	Sunshine cloud.
11 30	0·12	6	Clouds.
12 0	0·46	......	Unclouded.
12 30 P.M.	0·077	......	Clouded over.
1 45	0·090	......	"
3 0	0·061	6	"
4 0	0·075	......	"
4 30	......	......	Rain.
5 20	0·054	6	Light clouds.
6 10	0·010	6	Sun shining.

July 1st, 1864. Mean Temp. Dry bulb 14°·6.
Barom. = 758·2 millims. " Wet bulb 11°·1.

Solar time.	Chemical intensity of light.	Amount of cloud.	Sun's disk.
8 15 A.M.	0·067	9	Clouded over.
9 5	0·11	4	Light clouds.
9 40	0·12	9	"
10 0	......	8	Rain.
10 30	0·17	......	Light clouds.
11 0	0·19	7	Clouds.
11 45	0·086	......	Clouded over.
12 30 P.M.	0·040	......	"
1 0	0·20	......	Sunshine.
2 15	0·25	5	Unclouded.
3 45	0·085	......	Clouded.
4 30	0·063	......	"
5 30	0·050	......	"

July 2nd, 1864.
Barom. = 752 millims.

Solar time.	Chemical intensity of light.	Amount of cloud.	Sun's disk.
8 10 A.M.	0·042	10	Rain.
10 0	......	10	Rain.
12 0	0·028	......	"
3 45 P.M.	0·071	......	Fair, clouded.
4 20	0·046	......	"
4 50	0·043	......	Rain.

July 4th, 1864. Mean Temp. Dry bulb 20°·3.
Barom. = 759·5 millims. " Wet bulb 11°·8.

Solar time.	Chemical intensity of light.	Amount of cloud.	Sun's disk.
7 30 A.M.	0·076	8	Clouded.
8 0	0·11	6	"
8 30	0·077	9	"
9 0	0·041	10	Rain.
9 30	0·023	10	Clouded over.
10 10	0·055	9	"
10 30	0·056	9	"
11 0	0·038	9	Rain.
11 30	0·034	10	"

July 4th, 1864 (continued). Mean Temp. Dry bulb 20°·3.
Barom. = 759·5 millims. " Wet bulb 11°·8.

Solar time.	Chemical intensity of light.	Amount of cloud.	Sun's disk.
h m			
12 0	0·065	9	Rain.
12 30 P.M.	0·070	9	"
1 0	0·097	8	"
1 30	0·090	8	"
2 0	0·14	8	"
2 30	0·14	......	Clouds.
3 0	0·34	......	Unclouded.
3 30	0·25	5	"
4 0	0·11	7	Clouded.
4 30	0·095	7	"
5 0	0·074	6	"
5 30	0·072	6	"
6 0	0·056	6	"
6 30	0·067	2	Sunshine.
7 0	0·043	0	Unclouded.
7 30	0·023	0	"

July 5th, 1864. Mean Temp. Dry bulb 14°·0.
Barom. = 761·6 millims. " Wet bulb 10°·7.

Solar time.	Chemical intensity of light.	Amount of cloud.	Sun's disk.
8 10 A.M.	0·12	10	Clouds.
8 30	0·10	10	"
9 0	0·033	10	"
9 30	0·14	10	"
10 0	0·11	10	"
10 30	0·077	10	"
11 0	0·14	10	"
11 30	0·15	10	"
12 0	0·18	10	"
12 30 P.M.	0·12	10	"
1 0	0·10	10	"
1 45	0·32	7	Light clouds.
2 15	0·13	10	Clouded over.
2 45	0·28	6	Unclouded.
3 30	0·25	6	Clouds.
4 0	0·18	6	Light cloud
4 30	0·26	6	"
5 0	0·072	7	"
5 30	0·093	6	"
6 0	0·067	4	Clouds.
7 30	0·035	4	Unclouded.

July 6th, 1864. Mean Temp. Dry bulb 17°·6.
Barom. = 765 3 millims. " Wet bulb 13°·4.

Solar time.	Chemical intensity of light.	Amount of cloud.	Sun's disk.
7 30 A.M.	0·058	1	Hazy.
8 0	0·083	1	"
8 30	0·10	3	"
9 0	0·077	7	Clouds.
9 30	0·20	7	Hazy.
10 15	0·13	4	"
10 45	0·078	10	Clouded over.
11 20	0·071	6	Light clouds.
11 50	0·10	7	"

Daily Chemical Intensity, Manchester, 1864 (continued).

July 6th, 1864 (continued). Mean Temp. Dry bulb 17°·6.
Barom.=765 3 millims. „ Wet bulb 13°·4.

Solar time.	Chemical intensity of light.	Amount of cloud.	Sun's disk.
h m			
12 30 P.M.	0·22	3	Light clouds.
1 0	0·21	6	„
1 30	0·17	9	„
2 0	0·28	7	„
2 30	0·36	7	„
3 0	0·15	......	„
3 30	0·17	4	„
4 0	0·21	......	Unclouded.
4 30	0·24	4	Light clouds.
5 15	0·092	......	„
6 30	0·063	4	„

July 7th, 1864. Mean Temp. Dry bulb 16°·4.
Barom.=764·7 millims. „ Wet bulb 13°·2.

Solar time.	Chemical intensity of light.	Amount of cloud.	Sun's disk.
7 30 A.M.	0·040	10	Clouds above.
8 0	0·058	10	„
8 30	0·10	10	„
9 15	0·079	10	„
9 45	0·073	10	„
10 10	0·069	10	„
10 45	0·056	7	„
11 30	0·020	7	„
12 0	0·055	9	„
12 30 P.M.	0·021	10	„
1 0	0·12	9	„
1 45	0·064	......	„
2 25	0·022	10	„
3 0	0·15	7	Light clouds.
3 30	0·092	......	Clouded over.
4 0	0·070	......	„
4 30	0·11	......	„
5 0	0·10	......	Clouds.
7 20	0·025	......	

July 8th, 1864. Mean Temp. Dry bulb 19°·6.
Barom.=765·1 millims. „ Wet bulb 13°·8.

Solar time.	Chemical intensity of light.	Amount of cloud.	Sun's disk.
7 10 A.M.	0·055	8	Clouded over.
7 50	0·068	7	Clouds.
8 25	0·089	9	„
9 0	0·12	......	„
9 30	0·12	......	„
10 30	0·13	......	„
11 0	0·12	10	Clouded over.
11 30	0·13	......	„

July 8th, 1864 (continued). Mean Temp. Dry bulb 19°·6.
Barom.=765·1 millims. „ Wet bulb 13°·8.

Solar time.	Chemical intensity of light.	Amount of cloud.	Sun's disk.
h m			
12 0	0·11	.....	Clouded over.
12 30 P.M.	0·13	9	Light clouds.
1 10	0·15	7	„
1 40	0·16	......	„
2 15	0·26	4	Unclouded.
3 0	0·29	3	„
3 30	0·26	......	„
4 0	0·22	......	„
4 30	0·15	1	„
5 0	0·12	1	„
6 10	0·011	......	„

July 9th, 1864. Mean Temp. Dry bulb 15°·5.
Barom.=764·1 millims. „ Wet bulb 11°·7.

Solar time.	Chemical intensity of light.	Amount of cloud.	Sun's disk.
8 0 A.M.	0·060	3	Hazy.
9 0	0·15	......	„
10 0	0·14	......	„
11 0	0·18	......	Unclouded.
12 20 P.M.	0·15	......	„
1 30	0·23	......	„
2 30	0·22	......	„
3 30	0·22	......	„
4 30	0·14	......	„
5 30	0·10	......	„

September 26th, 1864.

Solar time.	Chemical intensity of light.	Amount of cloud.	Sun's disk.
8 50 A.M.	0·11	......	Cloudless sky.
9 25	0·13	......	„
10 0	0·070	......	„
10 30	0 071	......	„
11 0	0·11	......	„
11 30	0·12	......	„
12 10	0·10	......	„
12 40 P.M.	0·11	......	„
1 5	0·15	......	„
1 55	0·17	......	„
2 30	0·12	......	„
3 0	0·096	......	„
3 40	0·078	......	„
4 10	0·056	......	„
4 45	0·038	......	„
5 15	0·018	......	„

Daily Chemical Intensity, Heidelberg, Dingwall, and Manchester, 1864.

July 4, 1864.—Heidelberg.

Solar time.	Chemical intensity of light.	Amount of cloud.	Sun's disk.
h m			
6 56 A.M.	0·072	......	Clouded.
7 1	0·170	......	Unclouded.
8 6	0·208	......	Clouds.
8 21	0·206	......	Unclouded.
8 50	0·244	......	”
9 21	0·290	......	”
9 40	0·394	2	”
9 42	0·470	2	”
10 23	0·475	2	”
10 35	0·590	2	”
11 30	0·620	......	”
11 49	0·60	......	”
12 18 P.M.	0·52	......	”
1 5	0·516	......	”
2 21	0·248	......	Clouded.
3 5	0·300	......	Unclouded.
3 50	0·270	......	”
4 30	0·126	......	Overclouded.
4 50	0·163	.....	Unclouded.
5 25	0·124	0	”

September 27, 1864.—Dingwall, N.B.

Solar time.	Chemical intensity of light.	Amount of cloud.	Sun's disk.
9 16 A.M.	0·18	......	Unclouded.
9 26	0·17	......	”
9 36	0·16	......	”
10 0	0·17	......	”
10 5	0·19	......	”
10 10	0·19	......	”
h m			
10 23 A.M.	0·22	......	Unclouded.
10 30	0·18	......	Haze.
10 35	0·16	......	”
10 50	0·13	......	Cloudy.
11 25	0·16	......	Clouds.
11 26	0·15	......	”
12 45 P.M.	0·24	......	Unclouded.
2 37	0·19	......	”
2 45	0·13	......	Clouds.
2 58	0·18	......	Unclouded.
3 57	0·066	......	Clouded.

September 27, 1864.—Manchester.

Solar time.	Chemical intensity of light.	Amount of cloud.	Sun's disk.
8 50 A.M.	0·13	......	Unclouded.
9 30	0·16	......	”
10 0	0·13	......	”
10 40	0·18	......	”
10 50	0·18	......	”
11 30	0·13	......	”
12 0	0·098	......	Cloud.
12 40 P.M.	0·16	......	”
1 10	0·13	......	”
1 40	0·17	......	”
2 10	0·14	......	”
2 55	0·12	......	”
3 40	0·081	......	”
4 20	0·052	......	”

XIII. *On the Commissures of the Cerebral Hemispheres of the Marsupialia and Monotremata as compared with those of the Placental Mammals.* *By* William Henry Flower, *F.R.S.*, *F.R.C.S.*, *Conservator of the Museum of the Royal College of Surgeons of England.*

Received January 24,—Read February 9, 1865.

The terms used in describing the anatomy of the vertebrated animals have in most cases been originally bestowed on parts of the human body, being frequently derived from some quality, relation, or real or fancied resemblance to some known object, possessed by the structure in question in Man. It will therefore be most convenient to pass from the best to the least known, and to commence by a short recapitulation of the characters and relations, in the human brain, of those parts to the consideration of which, in the brains of the lower mammalia, this communication is specially devoted.

Plate XXXVI. fig. 1 is a view of the inner surface of one of the hemispheres of the human cerebrum, such parts as pass across the middle line to the other hemisphere having been divided, and those that do not belong to the hemisphere proper being removed. A convenient central point to start from in the description is the part cut through to make the section last referred to. A is the surface of the divided mass of fibres, by which the hemisphere is connected with the inferior parts of the encephalon, and with the spinal cord—the crus or peduncle of the brain. The section is made between the corpus striatum and the thalamus opticus. The thalamus, as not belonging to the hemisphere proper, and interfering with the view of essential parts, is removed.

The upper and posterior portions of the circumference of the part just described are surrounded by the narrow slit-like opening, the "ventricular aperture" (O O), which leads into the great cavity in the interior of the hemisphere, the lateral ventricle. The inferior margin of the ventricular aperture is formed by the "tænia semicircularis," the superior by a stronger and better marked band of white fibres, forming the free edge of the inner wall of the hemisphere, the "fornix." Leaving this for the present, we must next notice as an important landmark, the great or superior commissure, or "corpus callosum" (B), here seen in its entire length, slightly arched, thick and rounded behind (the "splenium" E), and curving downwards in front (the "genu" C), ending in a pointed "rostrum" or beak (D) directed backwards. The hinder edge is also curved upon itself, ending in a rounded edge (N) projecting downwards and forwards and folded under the main body of the corpus callosum, much in the same way, though in a less degree, as the rostrum in front. This part requires special attention in connexion with the present subject, as in the lower mammals it acquires a much greater relative importance.

Its fibres principally connect, across the middle line, the parts of the cerebral hemispheres forming the inner wall of the middle horn of the ventricle, especially the folded part constituting the hippocampus major. As its free edge forms the hinder boundary of the region called the "psalterium" in human anatomy, the fibres composing it may be distinguished as the "psalterial fibres" of the corpus callosum. At a little distance behind and rather lower than the point of the rostrum of the corpus callosum, is the very distinct oval outline of the section of the white "anterior commissure" (F), and between this and the under surface of the corpus callosum, and prolonged into the concavity of the genu, is a portion of the inner wall of the hemisphere (G) closing the lateral ventricle towards the middle line, and with the corresponding portion of the opposite side forming the median septum which divides the two cavities from each other, as will be better seen in the transverse section. This important region Professor HUXLEY has distinguished as the "septal area"*.

To return to the upper arched border of the ventricular aperture. The middle part, which when united to the corresponding portion of the other hemisphere constitutes the "body of the fornix" (K), is composed of a considerable number of white fibres closely adherent posteriorly to the under surface of the body of the corpus callosum, and running in a longitudinal direction. Tracing these fibres forwards, a small round white cord (L) is seen to pass down from them behind the anterior commissure, constituting the part commonly spoken of as the "anterior pillar" of the fornix, but which, to avoid confusion, had better be designated as the "column" of the fornix (*Columna fornicis*, REICHERT). The further course of this into the corpus albicans and optic thalamus need not be detailed here. But a large portion of the fibres (I) running forwards from the body of the fornix do not pass down into these cords, being continued *above* the anterior commissure, and then curve downwards in front of that structure to join the inner wall of the anterior lobe of the hemisphere. For these fibres the name of "precommissural fibres" has been suggested by Professor HUXLEY. The presence of the precommissural fibres, as well as that of much grey matter, gives to the lower part of the septal area a much greater thickness than the upper part (to which the name of "septum lucidum" is applied) possesses. But the two divisions of the area are perfectly continuous in structure, the upper thin part also containing fibres prolonged from the fornix, radiating forwards and upwards to the under surface of the corpus callosum†.

Posteriorly the fibres of the fornix, following the border of the aperture they encircle, change their longitudinal direction, and gradually turn outwards, downwards, and finally forwards, and even slightly inwards. Although in their anterior and middle portions the fibres of the fornix run at right angles with the fibres of the corpus callosum, this change of direction in their posterior part brings them parallel to, and allows them to blend with, the transverse fibres of that body. The prominent sharp free margin of the ventricular aperture formed by the "posterior pillars" of the fornix is called "corpus fimbriatum"

---

* Lectures at the Royal College of Surgeons, Medical Times and Gazette, March 5th, 1864.

† See SOLLY 'On the Human Brain,' 2nd Edit. 1847, p. 261.

(M). A little way external and parallel to this, on the surface of the hemisphere, is a deep sulcus, corresponding in direction and extent with the hinder third of the ventricular aperture. This is the "dentate" or "hippocampal" sulcus (Q). It terminates above under the posterior end of the corpus callosum. If the cortical grey matter of the hemisphere is traced from the external border of the hemisphere towards the ventricular aperture, it will be found to dip down into this sulcus, and rising again to the surface to terminate abruptly just external to the corpus fimbriatum. The free border in which it terminates, lying between the "hippocampal sulcus" and the "corpus fimbriatum," is called the "fascia dentata" (P), its surface being generally somewhat notched or indented at intervals. The cerebral wall folded inwards at the sulcus just described, forms a corresponding projection in the cavity of the ventricle called the "hippocampus major."

The relation of some of the parts above mentioned will be better understood by a reference to Plate XXXVI. fig. 2. It is drawn from a vertical transverse section of the human brain, at the point indicated by the line drawn across Plate XXXVI. fig. 1, viz., through the middle of the anterior commissure. B is the corpus callosum, passing from hemisphere to hemisphere, across the bottom of the great longitudinal fissure*. As its fibres pass outwards from the middle line, they curve slightly upwards before separating to radiate throughout the medullary substance of the hemispheres. Immediately underneath the corpus callosum lie the cavities of the hemispheres or "lateral ventricles," completely separated from each other in this section by a septum (G), attached above to the under surface of the corpus callosum, and below resting on the small transverse "anterior commissure" (F). This part, the "septal area" of the former section, may be demonstrated to consist throughout of two lateral portions, applied closely together in the middle line below, but in the upper part slightly separated, the interval constituting the fifth ventricle, or ventricle of the septum lucidum. The lower part of the septum, much thicker than the septum lucidum, contains the precommissural fibres of the fornix with much grey matter interposed. It seems never to have received any special name, or to have been sufficiently distinguished from the septum lucidum, although it is the most constant, and therefore important division of the septal area, as will be shown hereafter. The grey masses (R R) forming the outer boundaries of the ventricles are the "corpora striata." The anterior commissure is seen as a small cylindrical bundle of white fibres (F) passing between the corpora striata.

The true nature of these parts cannot be perfectly understood without a glance at their development. This is a subject confessedly still involved in some obscurity. I follow, however, the observations of F. SCHMIDT, who has given a detailed and apparently truthful account of the process†. Without entering into previous changes, it may be stated that each hemisphere consists, in a very early condition, of a hollow thin-walled body, with a fissure (O) in its inner surface, leading to the cavity within (Plate XXXVI. fig. 3, I).

---

* "— the cross portion of white substance which lies between the hemispheres at the bottom of the longitudinal fissure," QUAIN and SHARPEY's 'Anatomy,' 5th edit. vol. ii. p. 464.

† Zeitschrift fur Wissenschaftliche Zoologie, vol. xi. (1861) p. 43.

Through this a portion of the pia mater (afterwards developed into the choroid plexus) enters. The fissure is at first perpendicular in direction. In front of it (at G) the two hemispheres are united across the middle line, immediately behind it (A) they are connected with the parts formed by the second cerebral vesicle, the subsequent optic thalamus and crus cerebri. The last-named point (the *crus* or "*hirnstiel*") forms a pivot around which the whole hemisphere curves itself as development proceeds. The fissure undergoes a corresponding change of form and direction. The anterior edge becomes its upper convex border. The upper end gradually becomes depressed until it is finally the lowest part, and the characteristic form of the ventricular aperture is already recognized at this early age (Plate XXXVI. fig. 3, III). The point of union between the hemispheres is still confined to the part immediately in front of the anterior end of the fissure, the "septal area." About this time the wall of the hemisphere commences to undergo a folding upon itself, producing certain definite grooves or sulci on the outer surface, and corresponding elevations upon the interior. At a very early period an arched sulcus (*bogenfurche*) appears parallel to the upper border of the fissure, marking off an arched convolution or gyrus between it and the fissure, the "marginal arch" (*randbogen*, SCHMIDT). It is the hinder part of this groove which afterwards forms the "hippocampal sulcus." Into the further development of the convolutions and sulci it is unnecessary to enter. A more important subject in connexion with the present communication is the mode of formation of the corpus callosum, the fornix, and adjacent parts. KÖLLIKER* has given so good an abridgement of SCHMIDT's views, that I have thought it best to follow pretty closely his words.

The convolutions of the hemispheres are distinctly seen from the third month to consist of two layers, an external with perpendicular fibres, which at a later period constitutes the grey or cortical substance of the convolutions, and an inner layer with fibres running horizontally. The fibres of the inner layer, constituting the medullary substance of the hemispheres, are found already in the third month, before the corpus callosum exists, to converge towards two points; first, towards the crus (*hirnstiel*, A), where they form the so-called *stabkranz*; and secondly, towards a point situated immediately above the place of union of the two hemispheres. This last arrangement of fibres is the first indication of the radiation of the corpus callosum (*balkenstrahlung*). It is at this spot (B) that in the fourth month the horizontal fibres break through the cortical substance and unite with the corresponding fibres of the opposite hemisphere.

This is the commencement of the corpus callosum, which in its earliest form (see Plate XXXVI. fig. 3, IV) is a very small nearly cylindrical commissure, situated in the "marginal arch" immediately above the most anterior part of the ventricular aperture. In order to indicate more closely the relation of the marginal arch to the corpus callosum, it is to be noticed that the former separates into two parts, a lower division immediately bordering the ventricular aperture, consisting only of horizontal or antero-posterior fibres, without the cortical layer, and an upper division possessing both layers.

---

* Entwicklungsgeschichte des Menschen und der höheren Thiere, p. 237, Leipzig 1861.

Now the corpus callosum breaks through just at the limit between these two divisions, and by its further growth backwards, the upper division comes to lie on its outer surface and is converted into the stria alba Lancisi and stria obtecta of the corpus callosum, and into the fascia dentata of the hippocampus major; whilst the inferior or inner arch, with its longitudinal fibres, forms the fornix and septum (*scheidewand*). The fornix is thus, as was known to ARNOLD and RETZIUS, a transformation of the upper margin of the transverse fissure. The lower margin of the fissure is formed into the tænia semicircularis or stria cornea, which, as is well known, is connected at each end with the extremities of the fornix. It will be seen from the preceding observations that the anterior perpendicular part of the fornix is originally united with the corresponding part of the other side, and the body of the fornix developes itself out of the uppermost part of this spot, adjoining the primitive corpus callosum. Lower down the parts separate and then resolve themselves into the columnæ fornicis, or anterior crura, and the two halves of the septum lucidum, the ventricle of which is thus no primitive formation. In this part also originates, not by growing together from opposite sides, but by histological differentiation, the anterior commissure (F), which is evident a short time before the corpus callosum. The septum lucidum and body of the fornix, in the beginning very small, gradually increase in extent with the development of the corpus callosum.

According to SCHMIDT, the opinion formerly entertained that the genu of the corpus callosum was the part first formed, and that the hinder part developed afterwards, is not correct. The rudimentary corpus callosum on its first appearance already contains the elements of all its subsequent parts, as from the very first, fibres radiate from it into the hinder and middle, as well as the anterior lobes, and the intimate connexion of the former with the posterior crura of the fornix can already be recognized. It increases, with the rest of the hemisphere, chiefly in longitudinal extent, spreading both backwards and forwards from the point of its first appearance, but principally in the former direction. The curved part in front, called the genu, is not formed until the end of the fifth month, and about a month later, the thickening and extension of the hinder end over the corpora quadrigemina gives the permanent form to this part of the brain.

I will next proceed to trace the modifications of the parts of the brain above indicated, in certain of the placental mammalia. The preparations from which the figures are taken were all made in the same manner as that adopted in the case of the human brain, viz., (I.) a vertical longitudinal section in the middle line, exhibiting the inner surface of a single (the right) hemisphere, the thalamus opticus and crus having been removed so as to show clearly the whole surface with the parts forming the upper boundary of the ventricular aperture; (II.) a vertical transverse section through the middle of the anterior commissure.

The Sheep.—In the longitudinal section of the sheep's brain (Plate XXXVII. fig. 1), the elongated narrow corpus callosum (B) is seen lying in a line nearly horizontal, or corresponding with the long axis of the hemisphere; slightly concave in the middle above, with a thickened posterior end (E) turned somewhat downwards, and a distinct genu (C) and rostrum (D) in front. The latter has a smaller proportional development than in the human brain. On the other hand, the slightly projecting posterior fold observed in the human corpus callosum is prolonged forwards as a thin layer of transverse fibres (N) arching across the under surface of the longitudinal fibres of the fornix, and ending in no abrupt edge in front. The difference in the form and extent of this part of the great transverse commissure may be clearly seen to depend upon the difference in the form, and more extensive proportions of the parts that have to be brought into relation to each other by it, viz. those forming the inner wall of the descending cornu of the lateral ventricle. At a considerable distance below the anterior part of the corpus callosum the small anterior commissure (F) is seen, with the wide septal area (G) in front of and above it. The portion of this part to which the term "septum lucidum" can be applied, is reduced to a small strip beneath the anterior third of the corpus callosum, exactly defined below and in front by the extent of the rostrum of that body. The greater part of the septum is formed by a thick layer, consisting of a great development of the precommissural fibres of the fornix, associated with much grey matter. The small white column (L) of the fornix is seen passing down behind the anterior commissure. The ventricular aperture is less regularly curved than in man, being bent almost at a right angle. Above and behind it is seen a broad corpus fimbriatum (M), behind which the abrupt termination of the cortical substance of the hemisphere in the fascia dentata (P) is very distinctly seen. The regularly curved hippocampal sulcus (Q) ends beneath the hinder end of the corpus callosum, the grey matter of the fascia dentata being continued superficially round its extremity into that of the next succeeding gyrus.

In the transverse section (Plate XXXVII. fig. 2), at the bottom of the deep longitudinal fissure, is seen the corpus callosum (B), a transverse white band of moderate thickness, and slightly arched upwards externally, where its fibres radiate out in the medullary substance of the hemisphere. The anterior commissure (F) is readily recognized near the lower part of the section. The cavities of the lateral ventricles are somewhat triangular in form and bounded above by the under surface of the corpus callosum, towards the middle line by the septum, and externally by the corpora striata. The septum obviously consists of two halves, one belonging to each hemisphere, but more or less joined together in the middle line. The upper part (septum lucidum) is extremely thin, and here the absence of union between the two halves allows the existence of a minute cavity, the fifth ventricle. The lower and larger part is very thick, with rounded outer surface. It contains much grey matter, with white longitudinal fibres externally. Within it, near the middle line, on each side, can be seen two bundles of white fibres, standing nearly perpendicularly and slightly diverging from each other below; they are the upper part of the columns of the fornix.

The most essential deviations in the commissures of this brain from those of Man consist in the reduction of the rostrum of the corpus callosum and the septum lucidum, and the augmentation of the inferior thick part of the septal area and of the psalterial fibres.

The Rabbit.—Plate XXXVII. fig. 3 represents the inner surface of the cerebral hemisphere of a rabbit. The corpus callosum (B) is no longer horizontal in its general direction, but, like the upper margin of the hemisphere, is elevated at the posterior end. In front it is slightly thickened, but the rostrum is scarcely perceptible. Although this commissure in its median section appears elongated from before backwards, it is very thin from above downwards. The inferior layer of transverse (psalterial) fibres are well developed, and, except posteriorly, distinct from the main part of the great transverse commissure. The septal area is large in extent. The anterior commissure is proportionally larger than in man or in the sheep. The hippocampal sulcus, corresponding with the large size of its internal projection into the ventricle, is deep, and prolonged for some distance beneath the hinder end of the corpus callosum. The hollow for the reception of the optic thalamus and corpora quadrigemina is very large, and the fascia dentata (P) lying in it very broad. The smooth inner wall of the hemisphere shows no other sulcus than that of the hippocampus.

The transverse section (Plate XXXVII. fig. 4) shows the corpus collosum at the bottom of the longitudinal fissure, curving up at the two extremities, in consequence of the form of the lateral ventricles. The anterior commissure is of actual greater depth in the section than the corpus callosum. Between the two is the septum, now only represented by the thick lower portion, very considerably increased in development. The thin upper part, together with the fifth ventricle, has entirely disappeared with the rostrum of the corpus callosum.

In the Two-toed Sloth (*Cholœpus didactylus*), Plate XXXVII. fig. 5, the same parts can be recognized, though somewhat changed in proportions. As compared with the sheep especially, the whole hemisphere is greatly shortened in the antero-posterior direction, and a greater shortening still has taken place in the corpus callosum. Instead of bearing, as in the sheep, the proportion to the hemisphere of 53 to 100, it is but as 32 to 100. It rises at the posterior part, where it is slightly enlarged. The anterior end is simple and obtusely pointed, without a trace of the reflected rostrum. The anterior commissure is considerably larger, relatively to the hemisphere, than in the sheep. The ventricular aperture is nearly vertical in general direction. At the posterior edge of the body of the fornix there is a considerable thickening, caused by the transverse psalterial fibres of the corpus callosum. The hippocampal sulcus may be traced upwards to near the hinder end of the corpus callosum; it then makes a sudden curve backwards, and almost immediately after another nearly equally sudden bend forwards, then arches over the end of the corpus callosum, and gradually approaching the upper surface of that body, at about its middle disappears in the lower margin of the callosal gyrus. Thus a thin portion of the dentate gyrus (fascia dentata) is continued over the hinder edge, on to the upper surface of the corpus callosum. In its principal part the gyrus itself is longitudinally grooved by a shallow sulcus, anterior and parallel to the hippocampal sulcus. The characteristic indentations are faintly indicated on the posterior edge.

The transverse section (Plate XXXVII. fig. 6) shows the corpus callosum curving up at the outer extremities owing to the upward development of the lateral ventricles, as in the rabbit, and in the fœtal condition of the higher mammals. The corpora striata (R, R) are very large. The anterior commissure exceeds in vertical depth the corpus callosum. The septum, broad below where it rests on the anterior commissure, diminishes above to a narrow edge, where it touches the under surface of the corpus callosum; but there is no part which can properly be called septum lucidum. On each side of the middle line are seen the vertical white fibres, forming the commencement of the columns of the fornix.

Plate XXXVII. figs. 7 & 8 are taken from the brain of the Common Hedgehog (*Erinaceus europæus*). The transition from the Sloth's brain to this is easy, although it presents a wide difference from that of the Rabbit. The inner surface of the cerebrum shows no trace of any sulcus, except the deep one of the hippocampus (Q), which is placed very near the hinder border of the truncated hemisphere, and terminates a little way behind and below the posterior end of the corpus callosum. The last named body is extremely reduced in size, its length being but one fifth that of the entire hemisphere. Its obliquity is so much increased that its general direction is rather vertical than horizontal. The psalterial fibres form a distinct projection (N) in the section closer to the body of the corpus callosum than in the two previously described brains. The septal area is much reduced, and the anterior commissure increased in bulk. The great size of the olfactory ganglion is very remarkable.

The transverse section shows a corresponding simplicity, and agrees in all its essential characters with that of the Sloth. The oblique position of the corpus callosum gives its section an apparent thickness, which it would not possess if divided, as in the higher mammals, at a right angle to the plane of its upper surface.

These are examples of some of the modifications of the commissural apparatus of the cerebral hemispheres among the placental mammals. They might be considerably multiplied, but they are sufficient for the purpose of affording a basis of comparison with the same parts in the Marsupials and Monotremes.

Before entering upon this part of the subject, it may be desirable to give an outline of the present condition of knowledge upon it. A reference to the works of comparative anatomists who wrote before the year 1837, shows that up to that period no important distinction had been suspected to exist in the cerebral organization of the placental and the implacental mammals. In the Philosophical Transactions of that year, however, appeared the memoir of Professor OWEN "On the Structure of the Brain in Marsupial Animals," in which was announced the absence in these animals, of the "corpus callosum and septum lucidum." A transverse commissure between the hemispheres superior to the anterior commissure is described, but called by Professor OWEN "fornix" or "hippocampal commissure." Of this it is stated, "This commissure may, nevertheless, be regarded as representing, besides the fornix, the rudimental commencement of the corpus callosum; but this determination does not invalidate the fact that the great commissure which unites the supraventricular masses of the hemispheres in the Beaver and all other placentally developed Mammalia, and which exists in addition to the hippocampal commissure, is wanting in the brain of the Wombat: and as the same deficiency exists in the brain of the Great and Bush Kangaroos, the Vulpine Phalanger, the Ursine, and MAUGE's Dasyures, and the Virginian Opossum, it is most probably the characteristic of the marsupial division of Mammalia." The relatively large size of the anterior commissure in the marsupials is referred to in the paper as worthy of notice, as also is the proportionally very large size of the hippocampi majores.

The description given in this important memoir was subsequently reproduced in the Cyclopædia of Anatomy and Physiology, art. Marsupialia, and it was shown that the same peculiarity also existed in the Monotremata, and therefore was characteristic of the whole implacental division. In the paper by the same author "On the Characters, Principles of Division and Primary Groups of the Class Mammalia"*, the Subclass *Lyencephala* ("loose" or "disconnected" brain), equivalent to the Implacentalia, are characterized as having "the cerebral hemispheres but feebly and partially connected together by the 'fornix' and 'anterior commissure,' while in the rest of the class a part called 'corpus callosum' is added, which completes the connecting or commissural apparatus"†. The views of Professor OWEN have been adopted without hesitation or qualification, in this country at least, and have been incorporated in almost every text-book on Anatomy and Physiology subsequently published. The same has been the case to a great extent upon the continent, and what is more important, they have received confirmation apparently from original dissections of several of the marsupials by the editors of the third edition of CUVIER'S 'Anatomie Comparée,' MM. F. CUVIER and LAURILLARD (1844), and in the case of the Echidna by MM. EYDOUT and LAURENT (Voyage de la Favorite, 1839).

But expressions of dissent have also been raised. LEURET, speaking of the brain of the Kangaroo, says,* "J'y ai vu bien manifestement un corps calleux, situé entre les deux lobes cérébraux, comme chez les autres mammifères."

---

* Proc. Linn. Soc. 1858.

† [The necessity of doing full justice to the labours of one who has made this subject so peculiarly his own, will excuse my quoting the following succinct account of the distinctive characteristics of the views of this eminent anatomist, as set forth in his most recent publication bearing upon the question.

"In investigating and studying the value and application of the cerebral characters of Man in the classification of the Mammalia, I have been led to note the relations of equivalent modifications of cerebral structure to the extent of the groups of mammals respectively characterized by such conditions of brain. The Monotremes and Marsupials, which offer numerous extreme modifications of the limbs, all agree in possessing a brain in which there is no connecting or commissural mass of fibres overarching the lateral ventricles of the cerebrum. The surface of this part shows, however, a few symmetrical convolutions in *Echidna* and *Macropus*, especially the largest species; but in the majority of marsupials the hemispheres are smooth. The 'corpus callosum,' or great commissure, makes its appearance abruptly in the Rats, Shrews, Bats, and Sloths, which in general organization and powers are next the 'loose-brained' marsupials or *Lyencephala*: but this commissure is associated with a similarly smooth unconvolute cerebrum, and with so small a size of the cerebrum as leaves uncovered the cerebellum and in most the optic lobes."—Contributions to the Natural History of the Anthropoid Apes, No. VIII., by Professor OWEN, Trans. Zool. Soc. vol. v. part 4, 1865, p. 270.—April 1865.]

FOVILLE, in a note to p. 172 of his well-known treatise on the Nervous System (1844), says, "M. DE BLAINVILLE a toujours soutenu l'existence du corps calleux chez les didelphes, et me l'a fait voir de la manière la plus manifeste chez plusieurs de ces animaux. Il a si peu de volume qu'on s'explique facilement comment on a pu croire à son absence."

F. J. C. MAYER†, speaking of the brain of the Common Opossum (*Didelphis virginiana*), says, "Das *corpus callosum* betreffend, so ist dasselbe ebenfalls und namentlich bei *Didelphis* vorhanden, nur schmal oder kurz, allerdings etwas schmäler oder kürzer, als bei den Nagern, allein noch kürzer ist das *corpus callosum* beim Igel [hedgehog] wo es ebenfalls nur ein vorderes schmales Markblatt bildet. Aber schon bei den Nagern treten der Eingang in den dritten Ventrikel und der Sehhügel hinter dem *corpus callosum* zu Tage, am meisten aber bei dem Igel, und die Beutelthiere stehen nur zwischen beiden, den Nagern und dem Igel in der Mitte, und es ist somit im Gehirne derselben keine abweichende Organisation wahrzunehmen, welche mit der Geschlechtstheile etwa eine Parallele liefern könnte"‡.

The more detailed description of this structure in the brain of the same animal, given by PAPPENHEIM§ in language remarkable for its precision, deserves to be quoted in full, as it has received little attention from subsequent authors. It agrees in the main with the observations recorded in this paper.

"Mais je crois devoir m'occuper, avant tout, de la nature du *corps calleux*. C'est une opinion très-répandue, que ce corps n'existe pas chez les Marsupiaux. Cependant les dessins et la description de M. OWEN prouvent que ce corps à été très-bien vu par cet anatomiste habile; mais que, d'un coté, il n'a pas reconnu sa marche entière, et que, de l'autre, il à été frappé par la situation de cette commissure, qu'il à considérée plutôt comme un fornix (voûte à trois piliers). Comme cet organe se trouve dessiné en partie dans le paquet cacheté que l'Académie a bien voulu me faire l'honneur d'accepter, je me bornerai aujourd'hui à signaler quelques faits qui, rapprochés de mes observations anciennes, prouveront que le corps en question est bien un corps calleux.

"1°. La commissure dont je parle est située en avant des couches optiques, là où leur premier développement s'opère, au-dessus de la commissure antérieure du cerveau. Toutes ses fibres rayonnent au-dessus du corps strié, dans les hémisphères, où elles se terminent en faisceaux parallèles aux fibres des pédoncules cérébraux.

---

* Anat. Comp. du Système Nerveux, t. i. p. 412 (1839).

† Neue Untersuchungen aus dem Gebiete der Anatomie und Physiologie. Bonn, 1842, p. 24.

‡ Professor OWEN (Annals and Mag. Nat. Hist. vol. xvi. p. 101, 1845), in replying to MAYER's statement, says, "The great transverse band or commissure which unites the two hemispheres, spanning from one to the other *above the lateral ventricle*—which is plainly visible, as such, in the lowest Rodent or other placental mammal, with the smoothest, and, to outward appearance, simplest brain,—this great commissure or corpus callosum, I again affirm, after reiterated dissections, to be absent in all the known genera of Marsupials. If the narrow transverse band, which unites together the hippocampi majores, at the front part of the fornix, be regarded, as I originally stated it might be, a rudiment of the 'corpus callosum,' the comparative anatomist is at liberty to apply that name to it."

§ "Notice préliminaire sur l'anatomie du sarigue femelle (*Didelphis virginiana*)," Comptes Rendus, tom. xxiv. p. 186 (1847).

"2°. Elle s'allonge en avant dans un corps genouillé, qui ne peut être comparé aux pédoncules du fornix, lesquels entrent dans les couches optiques, tandis que ce dernier corps rayonne dans les hémisphères.

"3°. Les fibres de cette commissure sont purement transversales, direction qui n'a aucun rapport avec celles des fibres du fornix.

"4°. Les fibres du fornix ne s'étalent jamais dans les parois des ventricules; aussi n'occupent-elles pas toute la longueur du ventricule latéral.

"Cette commissure n'est donc ni un fornix, ni un mélange du fornix avec le corps calleux.

"La partie postérieure est composée de fibres accumulées en un faisceau très-épais, tandis que les fibres antérieures du corps calleux sont étalées dans une couche large, mais extrêmement mince et tellement transparente, que l'on voyait à travers le corps strié. Du reste, quand on écartait les hémisphères, les fibres du corps calleux, étalées, se laissaient détacher facilement de l'autre substance blanche, sous forme de feuillet mince, tapissant, pour ainsi dire, la paroi du ventricule latéral dans chaque hémisphère.

"Les hémisphères étaient composés d'une manière trés-simple, savoir; des fibres des pédoncules cérébraux, qui étaient les plus externes; des fibres de la commissure antérieure, en avant et en dedans, et d'un feuillet appartenant au corps calleux, situé en dedans du rayonnement des fibres du pédoncle; tout autour, enfin, était une couche corticale très-épaisse et peut-être plus considérable que toutes les fibres blanches."

Such are the main results of the researches of those anatomists to whom we are indebted for all that is known upon the cerebral commissure of the Implacental Mammals. I will next give an account of these structures as actually observed in several of the leading types of the group, and afterwards discuss the relation which the conclusions derived from the present examination (differing somewhat in method from those previously used) bear to the opinions most generally received.

Kangaroo.—Several specimens of the brains of both *Macropus major* and *Macropus Bennettii* have been examined. They agree so closely in all essential points that one description will suffice for either, unless otherwise specially stated.

On looking at the upper surface of the brain (Plate XXXVI. fig. 4), the two hemispheres being partly separated, a transverse white band (B) is seen extending across the bottom of the longitudinal fissure, roofing over the anterior portion of the third ventricle, and occupying the same general position as the corpus callosum in the ordinary mammal, but developed to a smaller extent even than in the Hedgehog. In a brain of *Macropus Bennettii* it was found to cover, when still undisturbed by removal from the cranial cavity or contracted by spirit, about half the optic thalamus, and to measure from before backwards in the middle line, a quarter of an inch, or one-sixth of the entire length of the hemisphere. It is situated deeply in the great longitudinal fissure, is thickened and most elevated posteriorly, where the margin, slightly and evenly concave, crosses the cavity of the third ventricle (S), the peduncles of the pineal gland (T), and the optic thalami (U). The anterior margin is also concave, but extremely narrow, the white substance being continued on each side of a longitudinal median cleft for some distance towards the front of the cerebral hemisphere, as if in this anterior part the two lateral halves of the commissure had not been joined together in the middle line. On close examination it is seen to be composed of fibres of which the general direction is transverse, but on its upper surface can be distinguished a longitudinal median raphe, and on each side of this a few longitudinal white fibres, corresponding to the "striæ laterales" of other mammals.

On either side, the transverse fibres are lost beneath the overlapping grey matter constituting the margin of the convolution of the corpus callosum, the "labia cerebri" of some authors. To follow them further, the last named parts must be carefully removed with the handle of a scalpel or some similar instrument, when a delicate broad lamina formed by the lateral expansion of the narrow transverse band will come into view, passing at first horizontally outwards and then curving upwards above the precommissural fibres of the fornix (I), the cavity of the lateral ventricle, and the corpus striatum (R), and finally losing themselves in the medullary substance of the upper part of the cerebral hemispheres. The fibres radiate extensively forwards and backwards but forming a continuous lamina, posteriorly conterminous with those on the surface of the hippocampus major, anteriorly becoming much more delicate, so much so, indeed, that it is not easy to make a complete dissection of them without causing some rents, like that on the left side shown in the figure, through which the cavity of the ventricle below is exposed. This expansion of the transverse commissure in the hemisphere, though described by Pappenheim in the Opossum, appears not to have been observed by Owen in any of his dissections.

Plate XXXVIII. fig. 1 is a view of the inner surface of the right hemisphere of the Great Kangaroo. The hemisphere is short, and deep from above downwards, obtusely pointed in front and flattened or abruptly truncated behind. The temporal lobe is largely developed. Several well-marked sulci are seen upon the surface of the hemisphere. One of the most striking characteristics presented by this section is the great development of the anterior commissure (F), far exceeding that seen in any placental mammal. The form of its section is oval, with the long diameter nearly vertical, or inclining slightly forwards at the upper end. It consists of firm, white, transverse fibres, distinctly defined from the surrounding part, and forms a good landmark to the adjoining structures, as about its homologies there can be no question. At a very short distance above this is seen the section of the median part of that transverse band before described (B). This is oval, elongated from before backwards, slightly arched on its upper border. Its anterior and posterior extremities are rounded, the former is the narrowest. To the under surface of the latter, a body of transverse fibres (N), almost equal in size to the upper portion of the commissure, is intimately united. Beneath the anterior part of this, close to the middle line, a distinct white cylindrical band of fibres is seen to pass down, behind and in close contact with the anterior commissure, at first directed somewhat backwards and afterwards downwards until it loses itself in the thalamus opticus. This evidently answers to one of the columns of the fornix, its position being somewhat disturbed by the immense development of the anterior commissure. Between the superior transverse commissure (by which name I propose for the present to call the part marked B) and the anterior commissure are some fibres continued forwards from above the anterior end of the ventricular aperture, and mixed in this region with much grey matter, forming the greatly reduced septal area (G). They curve forwards and downwards, encircling the anterior half of the anterior commissure, and represent, doubtless, those designated as "precommissural" fibres in the higher mammals. The ventricular aperture is seen to occupy its ordinary position. Its upper margin is formed by the edge of a broad white band, corpus fimbriatum (M). On tracing this band forwards, it is found to be continuous with the hinder edge of the whole of the upper transverse commissure. The superficial grey layer (P) external to the corpus fimbriatum is readily recognized as the fascia dentata. This is bounded on the outer side by the hippocampal sulcus; but in respect to this sulcus a great peculiarity presents itself. On tracing it forwards, instead of stopping short beneath the projecting posterior rounded end of the corpus callosum, as in most, if not all placental mammals *, it is continued on, passing over the top in close contact with the upper transverse commissure, and is not lost until it reaches the inner surface of the anterior lobe, considerably in advance of both the upper and anterior commissures. The remarkable disposition of this sulcus must be particularly noted in reference to the nature of the commissure in close relation with it.

In the transverse section (Plate XXXVIII. fig. 2) the immense size of the anterior commissure (F) is as conspicuously seen as in the longitudinal section. It occupies one-fourth of the whole height of the brain in the middle line. Its fibres spread themselves outwards, the lower ones sweeping first slightly downwards, then curving up into the white medullary substance of the middle of the hemisphere. The higher fibres, taking a course more directly upwards, penetrate the grey matter of the corpora striata (R R), which they here divide into two distinct masses, and finally reach the medullary substance of the upper part of the hemisphere. Lying immediately upon the anterior commissure, close to the median line, are two bodies, which, taken together, present a surface broad from side to side, slightly concave above, nearly flat below, and rounded off at the outer inferior angles. These consist mostly of grey substance, with some white fibres, especially collected into two bands close to the median line (the roots of the columns of the fornix). These bodies are the two lateral halves of the very much thickened and depressed ventricular septum. Below they are in contact with the anterior commissure, on each side with the cavity of the lateral ventricle, above with a white

---

* A partial exception was shown in the Two-toed Sloth.

transverse band. This band, lying at the bottom of the great longitudinal fissure of the cerebrum, is the one previously mentioned as the superior transverse commissure. Traced outwards, its fibres, spreading into an extremely thin layer, form the upper and inner boundary of the superior portion of the lateral ventricle. They have a regular curve, outwards, upwards, and finally inwards, losing themselves in the medullary substance of the hemisphere at its upper and inner angle. Their internal concave border is in contact with a fold of cortical grey matter, surrounding a deeply penetrating sulcus, which from the very bottom of the longitudinal fissure runs outwards and then upwards in the hemisphere, and which, as shown in the previous section, is continuous with the hippocampal sulcus in the posterior part of the hemisphere. The lateral ventricle, as seen in this section, is prolonged to a considerable height in the hemisphere, but otherwise its relations are similar to those of the same part in the placental mammals.

Figs. 3 & 4, Plate XXXVIII. are taken from the brain of the Wombat (*Phascolomys vombatus*). In general form the cerebral hemispheres are more depressed and elongated than those of the Kangaroo, and the temporal lobe obtains a comparatively slight development. Corresponding with this general elongation, the ventricular aperture and the surrounding parts have a wider curve backwards. The essential characters are, however, precisely the same. The anterior commissure attains an equal magnitude. The superior transverse commissure has the same form and relations, and the continuation of the hippocampal sulcus extends above it, though it is not prolonged to quite the same extent on the anterior lobe. Seen in transverse section, the septum is narrower from side to side.

The large carnivorous Marsupial, the Thylacine (*Thylacinus cynocephalus*), so widely separated in external characters from both the Kangaroo and Wombat, shows the same general peculiarities of cerebral organization, but attended with a smaller development of the superior transverse commissure, especially of its anterior part, and a greater reduction of the thickness of the interventricular septum (see Plate XXXVIII. figs. 5 & 6).

Dissections of the brains of *Phalangista vulpina* and of *Didelphis virginiana* have yielded similar results, so that it may be presumed that the principle upon which the cerebral commissures are arranged is uniform throughout the Marsupial Order.

Of the two genera of Monotremes, I have only had the opportunity of dissecting the brain of one, the Echidna. This most remarkable brain, with its largely developed and richly convoluted hemispheres, conforms in the main with the Marsupial type in the disposition of the commissures, but in detail presents a still further deviation from the ordinary mammalian form. As seen in Plate XXXVIII. fig. 7, the anterior commissure is as large relatively as in the Marsupials. Above it is seen the section of the superior transverse commissure, very much reduced in extent, and in which the two portions, upper and lower, observed in the Kangaroo are no longer distinguishable. Its relations to the hippocampal sulcus, to the ventricular aperture, to the columns of the fornix, to the precommissural fibres, and to the lateral ventricles are however the same, so that whatever parts of the placental mammalian brain are represented by this commissure in the Kangaroo, are also represented by it, though in a reduced degree, in the Echidna. Perhaps the greatest change is in the extreme reduction of the septum, as best seen in the transverse section (Plate XXXVIII. fig. 8). In dissecting the brain from above, the fibres of the superior commissure are found to spread out into a delicate layer roofing in the ventricles quite to the anterior part of the hemisphere, as described in the Kangaroo.

Having described the actual condition of an important and well-marked region of the cerebrum in several members of the two great groups of the Mammalia, it now remains to trace out the relation that the several structures entering into the formation of this region bear to one another in each of the two groups. It will be necessary also to inquire how far the results brought out by the present method of examination are in accordance with the views generally received.

At the outset a distinct confirmation is afforded by the dissections recorded in this paper, of the great fact, first observed by Professor OWEN, that the brains of animals of the orders Marsupialia and Monotremata present certain special and peculiar characters, by which they may be at once distinguished from those of other mammals. The appearance of either a transverse or longitudinal section would leave no doubt whatever as to which group the brain belonged. In the differentiating characters to be enumerated, some members of the higher section present a considerable approximation to the lower; but, as far as is known at present, there is still an interval between them unconnected by any intermediate link.

The differences are manifold, but all have a certain relation to, and even a partial dependence on, each other.

They may be enumerated under the following heads:—

1. The peculiar arrangement of the folding of the inner wall of the cerebral hemisphere. A deep fissure, with corresponding projection within, is continued forwards from the hippocampal fissure, almost the whole length of the inner wall. In other words, the hippocampus major, instead of being confined as it is, at least in the higher forms of placental mammals, to the middle or descending cornu of the lateral ventricle, extends up into the body of the ventricle, constituting its inner wall.

2. The altered relation (consequent upon this disposition of the inner wall) and the very small development of the upper transverse commissural fibres (corpus callosum).

3. The great increase in amount, and probably in function, of the inferior set of transverse commissural fibres (anterior commissure).

These propositions must now be considered a little more closely. Arguing from our knowledge of the development of the brain in placental mammals (for of that of the marsupials we have at present no information), it may be supposed that the first-named is also first in order of time in the gradual evolution of the cerebral structures. Before any trace of the budding out of the fibres which shoot across the chasm separating the two hollow sac-like hemispheres, before the differentiation of a portion of the septal area into the anterior commissure, that remarkable folding of the inner wall, indicated by the deep furrow on the surface and the corresponding rounded projection in the interior, has already become distinctly manifest, and the future form of the ventricular cavity, with its elevations and depressions, has been sketched out. Now the first rudiment of the upper transverse commissure is found undoubtedly at the spot afterwards situated near its middle—that part to which in the lowest placental mammals it is almost entirely confined. This spot is situated a little way above and in front of the anterior end of the ventricular aperture, at the upper edge of the region of adherence of the two hemispheres (the future septal area). In the placental mammals this part is in direct relation to the great mass of the internal medullary substance of the hemispheres, which have to be brought into communication. In the Marsupial, on the other hand, the prolonged internal convolution or hippocampus extending up to and beyond this part, forms the inner wall of the hemisphere from which the fibres pass across, and it is necessarily through the medium of this convolution, and following the circuitous course of its relief in the ventricle, that the upper part of the hemisphere alone can be brought into connexion.

Can this transverse commissure, of which the relation is so disturbed by the disposition of the inner wall of the hemisphere, be regarded as homologous with the entire corpus callosum of the placental mammals? or is it, as has been suggested by Professor OWEN, to be looked upon as only representing the psalterial fibres or transverse commissure of the hippocampi? Undoubtedly a large proportion of its fibres do come under the latter category. But even if they should nominally be all so included, it is important to bear in mind that we have still a disposition in the marsupial brain very different from that which would remain in the brain of any placental mammal after the upper and main part of the corpus callosum had been cut away. In the latter case the commissure of a very small part of the inner wall of the hemisphere alone is left, that part folded into the hippocampus. In the former there is a commissure, feeble it may be, but radiating over *the whole of the inner wall*, from its most anterior to its posterior limits. Granted that only the psalterial fibres are represented in the upper commissure of the marsupial brain, why should the name of "corpus callosum" be refused to it? These fibres are part of the great system of transverse fibres bringing the two hemispheres into connexion with each other; they are inseparably mingled at the points of contact with the fibres of the main body of the corpus callosum, and are only separated from it in consequence of the peculiar form of the special portions of the hemisphere they unite. Indeed, as mentioned before, they are not more distinct than is the part called "rostrum" in front. And although they blend at each extremity with the fibres of the diverging posterior crura of the fornix, they certainly cannot be in any sense confounded with that body, the essential character of which is that it is a longitudinal commissure consisting of two halves closely applied in the middle, but each composed of fibres belonging to a single hemisphere only.

But is the main part of the corpus callosum of the placental mammal not also represented by the upper and anterior part of the transverse band passing between the hemispheres of the marsupial brain? The most important and indeed crucial test in determining this question, is its position in regard to the septum ventriculorum, and especially the precommissural fibres of the fornix. Without any doubt in all marsupial and monotreme animals examined (sufficient to enable us to affirm without much hesitation that it is the character common to all) it lies *above* them, as distinctly seen in the transverse sections. Moreover, passing outwards into the hemispheres, it overarches or forms the roof of the lateral ventricles of the cerebrum. This is precisely the same relationship as that which occurs in Man and all other mammalia.

The defective proportions of the part representing the great transverse commissure of the placental mammal, which appears to me to result from, or, at all events, to be related to the peculiar conformation of the wall of the hemisphere, must not lead to the inference that the great medullary masses of the two halves of the cerebrum are by any means "disconnected." The want of the upper fibres is compensated for in a remarkable manner by the immense size of the anterior commissure, the fibres of which are seen radiating into all parts of the interior of the hemisphere. There can be little doubt but that the development of this commissure is, in a certain measure, complementary to that of the corpus callosum. That it is not simply correspondent with the large size of the olfactory ganglion, as Professor OWEN has suggested, is shown by the fact that in the Hedgehog and some other placental mammals this ganglion attains a far greater proportionate volume than in many marsupials, and yet the commissure is very considerably smaller.

In descending the series from Man to the Placental Mammals of lowest cerebral organization, the great change in the condition of the corpus callosum has been seen to be, the disappearance of the rostral portion, and the coincident greater development of the posterior folded or psalterial portion; the latter being connected with the relative increase of the hippocampal region of the cerebrum. In the brain of the marsupial a change of precisely the same nature is carried to an excess. There is, however, as far as my observations show, no structure characteristic of the higher group which is absent in the lower.

The step from the marsupial or monotreme brain to that of an animal belonging to one of the lower vertebrate classes is very great. Indeed it is difficult to see in many of the peculiarities of their brain even an approach in the direction of that of the bird. We may allow that the diminution of the volume of the corpus callosum leads on to its entire absence; but in the great development of the anterior commissure is presented a special characteristic of the lowest group of mammalia, most remarkable because it is entirely lost in the next step of descent in the vertebrate classes. The same may be said of the cerebral folding constituting the hippocampus major.

Plate XXXVI. figs. 5 & 6 are views of the brain of a Goose, corresponding to those given of the various mammals. The smooth, thin, inner wall has no trace of that folding upon itself which gives rise to the hippocampus major in the mammal. In this respect there is a vast difference from the brain of the marsupial. The ventricular aperture (O O) is extremely reduced. Its upper border may be properly compared to the fornix, and the thickened part of the inner wall (G), above and in front of the small anterior commissure (F), evidently corresponds to the lower part of the septal area and precommissural fibres, as well seen in the transverse section. The walls of the hemispheres are in close apposition at this part, as the two lateral halves of the septum are in the mammals; but a distinct band of fibres passing across the middle line from one hemisphere to the other, above the anterior commissure, has never yet been satisfactorily demonstrated. The homology of the minute and delicate transverse lamella of nerve-substance, described by A. MECKEL as situated above the ventricular aperture posterior to the anterior commissure, is very questionable.

Great as is the difference between the placental and implacental mammal in the mode and extent of the connexion between the two lateral hemispheres of the cerebrum, it is not to be compared with that which obtains between the latter and the oviparous vertebrate.

DESCRIPTION OF THE PLATES.

All, except fig. 3, Plate XXXVI., are from original dissections. For convenience of comparison the cerebral hemispheres are reduced to the same absolute length.

PLATE XXXVI.

Fig. 1. Inner surface of the right cerebral hemisphere, Human brain.

Fig. 2. Vertical transverse section (through the anterior commissure), Human brain.

Fig. 3. Development of the Human brain (after F. SCHMIDT). I. Sixth week. II. Eighth week. III. Tenth week. IV. Sixteenth week. V. Sixth month.

Fig. 4. Brain of Kangaroo (*Macropus Bennettii*) dissected from above, natural size. A portion of the extremely delicate great transverse commissure (B) has been removed on the left side to show the structures lying beneath it.

Fig. 5. Brain of Goose. Inner surface of right hemisphere.

Fig. 6. Brain of Goose. Vertical transverse section.

PLATE XXXVII.

Fig. 1. Brain of Sheep. Inner surface of cerebral hemisphere.

Fig. 2. Brain of Sheep. Vertical transverse section.

Figs. 3 & 4. Brain of Rabbit.

Figs. 5 & 6. Brain of Sloth (*Cholœpus didactylus*).

Figs. 7 & 8. Brain of Hedgehog (*Erinaceus europæus*).

PLATE XXXVIII.

Figs. 1 & 2. Brain of Kangaroo (*Macropus major*).
Figs. 3 & 4. Brain of Wombat (*Phascolomys vombatus*).
Figs. 5 & 6. Brain of Thylacine (*Thylacinus cynocephalus*).
Figs. 7 & 8. Brain of Echidna (*Echidna hystrix*).

EXPLANATION OF THE LETTERS USED IN ALL THE FIGURES.

A. Crus cerebri, divided between thalamus opticus and corpus striatum.
B. Body of corpus callosum.
C. Genu of corpus callosum.
D. Rostrum of corpus callosum.
E. Splenium of corpus callosum.
F. Anterior commissure.
G. Septal area.
H. Septum lucidum.
I. Precommissural fibres.
K. Body of fornix.
L. Columns of fornix.
M. Corpus fimbriatum. Edge of posterior crura of fornix.
N. Psalterial fibres of corpus callosum.
O. Ventricular aperture.
P. Fascia dentata.
Q. Hippocampal sulcus.
R. Corpus striatum.
S. Third ventricle.
T. Peduncles of pineal body.
U. Thalamus opticus.
V. Corpora quadrigemina.

XIV. *On the Sextactic Points of a Plane Curve.*

By WILLIAM SPOTTISWOODE, *M.A.*, *F.R.S.*, *&c.*

Received June 15,—Read June 15, 1865.

THE beautiful equation given by Professor CAYLEY (Proceedings of the Royal Society, vol. xiii. p. 553) for determining the sextactic points of a plane curve, and deduced, as I understand, by the method of his memoir "On the Conic of Five-pointic Contact" (Philosophical Transactions, vol. cxlix. p. 371), led me to inquire how far the formulæ of my own memoir "On the Contact of Curves" (Philosophical Transactions, vol. clvii. p. 41) were applicable to the present problem.

The formulæ in question are briefly as follows: If $U=0$ be the equation of the curve, $H=0$ that of its Hessian, and $V=(a, b, c, f, g, h)(x, y, z)^2=0$ that of the conic of five-pointic contact; and if, moreover, α, β, γ being arbitrary constants,

$$\left.\begin{aligned} \delta&=\alpha x+\beta y+\gamma z,\\ \square&=(\gamma\partial_y U-\beta\partial_z U)\partial_x+(\alpha\partial_z U-\gamma\partial_x U)\partial_y+(\beta\partial_x U-\alpha\partial_y U)\partial_z, \end{aligned}\right\} \quad . \quad . \quad . \quad (1)$$

then, writing as usual

$$\left.\begin{array}{c} \partial_x U=u,\ \partial_y U=v,\ \partial_z U=w;\ \partial_x H=p,\ \partial_y H=q,\ \partial_z H=r,\\ \partial_x^2 U=u_1,\ ..\ \partial_y\partial_z U=u',\ ..\quad \partial_x^2 H=p_1,\ ..\ \partial_y\partial_z H=p',\ ..\\ \mathfrak{A}=v_1w_1-u'^2,\ ..\quad \mathfrak{F}=v'w'-u_1u',\ ..\\ v\gamma-w\beta=\lambda,\ w\alpha-u\gamma=\mu,\ u\beta-v\alpha=\nu, \end{array}\right\} \quad . \quad . \quad . \quad . \quad (2)$$

the values of the ratios $a : b : c : f : g : h$ are determined by the equations

$$V=0,\ \square V=0,\ \square^2 V=0,\ \square^3 V=0,\ \square^4 V=0. \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Now, if at the point in question the curvature of U be such that a sixth consecutive point lies on the conic V, the point is called a sextactic point; and the condition for this will be (in terms of the above formulæ) $\square^5 V=0$. From the six equations $V=0$, $\square V=0, .. \square^5 V=0$, the quantities a, b, c, f, g, h can be linearly eliminated; and the result will be an equation which, when combined with $U=0$, will determine the ratios $x:y:z$, the coordinates of the sextactic points of U. But the equation so derived contains (beside other extraneous factors) the indeterminate quantities α, β, γ, to the degree 15, which consequently remain to be eliminated. Instead therefore of proceeding as above, I eliminate α, β, γ beforehand, in such a way that ($W=0$ representing any one of the series $V=0$, $\square V=0, ..$ from which α, β, γ have been already eliminated) the equations $W=0$, $\square W=0$, $\square^2 W=0$ are replaced by

$$\frac{\partial_x W}{u}=\frac{\partial_y W}{v}=\frac{\partial_z W}{w}=\frac{\Delta W}{\varpi H} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (4)$$

where ϖ is a numerical factor, and

$$\Delta=(\mathfrak{A},\ \mathfrak{B},\ \mathfrak{C},\ \mathfrak{F},\ \mathfrak{G},\ \mathfrak{H})(\partial_x,\ \partial_y,\ \partial_z)^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (5)$$

To this preliminary transformation the first section of the paper is devoted *. The second section contains the actual elimination of the constants of the conic, and the reduction of the resultant to six forms, $\mathfrak{L}=0$, $\mathfrak{M}=0$, $\mathfrak{N}=0$, $\mathfrak{L}'=0$, $\mathfrak{M}'=0$, $\mathfrak{N}'=0$; of which $\mathfrak{L}$ and $\mathfrak{L}'$, $\mathfrak{M}$ and $\mathfrak{M}'$, $\mathfrak{N}$ and $\mathfrak{N}'$ differ respectively only by one and the same numerical factor, viz. $(n-2)^3$. All these forms, however, contain extraneous factors, the determination of which is the object of the remainder of the paper. The third section is devoted to the establishment of some formulæ of reduction, the demonstrations of which are rather too long to be conveniently inserted in what would otherwise be their more natural place (§ 4). Besides these I have established many others of a like nature; but the specimens here given will doubtless suffice to suggest the mode of proof of the rest to any one desirous of pursuing the subject further. In the fourth section it is shown that all six forms $\mathfrak{L}$, .. $\mathfrak{L}'$, .. are divisible by the Hessian of U, and that $\mathfrak{L}$, $\mathfrak{L}'$ are also divisible by u^3, $\mathfrak{M}$, $\mathfrak{M}'$ by v^3, and $\mathfrak{N}$, $\mathfrak{N}'$ by w^3, and that the result of these divisions is a single expression of the degree $12n-27$.

§ 1. *Preliminary Transformation.*

The first two equations of the system (3) are, as is well known, equivalent to the following, viz.

$$\frac{\partial_x V}{u}=\frac{\partial_y V}{v}=\frac{\partial_z V}{w}=\theta, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (6)$$

where θ is indeterminate. The third equation, viz. $\square^2 V=0$, when written in full, is

$$0=\square\lambda\partial_x V+\square\mu\partial_y V+\square\nu\partial_z V+\lambda^2\partial_x^2 V+\mu^2\partial_y^2 V+\nu^2\partial_z^2 V+2(\mu\nu\partial_y\partial_z V+\nu\lambda\partial_z\partial_x V+\lambda\mu\partial_x\partial_y V). \qquad (7)$$

Now n being the degree of U, we may without difficulty establish the following formulæ given by CAYLEY (*l. c.* p. 381):

$$\left.\begin{aligned}(n-1)u^2&=-\mathfrak{B}z^2+2\mathfrak{F}zy-\mathfrak{C}y^2,\\(n-1)v^2&=-\mathfrak{C}x^2+2\mathfrak{G}xz-\mathfrak{A}z^2,\\(n-1)w^2&=-\mathfrak{A}y^2+2\mathfrak{H}yx-\mathfrak{B}x^2,\\(n-1)vw&=-\mathfrak{F}x^2-\mathfrak{G}xy-\mathfrak{H}xz+\mathfrak{A}yz,\\(n-1)wu&=-\mathfrak{F}yx+\mathfrak{G}y^2-\mathfrak{H}yz+\mathfrak{B}zx,\\(n-1)vw&=-\mathfrak{F}zx-\mathfrak{G}zy+\mathfrak{H}z^2+\mathfrak{C}xy,\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (8)$$

---

[*] In a paper recently published in the 'Quarterly Journal of Mathematics,' vol. vii. p. 114, I have given a transformation having the same object in view; but its form is partial and in some sense incomplete, and the mode of proof less direct and obvious than that given in the text.

whence writing

$$\Phi=(a,\ b,\ c,\ f,\ g,\ h)(\alpha,\ \beta,\ \gamma)^2, \quad \ldots \quad (9)$$

we may derive

$$\left.\begin{aligned}
(n-1)\lambda^2 &=-\delta^2\mathfrak{A}+2\delta x(\mathfrak{A}\alpha+\mathfrak{H}\beta+\mathfrak{G}\gamma)-x^2\Phi,\\
(n-1)\mu^2 &=-\delta^2\mathfrak{B}+2\delta y(\mathfrak{H}\alpha+\mathfrak{B}\beta+\mathfrak{F}\gamma)-y^2\Phi,\\
(n-1)\nu^2 &=-\delta^2\mathfrak{C}+2\delta z(\mathfrak{G}\alpha+\mathfrak{F}\beta+\mathfrak{C}\gamma)-z^2\Phi,\\
(n-1)\mu\nu &=-\delta^2\mathfrak{F}+\delta z(\mathfrak{H}\alpha+\mathfrak{B}\beta+\mathfrak{F}\gamma)+\delta y(\mathfrak{G}\alpha+\mathfrak{F}\beta+\mathfrak{C}\gamma)-yz\Phi,\\
(n-1)\nu\lambda &=-\delta^2\mathfrak{G}+\delta x(\mathfrak{G}\alpha+\mathfrak{F}\beta+\mathfrak{C}\gamma)+\delta z(\mathfrak{A}\alpha+\mathfrak{H}\beta+\mathfrak{G}\gamma)-zx\Phi,\\
(n-1)\lambda\mu &=-\delta^2\mathfrak{H}+\delta y(\mathfrak{A}\alpha+\mathfrak{H}\beta+\mathfrak{G}\gamma)+\delta x(\mathfrak{H}\alpha+\mathfrak{B}\beta+\mathfrak{F}\gamma)-xy\Phi.
\end{aligned}\right\} \quad \ldots \quad (10)$$

But, as will be found on calculating the expressions,

$$\left.\begin{aligned}
(n-1)\square\lambda &=\delta(\mathfrak{A}\alpha+\mathfrak{H}\beta+\mathfrak{G}\gamma)-x\Phi,\\
(n-1)\square\mu &=\delta(\mathfrak{H}\alpha+\mathfrak{B}\beta+\mathfrak{F}\gamma)-y\Phi,\\
(n-1)\square\nu &=\delta(\mathfrak{G}\alpha+\mathfrak{F}\beta+\mathfrak{C}\gamma)-z\Phi,
\end{aligned}\right\} \quad \ldots \quad (11)$$

so that

$$\left.\begin{aligned}
(n-1)^2\lambda^2 &=-\delta^2\mathfrak{A}+2(n-1)x\square\lambda+x^2\Phi,\\
(n-1)^2\mu^2 &=-\delta^2\mathfrak{B}+2(n-1)y\square\mu+y^2\Phi,\\
(n-1)^2\nu^2 &=-\delta^2\mathfrak{C}+2(n-1)z\square\nu+z^2\Phi,\\
(n-1)^2\mu\nu &=-\delta^2\mathfrak{F}+(n-1)(y\square\nu+z\square\mu)+yz\Phi,\\
(n-1)^2\nu\lambda &=-\delta^2\mathfrak{G}+(n-1)(z\square\lambda+x\square\nu)+zx\Phi,\\
(n-1)^2\lambda\mu &=-\delta^2\mathfrak{H}+(n-1)(x\square\mu+y\square\lambda)+xy\Phi.
\end{aligned}\right\} \quad \ldots \quad (12)$$

Hence, if m be the degree of V,

$$\begin{aligned}
&(n-1)^2\{\lambda^2\partial_x^2\mathrm{V}+\mu^2\partial_y^2\mathrm{V}+\nu^2\partial_z^2\mathrm{V}+2(\mu\nu\partial_y\partial_z\mathrm{V}+\nu\lambda\partial_z\partial_x\mathrm{V}+\lambda\mu\partial_x\partial_y\mathrm{V})\}\\
&\quad=-\delta^2(\mathfrak{A},\ \mathfrak{B},\ \mathfrak{C},\ \mathfrak{F},\ \mathfrak{G},\ \mathfrak{H})(\partial_x,\ \partial_y,\ \partial_z)^2\mathrm{V}+2(n-1)(m-1)(\square\lambda\partial_x\mathrm{V}+\square\mu\partial_y\mathrm{V}+\square\nu\partial_z\mathrm{V}),
\end{aligned}$$

whence, substituting in (7), and bearing in mind that

$$\left.\begin{aligned}
(n-1)(\mathfrak{A}u+\mathfrak{H}v+\mathfrak{G}w) &=\mathrm{H}x,\\
(n-1)(\mathfrak{H}u+\mathfrak{B}v+\mathfrak{F}w) &=\mathrm{H}y,\\
(n-1)(\mathfrak{G}u+\mathfrak{F}v+\mathfrak{C}w) &=\mathrm{H}z,
\end{aligned}\right\} \quad \ldots \quad (13)$$

we have

$$(n-1)^2\left(1+\frac{2(m-1)}{n-1}\right)(\square\lambda\partial_x\mathrm{V}+\square\mu\partial_y\mathrm{V}+\square\nu\partial_z\mathrm{V})-\delta^2(\mathfrak{A},\mathfrak{B},\mathfrak{C},\mathfrak{F},\mathfrak{G},\mathfrak{H})(\partial_x,\partial_y,\partial_z)^2\mathrm{V}=0.$$

But

$$\begin{aligned}
\square\lambda\partial_x\mathrm{V}+\square\mu\partial_y\mathrm{V}+\square\nu\partial_z\mathrm{V} &=\theta(u\square\lambda+v\square\mu+w\square\nu)\\
&=\frac{\theta\delta}{n-1}\{(\mathfrak{A}u+\mathfrak{H}v+\mathfrak{G}w)\alpha+(\mathfrak{H}u+\mathfrak{B}v+\mathfrak{F}w)\beta+(\mathfrak{G}u+\mathfrak{F}v+\mathfrak{C}w)\gamma\}\\
&=\frac{\theta\delta^2}{(n-1)^2}\mathrm{H},
\end{aligned}$$

so that (7) finally takes the forms

$$(\mathfrak{A}, \mathfrak{B}, \mathfrak{C}, \mathfrak{F}, \mathfrak{G}, \mathfrak{H})(\partial_x, \partial_y, \partial_z)^2 V - \left(1 + \frac{2(m-1)}{n-1}\right)\theta H = 0, \quad . \quad . \quad . \quad (14)$$

or, in the case where V is a conic, and consequently $m=2$,

$$\mathfrak{A}a + \mathfrak{B}b + \mathfrak{C}c + 2(\mathfrak{F}f + \mathfrak{G}g + \mathfrak{H}h) - \tfrac{1}{2}\frac{n+1}{n-1}\theta H = 0; \quad . \quad . \quad . \quad . \quad . \quad (15)$$

and in general making $\varpi = 1 + \frac{2(m-1)}{n-1}$, (14) takes the form indicated above, viz.

$$\left.\begin{array}{l} \Delta V - \varpi\theta H = 0, \\ \text{or} \quad \dfrac{\partial_x V}{u} = \dfrac{\partial_y V}{v} = \dfrac{\partial_z V}{w} = \dfrac{\Delta V}{\varpi H}. \end{array}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

§ 2. *Elimination of the Constants of the Conic of Five-pointic Contact.*

Before proceeding to the application of the formulæ (16) to the investigation of the sextactic points, it will be convenient to premise that if s, t be any two homogeneous functions of x, y, z, the nature of the operation Δ is such that

$$\Delta st = s\Delta t + t\Delta s + 2(\mathfrak{A}, \mathfrak{B}, \mathfrak{C}, \mathfrak{F}, \mathfrak{G}, \mathfrak{H})(\partial_x s, \partial_y s, \partial_z s)(\partial_x t, \partial_y t, \partial_z t), \quad . \quad . \quad (17)$$

and also that

$$\Delta V = 3H, \quad \Delta u = p, \quad \Delta v = q, \quad \Delta w = r. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

This being premised, our first object is to establish an equivalent for $\square^3 V = 0$, divested of the extraneous quantities α, β, γ. Now, since

$$\begin{aligned} \delta(v\partial_z V - w\partial_y V) &= x\square V, \\ \delta(w\partial_x V - u\partial_z V) &= y\square V, \\ \delta(u\partial_y V - v\partial_x V) &= z\square V, \end{aligned}$$

and $\square\delta = 0$, it follows that

$$\begin{aligned} \delta\square(v\partial_z V - w\partial_y V) &= \lambda\square V + x\square^2 V, \\ \delta\square(w\partial_x V - u\partial_z V) &= \mu\square V + y\square^2 V, \\ \delta\square(u\partial_y V - v\partial_x V) &= \nu\square V + z\square^2 V; \end{aligned}$$

and consequently not only do $v\partial_z V - w\partial_y V$, $w\partial_x v - u\partial_z V$, $u\partial_y V - v\partial_x V$ vanish with $\square V$, but, when this is the case, $\square(v\partial_z V - w\partial_y V)$, .. vanish with $\square^2 V$. The same will obviously be the case if the operation $\square$ be continued; so that, in general terms, we may, by operating upon $v\partial_z V - w\partial_y V$, .. with the symbol $\square$, 0, 1, 2, .. times,. form a system of equations equivalent to that formed by operating on V with the same symbol 1, 2, 3, .. times. And if we represent any of the three quantities $v\partial_z V - w\partial_y V$, .. by W, the equations $W=0$, $\square W = 0$, $\square^2 W = 0$ will be equivalent to the system

$$\frac{\partial_x W}{u} = \frac{\partial_y W}{v} = \frac{\partial_z W}{w} = \frac{\Delta W}{\varpi_1 H}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

analogous to (16). More generally, if

$$\begin{aligned}\Delta_1&=u\Delta-\varpi\mathrm{H}\partial_x,\\ \Delta_2&=v\Delta-\varpi\mathrm{H}\partial_y,\\ \Delta_3&=w\Delta-\varpi\mathrm{H}\partial_z,\end{aligned}$$

and if Δ' stands for any of the three symbols Δ_1, Δ_2, Δ_3, then the equations $\mathrm{V}=0$, $\square\mathrm{V}=0$ are equivalent to

$$\frac{1}{u}\partial_x\mathrm{V}=\frac{1}{v}\partial_y\mathrm{V}=\frac{1}{w}\partial_z\mathrm{V};$$

the equations $\square^2\mathrm{V}=0$, $\square^3\mathrm{V}=0$ are equivalent to

$$\frac{1}{u}\partial_x\Delta'\mathrm{V}=\frac{1}{v}\partial_y\Delta'\mathrm{V}=\frac{1}{w}\partial_z\Delta'\mathrm{V}.$$

Similarly, if Δ'' stands for any one of the symbols Δ_1, Δ_2, Δ_3, either the same as Δ' or not, then $\square^4\mathrm{V}=0$, $\square^5\mathrm{V}=0$ are equivalent to

$$\frac{1}{u}\partial_x\Delta''\Delta'\mathrm{V}=\frac{1}{v}\partial_y\Delta''\Delta'\mathrm{V}=\frac{1}{w}\partial_z\Delta''\Delta'\mathrm{V},$$

and so on indefinitely, for $\square^{2i}\mathrm{V}=0$, $\square^{2i+1}\mathrm{V}=0$. If the series should terminate with $\square^{2i}\mathrm{V}=0$, *e. g.* $\square^6\mathrm{V}=0$, then the last equivalent would be $\Delta'''\Delta''\Delta'\mathrm{V}=0$, where Δ''' stands, like Δ'', for any one of the symbols Δ_1, Δ_2, Δ_3 indifferently. The form W, however, presents peculiar advantages for the application of the operations Δ, as will be more fully seen in the sequel. And it follows from what has been said above that, if W retain the same signification as before, we may replace the equations $\mathrm{W}=0$, $\square\mathrm{W}=0$ (and consequently the equations $\square\mathrm{V}=0$, $\square^2\mathrm{V}=0$) by

$$\frac{1}{u}\partial_x\mathrm{W}=\frac{1}{v}\partial_y\mathrm{W}=\frac{1}{w}\partial_z\mathrm{W},$$

and in the same way the equations $\square^2\mathrm{W}=0$, $\square^3\mathrm{W}=0$ (and consequently $\square^3\mathrm{V}=0$, $\square^4\mathrm{V}=0$) by

$$\frac{1}{u}\Delta'\partial_x\mathrm{W}=\frac{1}{v}\Delta'\partial_y\mathrm{W}=\frac{1}{w}\Delta'\partial_z\mathrm{W},$$

and so on. I do not, however, propose on the present occasion to pursue the general theory further.

Returning to the problem of the sextactic points, and forming the equations in W (19), we have

$$\left.\begin{aligned}\frac{1}{u}\partial_x(v\,\partial_z\mathrm{V}-w\partial_y\mathrm{V})&=\frac{1}{v}\partial_y(v\,\partial_z\mathrm{V}-w\partial_y\mathrm{V})=\frac{1}{w}\partial_z(v\,\partial_z\mathrm{V}-w\partial_y\mathrm{V})=\frac{1}{\varpi_1\mathrm{H}}\Delta(v\,\partial_z\mathrm{V}-w\partial_y\mathrm{V})\\ \frac{1}{u}\partial_x(w\partial_x\mathrm{V}-u\,\partial_z\mathrm{V})&=\frac{1}{v}\partial_y(w\partial_x\mathrm{V}-u\,\partial_z\mathrm{V})=\frac{1}{w}\partial_z(w\partial_x\mathrm{V}-u\,\partial_z\mathrm{V})=\frac{1}{\varpi_1\mathrm{H}}\Delta(w\partial_x\mathrm{V}-u\,\partial_z\mathrm{V})\\ \frac{1}{u}\partial_x(u\,\partial_y\mathrm{V}-v\,\partial_x\mathrm{V})&=\frac{1}{v}\partial_y(u\,\partial_y\mathrm{V}-v\,\partial_x\mathrm{V})=\frac{1}{w}\partial_z(u\,\partial_y\mathrm{V}-v\,\partial_x\mathrm{V})=\frac{1}{\varpi_1\mathrm{H}}\Delta(u\,\partial_y\mathrm{V}-v\,\partial_x\mathrm{V}).\end{aligned}\right\}\quad(20)$$

But since W is of the degree n, $\varpi_1=1+\frac{2(n-1)}{n-1}=3$. Also since $\partial_x\mathrm{V}$, $\partial_y\mathrm{V}$, $\partial_z\mathrm{V}$ are linear in x, y, z, it follows that $\Delta\partial_x\mathrm{V}=0$, $\Delta\partial_y\mathrm{V}=0$, $\Delta\partial_z\mathrm{V}=0$; hence, applying the formulæ (17), (18),

$$\Delta v\partial_z\mathrm{V}=q\partial_z\mathrm{V}+2(\mathfrak{A}\,..\,\mathfrak{F}\,..\,)(w',\ v_1,\ u')(\partial_x\partial_z\mathrm{V},\ \partial_y\partial_z\mathrm{V},\ \partial_z^2\mathrm{V}).$$

But since

$$\mathfrak{A}w'+\mathfrak{H}v_1+\mathfrak{G}u'=0,\quad \mathfrak{H}w'+\mathfrak{B}v_1+\mathfrak{G}u'=\mathrm{H},\quad \mathfrak{G}w'+\mathfrak{F}v_1+\mathfrak{C}u'=0,$$

it follows that

$$\Delta v\partial_z\mathrm{V}=q\partial_z\mathrm{V}+2\mathrm{H}\partial_y\partial_z\mathrm{V}.$$

Similarly,

$$\Delta w\partial_y\mathrm{V}=r\partial_y\mathrm{V}+2\mathrm{H}\partial_z\partial_y\mathrm{V},$$

so that (20) become

$$\left.\begin{aligned}
q\partial_z\mathrm{V}-r\partial_y\mathrm{V}&=\frac{3\mathrm{H}}{u}(w'\partial_z\mathrm{V}-v'\,\partial_y\mathrm{V}+2vg-2wh)=\,..\\
r\partial_x\mathrm{V}-p\partial_z\mathrm{V}&=\frac{3\mathrm{H}}{u}(v'\,\partial_x\mathrm{V}-u_1\partial_z\mathrm{V}+2wa-2ug)=\,..\\
p\partial_y\mathrm{V}-q\partial_x\mathrm{V}&=\frac{3\mathrm{H}}{u}(u_1\partial_y\mathrm{V}-w'\partial_x\mathrm{V}+2uh-2va)=\,..\,,
\end{aligned}\right\}\quad\dots\dots\quad(21)$$

whence, multiplying by p, q, r respectively, and adding, we have

$$0=\left.\left\{\begin{vmatrix} p & u_1 & \partial_x\mathrm{V}\\ q & w' & \partial_y\mathrm{V}\\ r & v' & \partial_z\mathrm{V}\end{vmatrix}+2\begin{vmatrix} p & u & a\\ q & v & h\\ r & w & g\end{vmatrix}\right\}\right\}\quad\dots\dots\dots\quad(22)$$

Substituting for $\partial_x\mathrm{V}=\theta u$, $\partial_y\mathrm{V}=\theta v$, $\partial_z\mathrm{V}=\theta w$, (22) becomes

$$\left.\begin{vmatrix} p & u & 2a-\theta u_1\\ q & v & 2h-\theta w'\\ r & w & 2g-\theta v'\end{vmatrix}=0;\right\}\quad\dots\dots\dots\dots\dots\quad(23)$$

and writing

$$vr-wq=\mathrm{X},\quad wp-ur=\mathrm{Y},\quad uq-vp=\mathrm{Z},$$

$$\left.\begin{aligned}
u_1\mathrm{X}+w'\mathrm{Y}+v'\,\mathrm{Z}&=\mathrm{P}\\
w'\mathrm{X}+v_1\,\mathrm{Y}+u'\,\mathrm{Z}&=\mathrm{Q}\\
v'\,\mathrm{X}+u'\,\mathrm{Y}+w_1\mathrm{Z}&=\mathrm{R},
\end{aligned}\right\}\quad\dots\dots\dots\dots\dots\quad(24)$$

(23) takes the form

$$2(a\mathrm{X}+h\mathrm{Y}+g\mathrm{Z})-\theta\mathrm{P}=0;$$

or finally substituting $2(ax+hy+gz)=\theta u$, and forming similar equations in Q and R, we have the system

$$\left.\begin{aligned}
a(u\mathrm{X}-x\mathrm{P})+h(u\mathrm{Y}-y\mathrm{P})+g(u\mathrm{Z}-z\mathrm{P})&=0\\
h(v\,\mathrm{X}-x\mathrm{Q})+b(v\,\mathrm{Y}-y\mathrm{Q})+g(v\,\mathrm{Z}-z\mathrm{Q})&=0\\
g(w\mathrm{X}-x\mathrm{R})+f(w\mathrm{Y}-y\mathrm{R})+c(w\mathrm{Z}-z\mathrm{R})&=0,
\end{aligned}\right\}\quad\dots\dots\quad(25)$$

which may be regarded as the three forms by any one of which $\square^3V=0$ may be replaced. Before proceeding further, it will be convenient to notice that the quantities $uX-xP, \ldots$ are capable of being transformed in a manner which will be useful hereafter, as follows:—

$$\begin{aligned} Px &= Xu_1x+(ww'-vv')px+(v'q-w'r)ux \\ &= Xu_1x+(ww'-vv')(3\overline{n-2}H-qy-rz)-(v'q-w'r)(vy-wz) \\ &= X(u_1x+w'y+v'z)+3(n-2)H(ww'-vv') \\ &= (n-1)uX+3(n-2)H(ww'-vv'), \end{aligned}$$

i. e.

$$\left.\begin{aligned} -uX+xP &= (n-2)\{uX-3H(vv'-ww')\} \\ -uY+yP &= (n-2)\{uY-3H(wu_1-uv')\} \\ -uZ+zP &= (n-2)\{uZ-3H(uv'-vu_1)\}. \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (26)$$

Returning to (25), and taking any one of the three as W, we shall have for $\square^3V=0$, $\square^4V=0$, $\square^5V=0$,

$$\left.\begin{aligned} a\partial_x(uX-xP)+h\partial_x(uY-yP)+g\partial_x(uZ-zP)-\quad\theta_2u &= 0 \\ a\partial_y(uX-xP)+h\partial_y(uY-yP)+g\partial_y(uZ-zP)-\quad\theta_2v &= 0 \\ a\partial_z(uX-xP)+h\partial_z(uY-yP)+g\partial_z(uZ-zP)-\quad\theta_2w &= 0 \\ a\Delta(uX-xP)+h\Delta(uY-yP)+g\Delta(uZ-zP)-\varpi_2\theta_2H &= 0\,; \end{aligned}\right\} \quad \ldots \quad (27)$$

and similar groups may be formed from the other two equations of (25). Now as (27) contain only three out of the six constants $a, \ldots f_1 \ldots$, and the single indeterminate θ_2, they are sufficient for the elimination in view, and give for the equation whereby the sextactic points are to be determined,

$$\left.\begin{vmatrix} \partial_x(uX-xP) & \partial_x(uY-yP) & \partial_x(uZ-zP) & u \\ \partial_y(uX-xP) & \partial_y(uY-yP) & \partial_y(uZ-zP) & u \\ \partial_z(uX-xP) & \partial_z(uY-yP) & \partial_z(uZ-zP) & w \\ \Delta(uX-xP) & \Delta(uY-yP) & \Delta(uZ-zP) & \varpi_2H \end{vmatrix}=0,\right\} \quad \ldots \quad (28)$$

which, in virtue of (26), may also be written in the form

$$\left.\begin{vmatrix} \partial_x\{uX-3H)vv'-ww')\} & \partial_x\{uY-3H(wu_1-uv')\} & \partial_x\{uZ-3H(uv'-vu_1)\} & u \\ \partial_y\{uX-3H)vv'-ww')\} & \partial_y\{uY-3H(wu_1-uv')\} & \partial_y\{uZ-3H(uv'-vu_1)\} & v \\ \partial_z\{uX-3H)vv'-ww')\} & \partial_z\{uY-3H(wu_1-uv')\} & \partial_z\{uZ-3H(uv'-vu_1)\} & w \\ \Delta\{uX-3H)vv'-ww')\} & \Delta\{uY-3H(wu_1-uv')\} & \Delta\{uZ-3H(uv'-vu_1)\} & \varpi_2H \end{vmatrix}=0,\right\} \quad (29)$$

with similar expressions in v, Q; w, R. Calling (28) and (29) $\mathfrak{L}$, $\mathfrak{L}'$ respectively, we may designate the entire group of six forms, three of the form (28), and three of the form (29) by

$$\mathfrak{L}=0,\ \mathfrak{M}=0,\ \mathfrak{N}=0,\ \mathfrak{L}'=0,\ \mathfrak{M}'=0,\ \mathfrak{N}=0. \quad \ldots\ldots \quad (30)$$

And as $\mathfrak{L}$, $\mathfrak{L}'$ differ only in respect of a numerical factor, any other factor that can be predicated of $\mathfrak{L}$ may be affirmed of $\mathfrak{L}'$, and *vice versâ*; and similarly for the other pairs $\mathfrak{M}$, $\mathfrak{M}'$; $\mathfrak{N}$, $\mathfrak{N}'$.

§ 3. *Formulæ of Reduction.*

The degree of the expressions (28) or (29) is $18n-36$; it remains to show that existence of certain extraneous factors, which when divided out will reduce the degree to $12n-27$, and at the same time render the three forms identical. But before entering upon this, it will be convenient to premise the following formulæ, the first group of which are easily verified.

$$\left.\begin{aligned}
yZ-zY&=3(n-2)Hu\\
zX-xZ&=3(n-2)Hv\\
xY-yX&=3(n-2)Hw\\[6pt]
y\partial_xZ-z\partial_xY&=(3n-7)up-(n-1)up+3(n-2)Hu_1\\
y\partial_yZ-z\partial_yY&=(3n-7)uq-(n-1)vp+3(n-2)Hw'\\
y\partial_zZ-z\partial_zY&=(3n-7)ur-(n-1)wp+3(n-2)Hv'\\[6pt]
z\partial_xX-x\partial_xZ&=(3n-7)vp-(n-1)uq+3(n-2)Hw'\\
z\partial_yX-x\partial_yZ&=(3n-7)vq-(n-1)vq+3(n-2)Hv_1\\
z\partial_zX-x\partial_zZ&=(3n-7)vr-(n-1)wq+3(n-2)Hu'\\[6pt]
x\partial_xY-y\partial_xX&=(3n-7)wp-(n-1)ur+3(n-2)Hv'\\
x\partial_yY-y\partial_yX&=(3n-7)wq-(n-1)vr+3(n-2)Hu'\\
x\partial_zY-y\partial_zX&=(3n-7)wr-(n-1)wr+3(n-2)Hw_1.
\end{aligned}\right\}\quad\ldots\ldots\quad(31)$$

And writing

$$\left.\begin{aligned}
-P_1&=Xp_1+Yr'+Zq'\\
-Q_1&=Xr'+Yq_1+Zp'\\
-R_1&=Xq'+Yp'+Zr_1,
\end{aligned}\right\}\quad\ldots\ldots\ldots\ldots\quad(32)$$

then also

$$\left.\begin{aligned}
&Y\partial_xZ-Z\partial_xY=-(pP+uP_1)\quad Z\partial_xX-X\partial_xZ=-(qP+vP_1)\quad X\partial_xY-Y\partial_xX=-(rP+wP_1)\\
&Y\partial_yZ-Z\partial_yY=-(pQ+uQ_1)\quad Z\partial_yX-X\partial_yZ=-(qQ+vQ_1)\quad X\partial_yY-Y\partial_yX=-(rQ+wQ_1)\\
&Y\partial_zZ-Z\partial_zY=-(pR+uR_1)\quad Z\partial_zX-X\partial_zZ=-(qR+vR_1)\quad X\partial_zY-Y\partial_zX=-(rR+wR_1),
\end{aligned}\right\}(33)$$

Moreover, writing with Professor CAYLEY,

$$\left.\begin{aligned}
&(\mathfrak{A},\mathfrak{B},\mathfrak{C},\mathfrak{F},\mathfrak{G},\mathfrak{H})(\partial_x,\partial_y,\partial_z)^2H=\Omega\\
&\partial_x\Omega_{\overline{U}}=(\mathfrak{A},\mathfrak{B},\mathfrak{C},\mathfrak{F},\mathfrak{G},\mathfrak{H})(\partial_x,\partial_y,\partial_z)^2p,\ \partial_y\Omega_{\overline{U}}=\ldots,\ \partial_z\Omega_{\overline{U}}=\ldots\\
&\partial_x\Omega_{\overline{H}}=(\partial_x\mathfrak{A},\partial_x\mathfrak{B},\partial_x\mathfrak{C},\partial_x\mathfrak{F},\partial_x\mathfrak{G},\partial_x\mathfrak{H})(\partial_x,\partial_y,\partial_z)^2H,\ \partial_y\Omega_{\overline{H}}=\ldots,\partial_z\Omega_{\overline{H}}=\ldots,
\end{aligned}\right\}(34)$$

and noticing that

$$\mathrm{X}\partial_x\Omega_{\mathrm{U}}+\mathrm{Y}\partial_y\Omega_{\mathrm{U}}+\mathrm{Z}\partial_z\Omega_{\mathrm{U}}=\text{Jac.}\,(\mathrm{U},\ \mathrm{H},.\ \Omega_{\mathrm{U}}), \quad . \quad . \quad . \quad . \quad . \quad . \quad (35)$$

and that

$$\left.\begin{aligned}\Delta\mathrm{X}&= v\partial_z\Omega_{\mathrm{U}}-w\partial_y\Omega_{\mathrm{U}}\\ \Delta\mathrm{Y}&=w\partial_x\Omega_{\mathrm{U}}-u\partial_z\Omega_{\mathrm{U}}\\ \Delta\mathrm{Z}&= u\partial_y\Omega_{\mathrm{U}}-v\partial_x\Omega_{\mathrm{U}},\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (36)$$

then we have

$$\left.\begin{aligned}\mathrm{Y}\,\Delta\mathrm{Z}-\mathrm{Z}\,\Delta\mathrm{Y}&=u\,\text{Jac.}\,(\mathrm{U},\ \mathrm{H},\ \Omega_{\mathrm{U}}) \quad & y\Delta\mathrm{Z}-z\,\Delta\mathrm{Y}&=(5n-12)\Omega u\\ \mathrm{Z}\,\Delta\mathrm{X}-\mathrm{X}\Delta\mathrm{Z}&=v\,\text{Jac.}\,(\mathrm{U},\ \mathrm{H},\ \Omega_{\mathrm{U}}) \quad & z\Delta\mathrm{X}-x\Delta\mathrm{Z}&=(5n-12)\Omega v\\ \mathrm{X}\Delta\mathrm{Y}-\mathrm{Y}\Delta\mathrm{X}&=w\,\text{Jac.}\,(\mathrm{U},\ \mathrm{H},\ \Omega_{\mathrm{U}}) \quad & x\Delta\mathrm{Y}-y\Delta\mathrm{X}&=(5n-12)\Omega w.\end{aligned}\right\} \quad . \quad . \quad (37)$$

Again, if $\mathfrak{A}'$, $\mathfrak{B}'$, $\mathfrak{C}'$, $\mathfrak{F}'$, $\mathfrak{G}'$, $\mathfrak{H}'$ be the same quantities with respect to H that $\mathfrak{A}$, $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{F}$, $\mathfrak{G}$, $\mathfrak{H}$ are with respect to U, *i. e.* if $\mathfrak{A}'=q_1r_1-p'^2, \ldots \mathfrak{F}'=q'r'-p_1p', \ldots$, and if

$$\left.\begin{aligned}\Theta&=(\mathfrak{A}',\ \mathfrak{B}',\ \mathfrak{C}',\ \mathfrak{F}',\ \mathfrak{G}',\ \mathfrak{H}')(u,\ v,\ w)^2\\ \Psi&=(\mathfrak{A},\ \mathfrak{B},\ \mathfrak{C},\ \mathfrak{F},\ \mathfrak{G},\ \mathfrak{H})(p,\ q,\ r)^2,\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (38)$$

then

$$\begin{matrix} u_1 & \partial_x\mathrm{Y} & \partial_x\mathrm{Z}\\ w' & \partial_y\mathrm{Y} & \partial_y\mathrm{Z}\\ v' & \partial_z\mathrm{Y} & \partial_z\mathrm{Z}\end{matrix} =\text{Jac.}\,(u,\ \mathrm{Y},\ \mathrm{Z},)= \begin{matrix} u_1 & v'p-u_1r+wp_1-uq' & u_1q-w'p+ur'-vp_1\\ w' & u'p-w'r+wr'-up' & w'q-v_1p+uq_1-vr'\\ v' & w_1p-v'r+wq'-ur_1 & v'q-u'p+up'-vq'\end{matrix}$$

$$\begin{array}{lll} =\mathrm{H}p^2 & =\mathrm{H}p^2-\Omega pu+(\mathfrak{A}p_1+\mathfrak{H}r'+\mathfrak{G}q')pu & =\mathrm{H}p^2-\Omega pu+\frac{3n-7}{n-1}\mathrm{H}p^2+\frac{1}{2}u\partial_x\Theta_{\mathrm{U}}\\ \quad -(\mathfrak{H}r'+\mathfrak{B}q_1+\mathfrak{F}p')pu & \quad +(\mathfrak{H}p_1+\mathfrak{B}r'+\mathfrak{F}q')pv & \\ \quad +(\mathfrak{H}p_1+\mathfrak{B}r'+\mathfrak{F}q')pv & \quad +(\mathfrak{G}p_1+\mathfrak{F}r'+\mathfrak{C}q')pw & \\ \quad +(\mathfrak{G}p_1+\mathfrak{F}r'+\mathfrak{C}q')wp & \quad +(\mathfrak{A}'u_1+\mathfrak{H}'w'+\mathfrak{G}'v')u^2 & \\ \quad +(\mathfrak{G}'u_1+\mathfrak{F}'w'+\mathfrak{C}'v')wu & \quad +(\mathfrak{H}'u_1+\mathfrak{B}'w'+\mathfrak{F}'v')uv & \\ \quad -(\mathfrak{G}q'+\mathfrak{F}p'+\mathfrak{C}r_1)pu & \quad +(\mathfrak{G}'u_1+\mathfrak{F}'w'+\mathfrak{C}v')uw & \\ \quad +(\mathfrak{A}'u_1+\mathfrak{H}'w'+\mathfrak{G}'v')u^2 & & \\ \quad +(\mathfrak{H}'u_1+\mathfrak{B}'w'+\mathfrak{F}'v')uv & & \end{array}$$

Similarly,

$$\text{Jac.}\,(u,\ \mathrm{Z},\ \mathrm{X})=\begin{matrix} u_1 & u_1q-w'p+ur'-vp_1 & w'r-v'q+vq'-wr'\\ w' & w'q-v_1p+uq_1-vr' & v_1r-u'q+vp'-wq_1\\ v' & v'q-u'p+up'-vq' & u'r-w_1q+vr_1-wp'\end{matrix}$$

$$\begin{array}{lll}
=\mathrm{H}pq & =\mathrm{H}pq-\Omega pv-(\mathfrak{A}r'+\mathfrak{H}q_1+\mathfrak{G}p')up & =\mathrm{H}pq-\Omega pv+\tfrac{1}{2}v\partial_x\Theta_{\mathrm{U}}+\frac{3n-7}{n-1}\mathrm{H}pq \\
-(\mathfrak{G}q'+\mathfrak{F}p'+\mathfrak{C}r_1)vp & -(\mathfrak{H}r'+\mathfrak{B}q_1+\mathfrak{F}p')uq & +\tfrac{1}{2}v\partial_x\Psi_{\mathrm{U}}-\tfrac{1}{2}u\partial_y\Psi_{\mathrm{U}} \\
+(\mathfrak{G}r'+\mathfrak{F}q_1+\mathfrak{C}p')wp & -(\mathfrak{G}r'+\mathfrak{F}q_1+\mathfrak{C}p')ur & \\
-(\mathfrak{G}r'+\mathfrak{F}q_1+\mathfrak{C}p')ur & +(\mathfrak{A}r'+\mathfrak{H}q_1+\mathfrak{G}p')up & \\
-(\mathfrak{H}r'+\mathfrak{B}q_1+\mathfrak{F}p')uq & +(\mathfrak{H}r'+\mathfrak{B}q_1+\mathfrak{F}p')vp & \\
+(\mathfrak{A}'u_1+\mathfrak{H}'w'+\mathfrak{G}'v')uv & +(\mathfrak{G}r'+\mathfrak{F}q_1+\mathfrak{C}p')wp & \\
+(\mathfrak{G}p_1+\mathfrak{F}r'+\mathfrak{C}q')vr & +(\mathfrak{A}p_1+\mathfrak{H}r'+\mathfrak{G}q')pv & \\
+(\mathfrak{H}p_1+\mathfrak{B}r'+\mathfrak{F}q')vq & +(\mathfrak{H}p_1+\mathfrak{B}r'+\mathfrak{F}q')qv & \\
+(\mathfrak{H}'u_1+\mathfrak{B}'w'+\mathfrak{F}'v')v^2 & +(\mathfrak{G}p_1+\mathfrak{F}r'+\mathfrak{C}q')rv & \\
+(\mathfrak{G}'u_1+\mathfrak{F}'w'+\mathfrak{C}'v')vw \quad +\tfrac{1}{2}v\partial_x\Theta_{\mathrm{U}} & &
\end{array}$$

Hence

$$\begin{aligned}
&\text{Jac.}\,(u,\ \mathrm{Y},\ \mathrm{Z})=\frac{4(n-2)}{n-1}\,\mathrm{H}p^2-(\Omega p-\tfrac{1}{2}\partial_x\Theta_{\mathrm{U}})u \\
&\text{Jac.}\,(u,\ \mathrm{Z},\ \mathrm{X})=\frac{4(n-2)}{n-1}\,\mathrm{H}pq-(\Omega p-\tfrac{1}{2}\partial_x\Theta_{\mathrm{U}})v+\tfrac{1}{2}u\partial_x\Psi_{\mathrm{U}}-\tfrac{1}{2}u\partial_y\Psi_{\mathrm{U}} \\
&\text{Jac.}\,(u,\ \mathrm{X},\ \mathrm{Y})=\frac{4(n-2)}{n-1}\,\mathrm{H}pr-(\Omega p-\tfrac{1}{2}\partial_x\Theta_{\mathrm{U}})w+\tfrac{1}{2}w\partial_x\Psi_{\mathrm{U}}-\tfrac{1}{2}u\partial_z\Psi_{\mathrm{U}} \\[1ex]
&\text{Jac.}\,(v,\ \mathrm{Y},\ \mathrm{Z})=\frac{4(n-2)}{n-1}\,\mathrm{H}qp-(\Omega q-\tfrac{1}{2}\partial_y\Theta_{\mathrm{U}})u+\tfrac{1}{2}u\partial_y\Psi_{\mathrm{U}}-v\partial_x\Psi_{\mathrm{U}} \\
&\text{Jac.}\,(v,\ \mathrm{Z},\ \mathrm{X})=\frac{4(n-2)}{n-1}\,\mathrm{H}q^2-(\Omega q-\tfrac{1}{2}\partial_y\Theta_{\mathrm{U}})v \qquad\qquad (39) \\
&\text{Jac.}\,(v,\ \mathrm{X},\ \mathrm{Y})=\frac{4(n-2)}{n-1}\,\mathrm{H}qr-(\Omega q-\tfrac{1}{2}\partial_y\Theta_{\mathrm{U}})w+\tfrac{1}{2}w\partial_y\Psi_{\mathrm{U}}-v\partial_z\Psi_{\mathrm{U}} \\[1ex]
&\text{Jac.}\,(w,\ \mathrm{Y},\ \mathrm{Z})=\frac{4(n-2)}{n-1}\,\mathrm{H}rp-(\Omega r-\tfrac{1}{2}\partial_z\Theta_{\mathrm{U}})u+\tfrac{1}{2}u\partial_z\Psi_{\mathrm{U}}-\tfrac{1}{2}w\partial_x\Psi_{\mathrm{U}} \\
&\text{Jac.}\,(w,\ \mathrm{Z},\ \mathrm{X})=\frac{4(n-2)}{n-1}\,\mathrm{H}rq-(\Omega r-\tfrac{1}{2}\partial_z\Theta_{\mathrm{U}})v+\tfrac{1}{2}v\partial_z\Psi_{\mathrm{U}}-\tfrac{1}{2}w\partial_y\Psi_{\mathrm{U}} \\
&\text{Jac.}\,(w,\ \mathrm{X},\ \mathrm{Y})=\frac{4(n-2)}{n-1}\,\mathrm{H}r^2-(\Omega r-\tfrac{1}{2}\partial_z\Theta_{\mathrm{U}})w.
\end{aligned}$$

Again,

$$x\,\text{Jac.}\,(u,\ \mathrm{Y},\ \mathrm{Z})+y\,\text{Jac.}\,(v,\ \mathrm{Y},\ \mathrm{Z})+z\,\text{Jac.}\,(w,\ \mathrm{Y},\ \mathrm{Z})=(n-1)\,\text{Jac.}\,(\mathrm{U},\ \mathrm{Y},\ \mathrm{Z});$$

whence, bearing in mind that

$$\begin{aligned}
x\partial_x\Psi_{\mathrm{U}}+y\partial_y\Psi_{\mathrm{U}}+z\partial_z\Psi_{\mathrm{U}}&=2(3n-7)\Psi_{\mathrm{U}}, \\
x\partial_x\Theta_{\mathrm{U}}+y\partial_y\Theta_{\mathrm{U}}+z\partial_z\Theta_{\mathrm{U}}&=2(n-1)\Theta_{\mathrm{U}},
\end{aligned}$$

because in the differentiations $\mathfrak{A}, \dots \mathfrak{A}', \dots$ are supposed constant, it follows that

$$\left.\begin{aligned}\text{Jac.}\,(\mathrm{U}, \mathrm{Y}, \mathrm{Z})&=\frac{12(n-2)^2}{(n-1)^2}\mathrm{H}^2 p+\left\{\frac{3n-7}{n-1}\Psi-\frac{3(n-2)}{n-1}\mathrm{H}\Omega+\Theta\right\}u\\ \text{Jac.}\,(\mathrm{U}, \mathrm{Z}, \mathrm{X})&=\frac{12(n-2)^2}{(n-1)^2}\mathrm{H}^2 q+\left\{\frac{3n-7}{n-1}\Psi-\frac{3(n-2)}{n-1}\mathrm{H}\Omega+\Theta\right\}v\\ \text{Jac.}\,(\mathrm{U}, \mathrm{X}, \mathrm{Y})&=\frac{12(n-2)^2}{(n-1)^2}\mathrm{H}^2 r+\left\{\frac{3n-7}{n-1}\Psi-\frac{3(n-2)}{n-1}\mathrm{H}\Omega+\Theta\right\}w.\end{aligned}\right\} \quad . \quad . \quad (40)$$

Again,

$$\begin{matrix} u_1 & \partial_x\mathrm{Y} & \partial_x\mathrm{Z} \\ w' & \partial_y\mathrm{Y} & \partial_y\mathrm{Z} \\ v' & \partial_z\mathrm{Y} & \partial_z\mathrm{Z} \end{matrix} = \begin{matrix} u_1 & v'p-u_1r+wp_1-uq' & u_1q-w'p+ur'-vp_1 \\ w' & u'p-w'r+wr'-up' & w'q-v_1p+uq_1-vr' \\ v' & w_1p-v'r+wq'-ur_1 & v'q-u'p+up'-vq' \end{matrix}$$

$$\begin{aligned}&=p^2\mathrm{H}-(\mathfrak{H}r'+\mathfrak{B}q_1+\mathfrak{F}p')up+(\mathfrak{G}'u_1+\mathfrak{F}'w'+\mathfrak{C}'v')wu\\ &\quad+(\mathfrak{G}p_1+\mathfrak{F}r'+\mathfrak{C}q')wp+(\mathfrak{A}'u_1+\mathfrak{H}'w'+\mathfrak{G}'v')u^2\\ &\quad-(\mathfrak{G}q'+\mathfrak{F}p'+\mathfrak{C}r_1)up+(\mathfrak{H}'u_1+\mathfrak{B}'w'+\mathfrak{F}'v')uv\\ &\quad+(\mathfrak{H}p_1+\mathfrak{B}r'+\mathfrak{F}q')vp\end{aligned}$$

$$=p^2\mathrm{H}+\frac{3n-7}{n-1}p^2\mathrm{H}-\Omega up+(\mathfrak{A}', \dots \mathfrak{F}', \dots)(u, v, w)(u_1, w', v')u.$$

Whence

$$\left.\begin{aligned}\text{Jac.}\,(u, \mathrm{Y}, \mathrm{Z})&=\frac{4(n-2)}{n-1}\mathrm{H}p^2-\Omega up+(\mathfrak{A}', \dots \mathfrak{F}', \dots)(u, v, w)(u_1, w', v')u\\ \text{Jac.}\,(u, \mathrm{Z}, \mathrm{X})&=\frac{4(n-2)}{n-1}\mathrm{H}pq-\Omega uq+(\mathfrak{A}', \dots \mathfrak{F}', \dots)(u, v, w)(w', v_1, u')u\\ \text{Jac.}\,(u, \mathrm{X}, \mathrm{Y})&=\frac{4(n-2)}{n-1}\mathrm{H}pr-\Omega ur+(\mathfrak{A}', \dots \mathfrak{F}', \dots)(u, v, w)(v', u', w_1)u.\end{aligned}\right\} \quad (41)$$

A similar process of reduction conducts to the relation

$$\begin{aligned}\text{Jac.}\,(\mathrm{X}, \mathrm{Y}, \mathrm{Z})=&-(\mathfrak{A}, \dots \mathfrak{F}, \dots)(p, q, r)(p_1, r', q')\mathrm{X}-(\mathfrak{A}', \dots \mathfrak{F}', \dots)(u, v, w)(u_1, w', v')\mathrm{X}\\ &-(\mathfrak{A}, \dots \mathfrak{F}, \dots)(p, q, r)(r', q_1, p')\mathrm{Y}-(\mathfrak{A}', \dots \mathfrak{F}', \dots)(u, v, w)(w', v_1, u')\mathrm{Y}\\ &-(\mathfrak{A}, \dots \mathfrak{F}, \dots)(p, q, r)(q', p', r_1)\mathrm{Z}-(\mathfrak{A}', \dots \mathfrak{F}', \dots)(u, v, w)(v', u', w_1)\mathrm{Z}\\ =&-\text{Jac.}\,(\mathrm{U}, \mathrm{H}, \Psi_{\mathrm{U}})-\text{Jac.}\,(\mathrm{U}, \mathrm{H}, \Theta_{\mathrm{U}}).\end{aligned}$$

Whence also

$$\left.\begin{aligned}\text{Jac.}\,(u\mathrm{X}, u\mathrm{Y}, u\mathrm{Z})&=u^3\text{Jac.}\,(\mathrm{X}, \mathrm{Y}, \mathrm{Z})+u^2\{\mathrm{X}\,\text{Jac.}\,(u, \mathrm{Y}, \mathrm{Z})+\mathrm{Y}\,\text{Jac.}\,(\mathrm{X}, u, \mathrm{Z})+\mathrm{Z}\,\text{Jac.}\,(\mathrm{X}, \mathrm{Y}, u)\}\\ &=-u^3\text{Jac.}\,(\mathrm{U}, \mathrm{H}, \Psi_{\mathrm{U}}).\end{aligned}\right\}(42)$$

§ 4.

The resultant equation which, when combined with that of the original curve, will determine the sextactic points, was exhibited in § 2 under six different forms, there designated by

$$\mathfrak{L}=0,\quad \mathfrak{M}=0,\quad \mathfrak{N}=0,\quad \mathfrak{L}'=0,\quad \mathfrak{M}'=0,\quad \mathfrak{N}'=0.$$

Now since $\mathfrak{L}$ and $\mathfrak{L}'$, $\mathfrak{M}$ and $\mathfrak{M}'$, $\mathfrak{N}$ and $\mathfrak{N}'$ respectively differ only by the numerical factor $(n-2)^3$, we shall, in seeking to discover the extraneous factors, employ either $\mathfrak{L}, \ldots$ or $\mathfrak{L}', \ldots$ as most convenient for the purpose. And in the first place it will be shown that H is a factor of all these expressions. Putting $\mathrm{H}=0$, $\mathfrak{L}'$ becomes

$$\left.\begin{array}{llll} \partial_x u\mathrm{X} & \partial_x u\mathrm{Y} & \partial_x u\mathrm{Z} & u \quad =0\,; \\ \partial_y u\mathrm{X} & \partial_y u\mathrm{Y} & \partial_y u\mathrm{Z} & v \\ \partial_z u\mathrm{X} & \partial_z u\mathrm{Y} & \partial_z u\mathrm{Z} & w \\ \Delta u\mathrm{X} & \Delta u\mathrm{Y} & \Delta u\mathrm{Z} & \varpi_2\mathrm{H} \end{array}\right\} \quad \ldots \ldots \ldots \quad (43)$$

also

$$\left.\begin{array}{l} \Delta u\mathrm{X}=p\mathrm{X}+u\Delta\mathrm{X}+2\mathrm{H}\partial_x\mathrm{X} \\ \Delta u\mathrm{Y}=p\mathrm{Y}+u\Delta\mathrm{Y}+2\mathrm{H}\partial_x\mathrm{Y} \\ \Delta u\mathrm{Z}=p\mathrm{Z}+u\Delta\mathrm{Z}+2\mathrm{H}\partial_x\mathrm{Z}\,; \end{array}\right\} \quad \ldots \ldots \ldots \quad (44)$$

so that the above equation, written in full, is

$$\begin{array}{llll} u_1\mathrm{X}+u\partial_x\mathrm{X} & u_1\mathrm{Y}+u\partial_x\mathrm{Y} & u_1\mathrm{Z}+u\partial_x\mathrm{Z} & u \\ w'\mathrm{X}+u\partial_y\mathrm{X} & w'\mathrm{Y}+u\partial_y\mathrm{Y} & w'\mathrm{Z}+u\partial_y\mathrm{Z} & v \\ v'\mathrm{X}+u\partial_z\mathrm{X} & v'\mathrm{Y}+u\partial_z\mathrm{Y} & v'\mathrm{Z}+u\partial_z\mathrm{Z} & w \\ p\,\mathrm{X}+u\Delta\mathrm{X}+2\mathrm{H}\partial_x\mathrm{X} & p\,\mathrm{Y}+u\Delta\mathrm{Y}+2\mathrm{H}\partial_x\mathrm{Y} & p\,\mathrm{Z}+u\Delta\mathrm{Z}+2\mathrm{H}\partial_x\mathrm{Z} & \varpi_2\mathrm{H}. \end{array}$$

Although this expression contains terms explicitly multiplied by H, which might on the present supposition be omitted, it will still perhaps be worth while to develope it completely. Expanding in the usual way, it becomes

$$u^2\mathrm{X}\begin{array}{llll} u_1 & \partial_x\mathrm{Y} & \partial_x\mathrm{Z} & u \\ w' & \partial_y\mathrm{Y} & \partial_y\mathrm{Z} & v \\ v' & \partial_z\mathrm{Y} & \partial_z\mathrm{Z} & w \\ p & \Delta\mathrm{Y} & \Delta\mathrm{Z} & \varpi_2\mathrm{H} \end{array} +u^2\mathrm{Y}\begin{array}{llll} u_1 & \partial_x\mathrm{Z} & \partial_x\mathrm{X} & u \\ w' & \partial_y\mathrm{Z} & \partial_y\mathrm{X} & v \\ v' & \partial_z\mathrm{Z} & \partial_z\mathrm{X} & w \\ p & \Delta\mathrm{Z} & \Delta\mathrm{X} & \varpi_2\mathrm{H} \end{array} +u^2\mathrm{Z}\begin{array}{llll} u_1 & \partial_x\mathrm{X} & \partial_x\mathrm{Y} & u \\ w' & \partial_y\mathrm{X} & \partial_y\mathrm{Y} & v \\ v' & \partial_z\mathrm{X} & \partial_z\mathrm{Y} & w \\ p & \Delta\mathrm{X} & \Delta\mathrm{Y} & \varpi_2\mathrm{H} \end{array} +u^3\begin{array}{llll} \partial_x\mathrm{X} & \partial_x\mathrm{Y} & \partial_x\mathrm{Z} & u \\ \partial_y\mathrm{X} & \partial_y\mathrm{Y} & \partial_y\mathrm{Z} & v \\ \partial_z\mathrm{X} & \partial_z\mathrm{Y} & \partial_z\mathrm{Z} & w \\ \Delta\mathrm{X} & \Delta\mathrm{Y} & \Delta\mathrm{Z} & \varpi_2\mathrm{H} \end{array}$$

$$+\mathrm{H}\begin{array}{llll} u_1\mathrm{X}+u\partial_x\mathrm{X} & u_1\mathrm{Y}+u\partial_x\mathrm{Y} & u_1\mathrm{Z}+u\partial_x\mathrm{Z} & u \\ w'\mathrm{X}+u\partial_y\mathrm{X} & w'\mathrm{Y}+u\partial_y\mathrm{Y} & w'\mathrm{Z}+u\partial_y\mathrm{Z} & v \\ v'\mathrm{X}+u\partial_z\mathrm{X} & v'\mathrm{Y}+u\partial_z\mathrm{Y} & v'\mathrm{Z}+u\partial_z\mathrm{Z} & w \\ 2\partial_x\mathrm{X} & 2\partial_x\mathrm{Y} & 2\partial_x\mathrm{Z} & \varpi_2. \end{array}$$

In this the coefficient of $-p$

$$=\tfrac{1}{2}\left\{\begin{matrix}\partial_x\mathrm{X} & \mathrm{Z}\partial_x\mathrm{Y}-\mathrm{Y}\partial_x\mathrm{Z} & u\\ \partial_y\mathrm{X} & \mathrm{Z}\partial_y\mathrm{Y}-\mathrm{Y}\partial_y\mathrm{Z} & v\\ \partial_z\mathrm{X} & \mathrm{Z}\partial_z\mathrm{Y}-\mathrm{Y}\partial_z\mathrm{Z} & w\end{matrix}\;+\begin{matrix}\partial_x\mathrm{Y} & \mathrm{X}\partial_x\mathrm{Z}-\mathrm{Z}\partial_x\mathrm{X} & u\\ \partial_y\mathrm{Y} & \mathrm{X}\partial_y\mathrm{Z}-\mathrm{Z}\partial_y\mathrm{X} & v\\ \partial_z\mathrm{Y} & \mathrm{X}\partial_z\mathrm{Z}-\mathrm{Z}\partial_z\mathrm{X} & w\end{matrix}\;+\begin{matrix}\partial_x\mathrm{Z} & \mathrm{Y}\partial_x\mathrm{X}-\mathrm{X}\partial_x\mathrm{Y} & u\\ \partial_y\mathrm{Z} & \mathrm{Y}\partial_y\mathrm{X}-\mathrm{X}\partial_y\mathrm{Y} & v\\ \partial_z\mathrm{Z} & \mathrm{Y}\partial_z\mathrm{X}-\mathrm{X}\partial_z\mathrm{Y} & w\end{matrix}\right\}$$

$$=\tfrac{1}{2}\left\{\begin{matrix}p\partial_x\mathrm{X}+q\partial_x\mathrm{Y}+r\partial_x\mathrm{Z} & \mathrm{P} & u\\ p\partial_y\mathrm{X}+q\partial_y\mathrm{Y}+r\partial_y\mathrm{Z} & \mathrm{Q} & v\\ p\partial_z\mathrm{X}+q\partial_z\mathrm{Y}+r\partial_z\mathrm{Z} & \mathrm{R} & w\end{matrix}\;+\begin{matrix}u\partial_x\mathrm{X}+v\partial_x\mathrm{Y}+w\partial_x\mathrm{Z} & \mathrm{P}_1 & u\\ u\partial_y\mathrm{X}+v\partial_y\mathrm{Y}+w\partial_y\mathrm{Z} & \mathrm{Q}_1 & v\\ u\partial_z\mathrm{X}+v\partial_z\mathrm{Y}+w\partial_z\mathrm{Z} & \mathrm{R}_1 & w\end{matrix}\right\}=\begin{matrix}\mathrm{P}_1 & \mathrm{P} & u\\ \mathrm{Q}_1 & \mathrm{Q} & v\\ \mathrm{R}_1 & \mathrm{R} & w.\end{matrix}$$

Now

$$\begin{matrix}u & u_1 & \mathrm{P}_1\\ v & w' & \mathrm{Q}_1\\ w & v' & \mathrm{R}_1\end{matrix}=\frac{1}{n-1}\{z(\mathfrak{H}\mathrm{P}_1+\mathfrak{B}\mathrm{Q}_1+\mathfrak{F}\mathrm{R}_1)-y(\mathfrak{G}\mathrm{P}_1+\mathfrak{F}\mathrm{Q}_1+\mathfrak{C}\mathrm{R}_1)\}$$

$$\begin{matrix}u & w' & \mathrm{P}_1\\ v & v_1 & \mathrm{Q}_1\\ w & u' & \mathrm{R}_1\end{matrix}=\frac{1}{n-1}\{x(\mathfrak{G}\mathrm{P}_1+\mathfrak{F}\mathrm{Q}_1+\mathfrak{C}\mathrm{R}_1)-z(\mathfrak{A}\mathrm{P}_1+\mathfrak{H}\mathrm{Q}_1+\mathfrak{G}\mathrm{R}_1)\}$$

$$\begin{matrix}u & v' & \mathrm{P}_1\\ v & u' & \mathrm{Q}_1\\ w & w_1 & \mathrm{R}_1\end{matrix};\quad=\frac{1}{n-1}\{y(\mathfrak{A}\mathrm{P}_1+\mathfrak{H}\mathrm{Q}_1+\mathfrak{G}\mathrm{R}_1)-x(\mathfrak{H}\mathrm{P}_1+\mathfrak{B}\mathrm{Q}_1+\mathfrak{F}\mathrm{R}_1)\}$$

so that multiplying these equations by **X, Y, Z** respectively, and adding,

$$\left.\begin{aligned}\begin{matrix}u & \mathrm{P} & \mathrm{P}_1\\ v & \mathrm{Q} & \mathrm{Q}_1\\ w & \mathrm{R} & \mathrm{R}_1\end{matrix}&=\frac{1}{n-1}\{(\mathfrak{A}\mathrm{P}_1+\mathfrak{H}\mathrm{Q}_1+\mathfrak{G}\mathrm{R}_1)(y\mathrm{Z}-z\mathrm{Y})\\ &\qquad+(\mathfrak{H}\mathrm{P}_1+\mathfrak{B}\mathrm{Q}_1+\mathfrak{F}\mathrm{R}_1)(z\mathrm{X}-x\mathrm{Z})\\ &\qquad+(\mathfrak{G}\mathrm{P}_1+\mathfrak{F}\mathrm{Q}_1+\mathfrak{C}\mathrm{R}_1)(x\mathrm{Y}-y\mathrm{X})\}\\ &=\frac{3(n-2)}{n-1}\mathrm{H}\{(\mathfrak{A}u+\mathfrak{H}v+\mathfrak{G}w)\mathrm{P}_1+(\mathfrak{H}u+\mathfrak{B}v+\mathfrak{F}w)\mathrm{Q}_1+(\mathfrak{G}u+\mathfrak{F}v+\mathfrak{C}w)\mathrm{R}_1\}\\ &=\frac{3(n-2)}{(n-1)^2}\mathrm{H}^2(\mathrm{P}_1x+\mathrm{Q}_1y+\mathrm{R}_1z)\\ &=\frac{3(n-2)(3n-7)}{(n-1)^2}\mathrm{H}^2(\mathrm{X}p+\mathrm{Y}q+\mathrm{Z}r)\\ &=0.\end{aligned}\right\}\quad(45)$$

Hence the whole expression[*]

$$=u^2\left\{\begin{array}{cccc} u_1 & u & Y\partial_x Z-Z\partial_x Y & \partial_x X \\ w' & v & Y\partial_y Z-Z\partial_y Y & \partial_y X \\ v' & w & Y\partial_z Z-Z\partial_z Y & \partial_z X \\ . & \varpi_2 H & Y\Delta Z-Z\Delta Y & \Delta X \end{array} + \begin{array}{cccc} u_1 & u & Z\partial_x X-X\partial_x Z & \partial_x Y \\ w' & v & Z\partial_y X-X\partial_y Z & \partial_y Y \\ v' & w & Z\partial_z X-X\partial_z Z & \partial_z Y \\ . & \varpi_2 H & Z\Delta X-X\Delta Z & \Delta Y \end{array} + \begin{array}{cccc} u_1 & u & X\partial_x Y-Y\partial_x X & \partial_x Z \\ w' & v & X\partial_y Y-Y\partial_y X & \partial_y Z \\ v' & w & X\partial_z Y-Y\partial_z X & \partial_z Z \\ . & \varpi_2 H & X\Delta Y-Y\Delta X & \Delta Z \end{array}\right\}$$

$$+u^3\begin{array}{cccc} \partial_x X & \partial_x Y & \partial_x Z & u \\ \partial_y X & \partial_y Y & \partial_y Z & v \\ \partial_z X & \partial_z Y & \partial_z Z & w \\ \Delta X & \Delta Y & \Delta Z & \varpi_2 H; \end{array}$$

or in virtue of (33),

$$=u^2\left\{\begin{array}{cccc} u_1 & u & -(pP+uP_1) & \partial_x X \\ w' & v & -(pQ+uQ_1) & \partial_y X \\ v' & w & -(pR+uR_1) & \partial_z X \\ . & \varpi_2 H & u\,\mathrm{Jac.}(U,H,\Omega_U) & \Delta X \end{array} + \begin{array}{cccc} u_1 & u & -(qP+vP_1) & \partial_x Y \\ w' & v & -(qQ+vQ_1) & \partial_y Y \\ v' & w & -(qR+vR_1) & \partial_z Y \\ . & \varpi_2 H & v\,\mathrm{Jac.}(U,H,\Omega_U) & \Delta Y \end{array} + \begin{array}{cccc} u_1 & u & -(rP+wP_1) & \partial_x Z \\ w' & v & -(rQ+wQ_1) & \partial_y Z \\ v' & w & -(rR+wR_1) & \partial_z Z \\ . & \varpi_2 H & w\,\mathrm{Jac.}(U,H,\Omega_U) & \Delta Z \end{array}\right\}$$

$$+u^3\begin{array}{cccc} \partial_x X & \partial_x Y & \partial_x Z & u \\ \partial_y X & \partial_y Y & \partial_y Z & v \\ \partial_z X & \partial_z Y & \partial_z Z & w \\ \Delta X & \Delta Y & \Delta Z & \varpi_2 H \end{array}$$

$$=2u^2\varpi_2 H\begin{array}{ccc} u_1 & P_1 & P \\ w' & Q_1 & Q \\ v' & R_1 & R \end{array}+u^2\,\mathrm{Jac.}(U,H,\Omega_U)\begin{array}{ccc} u_1 & u & P \\ w' & v & Q \\ v' & w & R \end{array}+u^2\,\mathrm{Jac.}(U,H,\Omega_U)\begin{array}{ccc} u_1 & u & P \\ w' & v & Q \\ v' & w & R \end{array}+u^3\begin{array}{cccc} \partial_x X & \partial_x Y & \partial_x Z & u \\ \partial_y X & \partial_y Y & \partial_y Z & v \\ \partial_z X & \partial_z Y & \partial_z Z & w \\ \Delta X & \Delta Y & \Delta Z & \varpi_2 H. \end{array}$$

But

$$\begin{array}{ccc} u_1 & P_1 & P \\ w' & Q_1 & Q \\ v' & R_1 & R \end{array}=Z(\mathfrak{H}P_1+\mathfrak{B}Q_1+\mathfrak{F}R_1)-Y(\mathfrak{G}P_1+\mathfrak{F}Q_1+\mathfrak{C}R_1)$$

$$=u(\mathfrak{A}\ \mathfrak{B}\ \mathfrak{C}\ \mathfrak{F}\ \mathfrak{G}\ \mathfrak{H})(p\ q\ r)(P_1Q_1R_1)-p(\mathfrak{A}\ \mathfrak{B}\ \mathfrak{C}\ \mathfrak{F}\ \mathfrak{G}\ \mathfrak{H})(u\ v\ w)(P_1Q_1R_1)$$

$$=u(\mathfrak{A}\ \mathfrak{B}\ \mathfrak{C}\ \mathfrak{F}\ \mathfrak{G}\ \mathfrak{H})(p\ q\ r)(P_1Q_1R_1),$$

since

$$(\mathfrak{A}\ \mathfrak{B}\ \mathfrak{C}\ \mathfrak{F}\ \mathfrak{G}\ \mathfrak{H})(u,\ v,\ w)P_1Q_1R_1)=\frac{1}{n-1}H(P_1x+Q_1y+R_1z)=0,$$

also

$$\begin{aligned}(\mathfrak{A}\ \mathfrak{B}\ \mathfrak{C}\ \mathfrak{F}\ \mathfrak{G}\ \mathfrak{H})(P_1Q_1R_1) &= (\mathfrak{A}\,..)(p\ q\ r)(p_1\ r'\ q')X \\ &\quad +(\mathfrak{A}\,..)(p\ q\ r)(r'\ q_1\ p')Y \\ &\quad +(\mathfrak{A}\,..)(p\ q\ r)(q'\ p'\ r_1)Z \\ &= \mathrm{Jac.}(U,\ H,\ \Psi_U).\end{aligned}$$

Hence the whole expression above written

$$=2u^3\left\{\varpi_2 \,\text{Jac.}\,(\text{U}, \text{H}, \Psi_{\text{U}})+\frac{3(n-2)}{n-1}\text{H}\,\text{Jac.}\,(\text{U}, \text{H}, \Omega_{\text{U}})\right\}\text{H}+u^3\left|\begin{array}{l}\partial_x\text{X}\,..\,u\\ \partial_y\text{X}\,..\,v\\ \;\;\cdot\quad\;\cdot\end{array}\right|.$$

But

$$\begin{array}{cccc}\partial_x\text{X} & \partial_x\text{Y} & \partial_x\text{Z} & u\\ \partial_y\text{X} & \partial_y\text{Y} & \partial_y\text{Z} & v\\ \partial_z\text{X} & \partial_z\text{Y} & \partial_z\text{Z} & w\\ \Delta\text{X} & \Delta\text{Y} & \Delta\text{Z} & .\end{array}=-\frac{12(n-2)^2}{(n-1)^2}\text{H}^2\,\text{Jac.}\,(\text{U}, \text{H}, \Omega_{\text{U}}),$$

and

$$\begin{array}{ccc}\partial_x\text{X} & \partial_x\text{Y} & \partial_x\text{Z}\\ \partial_y\text{X} & \partial_y\text{Y} & \partial_y\text{Z}\\ \partial_z\text{X} & \partial_z\text{Y} & \partial_z\text{Z}\end{array}=-\,\text{Jac.}\,(\text{U}, \text{H}, \Psi_{\text{U}})-\text{Jac.}\,(\text{U}, \text{H}, \Theta_{\text{U}}).$$

Hence, finally, the whole expression

$$\left.\begin{array}{l}=u^3\text{H}\left\{\varpi_2\,\text{Jac.}\,(\text{U}, \text{H}, \Psi_{\text{U}})+\left(\frac{6(n-2)}{n-1}-\frac{12(n-2)^2}{(n-1)^2}\right)\text{H}\,\text{Jac.}\,(\text{U}, \text{H}, \Omega)-\varpi_2\,\text{Jac.}\,(\text{U}, \text{H}, \Theta_{\text{U}})\right\}\\ =u^3\text{H}\left\{\varpi_2\Big(\text{Jac.}\,(\text{U}, \text{H}, \Psi_{\text{U}})-\text{Jac.}\,(\text{U}, \text{H}, \Theta_{\text{U}})\Big)-\frac{6(n-2)(n-3)}{(n-1)^2}\text{H}\,\text{Jac.}\,(\text{U}, \text{H}, \Omega_{\text{U}})\right\},\end{array}\right\}(46)$$

which is therefore divisible by $\text{H}u^3$. Consequently H is a factor of all the expressions $\mathfrak{L}_1\,..\,\mathfrak{L}'\,..$, which was to be proved.

Although not absolutely necessary to our argument, it is perhaps worth while to show, as may readily be done, that $\mathfrak{L}$ is divisible by u. Omitting the terms explicitly multiplied by u in the first three columns, the equation becomes

$$\left.\begin{array}{lllll}u_1\text{X}-\partial_x x\text{P} & u_1\text{Y}-\partial_x y\text{P} & u_1\text{Z}-\partial_x z\text{P} & u & =0.\\ w'\text{X}-\partial_y x\text{P} & w'\text{Y}-\partial_y y\text{P} & w'\text{Z}-\partial_y z\text{P} & v & \\ v'\,\text{X}-\partial_z x\text{P} & v'\,\text{Y}-\partial_z y\text{P} & v'\,\text{Z}-\partial_z z\text{P} & w & \\ p\,\text{X}-\Delta x\text{P}+2\text{H}\partial_x\text{X} & p\,\text{Y}-\Delta y\text{P}+2\text{H}\partial_x\text{Y} & p\,\text{Z}-\Delta z\text{P}+2\text{H}\partial_x\text{Z} & \varpi_z\text{H} & \end{array}\right\}(47)$$

In this the coefficient of $\varpi_2\text{H}$,

$$\text{P}=(\text{Y}z-\text{Z}y)\left|\begin{array}{l}w'\partial_y\text{P}\\ v'\partial_z\,\text{P}\end{array}\right|+\text{P}(\text{Z}x-\text{X}z)\left|\begin{array}{l}v'\partial_z\text{P}\\ u_1\partial_x\text{P}\end{array}\right|+\text{P}(\text{X}y-\text{Y}x)\left|\begin{array}{l}u_1\partial_x\text{P}\\ w'\partial_y\text{P}\end{array}\right|$$

$$+\text{P}^2(u_1\text{X}+w'\text{Y}+v'\text{Z})-\text{P}^2(x\partial_x\text{P}+y\partial_y\text{P}+z\partial_z\text{P})-\text{P}^3,$$

which, writing

$$\text{K}=\begin{array}{lll}u & u_1 & \partial_x\text{P}\\ v & w' & \partial_y\text{P}\\ w & v' & \partial_z\text{P}\end{array}$$

$$=-(n-2)(3\text{HK}+5\text{P}^2)\text{P}.$$

Similarly, it will be found that the coefficients of

$$(pX-\Delta x P+2H\partial_x X)$$
$$(pY-\Delta y P+2H\partial_x Y)$$
$$(pZ-\Delta z P+2H\partial_x Z)$$

are

$$-(n-2)(3HK+5P^2)u,$$
$$-(n-2)(3HK+5P^2)v,$$
$$-(n-2)(3HK+5P^2)w$$

respectively; and consequently the whole expression

$$\begin{aligned}&=-(n-2)(3HK+5P^2)\{(pX-\Delta x P+2H\partial_x X)u\\ &\qquad\qquad+(pY-\Delta y P+2H\partial_x Y)v\\ &\qquad\qquad+(pZ-\Delta z P+2H\partial_x Z)w+\varpi_2 HP\}\\ &=-(n-2)(3HK+5P^2)\{-2HP-2(\mathfrak{A}\,..)(u, v, w)(\partial_x P, \partial_y P, \partial_z P)+\varpi_2 HP\}\\ &=-(n-2)(3HK+5P^2)\left\{-2-\frac{10(n-2)}{n-1}+\varpi_2\right\}HP.\end{aligned}$$

But $\varpi_2=1+\frac{10(n-2)}{n-1}$, so that the above expression

$$=(n-2)(3HK+P^2)HP.$$

Now

$$\left.\begin{aligned}-(n-2)(3HK+P^2)&=\begin{array}{llll} u & v & w & nU \\ u_1 & w' & v' & (n-1)(u-u) \\ p & q & r & 3(n-2)H \\ \partial_x P & \partial_y P & \partial_z & 5(n-2)P \end{array}\\ &=\begin{array}{lllll} u & u & w & xu\ +yv\ +zw & \\ u_1 & w' & v' & xu_1\ +yw'\ +zv' & -(n-1)u \\ p & q & r & xp\ +yq\ +zr & \\ \partial_x P & \partial_y P & \partial_z P & x\partial_x P+y\partial_y P+z\partial_z P & \end{array}\\ &=-(n-1)u\ \begin{array}{lll} u & p & \partial_x P \\ v & q & \partial_y P \\ w & r & \partial_z P; \end{array}\end{aligned}\right\} \quad . \quad (48)$$

so that the whole expression is divisible by u. Similarly, it might be shown that M_1 or M' is divisible by v, and N or N' by w.

It follows from what has gone before that $\mathfrak{L}$, $\mathfrak{M}$, $\mathfrak{N}$, $\mathfrak{L}'$, $\mathfrak{M}'$, $\mathfrak{N}'$ are all divisible by H, that $\mathfrak{L}$, $\mathfrak{L}'$ are divisible by u, $\mathfrak{M}$, $\mathfrak{M}'$ by v, $\mathfrak{N}$, $\mathfrak{N}'$ by w, and consequently dividing out those factors, the three expressions $\mathfrak{L}$, $\mathfrak{M}$, $\mathfrak{N}$ are of the form

$$\left.\begin{aligned}&\mathrm{A}u^2+\mathrm{B}_1u+\mathrm{C}_1=0,\\&\mathrm{A}v^2+\mathrm{B}_2v+\mathrm{C}_2=0,\\&\mathrm{A}w^2+\mathrm{B}_3w+\mathrm{C}_3=0,\end{aligned}\right\}\quad\ldots\ldots\ldots\ldots\quad(49)$$

in which the coefficients of u^2, v^2, w^2 are the same, viz. the expressions given in (46). From these equations it follows that

$$\frac{\mathrm{B}_1u+\mathrm{C}_1}{u^2}=\frac{\mathrm{B}_2v+\mathrm{C}_2}{v^2}=\frac{\mathrm{B}_3w+\mathrm{C}_3}{w^2}.\quad\ldots\ldots\ldots\quad(50)$$

But as u, v, w do not in general vanish simultaneously, these relations can hold good only in virtue of B_1 being divisible by u, and C_1 by u^2; B_2 by v, and C_2 by v^2; B_3 by w and C_3 by w^2. Whence, finally, $\mathfrak{L}$ is divisible by $\mathrm{H}u^3$, $\mathfrak{M}$ by $\mathrm{H}v^3$, $\mathfrak{N}$ by $\mathrm{H}w^3$; and the degree of the equation is reduced to

$$(18n-36)-3(n-2)-3(n-1)=12n-27.$$

Also, since the ratios $(\mathrm{B}_1u+\mathrm{C}_1):u^2$, $(\mathrm{B}_2v+\mathrm{C}_2):v^2$, $(\mathrm{B}_3w+\mathrm{C}_3):w^2$ are in virtue of (50) equal (say $=\mathrm{B}$), it follows that $\mathfrak{L}$, $\mathfrak{M}$, $\mathfrak{N}$, $\mathfrak{L}'$, $\mathfrak{M}'$, $\mathfrak{N}'$ all lead to the same result, viz. $\mathrm{A}+\mathrm{B}=0$, which it was our object to prove.

XV. *On the Marsupial Pouches, Mammary Glands, and Mammary Fœtus of the* Echidna Hystrix. *By* Professor OWEN, *F.R.S.*, *&c.*

Received February 18,—Read March 2, 1865.

IN the year 1834* it was known that the ovum of the *Ornithorhynchus paradoxus* left the ovarium with a spherical yelk or vitellus about $1\frac{1}{2}'''$ (lines) in diameter, and that, having reached the uterine portion of the oviduct, it had acquired a smooth subtransparent chorion or outer tunic separated from the proper membrana vitelli by a clear fluid. Such ova, usually two in number, had been detected in females killed in the month of October, in the left uterus, of sizes ranging from $2\frac{1}{2}'''$ to $3\frac{1}{2}'''$ (lines) in diameter, without any sign of organization of the chorion, or of preparation for placental adhesion on the uterine wall.

The increase of size in the uterine over the ripe ovarian ovum was due to increase of fluid between the chorion and vitelline tunics.

This fluid, homologous with the albumen of the egg of oviparous vertebrates, did not coagulate in alcohol, and the only change presented by the vitellus of the largest observed ovum was a separation from the "food-yelk" of a "germ-yelk" in the form of a stratum of very minute granules, adhering to part of the membrana vitelli. There was no trace of decidua in such impregnated uteri; the smooth chorion was firmer than that of uterine ova of *Rodentia*; whence, and for other reasons given in the paper above cited, it was inferred "that the *Monotremata* are essentially ovo-viviparous."

In the same year (1834) I received a young of the *Ornithorhynchus paradoxus* from a nest of that animal, discovered by Lieut. the Hon. LAUDERDALE MAULE in the banks of the "Fish River," Australia. This progeny, Plate XLI. fig. 5, measured in a straight line about 2 inches (other admeasurements will be subsequently given); it was naked, blind, with short, broad, flexible, and softly labiate mandibles; the tongue was proportionally large, and reached to near the end of the mandibles; the mouth was not round, as in the mammary fœtus of marsupials, but in the form of a wide transverse slit; a pair of small nostrils (*a*) opened upon the upper mandible, and between them was a small prominence (*e*), resembling the knob on the beak of the newly-hatched chick, but softer, and lacking the cuticle which had been torn off. There was no trace of navel or umbilical cicatrix†. The mouth of this young Platypus, or *Ornithorhynchus*, was adapted to be applied to the flat teatless areola upon which the numerous lactiferous ducts of the parent opened‡, and it was inferred that thus it received the lacteal nourishment with the aid of the compressor muscle of the large mammary gland.

---

* "On the Ova of the *Ornithorhynchus paradoxus*," Philosophical Transactions, vol. cxxiv. p. 555.

† "On the Young of the *Ornithorhynchus paradoxus*," Zoological Transactions, vol. i. p. 221.

‡ "On the Mammary Glands of the *Ornithorhynchus paradoxus*," Philosophical Transactions, vol. cxxii. p. 517.

The principal points in the generation of the *Monotremata* which remained to be determined by actual observation were—

1st. The manner of copulation.
2nd. The period of gestation.
3rd. The nature and succession of the temporary structures developed for the support of the fœtus during gestation.
4th. The exact size, condition, and powers of the young at the time of birth.
5th. The period during which the young requires the lacteal nourishment.
6th. The age at which the animal attains its full size.

"Notes" of these desired facts, with indications of the times and places most likely to supply them, have been sent by me far and wide, through Australia and Tasmania; and after the lapse of thirty years, I have been favoured with materials for making some further advance in this interesting physiological problem—a small one, it is true, but such as seemed to me worthy of being submitted to the Society as an addition to former records on the subject contained in the Philosophical Transactions.

For these materials I am indebted to my friend the accomplished botanist, Dr. FERDINAND MUELLER, F.R.S., of Melbourne, Australia. They consist of a female *Echidna* (*Ornithorhynchus Hystrix* of Home, *Echidna Hystrix* of Cuvier, the "Porcupine Anteater" of the colonists) and her young one, or one of her young, which was observed, as the captor supposed, suspended to a nipple when the animal was first secured. After five days' confinement the young was found detached and dead, was put into a bottle of spirits, and, with the mother still living, was transmitted from "Colac Forest," Victoria, the place of capture, to Melbourne. Here the female *Echidna* was examined by Dr. MUELLER and Dr. RUDALL of Melbourne, and was then transmitted to me, together with the young animal, and the following "Notes" of their dissection.

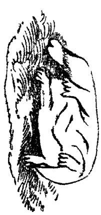
"*Brief Notes on the Generative Apparatus of the female* Echidna.

"The animal being excessively difficult to handle it was immersed in cold water, and by these means and the additional use of hydrocyanic acid its life was extinguished. A longitudinal incision was made from the orifice of the cloaca upwards to the length of about 5 inches. Five larger and some smaller ovules were found arranged in a grape-like manner, the largest measuring from $1'''$ to $1\frac{1}{2}'''$" [lines] "in diameter. Fine vessels expanded reticularly over the surface of the ovules. We vainly endeavoured to trace an opening at the ovarian end of the oviduct. Oviduct about $2''$" [inches] "long; its upper extremity expanded and attached to the ovarium. As a probable sign of recent functional activity, were noted a number of large distended veins lying between the layers of the peritoneum. Numerous oval mesenteric glands were seen. 'Meatus urinarius' lying in the inferior wall of the cloaca about $\frac{1}{4}''$ from the orifice. The ureter terminates in a conspicuous conical protuberance from $3'''$ to $4'''$ long. No other exit for the urine from the bladder being found but the point into which this conical protuberance fits, the ingress and egress of the urine, as far as we believe, takes place at the same aperture. In close proximity, and lateral to it, the oviducts terminate by slit-like openings. The mucous membrane of the thick walls of the oviducts are, at least in the lower portion, longitudinally folded. The oviducts are suddenly narrowed for about $\frac{1}{6}''$ from the lower orifice, offering some resistance to the passage of an ordinary sized probe.

"The upper portion of the oviduct seems of a structure capable of considerable expansion during gestation. The upper portion was dilated and thin, and a probe could be passed to near one of the ova. The lower portion of the rectum is so large and so capable of distension as to admit of the periodical inclusion of the young animal, in case its great size should possibly be provided for that purpose, as it is a receptacle large enough for a young animal twice the size of that found now with the mother. The fœtal young may possibly have been extruded prematurely after the capture of the animal. We found no cicatrix of an umbilical cord on the abdomen of the young animal.

"A rough sketch of the young as seen by us is appended (fig. 1). It was of a pale colour*; no apertures for the eyes were yet visible in the skin, nor were any tegumentary appendages formed. The finder contends that he saw the young external to the mother and alive. We purposely abstained from the internal examination of the young one, so as not to mutilate the only specimen available. The four mammary glands at this time are apparently quite rudimentary; they are destitute of nipples, as are those of the Ornithorhynchus. Nor was there the least appearance of milk in these glands. From the imperfect means of judging we had, we incline to the opinion that the Echidna cannot be oviparous.

Fig. 1.



Young Echidna.

(Signed) "JAMES T. RUDALL.
"FERD. MUELLER."

"Melbourne, August 25, 1864."

On receiving the specimens I proceeded to examine the female Echidna, and was gratified by finding unmistakeable evidences of marsupial structure. On each side of the abdominal integument, about two inches in advance of the cloaca, and about three inches and a half from the base of the tail, there was a semilunar pouch, with an aperture longitudinal and directed towards the median line, half an inch in depth and two-thirds of an inch in length of aperture, forming a symmetrical pair with their orifices opposite each other (Plate XXXIX. *a*, *b*).

These pouches were not at first apparent, being concealed by the hair which covers the under part of the body. It was in turning over this hair in quest of any rudiment of nipple, that I came, to my surprise, upon one of the pouches. The first doubt was whether it might have been produced by an accidental pressure of the end of a thumb or finger in the previous dissection of the animal, which depression had afterwards got hardened in the spirit; and to solve that doubt I proceeded to examine the opposite half of the ventral integument, when a pouch or inverted fold of precisely similar shape, depth, and dimensions appeared, but with the opening turned the opposite way; the folds were closer and less conspicuous on that side, the cavity of the pouch being flatter (see section, Plate XL. fig. 3), whence I inferred that the more open pouch (*ib.* section, fig. 2, *c*) had been the seat or nest of the very small and probably recently-born animal, whose position there, as in the figure, Plate XXXIX. *a*, had naturally led the original captor of the Echidna to conclude that it was hanging by a nipple.

---

* "Said originally to be bright red.—F. M."

No such projection, however, presented itself in any part of the inner surface of either pouch; but at the fundus of each was an "areola" or elliptic surface, about four lines in diameter (Plate XL. fig. 4), on which, with the pocket lens, could be discerned the orifices of about fifty ducts of a gland. The canals or roots of fine scattered hairs and several minute white papillæ (*ib.* fig. 5, *p*, *p*, magn.), about one or two lines apart, on which opened sebaceous follicles, were all the appearances characterizing the otherwise smooth and even surface of these inflexions of the abdominal integument.

The contrast which this pouch presents with that of a true marsupial quadruped containing the mammary fœtus* is great; for even in the uniparous species, *e. g.*, the larger Kangaroos, two, if not four, long slender nipples are conspicuous, to one of which the fœtus hangs, closely embracing the pendulous extremity of the nipple by its small, round, terminal, tubular mouth.

My next step was to test the statement in reference to the number and condition of the mammary glands.

I found, as in a former dissection of a younger unimpregnated female Echidna†, that these glands were two in number, forming, like the pouches, a symmetrical pair (Plate XL. fig. 1). Each gland (*a*, *a'*) was of a flattened, subelliptic form; the left (*a*) being 1 inch 10½ lines, the right (*a'*) 1 inch 8½ lines in long diameter, the left 1 inch 5 lines, the right 1 inch 3 lines in short diameter across the middle, and both glands about 5 lines in thickness at the middle part (figs. 2, 3). Each gland consists of about 100 long, narrow, flattened lobes, obtusely rounded at their free ends, and beginning, at about halfway towards the opposite side, to contract gradually to the duct which penetrates the corium (Plate XL. figs. 2 & 3, *b*), to terminate on the mammary areola (*ib.* *c*) at the fundus of the pouch. From the small size of the areola compared with that of the gland, the lobules have a convergent arrangement thereto, each terminating in its own duct, without blending with the substance of a contiguous lobe; and, as a general rule, without anastomosis of contiguous ducts to form a common canal. Each gland is enclosed in a loose capsule of cellular tissue (fig. 1, *e*, *e*) and lies between a thick "panniculus carnosus" (figs. 1, 2, 3, *d*, *d'*), adherent to the abdominal integument (*f*, *f'*) and the "obliquus externus abdominis" muscle, on a plane exterior or "lateral" to the pouch. The glands had not been exposed or disturbed by any dissection in the preliminary examination of the animal at Melbourne. The lobules of each gland converge toward the mesial line, in their course to terminate in the fundus of the pouch. Each lobe is a solid parenchymatous body; the duct is more directly continued from a canal which may be traced about halfway toward the fundus of the lobule; the canal gives off numerous short branches from its circumference, which subdivide and terminate in clusters of subspherical "acini" or secerning cellules. The structure is on the same general plan as that of the mammary glands in higher mammals, but the cellules are proportionally larger; it closely resembles the structure of the lobes of the same glands in the Ornithorhynchus, and in neither Monotreme can the elongated lobes be properly termed "pyriform cæcal pouches."

---

* For the signification of this term see "On the Generation of the Marsupial Animals," Philosophical Transactions, vol. cxxiv. p. 333.

† "On the Mammary Glands of the *Ornithorhynchus*," Phil. Trans., *tom. cit.* p. 537, Pl. XVII. figs. 2 & 3.

The converging termination of the lacteal ducts at the fundus of a pouch, or inverted fold of the skin, resembles the disposition of those parts in the *Cetacea*; save that here the ducts terminate on a prominence or nipple projecting from the fundus of the pouch into its cavity; whilst in the *Echidna* they terminate in the smooth and even concave surface of the fundus of the pouch.

Calling to mind Mr. MORGAN's observation of the concealed nipple in an inverted sac of the tegument at the fundus of the pouch in the young or non-breeding Kangaroo, where, instead of a nipple, there was seen only "a minute circular aperture, resembling in appearance the mouth of a follicle" *, I made sections of both the marsupial or mammary pouches and glands (Plate XL. figs. 2 & 3) satisfactorily demonstrating that no inverted or concealed nipple or any rudiment or beginning of such existed; and, indeed, had any such arrangement like that of the Kangaroo been characteristic of the mammary organization of the Echidna, the glands being functionally active and well developed in the female dissected, such nipple would have been everted, and would have served, as the first observer of the young animal in the pouch believed, to have attached and suspended it to the parent.

But it is evident that the young simply nestles itself within the marsupial fossa, clinging, it may be, by its precocious claws to the skin or hairs of that part, and imbibing by its broad, slit-shaped mouth the nutritious secretion as it is pressed by the muscles acting upon the gland from the areolar outlets of the ducts.

The skin of the abdomen, where it begins to be inverted, loses thickness, and at the fundus of the pouch (ib. fig. 1, *b*, fig. 3, *c*) is only half as thick as where it overspreads the abdomen (ib. fig. 1, *f*). This modification, and the relation of the pouches to the mammary glands, prove the structures shown in Plate XXXIX. *a*, *b*, and Plate XL. figs. 2 & 3, *c*, to be natural, not accidental.

The pair of lateral folds or clefts into the bottom of which the lacteal ducts open, in the *Echidna* are homologous with those similarly related to the mammary glands in Cetaceans, and also to the more developed folds or pouches in Marsupials. In Cetaceans the pair of tegumentary clefts have exclusive functional relations to the mammary organ; in Marsupials the superadded office of receiving and protecting the young is associated with so great a development of the inverted tegumentary fold, as to make the mammary relation seem a very subordinate and reduced one. But in the Marsupial series there is a gradation; and both in *Thylacinus* and in the small dorsigerous Opossums of South America (*Didelphys dorsigera*, *D. murina*, *D. pusilla*, &c.), the marsupial structure, if shown at all, is represented by a pair of shallow semilunar fossæ, with their concave outlets opposite to each other, as in *Echidna*.

---

* "A Description of the Mammary Organs of the Kangaroo," Linn. Trans., vol. xvi. p. 62, pl. 2. fig. 1, *b*.

In this comparison the distinctive peculiarity of the parts in the terrestrial Monotreme is the absence of a teat, or of any rudiment of such: no part of the fundus of the pouch is again everted, produced, or folded about the terminal ducts of the mammary gland, so as to form a pedicle by which the young could take hold with the mouth, and so suspend itself and suck.

The question remains, whether the marsupial pouches of the *Echidna* increase with the growth of the young? It is certain that they commence with the growth or enlargement of the mammary glands preliminary to birth.

In that young specimen of female *Echidna* in which the glands were first discovered*, their ducts opened upon a plane surface of the abdominal integument. In a nearly full-grown unimpregnated female, preserved in spirits, which I examined and compared with the breeding mother here described, there is also a total absence of inflected folds of the integument where the mammary ducts terminate.

Some movement, perhaps, of these ducts in connexion with the enlargement of the mammary lobes, under the stimulus of preparation for a coming offspring, may, with associated growth of the abdominal integument surrounding the areola, be amongst the physical causes of the first formation of the pouch.

It has already been remarked that the integument of the pouch, especially as it approaches the fundus, is thinner than that covering the abdominal surface of the body, from which the pouch is continued. Such tegumentary growth, continued with the pressure of the part of the growing young within, may lead to a marked increase of size; to be reduced, perhaps, by absorption and shrinking of the skin concomitantly with reduction of the mammary glands after the term of lactation has expired. I much doubt, however, whether the increase of size of the pouch would ever be such as to include and wholly conceal the young animal; it more probably, at the later period of lactation, serves only to admit the head or beak. Thus the ordinary condition of sucking would be reversed in these Australian Mammals; instead of the excretory ducts on an everted process of integument being taken into the mouth, this is received into an inverted pouch into which the milk is poured.

I have not hitherto met with any trace or beginning of such abdominal pouches in the various *Ornithorhynchi* in which I have had occasion to note different phases of the development of the ovaria and mammary glands†.

---

* Philosophical Transactions, 1832, p. 537, Pl. XVII. figs. 2 & 3.

† "On the Mammary Glands of the *Ornithorhynchus paradoxus*," Philosophical Transactions, 1832, p. 517. Pl. XV.–XVIII.

A warm-blooded air-breather, compelled to seek its food in water, could not safely carry the progeny it had brought forth in a pocket beneath its body during such quest; and all observers have noted the nest-making instinct of the *Platypus*, in which temporary and extraneous structures only the young have hitherto been found *. Mr. George Bennett states that the nest "appears to be found about the time of bringing forth the young, and consists merely of dried grass, weeds, &c." †

Whether the Echidna prepares any extraneous nest is not known. The specimen transmitted to me by Dr. Mueller was caught in the hollow of a prostrate "cotton tree." Being a terrestrial animal, she can carry her young about habitually concealed or partly sheltered in her pouches; and the present observations show the nearer affinity in this respect of the Echidna to the marsupial *Lyencephala*. The Echidna may further manifest this relationship by the more minute size of the young when born and transferred to the pouch, as compared with the Ornithorhynchus; but the size of the new-born or newly-excluded young of that monotreme is unknown. The smallest specimen of a young Ornithorhynchus which I have yet seen is that (Plate XLI. fig. 5) to which allusion has been already made as being about two inches in length in a straight line.

The following are the comparative dimensions of this, and of the young of the female Echidna (ib. fig. 3 (magn.), Plate XL. figs. 6–10 (nat. size)), the subject of the present communication:—

	Young Ornithorhynchus.		Young Echidna.	
	in.	lin.	in.	lin.
Length from the end of the upper jaw, over the curve of the back, to the end of the tail	3	9	1	10
Length from the same points in a straight line along the abdomen	2	1	1	1
Greatest circumference of the body	2	9	1	0 ‡
Length of the head	0	8½	0	4
Length of the upper mandible from the gape	0	3	0	1½
Breadth of the upper mandible at the base	0	4	0	1
Length of the tail from the vent	0	4½	0	1
Breadth of tail at the root	0	4	0	½
Length of the fore foot	0	3	0	2
Breadth of ditto	0	3½	0	1½
Length of the hind foot	0	4	0	1
Breadth of ditto	0	3	0	2⅓

The circumstances under which this young Echidna was obtained are given in a letter by the captor, Mr. G. O. Harris, to Dr. Mueller, dated "Colac Forest, August 31, 1864."

---

* *Tom. cit.* p. 533.

† Trans. Zool. Soc. vol. i. pp. 247 & 253.

‡ This might have been more before the body had become somewhat dried, or shrunk in parts.

It appears that Mr. HARRIS, being in Colac Forest, Victoria, on the 12th of August, 1864, his attention was attracted by his dogs to a fallen tree, in the hollow of which the Echidna had taken refuge. "On examining her I found the young one attached to one teat, presenting the appearance of a miniature Porcupine*, with an absence of quills, partially transparent, of a bright red colour." The mother was placed in a porter-cask with earth containing ants.

"On Wednesday the 17th of August it still remained attached to the teat, presenting the same appearance as when first captured, evidently in a living state. I avoided handling it more than necessary, as it evinced signs of terror by a protrusion of the vagina and frequently emitting urine.

"On Thursday, 18th of August, I emptied the earth out of the cask, to replace it with fresh earth containing ants, and to my surprise found the young one removed from the teat. I 'panned off' the earth, as for gold, and found the young considerably shrunk."

Mr. HARRIS thereupon placed it in a bottle of spirits, and transmitted it, with the mother alive, to Dr. MUELLER, Botanic Gardens, Melbourne. Mr. HARRIS concludes his letter by stating, "My dates are correct, as I keep a diary, and you may rely upon what I have stated being authentic."

The condition in which the young *Echidna* has reached me accords with the above account. It is naked, devoid of prickles, the integument thin, but with its transparency affected by the action of the alcohol, and somewhat wrinkled from contractions of the tissues through the same action. The new-born Kangaroo, of similar size and condition, described in the Philosophical Transactions for 1834, p. 344, Plate VII. fig. 5, was also red, like an earthworm, "resembling it not only in colour, but in the semi-transparency of the integument." Mr. HARRIS's observation of the young *Echidna* closely accords in this character with my own on the new-born living Kangaroo. Mr. HARRIS observed the young Echidna attached to the mother, and he concluded from analogy that the mode of attachment was as in the other land-quadrupeds of the colony and in mammalia generally; whereas it was kept *in situ* by the duplicature of the skin, and by clinging with the precociously-developed claws of the fore feet to the interior of the pouch. There was most assuredly no nipple: in that particular my own scrutiny accords with the results of the examination of the recent animal by Drs. MUELLER and RUDALL. What appearances suggested to them the idea of four quite rudimentary mammary glands I have been unable to discover; the pair of large mammary glands, together with the pouches into which they pour their secretion, had escaped their observation.

The young *Echidna* (Plate XLI. figs. 3 & 4), of which the admeasurements have been given, resembles the young *Ornithorhynchus* (ib. fig. 5) in the general shape and curvature of the body; it also resembles the new-born Kangaroo above cited in the proportions of the limbs to the body, in the inferior size of the hinder pair, in the degree of development of the digits, and in the feeble indication of eyes or eyelids.

---

* The name by which the Echidna is commonly known to the settlers and gold-seekers of the colony.

But the mouth is proportionally wider, and has the form of a transverse slit (Plate XL. fig. 9, Plate XLI. fig. 4, *n*); it is not circular. Upon the upper lip (ib. fig. 4, *m*), in the mid line between the two nostrils (*a*), is a small protuberance (*e*), corresponding to that in the young of the *Ornithorhynchus paradoxus* (ib. fig. 5, *e*), and wanting the cuticle. The tongue (ib. fig. 4, *l*) is broad and flat, extending to the "rictus oris," but very short in proportion to that of the parent, and of a very different shape.

The traces of ears are less conspicuous than in the young Kangaroo, the conch being little if at all developed in the mature Echidna. The tail is much shorter than in the young Kangaroo, and shows as much proportional size as in the full-grown Echidna, in which it is a mere stump (Plate XXXIX. *c*) concealed by the quills and hair.

The head is proportionally longer and more slender in the marsupial fœtus of the Echidna than in that of the Kangaroo, and already, at this early period, foreshows the characteristic elongation and attenuation of that part in the mature animal.

The form of the mouth as a transverse slit, in *Echidna* as in *Ornithorhynchus*, is a good monotrematous character of the young at that period, since in all true or teated marsupials the mouth of the mammary fœtus has a peculiar circular and tubular shape.

A scarcely visible linear cicatrix at the middle of the lower part of the abdomen is the sole trace of umbilicus (Plate XL. fig. 9). A bifid, obtuse rudiment of penis or clitoris (Plate XLI. fig. 3, *d*) projects from the fore part of the single urogenital or cloacal aperture, and in advance of the base of the tail-stump (ib. *c*).

The brain, of which the largest part is the mesencephalon, chiefly consisting of a vesicular condition of the optic lobes, has collapsed, leaving a well-defined elliptical fossa of the integument indicative of the widely open "fontanelle" at the upper part of the cranium (Plate XL. fig. 10, Plate XLI. fig. 3, *o*). The skin of the shrunk body shows folds indicative of the originally plump, well-filled abdomen.

The fore limbs (Plate XL. figs. 11 & 12), in their shortness and breadth, foreshow the characteristics of those of the parent, which may be said, indeed, to retain in this respect the embryonic character with superinduced breadth and strength. The digits have already something of the adult proportions, the first or innermost of the five (fig. 12, *i*) being the shortest, the others retaining nearly equal length, but graduating shorter from the third to the fifth. The characteristic disposition of the digits is better marked in the hind limb (ib. figs. 13 & 14), the second (*i i*) already being the strongest and longest, the rest more rapidly shortening to the fifth (*v*) than in the fore leg; the innermost (*i*), agreeably with the law of closer retention of type in the embryo, though the shortest of the five, is less disproportionately so than in the adult.

It thus appears that the exterior characters of the young animal, figured in Plates XL. & XLI., accord with what might be expected, from the correspondingly immature characters in *Macropus* and *Ornithorhynchus*, in the offspring of the species alleged.

In a question of this kind, as the liberal transmitters of the specimens were not themselves the captors or original observers of the young with the mother, every possibility of error had to be considered. But I know of no pentadactyle ecaudate marsupial animal which could have afforded a mammary or marsupial fœtus with the characters of that which Mr. HARRIS affirms to have discovered attached to the female Echidna, and which he transmits to his correspondents in Melbourne as the young of that monotreme. The condition of the mammary glands, and the presence of heretofore unobserved marsupia, accord moreover with her alleged maternity and with the state of development of her offspring.

It occurred to me that an additional test might be afforded by the more essential parts of the female organs of generation. These had been examined in a general way by Drs. MUELLER and RUDALL, whose "Notes" have been already quoted. I proceeded, therefore, to remove these organs (Plate XLI. fig. 1), with the rectum (ib. *m*), urinary bladder (*r*), urogenital canal (*u*), and cloacal vestibule (*m'*).

The left ovarium (*o*), as in the *Ornithorhynchus paradoxus*, is of an oblong flattened form, developed from the posterior division of the ovarian ligament (*i*) and corresponding wall of the ovarian capsule (*c*); it consists of a rather lax stroma invested by a smooth, thin, firm "tunica propria," which glistens where stretched over the enlarged ovisacs. Of these there were five, of a spherical form, most of them suspended to the rest of the ovarium by a contracted part of the periphery, not stretched into a pedicle. The largest had a diameter of 1½ line, the least of the five had a diameter of rather less than one line. In the recent state, very fine vessels were spread reticularly, according to the original dissectors, over the ovisacs. Beneath these, or nearer the ovarian ligament, was a cluster of smaller ovisacs, the largest not exceeding ⅓rd of a line, the rest so small as to give a granular character to the part. External to this, at the end of the ovarium nearest the bifurcation of the ligament, was an empty ovisac (*g*), 2½ lines in length, and 2 lines in diameter, of a flattened pyriform shape, with a somewhat wrinkled exterior, attached by the base, with the apex slightly tumid, and showing a trace of a fine cicatrix. This is a "corpus luteum" or ovisac from which an ovarian ovum had been discharged.

The oviducal branch of the ovarian ligament passes, as in the *Ornithorhynchus*, to the outer angle of the wide oviducal slit or aperture (*e*), which occupies or forms the margin of the ovarian pouch (*c*), opposite to that to which the ovary is attached. The ligament spreads upon the inner wall of the infundibular part of the oviduct, and rejoins the ovarian division of the ligament, to be continued along the oviduct, puckering up its short convolutions into a small compass.

The "fallopian" aperture of the infundibulum (*e*), is a longitudinal slit of 9 lines in length, with a delicate membranous border extending about a line beyond the part where the muscular and mucous tissues of the oviduct make the thin wall of the infundibulum opake; its transparency against a dark ground, contrasting with the opake beginning of the proper tunics of the oviduct, which nevertheless are here very thin.

No part of this delicate free margin is produced into fimbriæ; in this respect the

Echidna accords with the Ornithorhynchus, and equally manifests the character by which the Monotremes differ from the Marsupials*.

The infundibular dilatation suddenly contracts about an inch from the opening into a "fallopian tube," about a line in diameter, which is puckered up into four or five short close coils. The oviduct, after a slight contraction, suddenly expands into the uterus (ib. *d*). This is about 2 inches long, and appears to have been about 6 lines in diameter, before being cut open. It commences by a short well-marked band, convex outwards, and then proceeds nearly straight, the pair converging to the urogenital compartment, slightly contracting at its termination, which projects, as an "os tincæ" (ib. *s'*), into the side of the fundus of that division of the cloaca.

The tunics of the uterus are, externally, the peritoneum (ib. fig. 2, *a*), which is attached by a lax cellulosity to the "tunica propria" (*b*); this, with its fibrous or muscular layer, is thin, not exceeding ⅛th of a line in the present specimen. The inner layer of the uterine wall (*c*) is the thickest, and chiefly composes it, consisting of delicate vascular lamellæ stretched transversely between the fibrous layer and the fine smooth lining membrane (*d*), the whole being of a pulpy consistence, and doubtless in the recent animal highly vascular, especially in the impregnated state.

The lining membrane was thrown into delicate irregular rugæ, which assumed the longitudinal direction at the "cervix" or contracted terminal part of the uterus. It is laid open in the left uterus; a style (*s*) is passed through it in the right uterus.

The orifice in the "os tincæ" was a puckered slit, about a line in extent; below it, on a produced or papillose part of the prominence, was the small circular orifice of the ureter; a fine hair is passed through each of these tubes in fig. 1, *u*, Plate XLI.

The right ovarium (*o'*), was proportionally more developed and larger than in the *Ornithorhynchus paradoxus*: three ovisacs were enlarged and attached to the stroma, as in the left ovarium; and there was also a compressed ovisac (*g*), similar in size and shape to that in the left side, and exhibiting an apical cicatrix; whence it is to be inferred that, in this instance, the right as well as the left ovarium had furnished an impregnated ovum; and the near equality of size and close similarity of structure and condition of the right oviduct and uterus equally evinced that they had participated in the last operations of the season of generation.

Figure 2 gives a magnified view of the structure of the right uterine walls, as seen in transverse section.

The urinary bladder (*r*), opened into the middle of the fundus of the urogenital compartment, as indicated by the stylet (*r*, fig. 1, Plate XLI.), the uterine orifices intervening between the vesicular one and those of the ureters, as in the *Ornithorhynchus paradoxus*.

---

* See Philosophical Transactions, 1834, Plate VI. fig. 1—"fimbriæ" of Kangaroo"; and art. *Marsupialia*, Cyclop. of Anatomy and Physiology, vol. iii. fig. 137, "fimbriæ" still more remarkably developed in the Wombat (*Phascolomys*). The absence of these fimbriæ, and the resemblance of the true abdominal orifice of the oviduct to that of the ovarian pouch, or to an ordinary duplicature of membrane, appear to have prevented its recognition by Drs. M. and R.

The urogenital canal is 1 inch 4 lines in length, and about 9 lines in diameter: its inner surface shows by some coarse wavy longitudinal rugæ its capacity for dilatation.

The rectum was here of great width; it terminated by a contracted puckered aperture (m'), in the back part of the beginning of the vestibule, behind the aperture of communication of the urogenital with the vestibular canal. The distal half of the vestibule is lined by a denser and less vascular epithelium than the proximal one.

I conclude from these appearances that the present Echidna had produced two young, of which one only was secured; and that, either, one was left in a nest in the fallen hollow tree, while the other was imbibing milk from the pouch; or that, if she had carried a mammary fœtus in each pouch prior to her capture, one had fallen out in the scuffle that drove her from her place of shelter and concealment. The slight difference in size between the right and left mammary glands may relate to the longer continuance of the left one in functional activity, after the loss of the young from the right pouch.

The chief points in the generative economy of the Monotremes which still remain to be determined by actual observation are—

1. The manner of copulation.
2. The season of copulation.
3. The period of gestation.
4. The nature and succession of the temporary structures for the nourishment and respiration of the fœtus prior to birth or exclusion.
5. The size, condition, and powers of the young at the time of birth or exclusion.
6. The period during which the young requires the lacteal nourishment.
7. The age at which the animal attains its full size.

In respect to the second point: as Mr. HARRIS caught the female Echidna with the young, about an inch in length, on the 12th of August, she may be impregnated at the latter end of June or in July. Females killed in the last week of July and the first week of August, in the Province of Victoria, would be most likely to afford the capital facts noted under the fourth head; viz. the impregnated ovum *in utero* showing some stage of embryonal development in the spiny terrestrial Monotreme. As to the hairy and aquatic *Ornithorhynchus*, the impregnated females in which ova were found in the uterus, of small size, and prior to the formation of the embryo, were caught on the 6th and 7th of October*. Young *Ornithorhynchi*, measuring in length in a straight line 1 inch and $\frac{7}{8}$ths, were found in the nest on the 8th of December. The period of impregnation is, therefore, in this species, in the locality of the Murrumbidgee River, probably the latter end of September or beginning of October. Females captured in the latter half of October and in the month of November, would be most likely to have ova *in utero* exhibiting stages of embryonal development.

On this point I have been favoured with the following letter, one of a kind including most which reach me from Australia on the subject, exciting, instead of allaying, curiosity.

---

* See figure of the impregnated specimen in Philosophical Transactions, 1834, Plate XXV. *a*, *a'*.

"Wood's Point, September, 21st, 1864.

"To Professor R. OWEN,

"SIR,—I have great pleasure in being able to inform you of a very interesting discovery in the economy of the *Ornithorhynchus paradoxus*, and one which I have no doubt you will hail with delight. About ten months ago, a female Platypus was captured in the River Goulburn by some workman who gave it to the Gold-Receiver of this district. He, to prevent its escape, tied a cord to its leg and put it into a gin-case, where it remained during the night. The next morning, when he came to look at it, he found that it had laid two eggs. They were about the size of a crow's egg, and were white, soft and compressible, being without shell or anything approaching to a calcareous covering.

"I had an opportunity of examining them externally, and I found no evidence of their having had any recent vascular connexion with the maternal organs; but I am sorry to say that I never had a chance of examining their contents, as, on inquiring for them a day or two afterwards, I found they had been thrown away, much to my chagrin and disappointment.

"The animal itself was afterwards killed (next day), and I was told that numerous ova [in the words of my informant 'eggs'] were found in it, in various stages of development, which in the aggregate somewhat resembled a bunch of grapes; but this I cannot personally vouch for.

"It may appear to you a matter of surprise that I did not examine more minutely this most interesting animal; but I am sorry to say that the same spirit that dictated the throwing away of the eggs, prevented me making a more detailed investigation.

"I am in hopes that I shall be able to get another pregnant specimen, if so, I shall have much pleasure in sending it to you for your inspection.

"I have the honour to be, Sir,
"Your obedient Servant,
"JNO. NICHOLSON, M.D., &c."

"Wood's Point, Victoria, Australia."

By a following mail I was favoured by my esteemed correspondent, Dr. MUELLER, with a letter from the "Gold-Receiver" referred to by Dr. NICHOLSON, in reply to inquiries which vague reports of the occurrence had induced Dr. MUELLER to make.

"Wood's Point, September 25, 1864.

"DEAR SIR,—In reply to your inquiries relative to the *Ornithorhynchus paradoxus*, I must in the first place correct an erroneous impression which the newspaper paragraph has conveyed.

"The Platypus is not now in my possession, and the eggs were layed the day after its capture. The animal was captured in the Goulburn and given to me. It was then fastened by a cord in a gin-case, and on examining it the next morning the two egg were found in the bottom of the box, both of them having undoubtedly been laid during the night. In the course of the day the creature was killed by a *would be* scientific friend of mine, with the intention of preserving its skin; and on opening the body the ovaries were found to be clustered with ova in different stages of growth; but none of them so large as the eggs which were laid. These eggs were white, soft, and without shell, easily compressible, and about the size of a crow's egg.

"Not being sufficiently versed in the subject I am not prepared to say whether these eggs might not have been abortions caused by fear, but there was no appearance on the surface of their ever having been vascularly connected with the maternal uterus, and reviewing all the facts observed I should undoubtedly say that the animal was oviparous.

"I am, dear Sir,
"Yours faithfully,
(Signed) "GEO. J. RUMBY."

Dr. MUELLER, in transmitting me the foregoing copy of the Gold-Receiver's letter, writes (November 25th, 1864), "Since writing to you by last mail I have received the enclosed letter respecting the *Ornithorhynchus* having proved to be '*oviparous.*' How are all these statements to be reconciled?"

Assuming the fact of the oviposition, in the month of December 1863 (Dr. NICHOLSON writes of the occurrence as having happened "about ten months" before the date of his letter, September 21, 1864) by a female *Ornithorhynchus*, of two ova, about the size of a crow's egg, "white, soft, compressible, without shell or anything approaching to a calcareous covering," the question is—What did they contain? Had the unvascular chorion been cut or torn open, an embryo or a yelk might have been seen. Better still would it have been if both ova had been at once immersed in a bottle of whatever colourless alcoholic liquor might be at hand. Probably no medical man had ever an opportunity or a chance of settling a point in Comparative Physiology of more interest, and with less trouble, than the gentleman who was privileged to be the first person to see and handle the new-laid eggs of the *Ornithorhynchus paradoxus*.

For the reasons given in my Memoir of 1834*, I concluded that the Monotremes were not "oviparous" in the sense of the author of the memoir in the 'Annales des Sciences Naturelles,' vol. xviii. (1829)†, but that they were ovo-viviparous, and in a way or degree more nearly resembling the generation of the Viper and Salamander than occurs in the

The young Viper is provided with a specially and temporarily developed premaxillary tooth for lacerating the soft, but tough, shell of its egg, and so liberating itself‡. From this analogy I imagine that the young Monotremes may be provided with a horny or epidermal process or spine upon the internasal tubercle, for the same purpose. This temporary tubercle is obviously homologous with the hard knob on the upper mandible of chelonians and birds, by which they break their way through the harder calcareous covering of their externally hatched embryo.

---

* "On the Ova of the *Ornithorhynchus paradoxus*," Philosophical Transactions, vol. cxxiv. p. 555.

† R. E. GRANT, "Œufs de l'Ornithorhynque," Ann. des Sciences Nat. 1829.

‡ WEINLAND, in MÜLLER's Archiv für Physiologie, 1841.

Some modification of epiderm has been removed from the tubercle in the young Echidna (Plate XLI. fig. 11, *e*), as in the young *Ornithorhynchus**.

DESCRIPTION OF THE PLATES.

PLATE XXXIX.

Female Echidna (*Echidna Hystrix*, Cuv.), two-thirds nat. size.

a. Left "Marsupial" or "Mammary" pouch, with young as seen therein.
b. Right ditto empty.
c. Tail-stump of Echidna.
d. Outlet of cloacal vestibule.
e. Young or "mammary fœtus," as removed from the pouch; two-thirds nat. size.

PLATE XL.

Fig. 1. Section of abdominal integument, with mammary glands of the Echidna exposed from the inner side.
    a. Left mammary gland; *a'*. Right mammary gland.
    b. Ducts converging to fundus of mammary pouch.
    d, *d'*. Part of "panniculus carnosus" acting as compressor of the gland.
    e. Fascia forming a capsule of the gland, reflected.
    f. Skin of abdomen.
Fig. 2. Section of abdominal integument, and left mammary gland and pouch.
Fig. 3. Section of abdominal integument, and right mammary gland and pouch.
    c. Cavity of pouch; the other letters as in figure 1.
Fig. 4. Orifice of mammary pouch, expanded to expose the mammary areola.
Fig. 5. Mammary areola magnified to show the orifices of the lacteal ducts, and *p*, sebaceous papillæ.
Fig. 6. Young or "mammary fœtus" of *Echidna Hystrix*: nat. size: side view.
Fig. 7. Ditto: front view.
Fig. 8. Ditto: back view.
Fig. 9. Ditto: under view.
Fig. 10. Ditto: upper view.
Figs. 11 & 12. Ditto: fore-foot magnified.
Figs. 13 & 14. Ditto: hind-foot magnified.

---

* Transactions of the Zoological Society, vol. i. pl. xxxiii. fig. 8.

PLATE XLI.

Fig. 1. Female organs of *Echidna Hystrix*; letters explained in the text.

Fig. 2. Section of uterus: magnified; ditto.

Fig. 3. Young of *Echidna Hystrix*: twice nat. size; ditto.

Fig. 4. Ditto: mouth and end of upper jaw: five times nat. size:—*a*, nostril; *e*, internarial tubercle; *m*, upper lip; *n*, lower lip; *l*, tip of tongue.

Fig. 5. Young of *Ornithorhynchus paradoxus*:—*a*, nostril; *b*, eye-orifice; *c*, ear-orifice; *e*, internarial tubercle; relatively smaller than in fig. 3, as being in progress of disappearance in a more advanced young one.

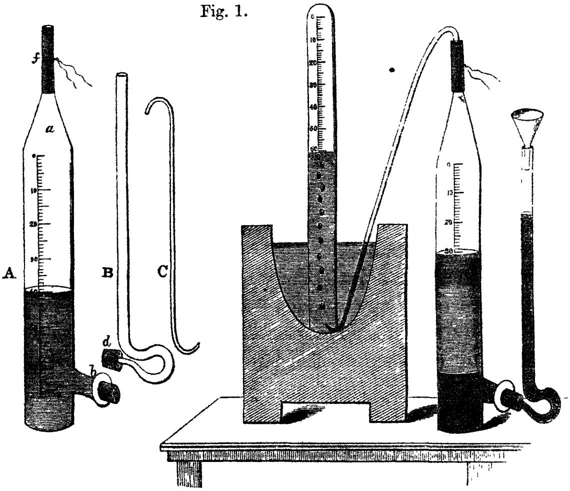
XVI. *On the Influence of Physical and Chemical Agents upon Blood; with special reference to the mutual action of the Blood and the Respiratory Gases.*

By GEORGE HARLEY, *M.D., Fellow of the Royal College of Physicians, Professor of Medical Jurisprudence in University College, London. Communicated by Professor* SHARPEY, *M.D., Sec. R.S.*

Received March 3,—Read March 10, 1864.

IN order to prevent repetition, as well as to facilitate the understanding of the researches about to be described, it is deemed advisable at once to give a brief explanation of the manner in which the experiments were conducted. In the first place, it may be mentioned that all the gas-analyses herein detailed were made in strict accordance with the justly celebrated method of Professor BUNSEN, so ably explained in his work on Gasometry. In the second place, the blood employed in the experiments was always obtained from apparently healthy animals, and with the few exceptions, presently to be alluded to, operated upon while still perfectly fresh. In the third place, the apparatus used in the majority of the experiments consisted of a graduated glass receiver of the shape represented in the accompanying figure (A), the neck of which was drawn out to a fine capillary tube, upon the end of which was placed a piece of caoutchouc tubing.

Fig. 1.



After a certain quantity of blood (usually 62 cubic centimetres) or other fluid was introduced at the mouth (*b*), the latter was firmly closed with a tightly fitting cork, and the remaining opening (*f*) secured by a ligature, so that all communication between the external atmosphere and the gas confined with the blood was effectually interrupted.

When the experiment was completed, the gas was obtained from the receiver by plunging the lower end of the vessel into mercury, and carefully removing the cork, while it was still retained in that position, so that neither the contained gas could find an exit, nor the external air obtain admittance. A tube (B) partly filled with mercury was now carefully adjusted to the mouth of the receiver by a well-fitting cork (*d*); the receiver was next removed from the mercury trough, and a fine capillary glass tube (C) inserted into the free end of its piece of caoutchouc tubing ; the end of this tube was dipped under the surface of mercury and the ligature at *f* removed. The mercury in B immediately descended and forced the atmospheric air out of the tube C, which in its turn became filled with gas from the receiver. The end of the tube C was then brought under an inverted eudiometer filled with mercury, and more of that liquid poured into B until sufficient gas was obtained from the receiver for analysis. In the fourth place, the temperature of the human body was imitated by employing an artificial digesting apparatus which could be readily kept at a constant heat of 38° C.

Lastly, the experiments were performed in a gas-laboratory, the temperature of which varied but slightly during the twenty-four hours, and their performance was thereby greatly facilitated. For the use of this laboratory I am deeply indebted to the President and Council of University College, London, who most liberally placed it at my entire disposal during a period of three years.

As indicated by the title of the paper, the series of researches about to be detailed is devoted to the influence of some physical and chemical agents on the blood with reference to its action on the respiratory gases. For the sake of convenience, the communication is divided into two parts.

The first includes the influence of the following physical agents.

a. The effect of simple diffusion in producing a change in the mixture of gases confined with blood.

b. The influence of motion on the changes reciprocally exerted upon each other by blood and atmospheric air.

c. The influence of time on the interchange of the respiratory gases.

d. The effect of temperature on the same, from 0° C. to 38° C.

e. The influence of the age of the blood, including the effect of the putrefaction.

The second part of the communication is devoted to the consideration of the influence of chemical agents, especially such as are usually denominated powerful poisons. These agents are selected from the three kingdoms.

a. Animal.
b. Vegetable, and
c. Mineral.

In relating the experiments, I have sedulously avoided advancing any theories with regard to the mode of action of any of the agents studied, and on one or two occasions only has even as much as a hint been given that the results obtained might in any way tend to the elucidation of the action of remedies or the mode of death by poison. The reticence in this instance has arisen from the circumstance that several of the results are so novel and at the same time so pregnant with material for theorizing, that the individual facts might soon be lost sight of in a sea of speculation. It appears to me therefore that the ends of science will be much better served if I confine myself to a description of the bare data, rather than propound the numerous theories which the different results suggest, and which, although they might make the paper more interesting, could not in reality add to its true value.

I may also mention that the material is so arranged as to be easily accessible, each fact having been made as far as possible independent of its associates, in order that future inquirers may find no difficulty in isolating any particular result they may desire specially to investigate. Moreover, the progressive details of each experiment are given in the form of an appendix, so that the initiated investigator can follow it with facility through its different stages, either for the purposes of comparison or verification [*].

PART I.—INFLUENCE OF PHYSICAL AGENTS.

(a) *The effect of Diffusion in modifying the composition of atmospheric air confined with fresh blood.*

The influence of both venous and arterial blood was studied.

1st. As regards arterial blood.

A certain quantity of arterial blood was allowed to flow directly from the femoral artery of a healthy dog into a glass receiver, and after being carefully secured along with 100 per cent. of atmospheric air, was placed aside in a warm room during forty-eight hours. At the end of this time the receiver was opened in the manner already described, and a certain quantity of its gas removed for analysis.

---

[*] The Appendix is deposited for reference in the Archives of the Royal Society. The first analysis only is given in detail as a specimen.

No. 1.—Air from arterial blood of Dog.

	Volume.	Barometric pressure.	Temperature.	Vol. at 0° C. and 1 metre pressure.
For carbonic acid.				
Air employed	140·3	718·7	7·7	98·08
After absorption of carbonic acid	139·0	719·4	5·8	97·91
For oxygen.				
Air employed	244·2	359·0	6·2	85·72
After addition of hydrogen	331·8	449·9	6·1	146·00
After explosion	258·0	372·9	4·5	94·64

No. 1.—In 100 parts of air.

Oxygen	19·928	Total oxygen 20·111
Carbonic acid	0·183	
Nitrogen	79·889	

2nd. As regards venous blood.

A certain quantity of venous blood was allowed to flow directly from the jugular vein of an apparently healthy dog into a glass receiver. It was then secured along with 100 per cent. of atmospheric air, and kept, as in the previous case, in a room of moderate temperature during forty-eight hours. The gas from this blood gave the following result:—

No. 2.—In 100 parts of air.

Oxygen	18·400	Total oxygen 20·557
Carbonic acid	2·157	
Nitrogen	79·443	

As the composition of ordinary atmospheric air is supposed to be:—

In 100 parts.

Oxygen	20·960	Total oxygen 20·962
Carbonic acid	0·002	
Nitrogen	79·038	

it appears from the results of these experiments that both arterial and venous blood act in precisely the same manner, the amount alone of their action being different. As might have been expected, the venous blood has yielded by simple diffusion a much greater amount of carbonic acid than the arterial blood. Moreover, under the same circumstances it has absorbed a much larger quantity of oxygen.

In 100 parts.

	Oxygen.	Carbonic acid.	Nitrogen.	Total oxygen.
Atmospheric air operated upon............	20·960	0·002	79·038	20·962
Air after forty-eight hours' contact with—				
Arterial blood	19·928	0·183	79·889	20·111
Venous blood	18·400	2·157	79·443	20·557

The total amount of oxygen is in both cases slightly diminished, and with this diminution the proportion of nitrogen, which is calculated by "difference," is necessarily increased.

(b) *Effect of Motion on the action of blood on atmospheric air.*

The mere effect of motion was attempted to be ascertained in the following manner. Two portions of the same blood of a calf, after being thoroughly arterialized by being repeatedly shaken with renewed portions of air, were confined in receivers with 100 per cent. of air, and treated in a precisely similar manner during forty-eight hours, except that one blood had a small quantity of quicksilver added to it in order to render its agitation more complete. The following were the results obtained.

Pure blood of calf, forty-eight hours' action with 100 per cent. of atmospheric air:—

No. 3.—In 100 parts of air.

Oxygen . . .	15·14	Total oxygen 18·22
Carbonic acid . .	3·08	
Nitrogen . . .	81·78	

Same blood shaken with quicksilver, forty-eight hours' action with 100 per cent of air, yielded the following result:—

No. 4.—In 100 parts of air.

Oxygen	4·11	Total oxygen 11·64
Carbonic acid . .	7·53	
Nitrogen . . .	88·76	

	Oxygen.	Carbonic acid.	Nitrogen.
Ox-blood	15·14	3·08	81·78
Ox-blood plus quicksilver...	4·1	7·53	88·76

The difference between these results is very striking, so much so, that it was thought advisable to discover if the mercury had not exerted some undefined chemical action, either on the air or blood, in addition to its mere mechanical influence in facilitating their thorough mixing. With the view of solving this question, other two portions of blood were taken, and while to one a small quantity of quicksilver was added, the other had an equal amount of powdered glass mixed with it. Both receivers were put aside in a place where the temperature never exceeded 7° C. At the end of five days, during which period they were repeatedly shaken, the air was analyzed for carbonic acid.

No. 5.—In 100 parts of air.

Carbonic acid from blood, plus quicksilver . .	1·72
,, ,, ,, ,, ,, glass . . .	1·30

As it appeared from this and the foregoing that the action of the mercury was something more than merely mechanical, in order to ascertain the influence of motion alone, two equal portions of the same fresh venous blood from an ox were placed in receivers with similar proportions of atmospheric air (1 vol. of blood to 3 vols. air) and kept at a temperature of 30° C. during six hours. In each receiver was placed a small quantity of powdered glass, in order the more effectually, when the receivers were shaken, to mix the blood. The first receiver was shaken only three minutes at a time, the second five. In all other respects they were treated exactly alike*.

Air after being enclosed during six hours at a temperature of 30° with venous blood shaken with glass, three minutes at a time. Result:—

No. 6.—In 100 parts of air.

Oxygen	14·78	Total oxygen 18·20
Carbonic acid . .	3·42	
Nitrogen . . .	81·80	

Same blood as the preceding, under precisely the same circumstances, but shaken during five minutes at a time. Result:—

No. 7.—In 100 parts of air.

Oxygen	14·49	Total oxygen 18·93
Carbonic acid . .	4·44	
Nitrogen . . .	81·07	

It thus appears that the mere effect of motion has an influence on the amount of gases interchanged.

(c) *Influence of Time on the interchange of gases between the blood and air.*

It was found from a series of experiments (as might have been expected from our knowledge of the respiratory process) that the longer air is retained in contact with blood, the greater is the change worked in its chemical composition. Thus it was found that if the ordinary respiratory act was imitated as closely as possible, by simply passing a current of pure atmospheric air through a series of twenty-four blown glass bulbs, partly filled with defibrinated arterialized ox-blood, kept in a digestive apparatus so constructed as to be capable of being retained at the temperature of the human body, the air underwent the following change.

---

* It may be here mentioned that during the course of these experiments it was found necessary, in order to arrive at anything like correct results, not only to use (in the comparative experiments) the blood of the same species of animal, but of the same bleeding; as for some cause or other, the state of the digestion or the health of the animal, different bleedings invariably gave slight differences in result.

Air after passing through twenty-four bulbs half filled with blood, at a temperature of 38° C., gave the following results:—

No. 8.—In 100 parts of air.

Oxygen	20·61	Total oxygen 21·57
Carbonic acid . .	0·96	
Nitrogen . . .	78·43	

It is thus seen that the blood out of the body exerts a similar chemical action upon air brought in contact with it as it does in the lungs of the living animal, at least so far as the interchange of gases is concerned. The next point being to retain the air longer in contact with the blood at the same temperature, the following experiment was performed.

Defibrinated fresh ox-blood, after being well arterialized by shaking it with renewed portions of air, was kept during 1½ hour in contact with 100 per cent. of pure atmospheric air at a temperature of 38° C.

No. 9.—In 100 parts of air.

Oxygen	19·76	Total oxygen 22·68
Carbonic acid . .	2·92	
Nitrogen . . .	77·32	

Another portion of the same blood as the preceding was heated in precisely the same manner, but instead of being kept only 1½ hour in contact with the air it was retained 3¼ hours.

No. 10.—In 100 parts of air.

Oxygen	18·80	Total oxygen 22·87
Carbonic acid . .	4·07	
Nitrogen . . .	77·13	

The effect of time is well illustrated in these three examples, for with the single exception of the period during which the air was in contact with the blood, all the other factors were identical. By placing the results in a tabular form, the influence of time is more easily appreciated.

	Oxygen.	Carbonic acid.	Nitrogen.
Air employed....................................	20·96	00·00	79·04
After a few seconds' action by blood	20·61	00·96	78·57
After 1½ hour's action	19·76	02·92	77·32
After 3¼ hours' action	18·80	04·28	76·92

It is here seen that the reciprocal action of blood and air is gradual, and one requiring time, a fact which supports the view that the inspired air gradually combines with the constituents of the blood in the torrent of the circulation.

(d) *Influence of Temperature.*

1st. As regards the amount of carbonic acid exhaled.

Three equal portions of freshly-defibrinated ox-blood, after being well arterialized by repeated agitation, were put into receivers with 100 per cent. of air, and kept at the following different temperatures during 3¼ hours:—

1st. At 0° C.
2nd. At 26° C.
3rd. At 38° C.

No. 11.—The results when calculated yield in 100 parts of air,—

1st. Temperature 0° C.=0·00 carbonic acid.
2nd. „ 26° C.=3·08 „
3rd. „ 38° C.=4·07 „

Thus the higher the temperature, up to a certain point, the greater is the amount of carbonic acid exhaled.

In order to see if the same rule is applicable to the oxidation of the constituents of the blood, other three portions of defibrinated ox-blood were taken, and after being treated in the usual way, were kept at different temperatures during twenty-four hours.

(*a*) In an ice cellar.
(*b*) In a room at 12° C.
(*c*) In an artificial digesting apparatus heated to 38° C.

(*a*) Ox-blood with 100 per cent. of air, twenty-four hours' action at 0° C. Result:—

No. 12.—In 100 parts of air.

Oxygen	17·43	Total oxygen 18·02
Carbonic acid . .	0·59	
Nitrogen . . .	81·98	

This experiment was made in foggy weather.

(*b*) Ox-blood with 100 per cent. of air, twenty-four hours' action at 12° C. Result:—

No. 13.—In 100 parts of air.

Oxygen	12·54	Total oxygen 15·31
Carbonic acid . .	2·77	
Nitrogen . . .	74·69	

(*c*) Ox-blood with 100 per cent. of air, twenty-four hours' action at 38° C. Result:—

No. 14.—In 100 parts of air.

Oxygen	00·00	Total oxygen 22·40
Carbonic acid . .	22·40	
Nitrogen . . .	77·60	

The amount of carbonic acid exhaled in this case seems very extraordinary, nevertheless I believe that it is perfectly correct, for another portion of the same blood, used as a controlling experiment, yielded to within a fraction of the same amount of carbonic acid. The fraction of difference, too, was an excess, being 22·6 instead of 22·4. Thus 24 hours at 38° C. Result:—

No. 15.—In 100 parts of air.

Carbonic acid=22·6.

As the weather was exceedingly foggy at the time these experiments were made, it was deemed advisable to analyze the fog in order to ascertain how much carbonic acid it contained, lest the extraordinary results obtained in the last two experiments might be due to that cause, or to some disease in the blood.

No. 16.—Result of an analysis of fog in 100 parts of air.

Carbonic acid=0·52.

This is the greatest amount of carbonic acid I ever obtained from London fog, and large though it be, it is still far too small a quantity to account for the results in the last two cases.

By placing the different effects of temperature in a tabular form, the influence exerted by that factor over the chemical changes occurring in blood will be still better appreciated.

Defibrinated ox-blood.	Oxygen.	Carbonic acid.	Nitrogen.
Temperature 0° C. 24 hours	17·43	00·59	81·98
„ 12° C. „	12·54	02·77	74·69
„ 38° C. „	00·00	22·40	77·60

The influence of temperature on the interchange of gases is equally well illustrated by comparing the results of experiment 13 with that of experiment 10, when it will be seen that 3¼ hours' action at a temperature of 38° C. (the temperature of the animal body) yields much more carbonic acid than 24 hours' action at a temperature of 12° C.

100 per cent. of air with ox-blood.	Oxygen.	Carbonic acid.	Nitrogen.
24 hours' action at 12° C....................	12·54	2·77	74·69
3¼ „ „ 38° C....................	18·80	4·07	77·13

The effect of temperature on the individual constituents of the blood was also studied, but only with red coagulum was it found sufficiently well marked to merit being noticed here. Three equal portions of coagulum from fresh ox-blood were confined with 100 per cent. of atmospheric air during six hours at the following temperatures.

(*a*) At 21° C.; (*b*) at 30° C.; (*c*) at 36° C., with the following results:—

Amount of carbonic acid in 100 parts of air in

No. 17. (*a*) 6 hours at temperature of 21° C.=2·34 carbonic acid.
No. 18. (*b*) „ „ 30° C.=5·18 „
No. 19. (*c*) „ „ 36° C.=7·29 „

It is thus seen that the amount of carbonic acid exhaled by red-blood coagulum increases with the temperature as far as the experiment went, namely from 21° to 36° C.

2nd. As regards the influence of cold in retarding the reciprocal chemical changes which occur between atmospheric air and blood, a striking proof of which is to be found in the result of the following experiment.

Two ounces of arterial blood were allowed to flow directly from the carotid artery of a dog into a glass receiver, which in order still further to ensure its being thoroughly oxidized, as well as to prevent its coagulating into a solid mass, was shaken with renewed portions of air during two hours; a small quantity of fluid mercury being also employed to prevent the coagulation. After this treatment the receiver was firmly corked and kept (with occasional agitation) in a room the temperature of which never exceeded 7° C. during five whole days.

Dog's arterial blood five days at a temperature under 7° C.[*] Result:

No. 20.—In 100 parts of air.

Oxygen	12·62	Total oxygen 14·34
Carbonic acid . .	1·72	
Nitrogen . . .	85·66	

On its removal from the receiver, the blood, although dark in colour, had a perfectly fresh odour. The diminished temperature not only retarded the chemical changes, which for the sake of convenience we may term "respiratory," but also those decompositions and transformations so intimately connected with oxidation, to which the name "putrefaction" has been given.

(e) *Influence of the age of the blood.*

The putrefactive changes occurring in blood are exceedingly curious, and perhaps it may not be out of place if some of them be here alluded to.

The following series of experiments were made on sheep's blood. The first began within two hours after the blood was withdrawn from the animal, the last after it had stood 688 hours.

---

[*] The first part of this experiment has been already given, but it is here again repeated in order to save the time of the reader in referring back to it, and so it is occasionally done with some others.

Two ounces of well defibrinated sheep's blood, after being arterialized by constant agitation with renewed portions of air during twenty minutes, were put into a receiver with 100 per cent. of atmospheric air and kept during twenty-four hours in a room the temperature of which varied from 6° to 12° C. Result:—

No. 21.—In 100 parts of air.

Oxygen	13·76	Total oxygen 15·81
Carbonic acid . .	2·05	
Nitrogen . . .	84·19	

A similar portion of the same blood as the preceding, after being exposed to the air in an open glass vessel during sixty hours, was treated in an exactly similar manner, and then placed in a receiver with 100 per cent. of air. The temperature of the room during the time of the experiment varied, as before, from 6° to 12° C. The blood after the sixty hours' exposure had become of a dark venous hue, but it still arterialized readily on being agitated with fresh portions of air. It smelt slightly, as if putrefaction had begun. Under the microscope the red blood-corpuscles were perfectly distinct. Result:—

No. 22.—In 100 parts of air.

Oxygen	2·88	Total oxygen 6·57
Carbonic acid . .	3·69	
Nitrogen . . .	93·43	

This blood, which was of a bright arterial hue when put into the receiver with the air, at the end of the twenty-four hours had again resumed the venous colour. On shaking the vessel the blood looked as if it were decomposed. It remained of a dark purple colour on the sides of the glass, although the blood was at this time eighty-four hours old. On removing it from the receiver, and shaking it with renewed portions of atmospheric air, it again assumed the arterial tint. After the sheep's blood was 136 hours old it was of a dark purple colour, and when a thin layer was spread over a white plate it looked quite granular. When examined with the microscope, the blood-corpuscles were still found perfectly distinct in their outline, and on being measured they averaged $\frac{1}{400}$ millim. ($\frac{1}{10,000}$ inch) in diameter. The blood arterialized readily on being shaken with fresh air.

A third portion of this blood was taken and subjected in every respect to precisely the same treatment as in the two preceding cases. Result:—

No. 23.—In 100 parts of air.

Oxygen	1·01	Total oxygen 5·32
Carbonic acid . .	4·31	
Nitrogen . . .	94·68	

A fourth portion from the same blood, after it was 184 hours old, still became of an arterial hue when well shaken with air, although it had a film of fungi on its surface, and smelt strongly as if it were putrid. When once arterialized it looked exactly like freshly-drawn blood, and when examined microscopically it showed the red blood-corpuscles as well as if it had only been a day old. Indeed, by its previous history, and smell alone, could a stranger have had any idea of its having been drawn from the animal more than a few hours. The fourth portion was treated in a similar manner, and for the same length of time as the others.

In this case, for some cause or other, no explosion could be obtained, even after the addition of 50 per cent. of explosive gas. Result:—

No. 24.—In 100 parts of air.

Oxygen	0·00
Carbonic acid . .	4·91
Nitrogen . . .	95·09

The blood after 304 hours' exposure still arterialized when well agitated with air. On using the microscope, the corpuscles were found to be distinct, though not so numerous as at first. They were best seen without adding water. Indeed the addition of water almost totally destroyed them by instantly dissolving their attenuated walls and allowing their contents to escape.

A fifth portion of this blood was treated precisely as the preceding examples with 100 per cent. of air in one of the usual glass receivers, the temperature of the room varying, as before, from 6° to 12° C.

The oxygen, if there was any, was not estimated.

No. 25.—In 100 parts of air.

Carbonic acid . . 4·99

The blood after being kept 688 hours still arterialized on being thoroughly shaken with renewed portions of air. It was fearfully fetid, and contained numbers of living animalcules of the Vibrio class. The red corpuscles were still distinct, though in greatly diminished quantity, from numbers of them having become broken up and dissolved*.

The usual quantity of this blood was put into the receiver with 100 per cent. of air and treated during twenty-four hours in the ordinary manner.

No. 26.—In 100 parts of air†.

Carbonic acid . . 5·11

---

* This series of experiments was performed in the winter months, but in one conducted during the months of April, May, June, and July, I was able to detect blood-corpuscles in the putrid fluid after it was three months and seven days old; so that blood-corpuscles appear to be much more persistent bodies than is in general imagined.

† The oxygen was also estimated in this case, but unfortunately without a controlling experiment being at the same time performed, so it is of little value. The following is the result of the analysis.

No. 27.—In 100 parts of air.

Oxygen . . 2·10

The analysis of the gas after twenty-four hours' contact with the blood therefore stands thus:

In 100 parts of air.		
Oxygen	2·10	Total oxygen 7·21
Carbonic acid . .	5·11	
Nitrogen . . .	92·79	

As it is rather troublesome to carry the results of these analyses in the mind, I shall now give them in a tabular form, when it will be at once evident to any one who has given attention to the subject, that the chemical changes exerted upon air by putrefaction, in so far as they are here studied, are very different from the true respiratory ones previously alluded to.

In 100 parts of air.	Oxygen.	Carbonic acid.	Nitrogen.
1st portion of fresh blood....................	13·76	2·05	84·19
2nd " same " 60 hours old ...	2·88	3·69	93·43
3rd " " " 136 " ...	1·01	4·31	94·68
4th " " " 184 " ...	0·00	4·91	95·09
5th " " " 304 " ...	——	4·99	——
6th " " " 688 " ...	——	5·11	——

It is here seen that the process of putrefaction exerts, up to a certain extent, the same effect on the absorption of oxygen and exhalation of carbonic acid by the constituents of the blood, as was observed to be exercised by an increase of temperature. Thus we find that the older the blood becomes the more oxygen it extracts from the air, and the more carbonic acid does it at the same time yield. Here, however, the analogy stops. For we find that while in those cases where the normal respiratory action is such as to have produced the exhalation of more than 5 per cent. of carbonic acid, the oxygen does not entirely disappear from the air (see experiments 35 and 58, Part II.), and in those again where the oxygen has been entirely taken up by the blood it is again all returned to the atmosphere, as seen in the results of experiment 14 related at page 695. During the putrefactive process, on the other hand, the amount of oxygen absorbed is exceedingly great in proportion to the quantity of carbonic acid exhaled.

PART II.—INFLUENCE OF CHEMICAL AGENTS ON THE BLOOD.

EFFECT OF ANIMAL PRODUCTS.

Snake Poison.

For the purpose of studying the effect of animal poisons upon the reciprocal action of blood and atmospheric air, I obtained, through the kindness of the late Mr. MITCHELL, Secretary to the Zoological Gardens, the loan of two African Puff Adders. They were 3 feet in length, and about 8 inches in circumference at the thickest part.

The physiological action of animal poisons being as yet imperfectly understood, before alluding to the special action of the poison on the blood, I shall briefly relate the history of one of the experiments.

The experiments were performed at University College, in the presence of my colleagues, Professors SHARPEY, ELLIS, and WILLIAMSON. The serpents had eaten nothing during eight days, so it was supposed that their poison-bags were well charged with venom.

A large dog was bitten by one of the snakes over the right eye. The immediate appearance of a drop of blood indicated the position of the wound. In three minutes the dog became very restless, and gave a low whine as if in pain. After moving about the room for ten minutes searching for a comfortable place to lie down on, he placed himself in the coolest part of the chamber, and laid his head on the cold stones, as if to relieve headache. He moaned as if in distress. In a quarter of an hour after he received his wound the pulse had fallen from 100 to 64 per minute. As the effects of the poison passed away the pulse gradually recovered, and in twenty-five minutes it was again as high as 96 per minute.

In one hour after being bitten the dog had so far got over the effects of the poison as to be able to run about.

The serpent was once more allowed to bite him. The same train of symptoms again appeared, but in a more intense degree, and within twenty-five minutes he had become insensible. He looked as if in a profound sleep, from which he could not be roused. The respirations were 40 per minute, and the pulse so feeble in the femoral artery that it was found impossible to count it. The pupils were dilated.

Half an hour after being bitten the second time convulsive twitchings began to appear in the fore limbs and in the muscles of the neck. In ten minutes more the whole body became convulsed. The limbs were stretched out, and the head jerked backwards. During the convulsions the respirations rose to 90 per minute; but they subsided to 40 in the intervals. The temperature of the rectum gradually fell in the course of one hour and a half from 38° to 35° C. In two hours the respirations were reduced to 9 per minute, the animal temperature at the same time being 34° C. The pulse was completely imperceptible; even the heart's action could not be felt through the ribs.

In two hours and a quarter the animal appeared to be dead; but on making an incision into the thorax he gave a gasp. After waiting some time, without observing any further sign of life, another incision was made, when he again gasped, but only once. On opening the thorax the heart was found pulsating at the rate of 60 per minute; it was, however, more like a quivering than a true pulsation. The tissues of this and of the other animals killed by the puff adders presented a very strange appearance, namely, numerous extravasations of blood throughout the body, some small, some large. For example, in this animal there was an extravasation of blood into the anterior mediastinum, and into the tissue of the pericardium, but no effusion into the pericardium itself. There were extravasations along all the great veins, into the cellular tissue of the pancreas, throughout the diaphragm, beneath the peritoneum, and all over the abdomen. The interior of the latter, indeed, looked exactly as if it had been sprinkled over with blood. A similar condition also existed in the subcutaneous cellular tissue. In fact, had the history of the case not been known, it would have been supposed that the animal had laboured under a severe form of purpura hæmorrhagica.

In the neighbourhood of the wounds there was great swelling, as well as an extravasation of brownish putrid looking blood. Everything pointed to blood poisoning.

The state of the spleen merits special attention. It was of a dark bluish olive tint; quite peculiar. I have never met with a similar hue in any other case of poisoning. On exposure to the air the blood became arterialized, and the organ then lost the strange appearance. The muscles were darker than usual. In the course of a few hours they passed into a state of rigor mortis, which was quite distinct seventeen hours after death. The brain was very anæmic, and showed no signs of extravasation.

In the course of a few weeks after this experiment was made three of the puff adders died and were sent to me for examination. They were in exceedingly good condition, and beyond having fatty livers there was no apparent disease. On removing the poison from their poison bags and allowing it slowly to evaporate on a glass slide, beautiful crystals were observed to form in it similar to the specimens represented in the accompanying figure.

Fig. 2.

Crystals from puff-adder poison.

This crystalline body seems to be peculiar to this species of snake, as I failed to obtain it from the common adder, as well as from two specimens of Cobra, one from Morocco, and one from Egypt.

Examination of the Blood.

Under the microscope, the red corpuscles were in general normal in appearance. There were, however, a number of three-cornered ones to be seen, like what is sometimes met with in the half-putrid blood of fish. There was also an excess of white corpuscles, which might have been due to the animal being in full digestion.

After the blood had stood for some hours in a glass vessel, although not coagulated, it had deposited the corpuscles and left a layer of serum on the top*. Shaken with air it arterialized readily. It contained 0·235 gramme (3·64 grains) of urea per ounce. No sugar could be detected in it, yet after standing a couple of days it became quite acid. A quantity of this blood, after being thoroughly arterialized, was put into a receiver with 100 per cent. of air, and in order to make the experiment as exact as possible, a healthy dog was sacrificed, and a similar quantity of its blood treated in exactly the same manner. As this experiment was performed during the season of the year when the days were short, and I could not work in the laboratory after four o'clock, I carried the receivers home with me, and repeatedly agitated them during the evening, and pretty far on into the night.

After twenty-four hours' action the analyses of the gases gave the following results:—

1st. Blood of healthy dog. Result:—

No. 28.—In 100 parts of air.

Oxygen . . .	19·700	Total oxygen 20·109
Carbonic acid .	0·409	
Nitrogen . . .	79·891	

2nd. Blood of dog poisoned by puff adder. Result:—

No. 29.—In 100 parts of air.

Oxygen . . .	17·09	Total oxygen 18·18
Carbonic acid .	1·09	
Nitrogen . . .	81·82	

It is here observed that there has been a marked difference in the action of the two bloods. The puff-adder poison seems to have accelerated the transformations and decompositions upon which the absorption of oxygen and the exhalation of carbonic acid by the blood depend. By placing the results in the form of a Table, this fact is rendered still more apparent.

	Oxygen.	Carbonic acid.	Nitrogen.	Total oxygen.
In 100 parts of atmospheric air	20·960	0·002	79·038	20·962
Ditto, after being acted on by pure blood	19·700	0·409	79·891	20·109
Ditto, after being acted on by poisoned blood..	17·09	1·09	81·82	18·18

---

* On opening the other animals some hours after death the blood was found to be fluid, but it coagulated after its withdrawal from the body. It formed a jelly rather than a clot. There seemed to be a marked diminution in the amount of fibrin, as well as a thinning of the blood, in all the cases.

As these results are probably different from what most persons may have expected, it may be advisable briefly to relate the controlling experiments, at least so much of them as refer to the exhalation of carbonic acid. They were performed in a precisely similar manner, except that the proportion of blood to that of air was as one to three.

1st. Healthy dog. 1 volume of pure blood to 3 volumes of air. Twenty-four hours' action at temperature under 12° C. Result:—

No. 30.—In 100 parts of air.

Carbonic acid	0·38

2nd. Blood of dog poisoned by puff adder. 1 volume of blood to 3 volumes of air. Twenty-four hours' action at temperature under 12° C. Result:—

No. 31.—In 100 parts of air.

Carbonic acid	0·78

Here too it is seen that, although treated in every respect alike, the blood of the poisoned dog exhaled more carbonic acid than that of the healthy animal.

Uric Acid.

As uric acid, although a normal constituent of the animal body, may be regarded in the light of an animal poison, inasmuch as it is an effete product, it was experimented with in the following manner.

Two portions of well defibrinated sheep-blood, after being thoroughly arterialized, were placed in receivers with 100 per cent. of atmospheric air. To one of them was added 0·2 gramme (3·1 grains) of pure uric acid prepared from human urine (the uric acid was thoroughly pounded in distilled water and then mixed with the blood in a mortar; 62 grammes of blood was the quantity employed). The pure blood was treated in the same way, but with distilled water alone. After twenty-four hours' action under identical circumstances, the air of the receivers was analyzed.

Air after being in contact with pure blood of sheep during twenty-four hours. Result:—

No. 32.—In 100 parts of air.

Oxygen . . .	13·90	Total oxygen 15·85
Carbonic acid .	1·95	
Nitrogen . . .	84·15	

Air after being in contact with sheep's blood to which uric acid was added. Result:—

No. 33.—In 100 parts of air.

Oxygen . . .	13·17	Total oxygen 15·79
Carbonic acid .	2·62	
Nitrogen . . .	84·21	

It is thus seen that the presence of an abnormal amount of uric acid in blood hastens the chemical decompositions and transformations upon which the absorption of oxygen and exhalation of carbonic acid depend.

Animal Sugar.

As an illustration of the action of animal sugar upon blood, the following experiment may be cited. To a third portion (62 grammes) of the same blood as was used in the two preceding experiments, 0·4 gramme (6·2 grains) of sugar obtained from the urine of a diabetic patient were added. The sugar was first made into a syrup with a small quantity of distilled water, and then mixed in a mortar with the blood. In order to avoid all possibility of error, the pure blood, as before stated, was treated in the same way with distilled water alone. Result:—

No. 34.—In 100 parts of air.

$$\left.\begin{array}{ll}\text{Oxygen . . .} & 15{\cdot}01\\ \text{Carbonic acid .} & 1{\cdot}61\end{array}\right\}\text{Total oxygen } 16{\cdot}62$$

Nitrogen . . . 83·38

It is here seen that the animal sugar had the effect of retarding the respiratory changes produced in atmospheric air by blood, less carbonic acid being exhaled, and a smaller amount of oxygen absorbed; just the opposite effect as was observed to follow the addition of uric acid to blood.

The subjoined Table shows this more distinctly.

Sheep's blood. Twenty-four hours. 100 per cent. of air.

	Oxygen.	Carbonic acid.	Nitrogen.	Total oxygen.
Pure blood	13·90	1·95	84·15	15·85
Blood plus uric acid	13·17	2·62	84·21	15·79
Blood plus sugar	15·01	1·61	83·38	16·62

ACTION OF VEGETABLE PRODUCTS ON BLOOD.

Hydrocyanic Acid.

The following are examples of the influence of hydrocyanic acid on the action of blood on the respiratory gases.

A quantity of perfectly fresh ox-blood was taken and carefully switched until freed, as far as possible, of its fibrin. After being thoroughly arterialized, it was then divided into several portions of 62 grammes each, and treated in the usual manner in a room of moderate temperature during twenty-four hours.

Pure defibrinated ox-blood with 100 per cent. of atmospheric air. Twenty-four hours' action. Result:—

No. 35.—In 100 parts of air.

Oxygen . . .	10·42	Total oxygen 15·47
Carbonic acid. .	5·05	
Nitrogen . . .	84·53	

Defibrinated ox-blood with 6 drops (20 per cent. strength) of hydrocyanic acid. 100 per cent. of air. Twenty-four hours' action. Result:—

No. 36.—In 100 parts of air.

Oxygen . . .	16·32	Total oxygen 18·23
Carbonic acid .	1·91	
Nitrogen . . .	81·77	

It is thus seen that the effect of hydrocyanic acid is to retard those transformations and decompositions upon which the interchange of the respiratory gases depend. The effect is well marked in this case, but it is even more so in a case of poisoning in the human subject, which I shall immediately refer to; meanwhile the results of these two analyses are—

	Oxygen.	Carbonic acid.	Nitrogen.	Total oxygen.
In 100 per cent. of air from pure ox-blood......	10·42	5·05	84·53	15·47
Ditto plus hydrocyanic acid	16·32	1·91	81·77	18·23

Action of Hydrocyanic Acid on Human Blood.

A quantity of blood was removed from the heart and great vessels of a healthy well-developed young woman, aged 19 years, who died within half an hour after swallowing a couple of drachms of bitter almond oil. The blood was still fluid forty-eight hours after death, and yielded a small quantity of hydrocyanic acid by distillation. A portion of the blood, after being thoroughly arterialized by agitation with renewed portions of air, was put into a receiver with 100 per cent. of atmospheric air, and kept twenty-four hours (with occasional agitation) in a room of an average temperature of 15° C. At the end of the twenty-four hours the air confined with the blood was analyzed, with the subjoined result:—

No. 37.—In 100 parts of air.

Oxygen	19·56
Carbonic acid	0·00
Nitrogen	80·44

It is here seen that the effect of hydrocyanic acid is the same in the body as out of it, namely, to arrest respiratory changes.

Nicotine.

Various experiments were performed with nicotine, and it was invariably found to produce the same result; namely, to retard the normal oxidation processes in blood, and at the same time to diminish the exhalation of carbonic acid. The following experiment may be quoted as an illustration of the fact.

Two portions (62 grammes) of defibrinated ox-blood, after being thoroughly arterialized, were placed in receivers with 100 per cent. of atmospheric air, and both were treated during twenty-four hours exactly alike, except that to one was added 6 drops of chemically pure nicotine.

Gas from pure ox-blood after twenty-four hours' action with 100 per cent. of atmospheric air. Result:—

No. 38.—In 100 parts of air.

Oxygen . . .	14·66	Total oxygen 17·04
Carbonic acid . .	2·38	
Nitrogen . . .	82·96	

Gas from ox-blood after twenty-four hours' action with 6 drops of nicotine. 100 per cent. of atmospheric air. Result:—

No. 39.—In 100 parts of air.

Oxygen . . .	19·60	Total oxygen 21·09
Carbonic acid .	1·49	
Nitrogen . . .	78·91	

It is thus seen, as was before said, that nicotine diminishes the power of the blood to take up oxygen and give off carbonic acid, and thereby become fitted for the purposes of nutrition.

	Oxygen.	Carbonic acid.	Nitrogen.	Total oxygen.
In 100 per cent. of air from pure ox-blood......	14·66	2·38	82·96	17·04
Ditto plus nicotine	19·60	1·49	78·91	21·09

Woorara Poison.

Two portions of defibrinated sheep's blood, after being thoroughly arterialized, were placed in receivers with 100 per cent. of atmospheric air, and kept, with occasional shaking, at a temperature of 15° C. during twenty-four hours. The treatment of the two portions of blood only differed in this respect, that to one nothing was added, while 0·01 gramme of woorara was put into the other. The amount of blood in each case was 62 grammes.

Air from pure sheep's blood. Twenty-four hours' action. 100 per cent. of air. Result:—

No. 40.—In 100 parts of air.

Oxygen	12·42	Total oxygen 13·12
Carbonic acid . .	0·70	
Nitrogen . . .	86·88	

Air from sheep's blood plus woorara. Twenty-four hours' action. 100 per cent. of air. Result:—

No. 41.—In 100 parts of air.

Oxygen	16·68	Total oxygen 18·28
Carbonic acid . .	1·60	
Nitrogen . . .	81·72	

It is thus seen that woorara has the peculiar effect of diminishing oxidation, and at the same time increasing the exhalation of carbonic acid gas.

	Oxygen.	Carbonic acid.	Nitrogen.	Total oxygen.
In 100 per cent. of air from pure sheep's blood	12·42	0·70	86·88	13·12
Ditto plus woorara	16·68	1·60	81·72	18·28

For the purpose of studying the action of woorara upon the blood of the living animal, I injected under the skin of a dog an aqueous solution of five grains of the poison[*]. The animal soon became paralyzed and died, as is usual in those cases, from the cessation of the respiratory movements. The heart's action continued vigorous for some time after apparent death: a portion of this dog's blood was then taken and thoroughly arterialized by repeatedly shaking it with renewed quantities of air. The blood was then enclosed in a receiver with 100 per cent. of atmospheric air, and treated in the usual way during twenty-four hours. The result of the analysis was as follows:—

No. 42.—In 100 parts of air.

Oxygen	18·68	Total oxygen 20·19
Carbonic acid . .	1·51	
Nitrogen . . .	79·81	

If we compare this result with the analysis of air from the blood of a healthy dog (No. 28) already given (page 702), we shall find that the effect of the woorara has been like that of snake poison, to increase the chemical decompositions and transformations in the blood, upon which the exhalation of carbonic acid depend.

---

[*] For the woorara employed on this occasion I am indebted to the liberality of Charles Waterton, Esq., of Walton Hall, the well-known author of the 'Wanderings.' He obtained it in Guiana in 1812, and though it is consequently half a century old, it is still an exceedingly active poison.

In 100 parts of air.	Oxygen.	Carbonic acid.	Nitrogen.	Total oxygen.
Healthy blood of dog	19·700	0·409	79·891	20·109
Blood of dog poisoned with woorara ...	18·680	1·510	79·810	20·190

It will be observed that there is a slight discrepancy between the amount of oxygen absorbed in this and the other experiment on the action of woorara out of the body; for here the oxidation has been greater than in the healthy animal. This most probably arises, however, from some accidental cause, due to the blood being taken from different animals and not operated on in the same day. Unfortunately it is impossible to operate on both healthy and poisoned blood of the same animal at the same time, so that all our experiments of comparison on the blood of living animals are liable to the source of error arising from the state of the body and the constitutional peculiarity of the animal. My former statement regarding the action of woorara, namely, that it diminishes oxidation and increases the exhalation of carbonic acid, at least in sheep's blood, is I have little doubt correct, as I have invariably found it to be so. I might here quote other experiments in proof of this assertion, but in order to prevent unnecessary repetition, shall delay doing so till the action of woorara is compared with that of other substances.

Antiar and Aconitine.

For the sake of brevity I shall take these two poisons together. As is well known, their physiological action on the animal body is, as nearly as possible, identical. They are both powerful cardiac poisons. So powerfully, indeed, do they act in this way, that when given to frogs they stop the action of the heart while the animal is otherwise sufficiently well to be able to spring about. This is the reverse of woorara, which allows the heart's action to continue long after the rest of the body is dead. Hence arises the saying that we may have a dead heart in a living body with antiar and aconitine, and a dead body with a living heart with woorara.

The result of the following experiment forcibly illustrates the truth of the latter statement. A healthy full-grown frog was pricked with the point of a poisoned arrow, and in the course of a few minutes its limbs gradually became paralysed. The paralysis soon extended itself over the body, the animal ceased to breathe, and in the course of a few minutes more was dead. On examining the heart about an hour afterwards, that organ, and that organ alone, was found still alive. Death could not be said here to have usurped its power, for it slowly and regularly pulsated as in life. On the following day the heart still continued to beat although the tissues surrounding it had assumed the appearance of death. Forty-eight hours after the animal had been poisoned its heart still continued to act regularly, and even seventy-two hours afterwards the action of the ventricle and auricles, though feeble, was yet distinct. On the fourth day (ninety-six hours after death) part of the heart died, the left auricle alone continued to pulsate. But now, not only was the frog dead, but its lower limbs were already shrunk and withered. I then made an attempt at resuscitation, and exactly 100 hours after the animal died I put it into a moist warm atmosphere, and there retained it till the temperature of its body was slightly raised. This treatment had the effect of restoring the irritability of the heart, and on touching the ventricle with a point of my pen it resumed its pulsations, and during several minutes the contractions, first of the auricles and then of the ventricles, continued rhythmically; even the pulsations in the large vessels attached to the heart also became distinctly visible, and continued so with regularity for upwards of a quarter of an hour.

The chemical action of antiar and aconitine on the blood, like their physiological action on the nervous system, are as near as possible alike. First, as regards their influence on the exhalation of carbonic acid. Two portions of thoroughly defibrinated and well arterialized sheep's blood, 62 grammes each, were put into receivers with 100 per cent. of air. To the one 0·01 gramme of antiar dissolved in water was added; to the other a similar quantity of pure aconitine dissolved in faintly acid water. After twenty-four hours' action the air in the receivers was analyzed with the following results.

Antiar*, twenty-four hours' action, 100 per cent. of air. Result:—

No. 43.—In 100 parts of air.

Carbonic acid . . . 2·05.

No. 44.—Result of analysis of air from blood with aconitine in 100 parts of air.

Carbonic acid . . . 2·02.

It is thus seen that the influence of antiar and aconitine on the exhalation of carbonic acid is very similar. I shall now quote a series of experiments in which the influence of these substances with that of woorara is compared.

A quantity of defibrinated sheep's blood was taken seventeen hours after the death of the animal, and after being completely arterialized it was divided into four portions, each of which was put into a receiver with 100 per cent. of atmospheric air. They were all treated precisely alike, except that to one 0·092 gramme of antiar was added, to another 0·092 gramme of aconitine, and to a third 0·092 gramme of woorara. The fourth portion was retained pure in order to form a standard of comparison. After twenty-four hours' action the air was analyzed, with subjoined results.

No. 45.—Air from pure blood in 100 parts of air.

Oxygen	13·76	Total oxygen 15·81
Carbonic acid . .	2·05	
Nitrogen . . .	84·19	

---

* For the antiar employed in these experiments I am indebted to the kindness of Professor SHARPEY.

No. 46.—Air from blood plus woorara, in 100 parts of air.

Oxygen	16·85	Total oxygen 19·83
Carbonic acid . .	2·98	
Nitrogen . . .	80·17	

No. 47.—Air from blood plus antiar, in 100 parts of air.

Oxygen	12·98	Total oxygen 13·99
Carbonic acid . .	1·01	
Nitrogen . . .	86·01	

No. 48.—Air from blood plus aconitine, in 100 parts of air.

Oxygen	11·66	Total oxygen 12·96
Carbonic acid . .	1·30	
Nitrogen . . .	87·04	

By placing these results in a tabular form the comparative value of each of the factors will be made more apparent.

	Oxygen.	Carbonic acid.	Nitrogen.	Total oxygen.
In 100 parts of air from pure blood	13·76	2·05	84·19	15·81
Blood plus woorara	16·85	2·98	80·17	19·83
„ „ antiar	12·98	1·01	86·01	13·99
„ „ aconitine	11·66	1·30	87·04	12·96

The similarity in the action of antiar and aconitine, and the dissimilarity between their action and that of woorara, are well illustrated in the above Table. The woorara diminishes oxidation and increases the exhalation of carbonic acid. Antiar and aconitine increase oxidation and diminish the exhalation of carbonic acid gas.

Strychnine.

In order to ascertain the influence of strychnine, a quantity of fresh calf's blood was shaken with renewed portions of atmospheric air until it had become thoroughly saturated with oxygen. It was then enclosed in a receiver with 100 per cent. of ordinary air, corked up, and kept in a room of moderate temperature during twenty-four hours.

A second portion of the same blood (62 grammes) was similarly treated in every way except that it had 0·05 gramme of strychnine added to it. During the twenty-four hours the receivers were as usual frequently agitated to favour the mutual action of the blood and air. At the end of this period the composition of the gas in the receivers was found to be—

Gas from pure calf's blood, twenty-four hours' action with 100 per cent. of air:—

No. 49.—In 100 parts of air.

Oxygen. . . .	12·10	Total oxygen 18·04
Carbonic acid. .	5·94	
Nitrogen . . .	81·96	

Gas from calf's blood plus strychnine, dissolved in a minimum of very dilute hydrochloric acid, twenty-four hours' action with 100 per cent. of air:—

No. 50.—In 100 parts of air.

Oxygen. . . .	17·82	Total oxygen 20·55
Carbonic acid. .	2·73	
Nitrogen . . .	79·45	

Thus it is seen that strychnine is one of those substances possessing the strange property of preventing the chemical decompositions and transformations of the constituents of the blood upon which the absorption of oxygen and exhalation of carbonic acid depend.

	Oxygen.	Carbonic acid.	Nitrogen.	Total oxygen.
In 100 parts of gas from pure calf's blood	12·10	5·94	81·96	18·04
Ditto plus strychnine	17·82	2·73	79·45	20·55

The next point to determine is, does strychnine act in the same manner on blood in the living animal as out of it?

The results of the two following experiments seem to indicate this, but as they were performed with the view of solving an entirely different question not requiring any controlling experiments, they had none made with them, and therefore they can only be taken for what the results of single experiments are worth.

Into the peritoneal cavity of a healthy full-grown cat was injected a solution of $\frac{1}{20}$th of a grain of strychnine. In five minutes the animal became convulsed, and in four minutes more it died. On opening the body eight minutes after death, some of the blood was found already coagulated in the greater vessels, and the portion that was fluid coagulated as soon as it flowed into a capsule. The blood had a dark purple colour, and when shaken on the sides of a glass looked almost grumous and granular, as if the corpuscles were broken up, and had allowed their contents to escape. Under the microscope plenty of healthy red corpuscles were seen, many of them running into rolls; but besides these, although there were no broken-up cells to be seen yet there were an unusual number of small granules in the field. The animal was fasting, nevertheless there were also a considerable number of white corpuscles present. The blood contained 0·22 gramme of urea to the oz. (0·709 per cent.) and abundance of sugar.

Gas from blood of cat poisoned with strychnine, twenty-four hours' action with 100 per cent. of air in a room of moderate temperature:—

No. 51.—In 100 parts of air.

Oxygen	17·03	Total oxygen 17·63
Carbonic acid . .	0·60	
Nitrogen . . .	82·37	

It is thus seen that the blood of the poisoned animal yields even a smaller quantity of carbonic acid than the blood to which strychnine has been added out of the body, while the quantity of oxygen that has disappeared is the same in both cases.

Brucine.

Besides strychnine the alkaloid brucine is also obtained from nux vomica, and the following experiment was made with the view of testing if it had a similar action upon blood. The experiment in this case, however, was somewhat extended in order to compare its action with that of two other substances, namely, quinine and morphia, and as the results obtained form rather an interesting series, I shall give them consecutively.

A quantity of perfectly fresh calf's blood, after being defibrinated and thoroughly saturated with oxygen by repeatedly shaking it with renewed quantities of air, was divided into several portions of 62 grammes each. To the first nothing was added; to the second 0·005 gramme of brucine; to the third 0·005 gramme of quinine; and to the fourth 0·005 gramme of morphine: these alkaloids were all dissolved by the aid of a minimum quantity of hydrochloric acid. The different portions were then enclosed in receivers with 100 per cent. of air, and treated in the usual manner, with occasional agitation, in a room of moderate temperature during twenty-four hours. At the expiration of that period the air was analyzed, with the following results:—

No. 52.— The air from pure calf's blood contains in 100 parts of air—

Oxygen	6·64	Total oxygen 10·11
Carbonic acid . .	3·47	
Nitrogen . . .	89·89	

The air from the calf's blood plus brucine contained—

No. 53.—In 100 parts of air.

Oxygen	11·63	Total oxygen 13·97
Carbonic acid . .	2·34	
Nitrogen . . .	86·03	

It is thus seen that brucine acts like strychnine, but in a much less marked degree.

As has just been said, to another portion of the same blood as was employed in the two preceding cases, 0·005 gramme of quinine was added.

No. 54.—In 100 parts of air.

Oxygen	14·72	Total oxygen 16·77
Carbonic acid . .	2·05	
Nitrogen . . .	83·23	

To the fourth portion of the same blood 0·005 gramme of morphine dissolved in water acidulated with hydrochloric acid was added, and the result was as follows:—

No. 55.—In 100 parts of air.

Oxygen	17·17	Total oxygen 18·17
Carbonic acid . .	1·00	
Nitrogen . . .	81·83	

It is thus seen that these different substances, Brucine, Quinine, and Morphine, with hydrochloric acid as their solvent, have all acted on the blood in the same manner, retarding oxidation, and decreasing the exhalation of carbonic acid, but in very different degrees. By placing them in a tabular form, the difference in their respective results will be still better appreciated.

	Oxygen.	Carbonic acid.	Nitrogen.	Vol. at 0° C. and 1 metre pressure.
In 100 parts of air:—				
After being acted on by pure blood.........	6·64	3·47	89·89	10·11
Ditto by blood plus brucine..................	11·63	2·34	86·03	13·97
„ „ quinine..................	14·72	2·05	83·23	16·77
„ „ morphine..............	17·17	1·00	81·83	18·17
Composition of atmospheric air employed in the experiments........................	20·96	0·002	79·038	20·962

It ought not to be forgotten that the blood in all of these cases was not only taken from the same animal, and the product of one bleeding, but in every respect, both before and after being put into the receivers, subjected to precisely similar influences, under identical conditions. The difference in the results must therefore be regarded as entirely due to the effect of the alkaloids upon the blood.

Action of Anæsthetics on Blood.

Chloroform.

From the fact that of all anæsthetics at present employed chloroform holds the first rank, its action upon blood was carefully studied. The results obtained were exceedingly uniform and all tending to one conclusion, namely, that this substance has a powerful effect in retarding those chemical transformations and decompositions upon which the process of respiration depends.

1st. As regards the visible effect of chloroform upon blood.

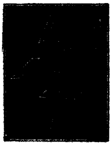
If 5 per cent. of pure chloroform be mixed with the freshly-drawn blood of a healthy animal, it will be found that within half an hour the blood will assume a brilliant scarlet hue. If the vessel containing it be now agitated, so as to mix the blood with atmospheric air, a quantity of colouring-matter adheres to the sides of the glass, and on allowing it again to stand for a few minutes, a red somewhat flocculent precipitate is deposited. This precipitate is not hæmatin alone. On the contrary, it consists of a dirty red-coloured protein substance, whereas the dissolved or suspended pigment has a vermilion hue. If the blood be kept at rest for some hours—laid aside during the night—it will to a certain extent lose its brilliant colour, and assume that of the red precipitate previously spoken of. At the same time it will be found to solidify into a gelatinous sticky paint-like mass. If instead of 5 per cent., 50, or still better 100 per cent. of chloroform, be added to venous blood either defibrinated or non-defibrinated, it causes it at once to assume the arterial hue, and this is still more marked if the vessel be well agitated. The blood rapidly solidifies and retains its vermilion tint for many hours, even days. It not unfrequently happens that blood to which chloroform has been added crystallizes on solidifying, more especially when only 5 per cent. of chloroform is used.

Serum is not solidified by chloroform in the same way, but it deposits a white precipitate.

2nd. Microscopical appearances presented by blood after being acted upon by chloroform.

If 5 per cent. of chloroform be added to blood, and the mixture well shaken, it will be found on examining it with the microscope that, although very many of the red corpuscles have disappeared, their walls having been dissolved, and their contents escaped, the great majority of them remain intact. Even 100 per cent. of chloroform fails to destroy totally the blood-cells. Great numbers of the red cells are, however, destroyed, and their contents diffused throughout the liquid. It is indeed the contents of the red corpuscles that crystallize. The crystals are in many cases quite red. They are prismatic in shape, and about four times as long as they are broad. The crystals are always most readily obtained from the blood of animals that have been poisoned with chloroform, but only after an additional quantity is added. They are insoluble in chloroform, ether, alcohol, and water.

Fig. 3.



Crystals obtained from blood by means of chloroform.

3rd. Chemical action of chloroform on blood.

Two equal portions of defibrinated and arterialized ox-blood, equal to 62 grammes each, were placed in receivers with 100 per cent. of atmospheric air, and kept in a room of moderate temperature during twenty-four hours. Both bloods were treated precisely alike, except that while the one was kept in its normal state, the other had three drops of chloroform added to it.

Gas from pure ox-blood, twenty-four hours' action with 100 per cent. of atmospheric air:—

No. 56.—In 100 parts of air.

Oxygen	10·42	Total oxygen 15·47
Carbonic acid . .	5·05	
Nitrogen . . .	84·53	

Gas from ox-blood plus chloroform, twenty-four hours' action, 100 per cent. of atmospheric air:—

No. 57.—In 100 parts of air.

Oxygen	18·76	Total oxygen 20·64
Carbonic acid . .	1·88	
Nitrogen . . .	79·36	

This result proves that chloroform possesses the property of diminishing the power of the constituents of the blood to unite with oxygen, and give off carbonic acid. A precisely similar result was obtained when the experiment was made on the blood of the young animal.

Perhaps as chloroform is so important an agent I may be pardoned quoting an experiment performed on the blood of the calf, which proves the correctness of the above assertion.

Equal parts of well-oxygenated freshly-defibrinated calf's blood were treated during twenty-four hours in receivers in the usual way. One was kept pure, and the other had three drops of chloroform added to it (as in the other cases the quantity of blood employed was 62 grammes).

Gas from pure calf's blood, twenty-four hours' action, with 100 per cent. of atmospheric air:—

No. 58.—In 100 parts of air.

Oxygen	12·10	Total oxygen 18·04
Carbonic acid . .	5·94	
Nitrogen . . .	81·96	

Gas from calf's blood plus chloroform, twenty-four hours' action, with 100 per cent. of atmospheric air. Result:—

No. 59.—In 100 parts of air.

Oxygen	18·05	Total oxygen 20·93
Carbonic acid . .	2·88	
Nitrogen . . .	79·07	

It is thus seen that chloroform acts in the same manner on the blood of the young as on that of the adult animal.

	Oxygen.	Carbonic acid.	Nitrogen.	Total oxygen.
In 100 parts of air from				
Pure ox-blood	10·42	5·05	84·53	15·47
Ditto plus chloroform	18·76	1·88	79·36	20·64
Pure calf's blood	12·10	5·94	81·96	18·04
Ditto plus chloroform	18·05	2·88	79·07	20·93

Ether.

The action of sulphuric ether, which is also used as an anæsthetic, upon blood is both chemically and physically different from that of chloroform, as shown by the result of the following experiments.

1st. Chemical effect of ether upon blood.

A quantity of ox-blood, after being defibrinated and well saturated with oxygen in the usual way, was divided into several portions, to one of which nothing was done, while to another 5 per cent. of sulphuric ether was added. After the different portions of blood had been kept with 100 per cent. of atmospheric air during twenty-four hours, in a room of moderate temperature, they yielded the subjoined results.

Gas from pure ox-blood, twenty-four hours' action, 100 per cent. of air yielded—

No. 60.—In 100 parts.

Oxygen	10·58	Total oxygen 14·91
Carbonic acid . .	3·33	
Nitrogen . . .	86·09	

Gas from ox-blood plus 5 per cent. of sulphuric ether, twenty-four hours' action, 100 per cent. of air. Result:—

No. 61.—In 100 parts of air.

Carbonic acid . .	3·40

In the experiments with ether the amount of oxygen absorbed by the blood could not be ascertained in consequence of the gas in the eudiometers refusing to explode. Even after the tubes were nearly filled with explosive gas the electric spark failed to ignite the gas, yet when the eudiometers were removed from the mercury trough, the gases instantly and violently exploded on the application of a lighted match.

2nd. Physical effects of ether upon blood.

When 5 per cent. of ether is added to fresh blood no marked effect is observed, except that the blood does not arterialize so readily as with chloroform. When ten, twenty, or more per cent. is added, the difference in the physical effect of the two anæsthetics upon blood is very striking. The etherized blood becomes clear but dark in colour, and cannot be made to assume the perfect arterial tint, not even after prolonged agitation with renewed portions of atmospheric air. The greater the percentage of ether the more visible is this effect. 100 per cent. of sulphuric ether gives to blood a beautifully rich transparent port-wine colour. When left some hours in repose, part of the ether separates from the blood and floats as a colourless liquid on the surface, while the blood itself still retains the rich dark hue, except the layer in immediate contact with the ether, which appears as if it had a vermilion tint. When examined with the microscope the blood-corpuscles are found to be completely destroyed, their colouring-matter being set free.

When non-defibrinated blood is employed, and the ether allowed to evaporate, the blood solidifies, and in so doing frequently crystallizes; but strange to say the crystals are quite different in form from those obtained by chloroform from the same blood. They are long needles, twelve times as long as broad, and are sometimes so abundant that they fill up the whole field of the microscope. The crystals are not usually so much coloured as those of chloroform. They too are most copious in the blood of animals poisoned by the anæsthetic. In some healthy bloods I have entirely failed in detecting them. The best are obtained from the blood of the dog*. Ether, as already said, destroys the corpuscles more than chloroform.

Fig. 4.

Crystals obtained from blood by means of ether.

It is curious to notice how the effects of different substances upon blood vary. I thought, for example, that alcohol would act like ether upon blood, whereas to my surprise its action much more closely resembled that of chloroform, although only in a mitigated degree. Notwithstanding that alcohol cannot properly be regarded in the light of an anæsthetic, I shall take the liberty of here introducing an experiment upon it, seeing that it was performed on a portion of the same blood as served for the last two examples, and was conducted under precisely similar circumstances. Five per cent. of pure alcohol was employed.

Alcohol.

Gas from ox-blood plus alcohol, after twenty-four hours' action, on 100 per cent. of atmospheric air:—

No. 62.—In 100 parts of air.

$$\left.\begin{array}{ll}\text{Oxygen . . . .} & 16{\cdot}59\\ \text{Carbonic acid . .} & 2{\cdot}38\end{array}\right\}\text{Total oxygen } 18{\cdot}97$$

Nitrogen . . . 81·03

By placing the results of these last three experiments in a tabular form the difference they present will be better seen.

	Oxygen.	Carbonic acid.	Nitrogen.	Total oxygen.
In 100 parts of air from				
Pure blood	**10·58**	**3·33**	**86·09**	**14·91**
Ditto plus ether	......	**3·40**		
Ditto plus alcohol	**16·59**	**2·38**	**81·03**	**18·97**

It is thus seen that while the action of ether is to increase, or at least not to diminish the transformations occurring in blood upon which the exhalation of carbonic acid depends, that of alcohol, on the other hand, is to restrain these, as well as to diminish the consumption of oxygen:—a similar effect, it will be remembered, to that which occurs with chloroform; the only difference being that the action of alcohol is very much less powerful, for a less quantity of chloroform produces a much greater effect.

---

* Magnificently large prismatic crystals are readily obtained by adding equal parts of ether to the blood of dogs poisoned by the vapour of chloroform. They are of a fine red colour, and many of them appear to be formed of bundles of needle-shaped crystals. Sometimes almost the whole blood crystallizes.

Physical effect of Alcohol upon Blood.

When blood is shaken with 10 per cent. or more of alcohol it becomes of a light brick-red hue. The albumen is coagulated and subsides to the bottom of the vessel. No amount of shaking with renewed portions of air will properly arterialize blood mixed with alcohol, nor have I ever obtained any crystals from blood so treated, not even from that of animals poisoned by chloroform. Alcohol does not destroy the blood-corpuscles nor set the hæmatin free.

Amylene.

Some years ago amylene was proposed as an anæsthetic for the purpose of annulling pain in surgical operations, but owing to its disagreeable odour, or some other cause, it has never come into general use. Several experiments were made with this substance.

1st. As regards its physical action upon blood.

When five per cent. of amylene is added to fresh blood, and the mixture well shaken, the blood assumes a dark-red tint, and does not arterialize readily. When 100 per cent. of the anæsthetic is employed, the blood becomes quite black, and when spread out in a thin layer has a dirty brownish-red appearance. It cannot now be made to arterialize at all. If the mixture be allowed to stand for twenty-four hours, the amylene in great part separates from the blood, and floats in a clear layer on its surface. The blood, however, still retains its black, thin, tarry-like aspect.

When examined with the microscope, the red corpuscles are found beautifully distinct; none appear to be destroyed, and no blood-crystals are to be found. Indeed the formation of the crystals seems to be in proportion to the destruction of the corpuscles.

2nd. Chemical action of amylene upon blood.

Two portions of defibrinated sheep's blood, after being saturated with oxygen in the usual manner, were placed in receivers, the one with nothing, the other with four drops of amylene to the 62 grammes of blood. After twenty-four hours' action the gases were analyzed in the usual way; but on attempting to estimate the oxygen in the air enclosed with the amylene, it was found impossible to obtain an explosion, not only after the mere addition of hydrogen, but after a large amount of explosive gas had been added to the mixture; and what was more extraordinary still, the electric spark even failed to produce any explosion after the sulphuric acid and potash balls had been employed. On inverting the eudiometer the gas was found to smell strongly of amylene, and there can be little doubt but that its presence prevented the explosion taking place. The analysis of the gas, as far as it went, was as follows:—

Gas from sheep's blood plus amylene, twenty-four hours' action, 100 per cent. of atmospheric air:—

No. 63.—In 100 parts of air.

Carbonic acid	0·62

Whereas the air from pure blood gave quite a different result.

Gas from pure sheep's blood after twenty-four hours' action, 100 per cent. of air:—

No. 64.—In 100 parts of air.

Carbonic acid	3·17

It thus appears that amylene has a marked effect in diminishing the exhalation of carbonic acid gas.

ACTION OF MINERAL SUBSTANCES ON BLOOD.

Chloride of Mercury (*Corrosive sublimate*).

The experiments with mineral products were in general conducted in the same manner as those with other substances. In the present instance, however, the experiment was like some of the exceptions previously related, slightly modified, and instead of employing defibrinated blood, the blood was put into the receivers direct from the animal. Calf's arterial blood was used in this case, and as it slightly coagulated in the vessels, it was found necessary to have them well shaken (before being definitely closed) until the coagula were all broken up. While to one of the portions of blood nothing was done, to the others 6 drops of a saturated aqueous solution of corrosive sublimate were added. The quantity of blood employed in each case amounted to 40 grammes, and the air confined with it to 150 per cent. The receivers were all treated alike, during twenty-four hours, in a room of moderate temperature. At the end of that time a marked difference was observed in the bloods. The pure blood still retained its arterial tint, while that to which corrosive sublimate had been added was of an intensely dark, almost black colour. Moreover the latter had separated into two layers, a thin dark red liquid, and a somewhat gelatinous coagulum. The dark liquid part of the blood felt quite sticky to the fingers.

Gas from pure calf's blood after twenty-hours' action with 150 per cent. of atmospheric air:—

No. 65.—In 100 parts of air.

Oxygen . . .	16·57	Total oxygen 18·72
Carbonic acid .	2·15	
Nitrogen . . .	81·28	

Gas from calf's blood plus corrosive sublimate, twenty-four hours' action, 150 per cent. of atmospheric air:—

No. 66.—In 100 parts of air.

Oxygen . . .	17·01	Total oxygen 20·59
Carbonic acid . .	3·58	
Nitrogen . . .	79·89	

It is thus seen that corrosive sublimate, while increasing the changes which develope carbonic acid, has an almost negative effect on those depending upon oxidation; if anything rather diminishing them than otherwise.

	Oxygen.	Carbonic acid.	Nitrogen.	Total oxygen.
In 100 parts of air from pure blood	16·57	2·15	81·28	18·72
Ditto plus corrosive sublimate	17·01	3·58	79·89	20·59

I may here take occasion to mention a fact in connexion with the physiological effects of corrosive sublimate on the animal body, which, as far as I am aware, has hitherto escaped notice, namely, *its cardiac action*. As we have already seen, there exist in the vegetable kingdom substances which, in consequence of their acting specially on the heart and lungs, have acquired the title of cardiac and respiratory poisons; few are, however, aware that in the mineral kingdom there are also substances to be met with, the peculiar action of which on the animal body is such as to entitle them with equal justice to the name of cardiac and respiratory poisons. Corrosive sublimate is an example of the former, protosulphate of iron of the latter.

In order not to be misunderstood, I shall briefly quote the following experiments to illustrate my meaning.

1st. As regards protosulphate of iron, a respiratory poison.

1st experiment. Into one of the jugular veins of a dog was slowly injected an aqueous solution of 15 grains of the protosulphate of iron. In sixty seconds from the commencement of the experiment (which of itself lasted about forty seconds) the animal manifested symptoms of impending suffocation. These speedily induced a convulsion, and the involuntary passage of the contents of the bladder and rectum, as is seen to occur in cases of true apnœa from a mechanical obstruction to the entrance of air into the lungs.

In eight minutes there was complete loss of sensation and voluntary motion. The limbs were paralysed, and the animal manifested no sign of pain on being pinched.

In ten minutes the symptoms of poisoning began to pass away, and in a few minutes more he was again upon his legs. When seen fifty minutes after the commencement of the operation, he was running about apparently quite well.

2nd experiment. Two days later, into the other jugular vein of the same dog, was injected an aqueous solution of 30 grains of the protosulphate of iron, double the quantity first used. Symptoms of suffocation instantly manifested themselves. The lungs did not act. The respiratory movements ceased. But the heart went on beating, and continued to do so for at least three or four minutes after all attempts at respiratory efforts had entirely stopped.

On opening the animal, the heart was found distended with fluid blood. The blood coagulated after its withdrawal from the body. On puncturing the right ventricle, a globule or two of air escaped; but the organ contained no frothy air, nor was there any reason to suppose that the air had been injected during the operation. On the contrary, it appeared as if it had been separated from the blood itself, as occasionally occurs in cases where the blood-vessels are unopened. The urine of the animal contained a large amount of the poison. It is on the above grounds that I consider that the protosulphate of iron merits the title of a respiratory poison. This will be made still more apparent by comparing the foregoing with the result of the following experiment.

2nd. As regards corrosive sublimate, a cardiac poison.

Into the femoral vein of a pregnant bitch was injected an aqueous solution of five grains of corrosive sublimate. In ten seconds the animal cried as if in pain; in sixty she became delirious; and in three and a quarter minutes after the operation was commenced the heart stopped. Neither was there an impulse to be felt on the application of the finger to the femoral artery, nor a sound to be heard on the application of the ear to the thoracic walls. The animal, however, still respired, and continued to make gasping respiratory efforts for thirty seconds more. They then ceased. In three-quarters of a minute after the cessation of respiration the thorax was opened, with the view of ascertaining the conditon of the heart. It was found still; and neither the stimulus of the cold air, of the point of the knife, nor of a feeble current from the galvanic forceps caused it to pulsate.

Ten minutes after death a stronger galvanic current was applied to the organ, but even then the portions between the points of the forceps alone contracted. No general pulsation could be reinduced. The fœtuses were alive and moving about in the uterus twelve and a half minutes after the death of the mother.

The corrosive sublimate had acted specially upon the heart; for the spontaneous peristaltic movements of the intestines were well marked, and continued to be so for twenty-two minutes. The thoracic muscles also contracted spontaneously, with a flickering movement, for no less than thirty minutes. They even responded to the direct application of galvanism for two hours and thirty-five minutes after the death of the animal.

Galvanism applied to the brachial plexus fifteen minutes after death caused violent muscular contractions in the limb supplied by it; yet, as was before said, the heart failed to respond to mechanical and galvanic stimuli applied within a single minute after death.

It appears to me, therefore, that corrosive sublimate merits the name of a cardiac poison quite as much as either aconitine or antiar.

Arsenic.

In testing the action of arsenic, as in the case of corrosive sublimate, non-defibrinated freshly-drawn arterial blood was employed, and the quantity of air with which it was enclosed also amounted to 150 per cent. In this instance, however, dog's instead of calf's blood was employed; and in order to give to the experiment all possible exactitude, while one of the portions of blood had 120 drops of a saturated aqueous solution (by boiling) of arsenious acid added to it, the other was treated to a similar amount of distilled water. In all other respects they were treated precisely alike, both before and after the twenty-four hours' action.

Gas from non-defibrinated fresh dog's blood plus 120 drops of distilled water, twenty-four hours' action with 150 per cent. of atmospheric air:—

No. 67.—In 100 parts of air.

Oxygen . . .	20·376	Total oxygen 21·357
Carbonic acid .	0·981	
Nitrogen . . .	78·643	

Gas from dog's blood plus arsenious acid, twenty-four hours' action with 150 per cent. of atmospheric air:—

No. 68.—In 100 parts of air.

Oxygen . . .	21·270	Total oxygen 21·538
Carbonic acid .	0·268	
Nitrogen . . .	78·562	

It is thus seen that arsenious acid is one of those substances which retard the transformation of the constituents of the blood on which the absorption of oxygen and exhalation of carbonic acid in the respiratory process depend.

	Oxygen.	Carbonic acid.	Nitrogen.	Total oxygen.
In 100 parts of air from pure dog's blood ...	20·376	0·981	78·643	21·357
Ditto plus arsenic	21·270	0·268	78·562	21·538
Pure atmospheric air	20·960	0·002	79·038	20·962

A precisely similar result was obtained with defibrinated calf's blood.

Tartrate of Antimony.

A quantity of well-defibrinated sheep's blood, after being thoroughly saturated with oxygen, was divided into several portions, and while one was left in its normal condition, 0·02 gramme of tartrate of antimony was added to another (the quantity of blood employed in each case was 62 grammes). The blood was treated in the usual manner, in receivers with 100 per cent. of air, during twenty-four hours.

Gas from pure sheep's blood, after twenty-four hours' action with 100 per cent. of atmospheric air:—

No. 69.—In 100 parts of air.

Oxygen . . .	19·262	Total oxygen 21·08
Carbonic acid .	1·818	
Nitrogen . . .	78·920	

Before treatment the blood contained 0·451 per cent. of urea; after treatment it contained 0·435 per cent.

Gas from sheep's blood plus tartrate of antimony, twenty-four hours' action, 100 per cent. of atmospheric air:—

No. 70.—In 100 parts of air.

Oxygen . . .	20·41	Total oxygen 22·96
Carbonic acid .	2·55	
Nitrogen . . .	77·04	

Before treatment this blood contained 0·451 per cent. of urea; after treatment it contained 0·354 per cent. In another portion of this blood, which was treated with sulphate of zinc, there remained only 0·28 per cent. of urea. In a series of experiments on the effects of antimony as a slow poison, I invariably found the urine loaded with urea, even when the animals were reduced to perfect skeletons. In the urine of a dog that died on the forty-third day after taking half a grain of antimony daily, there was such an amount of urea, that, on adding nitric acid, the whole urine solidified into one mass of crystals. The liver contained neither sugar nor glucogene.

In the above case tartrate of antimony is seen to diminish oxidation, and in a very marked degree to increase the exhalation of carbonic acid gas. The total amount of oxygen is also increased, making it thereby appear as if oxygen had been developed from some one or other of the constituents of the blood, either while they were being pulled down, or built up into new compounds. The apparent increase of the oxygen may be due, however, to another cause, namely, the disappearance of nitrogen from the air.

	Oxygen.	Carbonic acid.	Nitrogen.	Total oxygen.
In 100 parts of atmospheric air	20·960	0·002	79·038	20·962
Air from pure blood....................	19·262	1·818	78·920	21·080
Ditto plus antimony	20·41	2·55	77·04	22·96

This increase in the total amount of oxygen, or decrease in the amount of nitrogen, was even much more decided in another experiment with antimony on sheep's blood. In it the oxygen actually amounted to 24·69 per cent., and the nitrogen stood at 75·31 per cent.

In concluding this paper, it was my intention to make some remarks on the reciprocal action of hæmatin and atmospheric air; for, as stated in a communication on the condition of oxygen absorbed into the blood during respiration[*], which I had the honour of making to the Royal Society some years ago, the colouring-matter of the blood appears to possess a more powerful effect in altering the composition of atmospheric air than any other individual constituent of that liquid. The recent researches of Professor Stokes, however, cause me to pause before again publishing my views on animal colouring-matters. For the interesting results obtained by that gentleman with the prism, although in accordance with my facts, may nevertheless induce me to modify my theory; not regarding the action, but regarding the nature of these substances. I have hitherto held the view that all the animal pigments spring from one colourless radical, and that the difference in tint between hæmatin, urohæmatin, and biliverdin was simply due to the different stages of oxidation of the radical. It would appear, however, from the researches of Professor Stokes, that all these substances, although closely allied, are nevertheless chemically distinct. I consequently prefer reinvestigating the subject before communicating to the Society the data which are at present before me.

---

[*] Proceedings of the Royal Society, vol. viii. p. 82.

XVII. *On a New Geometry of Space.* *By* J. Plücker, *of Bonn, For. Memb. R.S.*

Received December 22, 1864,—Read February 2, 1865.

I. *On Linear Complexes of Right Lines.*

1. Infinite space may be considered either as consisting of points or transversed by planes. The points, in the first conception, are determined by their coordinates, by x, y, z for instance, taken in the ordinary signification; the planes, in the second conception, are determined in an analogous way by their coordinates, introduced by myself into analytical geometry, by t, u, v for instance.

The equation

$$tx+uy+vz+1=0$$

represents, in regarding x, y, z as variable and t, u, v as constant, a plane by means of its points. The three constants t, u, v are the coordinates of this plane. The same equation, in regarding t, u, v as variable, x, y, z as constant, represents a point by means of planes passing through it. The three constants are the coordinates of the point.

A point given by its coordinates and a point determined by its equation, or geometrically speaking by an infinite number of planes intersecting each other in that point, are quite different ideas, not to be confounded with one another. That is the case also with regard to a plane given by its coordinates and a plane represented by its equation, or considered as containing an infinite number of points. Hence is derived a double signification of a right line. It may be considered as the geometrical locus of points, or described by a point moving along it, and accordingly represented by two equations in x, y, z, each representing a plane containing that line. But it may likewise be considered as the intersection of an infinite number of planes, or as enveloped by one of these planes, turning round it like an axis; accordingly it is represented by two equations in t, u, v, each representing an arbitrary point of the line. The passage from one of the two conceptions to the other is a discontinuous one*.

2. The geometrical constitution of space, hitherto referred either to points or to planes, may as well be referred to right lines. According to the double definition of such lines, there occurs to us a double construction of space.

In the first construction we imagine infinite space to be transversed by lines themselves consisting of points. An infinite number of such lines pass in all directions through any given point; each of these lines may be regarded as described by a moving point. This constitution of space is admitted when, in optics, we consider luminous points as sending out in all directions luminous rays, or, in mechanics, forces acting on points in every direction.

---

* According to this discontinuity, a plane curve represented by ordinary coordinates may have a conjugate which disappears if the same curve be represented by means of line-coordinates. See "System der analytischen Geometrie," n. 330.

In the second construction infinite space is likewise regarded as transversed by right lines, but these lines are determined by means of planes passing through them. Every plane contains an infinite number of right lines having within it every position and direction, around each of which the plane may turn. We refer to this second conception when, in optics, we regard, instead of rays, the corresponding fronts of waves and their consecutive intersections, or when, in mechanics, according to POINSOT'S ingenious philosophical views, we introduce into its fundamental principles "couples," as well entitled to occupy their place as ordinary forces. The instantaneous axes of rotation are right lines of the second description.

3. In order to constitute a new geometry of space, we may fix the position of a right line, depending upon four constants, in a different way. We might do it by means of four given right lines, by determining, for instance, the shortest distance of any new line from each of the four given ones. But all such conceptions were rejected, and the ordinary system of axes adopted in order to fix the position in space of a right line. Thus the new researches, indicated by the foregoing remarks, are intimately connected with the usual methods of analytical geometry. The two fragments presented on this occasion are only calculated to give an exact idea of the new way of proceeding, and to show its importance, greater perhaps than it appears at first sight.

4. A right line of the first description, which we shall distinguish by the name of *ray*, may be determined by means of two of its projections. We may select the projections within the planes XZ and YZ, in order to get, without generalizing, the greatest symmetry obtainable, and give to their equations either the form

$$\left.\begin{aligned} x&=rz+\varrho, \\ y&=sz+\sigma, \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

or

$$\left.\begin{aligned} tx+v_xz&=1, \\ uy+v_yz&=1. \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

In adopting the first system of equations, the four constants r, s, ϱ, σ are *the coordinates of the ray*: two of them, r, s, indicating its direction, the remaining two, ϱ, σ, after its direction is determined, giving its position in space. The ray meets the plane XY in the point

$$x=\varrho, \qquad y=\sigma.$$

In adopting the second system of equations, we get, in order to determine the same ray, the four new constants t, u, v_x, v_y, which likewise may be regarded as its coordinates; t and u $\left(\text{equal to } \frac{1}{\varrho} \text{ and } \frac{1}{\sigma}\right)$ indicating the reciprocal values of the intercepts cut off on OX and OY by the two projections of the ray, v_x and v_y $\left(\text{equal to } \left(-\frac{r}{\varrho}\right) \text{ and } \left(-\frac{s}{\sigma}\right)\right)$ the reciprocal values of the two intercepts cut off both on OZ.

5. A right line of the second description, which we shall distinguish by the name of *axis*, is determined by any two of its points. We may select the intersection of the axis with the planes XZ and YZ as two such points, and represent them by the system of equations

$$\left.\begin{aligned} xt+z_t v&=1,\\ yu+z_u v&=1, \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

or by the following equally symmetrical,

$$\left.\begin{aligned} t&=pv+\varpi,\\ u&=qv+\varkappa. \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

In making use of the first two equations, the four constants x, y, z_t, z_u are *the coordinates of the axis*, indicating the position of the two points within the planes XZ, YZ.

In making use of the second system of equations, p, q, ϖ, $\varkappa$ are the four coordinates of the axis, this axis being fixed by the intersections of two planes, one of which is the plane projecting it on XY, and determined by two of the four coordinates,

$$t=\varpi=\frac{1}{x},\quad u=\varkappa=\frac{1}{y},$$

while the other plane determined by the two remaining ones,

$$t=pv=-\frac{z_t}{x}v,\quad u=qv=-\frac{z_u}{y}v,$$

and represented by the equation

$$px+qy+z=0,$$

passes through the axis and the origin.

6. If we consider the four coordinates of a ray as variable quantities, we may in attributing to them any given values successively obtain any ray whatever transversing space. But in admitting that an equation takes place between the four coordinates, rays are excluded: we say *that the remaining rays constitute a complex represented by the equation.*

In admitting *two* such equations existing simultaneously, those rays the coordinates of which satisfy both equations constitute a *congruency represented by the system of equations.* A "congruency" contains all congruent rays of two complexes, it may be regarded as their mutual intersection. If we admit that three equations are simultaneously verified by the four coordinates, the corresponding rays constitute a *configuration* (Strahlengebilde, surface reglée) *represented by the system of three equations.* A configuration may be regarded as the mutual intersection of three complexes, *i. e.* as the geometrical locus of congruent rays belonging to all three complexes. Four complexes or two configurations intersect each other in a limited number of rays. The number of rays constituting a configuration, a congruency, a complex, and space, are infinites of first, second, third, and fourth order.

7. If *rays* are replaced by axes, complexes, congruencies, and configurations of rays are replaced by *complexes, congruencies, and configurations of axes.*

8. A configuration of rays or axes, represented by three linear equations, is, according to the choice of coordinates, either a hyperboloid or a paraboloid. Let the three equations of a configuration of rays be

$$\left.\begin{array}{l} Ar+Bs+C+D\sigma+E\varrho=0, \\ A'r+B's+C'+D'\sigma+E'\varrho=0, \\ A''r+B''s+C''+D''\sigma+E''\varrho=0. \end{array}\right\} \quad \ldots\ldots\ldots \quad (5)$$

From these equations we derive by elimination six new ones, each containing two only of the four variables. Let them be

$$ar+bs=1, \quad \ldots\ldots\ldots\ldots \quad (6)$$

$$c\varrho+d\sigma=1, \quad \ldots\ldots\ldots\ldots \quad (7)$$

$$a'r+c'\varrho=1, \quad \ldots\ldots\ldots\ldots \quad (8)$$

$$b's+d'\sigma=1, \quad \ldots\ldots\ldots\ldots \quad (9)$$

$$a''r+d''\sigma=1, \quad \ldots\ldots\ldots\ldots \quad (10)$$

$$b''s+c''\varrho=1. \quad \ldots\ldots\ldots\ldots \quad (11)$$

In order to represent the configuration, the three primitive equations (5) may be replaced by any three of the six new ones.

The equation (7) may be written thus,

$$cx+dy=1, \quad \ldots\ldots\ldots\ldots \quad (7^*)$$

x and y replacing ϱ and σ. It represents a right line within XY, intersected by the rays of the configuration.

The equations (8) and (9) represent within XZ, YZ two points enveloped by the projections of the rays of the configuration; consequently the rays themselves meet two right lines passing through these points, and being parallel to OY, OZ. From the equations (8) and (9) if written thus,

$$\frac{1}{c'}=\frac{a'}{c'}\cdot r+\varrho,$$

$$\frac{1}{a'}=\frac{b'}{d'}\cdot s+\sigma,$$

we immediately derive

$$c'x=1, \quad c'z=a',$$

$$d'y=1, \quad d'z=b',$$

representing the two right lines.

Thus by selecting in order to represent the configuration the three equations (7), (8), (9), and interpreting them geometrically, we have proved that all its rays intersect three fixed right lines, one of which falls within XY, while the two remaining ones are parallel to OY and OX. Hence these rays, meeting three right lines parallel to the same plane, constitute a hyperbolic paraboloid.

In determining the paraboloid, we may replace any one of the three equations we made use of by the equation (6), which indicates that all rays are parallel to a given plane. This plane, if drawn through the origin, is represented by the equation

$$ax+by=z,$$

obtained from (6) by writing $\frac{x}{z}$, $\frac{y}{z}$ instead of r, s.

It may be sufficient here to state that a configuration of rays, if represented by three linear equations, in which the coordinates r, s, ϱ, σ are replaced by t, u, v_x, v_y, becomes a hyperboloid.

9. A configuration of *axes* represented by three linear equations would be a *paraboloid* if the coordinates x, y, z_t, z_u were employed, but becomes a *hyperboloid* if these coordinates are replaced by p, q, ϖ, $\varkappa$. We shall here consider the last case only, and may for that purpose directly replace the equations (6)–(11) by the following ones:—

$$ap+bq=1, \quad \ldots \tag{12}$$
$$c\varpi+d\varkappa=1, \quad \ldots \tag{13}$$
$$a'p+c'\varpi=1, \quad \ldots \tag{14}$$
$$b'q+d'\varkappa=1, \quad \ldots \tag{15}$$
$$a''p+d''\varkappa=1, \quad \ldots \tag{16}$$
$$b''q+c''\varpi=1. \quad \ldots \tag{17}$$

Any three of these equations, involving six constants, are sufficient to determine the configuration.

If, after having replaced p, q, ϖ, $\varkappa$ by

$$-\frac{z_t}{x},\quad -\frac{z_u}{y},\quad \frac{1}{x},\quad \frac{1}{y},$$

we regard x, y, z_t, z_u as variable, (14) and (15) may be written thus,

$$x=a'z+c',$$
$$y=b'z+d',$$

representing within the planes XZ, YZ two right lines (AA′, BB′) which are the locus of points (A, B) where the axes of the configuration meet the two planes.

In regarding ϖ and $\varkappa$ as coordinates of a right line, the equation (13), being written thus,

$$ct+du=1,$$

represents a given point (E),

$$x=c,\quad y=d,$$

enveloped within XY by the projections of axes. Therefore all axes of the configuration intersect a third right line (CC′) parallel to OZ and meeting XY in E.

Hence we conclude that the configuration represented by the three linear equations is a hyperboloid. Its axes meet three given lines, two of which, AA′, BB′, fall within XZ, YZ, while the third, CC′, is parallel to OZ.

The plane BOA passing through O and an axis AB is represented by the equation

$$z+qy+px=0.$$

The equation (12) being with regard to p and q of the first degree, indicates that all such planes, containing the different axes of the configuration, intersect each other along a given right line DD′ passing through O. Hence all axes meet a fourth right line, itself confined within the hyperboloid.

The complete determination of the hyperboloid presents no difficulties. We may for instance find its centre and its axes by determining the shortest distance of any two of the axes generating it.

10. Let a *congruency* either of rays or axes be represented by two linear equations. In adding to these equations two new ones, likewise of the first degree, there exists only one ray or axis the coordinates of which satisfy simultaneously the four linear equations. Two new equations of this description are obtained if, among the rays or axes of the congruency, we select those either passing through a given point, or confined within a given plane. In the case of rays, let (x', y', z') be a given point, then we get

$$x'=rz'+\varrho,$$
$$y'=sz'+\sigma$$

in order to express that all rays meet in that point. Let

$$t'x+u'y+v'z+1=0$$

be the equation of a given plane, then we get

$$t'r+u's+v=0,$$
$$t'\varrho+u'\sigma+1=0$$

in order to express that the rays lie within that plane. Again, in the case of axes, let (t', u', v') be a given plane, then we get the new linear equations

$$t'x+v'z_t=1, \qquad t'=pv'+\varpi,$$

or

$$u'x+v'z_u=1, \qquad u'=qv'+\varkappa,$$

in order to express that the axis is confined within that plane. Let in regarding x', y', z' as constant, t, u, v as variable,

$$x't+y'u+z'v+1=0$$

represent a given point, then we get

$$x'p+y'q+z'=0,$$
$$x'\varpi+y'\varkappa+1=0$$

in order to express that the axes pass through that point. Hence

In a congruency represented by the system of two linear equations, there is one single ray or axis passing through any given point of space, as there is one single ray or axis confined within a given plane.

11. In order to represent a congruency of rays, we shall here make use of the coordinates t, u, v_x, v_y. Let

$$At+Bu+Cv_x+Dv_y+1=0,$$
$$A't+B'u+C'v_x+D'v_y+1=0$$

be its two equations. By successively eliminating each coordinate, we get four equations of the following form,

$$at+bu+cv_x+1=0,$$
$$a't+b'u+dv_y+1=0,$$
$$a''t+c'v_x+d'v_y+1=0,$$
$$b''u+c''v_x+d''v_y+1=0,$$

any two of which involving six constants may replace the two primitive equations, the remaining two being derived from them.

The first two of these equations, if t, u, v_x and t, u, v_y be considered as plane coordinates, represent two points (U, V) the coordinates of which are

$$x=a, \qquad y=b, \qquad z=c, \quad \ldots\ldots\ldots \quad \text{(U)}$$
$$x=a', \qquad y=b', \qquad z=c', \quad \ldots\ldots\ldots \quad \text{(V)}$$

Consequently the six constants upon which the congruency depends, if referred to the three axes of coordinates OX, OY, OZ, are determined by means of the two points U and V. Hence is derived the following construction of rays of the congruency.

Trace through the two points U, V any two planes which intersect each other along a right line confined in the plane XY, and meeting OX, OY in the points D, F. Let E, G be the points where the two planes meet OZ. We shall get within the planes XZ, YZ the projections of a ray of the congruency by drawing DE, FG. The ray (AC) thereby completely determined will intersect the plane XY in the point C, the coordinates of which are

$$x=\frac{1}{t}=\text{OD}, \qquad y=\frac{1}{u}=\text{OF}.$$

If a plane be traced passing simultaneously through both points U, V, both intersections E, G falling into one point A′, the corresponding ray of the congruency A′C′ intersects OZ. If the right line UV be projected on YZ, XZ, the projections meet OZ in two points A″, A‴. In these points OZ is intersected by the rays of the congruency parallel to OX, OY. The ray parallel to OZ is obtained by the point C″ where it meets XY. The coordinates of C″ are

$$x=\text{OD}'', \qquad \text{Y}=\text{OF}'',$$

D″ and F″ being the points where the projection of UV intersects OX and OY.

Thus occurs to us the construction of rays passing through any point of OZ and any point of XY. We cannot go further into detail here.

12. Again, let a congruency of *axes* be represented by the equations

$$Ax+By+Cz_t+Dz_u+1=0,$$
$$A'x+B'y+C'z_t+D'z_u+1=0.$$

By successively eliminating z_u and z_t we may replace these equations by the following two,

$$ax+by+cz_t+1=0,$$
$$a'x+b'y+c'z_u+1=0,$$

the six new constants of which are derived from the primitive constants. In regarding x, y, z_t, z_u as point-coordinates (where z may be written instead of z_t and z_u), the last equations represent two planes. The six coordinates of both planes,

$$t=a, \qquad u=b, \qquad v=c,$$
$$t=a', \qquad u=b', \qquad v=c',$$

are the six constants of the congruency, consequently the congruency is determined by means of these two planes and the axes of coordinates.

Suppose both planes to be known. Draw any right line meeting them in M and M′, project M on XZ and M′ on YZ. The right line joining the two projections B and A is an axis of the congruency.

If we project on XZ and YZ any point of the right line JK along which both planes intersect each other, the right line joining both projections, B′, A′, is an axis parallel to XY. All axes obtained in that way meet, within XZ and YZ, both projections of JK. Hence the axes of the congruency parallel to XY constitute a paraboloid. The ray within XY is obtained by projecting the point where the traces of both planes meet on OX and OY and joining both projections, B″ and A″, by a right line, &c.

13. After these preliminary discussions we shall now proceed in a more systematic way, and henceforth exclusively make use of the coordinates r, s, ϱ, σ. When a complex of rays is represented by the linear equation

$$Ar+Bs+D\sigma+E\varrho+1=0, \quad \ldots\ldots\ldots \quad (1)$$

we may easily prove that the infinite number of rays passing through a given point of space are confined within the same plane, and, conversely, that the infinite number of rays confined within a given plane meet within the same point.

In order to select among the rays of the complex those passing through a given point (x', y', z'), the following two equations,

$$\left.\begin{aligned} x'&=rz'+\varrho, \\ y'&=sz'+\sigma, \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots \quad (2)$$

are to be added to the equation of the complex. By eliminating ϱ and σ we get

$$(A-Ez')r+(B-DZ')s+(1+Ex'+Dy')=0. \quad \ldots\ldots\ldots \quad (3)$$

This equation being of the first degree with regard to the remaining variables r and s, shows that all corresponding rays are parallel to a given plane, and therefore confined within the plane of that direction and passing through the point (x', y', z'). By replacing in the last equation r and s by $\frac{x-x'}{z-z'}$, and $\frac{y-y'}{z-z'}$, we obtain, in order to represent that plane, the following equation,

$$(A-Ez')(x-x')+(B-Dz')(y-y')+(1+Ex'+Dy')(z-z')=0. \quad . \quad . \quad . \quad (4)$$

14. Again, this equation being, with regard to (x', y', z'), of the first degree, proves that, conversely, all rays confined within a given plane meet in the same point of that plane.

15. A complex the rays of which are distributed through infinite space in such a way that in each point there meet an infinite number of rays constituting a plane, and, conversely, that each plane contains an infinite number of rays meeting in the same point, may be called a *linear complex of rays*. We may say, too, that, with regard to the complex, points and planes of the infinite space *correspond to each other*; each plane containing all rays which meet in the point placed within it, and each point being traversed by all rays which are confined within the plane passing through it.

16. A linear complex of rays is represented by the linear equation (1), but it is easily seen that this equation is not the *general* equation of a linear complex. The following considerations lead us to generalize the preceding developments and to render them by generalizing more symmetrical.

Hitherto we determined a ray by its two projections within XZ, YZ,

$$x=rz+\varrho,$$
$$y=sz+\sigma,$$

whence its third projection within XY is derived,

$$ry-sx=r\sigma-s\varrho. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

This equation furnishes the new term $(r\sigma-s\varrho)$, which, like ϱ and σ, depend upon r and s as well as upon x' and y' in a linear way.

Again, from the equations

$$tr+us+v=0,$$
$$t\varrho+u\sigma+w=0,$$

expressing that the ray (r, s, ϱ, σ) falls within the plane (t, u, v, w) represented by the equation

$$tx+uy+vz+w=0^{*},$$

we deduce

$$\frac{w}{t}\cdot s-\frac{v}{t}\sigma=(r\sigma-s\varrho). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

---

* Henceforth we shall make use of four plane-coordinates t, u, v, w, and accordingly represent a point by a homogeneous equation. Sometimes, where symmetry and brevity require it, likewise x, y, z shall be replaced by ξ/θ, η/θ, ζ/θ. Accordingly, by introducing the four point-coordinates ξ, η, ζ, θ, a plane is represented by a homogeneous equation.

17. After introducing a new term containing $(s\varrho - r\sigma)$, the equation of the complex may be written thus,

$$Ar+Bs+C+D\sigma+E\varrho+F(s\varrho-r\sigma)=0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (7)$$

When, after $(r\sigma - s\varrho)$ is eliminated by means of the equation

$$ry'-sx'=r\sigma-s\varrho,$$

we proceed as we did in the former case [14], the following equation is obtained in order to represent the plane corresponding to the given point (x', y', z'),

$$(A-Fy'-Ez')(x-x')+(B+Fx'-Dz')(y-y')+(C+Ex'+Dy')(z-z')=0. \quad . \qquad (8)$$

This equation may be expanded thus,

$$(A-Fy'-Ez')x+(B+Fx'-Dz')y+(C+Ex'+Dy')z=Ax'+By'+Cz', \quad . \quad . \qquad (9)$$

and reduced also to the following symmetrical form,

$$A(x-x')+B(y-y')+C(z-z')+D(y'z-z'y)+E(x'z-z'x)+F(x'y-y'x)=0. \qquad (10)$$

18. We may directly prove that all rays confined within a given plane meet in the same point. The equation of this plane being

$$t'x+u'y+v'z+w'=0, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (11)$$

we get, in order to express that a ray falls within that plane, the following three equations,

$$\begin{aligned} &t'r+u's+v'=0,\\ &t'\varrho+u'\sigma+w'=0,\\ &w's-v'\sigma-(r\sigma-s\varrho)t'=0, \end{aligned}$$

each of which results from the other two. Between these equations and the equation of the complex $(r\sigma - s\varrho)$, r and ϱ may be eliminated. The resulting equation,

$$(Bt'-Au'-Fw')s+(Dt'-Eu'+Fv')\sigma+Ct'-Av'-Ew'=0, \quad . \quad . \quad . \qquad (12)$$

being linear with regard to the two remaining variables s and σ, represents a right line parallel to OX and intersecting YZ in a point, the coordinates of which are

$$\left.\begin{aligned} z'&=\frac{Bt'-Au'-Fw'}{Dt'-Eu'-Fv'},\\ y'&=-\frac{Ct'-Av'-Ew'}{Dt'-Eu'+Fv'}. \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (13)$$

Hence all rays of the complex supposed to fall within the plane (11) intersect that right line, and consequently meet in the same point. Two coordinates of that point are given by the last equations, the third,

$$\left. x'=\frac{Cu'-Bv'-Dw'}{Dt'-Eu'+Fv'}, \right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (14)$$

is obtained by introducing the values of z' and y' into the equation of the plane.

We may represent the point corresponding to the given plane (t', u', v', w') by its equation,

$$(Cu'-Bv'-Dw')t-(Ct'-Av'-Ew')u+(Bt'-Au'-Fw')v+(Dt'-Eu'+Fv')w=0, \quad (15)$$

which may be written thus,

$$A(v'u-u'v)+B(t'v-v't)+C(u't-t'u)+D(t'w-w't)+E(w'u-u'w)+F(v'w-w'v)=0. \quad (16)$$

19. It is easily seen that both equations (12) and (16) are the most general ones, indicating the supposed correspondence between point and plane. Therefore (10) is the most general equation of a *linear complex.*

20. According to the fundamental relation which characterizes a linear complex, the plane corresponding to a given point is determined by means of any two rays passing through that point, as the point corresponding to a given plane is determined by any two rays confined within that plane.

Suppose P and P′ to be any two points of space, and p and p' the two corresponding planes. Let I be the right line joining both points, II the right line along which both planes intersect each other. Draw through I any plane intersecting II in Q, join Q to P and P′ by two right lines QP, QP′. These right lines, both passing through points (P, P′) and falling within planes (p, p') which pass through them, are rays of the complex. The plane PQP′, containing both rays and consequently containing I, corresponds to the point Q, whence we conclude that planes passing through any points Q, Q′ of II intersect each other along I. Likewise it may be proved that any plane drawn through II intersects I in the corresponding point. We shall call I and II *two right lines conjugate with regard to the linear complex*, or merely *conjugate lines.* The relation between two conjugate lines is a reciprocal one; each of them may be regarded as an axis in space around which a plane turns while the corresponding point describes the other; each also may be regarded as a ray, described by a moving point, the corresponding plane of which turns around the other.

Each right line meeting two conjugate right lines is a ray of the complex.

To each right line of space there is a conjugate one.

If a point move along a *ray of the complex*, the corresponding plane—containing each ray of the complex which passes through the point, and therefore especially the given one—turns around the ray.

Each ray of the complex may be regarded as two *coincident* conjugate lines.

21. We may also connect the preceding results with the general *principle of polar reciprocity.* Indeed the general equation (10), which represents the plane corresponding to a given point, is not altered if x', y', z' and x, y, z be replaced by one another. Consequently we may say, in introducing the denominations pole and polar plane instead of corresponding point and plane, that the polar planes of all points of a given plane pass through its pole, and conversely, that the poles of all planes passing through a given point fall within the polar plane of that point. In our particular case a plane,

containing its own pole, is determined by means of the poles of any two planes passing through that pole; likewise a point, falling within its polar plane, is determined by means of the polar planes of any two points of its polar plane. A right line joining any two points of space is *conjugate* to the right line, along which the polar planes of both points intersect each other. If one of two conjugate right lines envelopes within a given plane a curve, the other describes a conical surface; the vertex of the cone falls within the plane containing the enveloped curve. Generally if one of the two conjugate right lines describes a configuration, the other one likewise describes such a surface. If one of the two surfaces degenerates into a cone, the other degenerates into a plane curve*.

22. *A point of space being given, to construct the plane which contains all rays of the complex passing through the point.*

Each ray intersecting two conjugate lines is a ray of the complex. Accordingly the only right line starting from a given point and meeting any two conjugate is a ray of the complex. We obtain a new ray, starting from the same point, by means of each new pair of conjugate lines. All such lines constituting the plane corresponding to the given point, two pairs of conjugate lines are sufficient to determine that plane.

A plane of space being given, to construct the point where meet all rays of the complex confined within the plane.

Each right line joining the two points in which two conjugate right lines are intersected by a given plane being a ray of the complex, there will be obtained, within the given plane, as many rays as there are known pairs of conjugate lines. Any two such pairs are sufficient in order to determine the point within the plane corresponding to it where all rays meet.

A plane is intersected by the two lines of each conjugate pair in two points; the right lines joining two such points are rays of the complex converging all towards the point which corresponds to the plane. Again, the two planes passing through a point of space and meeting the two lines of a conjugate pair, intersect each other along a ray of the complex confined within the plane which corresponds to the point.

23. After this geometrical digression, immediately indicated by analysis, we resume the analytical way.

By putting in the general equation (9) of the plane corresponding to a given point (x', y', z'),

$$x'=0, \quad y'=0, \quad z'=0,$$

we obtain

$$\mathrm{A}x+\mathrm{B}y+\mathrm{C}z=0, \quad \ldots\ldots\ldots\ldots \quad (17)$$

in order to represent the plane corresponding to the origin.

---

* The peculiar kind of polar reciprocity we meet here was first noticed by M. Möbius in the 10th volume of 'Crelle's Journal,' and was afterwards expounded by L. F. Magnus in his valuable work 'Sammlung von Aufgaben und Lehrsätzen aus der analytischen Geometrie des Raumes,' pp. 139–145.

By putting successively

$$z' = \infty,$$
$$y' = \infty,$$
$$x' = \infty,$$

the same equation becomes

$$\left.\begin{aligned} C+Ex+Dy&=0,\\ B+Fx-Dz&=0,\\ A-Fy-Ez&=0.\end{aligned}\right\} \quad \ldots\ldots\ldots \quad (18)$$

Accordingly these equations represent the planes corresponding to points moved to an infinite distance along OZ, OY, OX.

By combining each of the equations (18) with (17), we get the rays conjugate to the axes of coordinates OZ, OY, OX, forming a triangle, the angles of which fall within the three planes of coordinates, XY, XZ, YZ, into the corresponding points.

24. By putting

$$w' = \infty,$$

the equation (15), representing a point corresponding to any given point $(x', y'\ z')$, becomes

$$Dt+Eu-Fv=0,$$

and then indicates that the point corresponding to the infinitely distant plane of space falls itself, at an infinite distance, along a direction which may be represented by the equations

$$\frac{x}{D}=\frac{y}{E}=\frac{z}{F}, \quad \ldots\ldots\ldots \quad (19)$$

while, if rectangular coordinates were supposed,

$$Dx+Ey+Fz=0$$

represents the plane perpendicular to it.

We shall call this direction *the characteristic direction of the complex.* It is invariably connected with the complex.

25. By putting successively

$$t' = \infty,$$
$$u' = \infty,$$
$$v' = \infty,$$

we get, in order to represent within the planes of coordinates YZ, XZ, XY, the points corresponding to these planes, the following equations:

$$\left.\begin{aligned} Cu-Bv-Dw&=0,\\ Ct-Av-Ew&=0,\\ Bt-Au-Fw&=0.\end{aligned}\right\} \quad \ldots\ldots\ldots \quad (20)$$

Accordingly the coordinates of these points are

$$\left.\begin{array}{lll} x=0, & y=-\frac{\mathrm{C}}{\mathrm{D}}\equiv y_t, & z=\frac{\mathrm{B}}{\mathrm{D}}\equiv z_t, \\ y=0, & x=-\frac{\mathrm{C}}{\mathrm{E}}\equiv x_u, & z=\frac{\mathrm{A}}{\mathrm{E}}\equiv z_u, \\ z=0, & x=-\frac{\mathrm{B}}{\mathrm{F}}\equiv x_v, & y=\frac{\mathrm{A}}{\mathrm{F}}\equiv y_v, \end{array}\right\} \quad \ldots\ldots\ldots \quad (21)$$

whence may be derived the following relation,

$$\frac{x_v y_t z_u}{x_u y_v z_t}=-1.$$

In putting $\mathrm{C}=-1$, the right line conjugate to OZ, if regarded as an axis, may be determined by its four coordinates [5],

$$p=\mathrm{A}, \quad q=\mathrm{B}, \quad \varpi=\mathrm{D}, \quad \varkappa=\mathrm{E}.$$

These coordinates therefore are four of the constants of the complex

$$\mathrm{A}r+\mathrm{B}s+\mathrm{D}\sigma+\mathrm{E}\varrho+\mathrm{F}(s\varrho-s\sigma)=1.$$

MN conjugate to OZ remains the same whatever may be the value of F. If by putting F equal to zero the last equation becomes a linear one, the complex is completely determined by MN conjugate to OZ.

26. The ratio of the three constants upon which the characteristic direction of the linear complex (1) depends,

$$\mathrm{D} : \mathrm{E} : \mathrm{F},$$

remains the same if the origin be changed or the complex moved parallel to itself. But if by turning the complex the characteristic direction simultaneously move, that ratio is altered. One of the three constants F, E, D becomes zero if the characteristic direction be confined within XY, XZ, YZ; two of them disappear, F and E, F and D; E and D if that direction fall within OX, OY, OZ. Here the general equation becomes

$$\left.\begin{array}{ll} \mathrm{A}r+\mathrm{B}s+\mathrm{C}+\mathrm{D}\sigma & =0, \\ \mathrm{A}r+\mathrm{B}s+\mathrm{C}+\mathrm{E}\varrho & =0, \\ \mathrm{A}r+\mathrm{B}s+\mathrm{C}+\mathrm{F}(s\varrho-r\sigma) & =0. \end{array}\right\} \quad \ldots\ldots\ldots \quad (22)$$

27. The ratio of the three constants

$$\mathrm{A} : \mathrm{B} : \mathrm{C}$$

varies if the complex be moved parallel to itself. If the plane corresponding to O pass through OZ, OY, OX, one of the three constants C, B, A becomes zero; if this plane be congruent with XY, XZ, YZ, *i. e.* if O be the point corresponding to XY, XZ, YZ, two constants A and B, A and C, B and C disappear, and the general equation of the complex becomes

$$\left.\begin{aligned} D\sigma+E\varrho+F(s\varrho-r\sigma)+C&=0,\\ D\sigma+E\varrho+F(s\varrho-r\sigma)+Bs&=0,\\ D\sigma+E\varrho+F(s\varrho-r\sigma)+Ar&=0.\end{aligned}\right\} \quad \dots\dots\dots \quad (23)$$

28. In order to represent a linear complex by equations of the utmost simplicity, let us take any plane XY, XZ, YZ perpendicular to the characteristic direction, and draw through its corresponding point O the axis OZ, OY, OX. The resulting equations will assume the following forms,

$$\left.\begin{aligned} F(s\varrho-r\sigma)+C&=0,\\ Bs\qquad+E\varrho&=0,\\ Ar\qquad+D\sigma&=0.\end{aligned}\right\} \quad \dots\dots\dots\dots \quad (23^*)$$

The planes corresponding to all points of a right line having the characteristic direction are parallel to each other; and conversely the locus of points corresponding to parallel planes is a right line of that direction. Hence we conclude that there is one fixed line, the points of which correspond to planes which are perpendicular to it. Consequently, on the supposition of rectangular coordinates, we may in only one way represent a linear complex by means of equations assuming the form of those above.

29. In order, for instance, to get the first of these equations, which by replacing $-\frac{C}{F}$ by k may be written thus,

$$s\varrho-r\sigma=k,$$

it will be sufficient to direct OZ along the fixed line. As no supposition is made either with regard to the position of the origin on OZ, or to the direction of OX and OY within the plane XY which is perpendicular to OZ, this equation will remain absolutely the same if the system of coordinates be moved parallel to itself along OZ, or turned round it. In other terms,

A linear complex of rays invariably remains the same if it be moved parallel to itself along a fixed right line or turned round it.

The fixed right line may be called the *axis of rotation*, or merely the *axis* of the complex.

30. We may give different geometrical interpretations to the last three equations, involving each a characteristic property of a linear complex of rays.

Any two planes XZ, YZ intersecting each other along OZ being given, rays of space may be determined either by their projections on both planes, or by the points where they meet them. In the first case, if a third plane intersecting XZ, YZ along OX, OY at right angles be drawn, there are two planes LMN, L′M′N′, parallel to each other, passing through the two projections LN, M′N, and meeting OZ, OY, OX in N and N′, M and M′, L and L′. In the second supposition, denote the two points of intersection by U and V, and their projections by U′ and V′. Accordingly U′U, V′V, and U′V′ may be regarded as the projections of UV on the planes XZ, YZ, and on OZ. If in the first case

$$\frac{\mathrm{LL}'.\,\mathrm{MM}'}{\mathrm{NN}'}=k,$$

in the second

$$\frac{\mathrm{UU}'.\,\mathrm{VV}'}{\mathrm{U}'\mathrm{V}'}=k,$$

all rays thus determined constitute the linear complex, represented by

$$s\varrho-r\sigma=k,$$

the axis of which is OZ.

If $k=0$, the linear complex is of a peculiar description, all its rays meet the same right line, the axis OZ.

31. The results of [29] may be derived in a direct way. Let (x', y', z') be any point of space; according to the general equation (10) its corresponding plane with regard to the complex

$$s\varrho-r\sigma=k \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

will be represented by

$$y'x-x'y=k(z-z'). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

In putting $x'=0$, $y'=0$, this equation shows that all planes corresponding to points of the axis of rotation OZ are perpendicular to this axis (in the case of oblique coordinates parallel to XY).

If the point fall within XY, we get by putting $z'=0$,

$$y'x-x'y=kz;$$

consequently the corresponding plane passes through O. In denoting the angle which it makes with the axis of rotation by λ, we obtain

$$\cos\lambda=\frac{k}{\sqrt{y'^2+x'^2+k^2}},$$

whence

$$y'^2+x'^2=k^2\tan^2\lambda. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

Hence we conclude,

Right lines parallel to and at an equal distance from the axis of the complex are met under the same angle by planes corresponding to their points.

32. The following results are immediately derived from (26).

The plane p corresponding to any given point P passes through OP, O being the projection of P on OZ. Let the plane p and the right line OM perpendicular to it in O turn round the axis OZ, through an angle $\frac{\pi}{2}$, and denote them after turning by p' and OM′. The projection of OP on OM′ is a constant, and equal to p. So is the perpendicular drawn from P to p'.

Again, k being given we may, by determining λ, construct the plane corresponding to a given point, and, conversely, by determining OP, construct the point corresponding to a given plane.

The following theorem is the geometrical interpretation of the equation (25).

Draw through a point P its corresponding plane p, and the plane XY perpendicular to the axis of the complex meeting that axis in O. Let R be an arbitrary point of p, and R′ its projection on XY. The double area of the triangle POR′ divided by R′R is a constant, and equal to k.

33. In order to generalize, we may start from the equation

$$\mathrm{A}r+\mathrm{B}s+\mathrm{C}+\mathrm{D}\sigma+\mathrm{E}\varrho+\mathrm{F}(s\varrho-r\sigma)=0 \quad . \quad (7),$$

and proceed in the following way. By replacing x, y, z by ξ, η, ζ, ϑ (see [16], note), and omitting the accents, we immediately derive from equation (10),

$$\left.\begin{aligned} \xi &= \mathrm{C}u-\mathrm{B}v-\mathrm{D}w, \\ \eta &= -\mathrm{C}t+\mathrm{A}v+\mathrm{E}w, \\ \zeta &= \mathrm{B}t-\mathrm{A}u-\mathrm{F}w, \\ \vartheta &= \mathrm{D}t-\mathrm{E}u+\mathrm{F}v, \end{aligned}\right\} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (27)$$

ξ, η, ζ, ϑ indicating any point, and t, u, v, w its corresponding plane. From the first three of these equations results the equation

$$\mathrm{A}\xi+\mathrm{B}\eta+\mathrm{C}\zeta=-(\mathrm{AD}-\mathrm{BE}+\mathrm{CF})w,$$

which, multiplied member by member by the fourth equation,

$$\mathrm{D}t-\mathrm{E}u+\mathrm{F}v=\vartheta,$$

and divided by ϑw, furnishes the following relation,

$$(\mathrm{A}x+\mathrm{B}y+\mathrm{C}z)\left(\mathrm{D}\frac{t}{w}-\mathrm{E}\frac{u}{w}+\mathrm{F}\frac{v}{w}\right)=-(\mathrm{AD}-\mathrm{BE}+\mathrm{CF}). \quad . \; . \; . \quad (28)$$

In a similar way we obtain

$$\left.\begin{aligned} & \frac{(\mathrm{C}\vartheta+\mathrm{E}\xi+\mathrm{D}\eta)}{\zeta}\cdot\frac{\mathrm{B}t-\mathrm{A}u-\mathrm{F}w}{v}, \\ = & \frac{\mathrm{B}\vartheta-\mathrm{D}\zeta+\mathrm{F}\xi}{\eta}\cdot\frac{-\mathrm{C}t+\mathrm{A}u+\mathrm{E}w}{u}, \\ = & \frac{\mathrm{A}\vartheta-\mathrm{E}\zeta-\mathrm{F}\eta}{\xi}\cdot\frac{\mathrm{C}u-\mathrm{B}v-\mathrm{D}w}{t}, \\ = & -(\mathrm{AD}-\mathrm{BE}+\mathrm{CF}). \end{aligned}\right\} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (29)$$

34. In starting again from the equation (26),

$$s\varrho-r\sigma=k,$$

and in supposing that there is a right line determined by means of the coordinates of any two of its points (x', y', z') and (x'', y'', z'') according to [31], its conjugate line will be represented by the system of equations,

$$\begin{aligned} y'x-x'y &= k(z-z'\,), \\ y''x-x''y &= k(z-z''), \end{aligned}$$

which, after eliminating successively y and x, may be replaced by the following ones:

$$x''y'-x'y''=k[(x''-x')z-(x''z'-x'z'')],$$
$$x''y'-x'y''=k[(y''-y')z-(y''z'-y'z'')].$$

In denoting the coordinates of the two conjugate lines by

$$r_0,\ s_0,\ \varrho_0,\ \sigma_0,\ \text{and}\ r^0,\ s^0,\ \varrho^0,\ \sigma^0,$$

the following relations are immediately obtained:

$$r_0=\frac{x''-x'}{z''-z'}, \qquad s_0=\frac{y''-y'}{z''-z'},$$
$$\varrho_0=\frac{x''z'-x'z''}{z''-z'}, \qquad \sigma_0=-\frac{y''z'-y'z''}{z''-z'},$$
$$s_0\varrho_0-r_0\sigma_0=-\frac{x''y'-x'y''}{z''-z'},$$
$$r^0=\quad k\frac{x''-x'}{x''y'-x'y''}, \qquad s^0=\quad k\cdot\frac{y''-y'}{x''y'-x'y''},$$
$$\varrho^0=-k\frac{x''z'-x'z''}{x''y'-x'y''}, \qquad \sigma^0=-k\frac{y''z'-y'z''}{x''y'-x'y''}.$$

Whence

$$\frac{r_0}{r^0}=\frac{s_0}{s^0}=\frac{\varrho_0}{\varrho^0}=\frac{\sigma_0}{\sigma^0}=\frac{(s_0\varrho_0-r_0\sigma_0)}{k}$$

and

$$(s_0\varrho_0-r_0\sigma_0)(s^0\varrho^0-r^0\sigma^0)=k^2.$$

Not any two conjugate right lines intersect each other; if congruent they belong to the complex.

35. A linear complex depends upon five constants, four of which fix in space the position of its axis. In the case of the equations (23), this axis falling within an axis of coordinates, there remains only one constant. The position of the axis of the complex and its remaining constant may be determined by means of the five independent constants of the general equation (7).

For that purpose we shall make use of the transformation of coordinates. If the axes of coordinates be changed, the coordinates of a ray change at the same time, and we get formulæ analogous to the formulæ in the case of ordinary coordinates, in order to express the coordinates of one system by means of the coordinates in the other.

36. Let

$$x=rz+\varrho,$$
$$y=sz+\sigma$$

be the equations of a ray referred to the system of coordinates (x, y, z). If referred to another system (x', y', z'), its coordinates will be replaced by new ones $(r', s', \varrho', \sigma')$, but their equations retain the same shape,

$$x'=r'z'+\varrho',$$
$$y'=s'z'+\sigma'.$$

If the primitive system of coordinates be only displaced parallel to itself, the coordinates of the new origin being (x^0, y^0, z^0), we obtain

$$x'=x-x^0, \quad y'=y-y^0, \quad z'=z-z^0;$$

and by substituting in the last equations,

$$\begin{aligned} x&=r'z+(\rho'+x^0-r'z),\\ y&=s'z+(\sigma'+y^0-s'z); \end{aligned}$$

whence, by comparison with the primitive equations,

$$\left.\begin{aligned} r&=r',\\ s&=s,\\ \rho&=\rho'+x^0-rz^0,\\ \sigma&=\sigma'+y^0-sz^0. \end{aligned}\right\} \quad \ldots \ldots \ldots \quad (30)$$

We have further

$$s\rho-r\sigma=(s'\rho'-r'\sigma')+x^0s-y^0r. \quad \ldots \ldots \quad (31)$$

If $x^0=0$, $y^0=0$, and accordingly the origin move along OZ, the expression $(s\rho-r\sigma)$ remains unaltered [29].

37. If OY and OX turn round OZ, forming in the new position OY$'$, OX$'$ the angles α' and α with OX, we have

$$\begin{aligned} x&=x'\cos\alpha+y'\cos\alpha'=rz+\rho,\\ y&=x'\sin\alpha+y'\sin\alpha'=sz+\sigma; \end{aligned}$$

whence, on putting $(\alpha'-\alpha)=\vartheta$,

$$\begin{aligned} x'&=\frac{r\sin\alpha'-s\cos\alpha'}{\sin\vartheta}z'+\frac{\rho\sin\alpha'-\sigma\sin\alpha'}{\sin\vartheta},\\ y'&=-\frac{r\sin\alpha-s\cos\alpha}{\sin\vartheta}\cdot z'-\frac{\rho\sin\alpha-\sigma\sin\alpha'}{\sin\vartheta}. \end{aligned}$$

We immediately derive from these equations of the ray in the new system (x', y', z'),

$$\left.\begin{aligned} r'\sin\vartheta&=r\sin\alpha'-s\cos\alpha',\\ \rho'\sin\vartheta&=\rho\sin\alpha'-\sigma\cos\alpha',\\ -s\sin\vartheta&=r\sin\alpha-s\cos\alpha,\\ -\sigma\sin\vartheta&=\rho\sin\alpha-\sigma\cos\alpha, \end{aligned}\right\} \quad \ldots \ldots \ldots \quad (31^*)$$

whence

$$\left.\begin{aligned} r&=r'\cos\alpha+s'\cos\alpha',\\ \rho&=\rho'\cos\alpha+\sigma'\cos\alpha',\\ s&=r'\sin\alpha+s'\sin\alpha',\\ \sigma&=\rho'\sin\alpha+\sigma'\sin\alpha', \end{aligned}\right\} \quad \ldots \ldots \ldots \quad (32)$$

and

$$(s\rho-r\sigma)=(s'\rho'-r'\sigma')\sin\vartheta. \quad \ldots \ldots \ldots \quad (33)$$

If especially $\vartheta=\frac{\pi}{2}$, the last four equations become

$$\left.\begin{aligned} r&=r' \cos\alpha-s' \sin\alpha,\\ \varrho&=\varrho' \cos\alpha-\sigma' \sin\alpha,\\ s&=r' \sin\alpha+s' \cos\alpha,\\ \sigma&=\varrho' \sin\alpha+\sigma' \cos\alpha, \end{aligned}\right\} \quad \dots\dots\dots \quad (34)$$

and the expression

$$s\varrho-r\sigma$$

will not be altered by the transformation of coordinates [29].

38. Again, let OX and OZ turn round OY; let α' and α be the angles formed by these axes in their new position, OX′ and OZ′, with OZ, and $\alpha'-\alpha=\vartheta$. In the new system of coordinates the primitive equations of the ray become

$$(z' \sin\alpha+x' \sin\alpha')=(z' \cos\alpha+x' \cos\alpha')r+\varrho,$$
$$y'=(z' \cos\alpha+x' \cos\alpha')s+\sigma.$$

From the first of these equations we derive

$$x'(\sin\alpha'-r\cos\alpha')=-z'(\sin\alpha-r\cos\alpha')+\varrho,$$

whence

$$r'=-\frac{\sin\alpha-r\cos\alpha}{\sin\alpha'-r\cos\alpha'}, \quad \dots\dots\dots \quad (35)$$

$$\varrho'=\frac{\varrho}{\sin\alpha'-r\cos\alpha'}. \quad \dots\dots\dots \quad (36)$$

After replacing in the second equation of this number x' by $(r'z'+\varrho')$, we obtain

$$y'=(\cos\alpha+r' \cos\alpha')sz'+(\sigma+s\varrho' \cos\alpha'),$$

whence

$$s'=(\cos\alpha+r' \cos\alpha')s,$$

$$\sigma'=\sigma+s\varrho' \cos\alpha';$$

and by eliminating r' and ϱ' by means of (35) and (36),

$$s'=\frac{s\sin\vartheta}{\sin\alpha'-r\cos\alpha'}, \quad \dots\dots\dots \quad (37)$$

$$\sigma'=\frac{(\sigma\varrho-r\sigma)\cos\alpha'+\sigma\sin\alpha'}{\sin\alpha'-r\cos\alpha'}. \quad \dots\dots\dots \quad (38)$$

From (35)-(38) we derive

$$s'\varrho'-r'\sigma'=\frac{(s\rho-r\sigma)\cos\alpha+\sigma\sin\alpha}{\sin\alpha'-r\cos\alpha'}; \quad \dots\dots\dots \quad (39)$$

from (36) and (37),

$$\frac{\varrho'}{s'}=\frac{\varrho}{s}\sin\vartheta. \quad \dots\dots\dots \quad (40)$$

On the supposition of rectangular axes of coordinates, the last equations become

$$\left.\begin{aligned} r' &= -\frac{\sin\alpha - r\cos\alpha}{\cos\alpha + r\sin\alpha}, \\ \varrho' &= \frac{\varrho}{\cos\alpha + r\sin\alpha}, \\ s' &= -\frac{s}{\cos\alpha + r\sin\alpha}, \\ \sigma' &= -\frac{(s\varrho - \varrho\sigma)\sin\alpha - \sigma\cos\alpha}{\cos\alpha + r\sin\alpha}, \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (41)$$

$$s'\varrho' - r'\sigma' = \frac{(s\varrho - r\sigma)\cos\alpha - \sigma\sin\alpha}{\cos\alpha + r\sin\alpha}, \quad \ldots\ldots\ldots \quad (42)$$

$$\frac{\varrho'}{s'} = \frac{\varrho}{s}. \quad \ldots\ldots\ldots\ldots\ldots \quad (43)$$

In order to pass from the first system of coordinates to the second, r, s, ϱ, σ and r', s', ϱ', σ' are to be replaced by one another, while the sign of α is to be changed. Thus we get the following formulæ:—

$$\left.\begin{aligned} r &= \frac{\sin\alpha + r'\cos\alpha}{\cos\alpha - r'\sin\alpha}, \\ \varrho &= \frac{\varrho'}{\cos\alpha - r'\sin\alpha}, \\ s &= -\frac{s'}{\cos\alpha - r'\sin\alpha}, \\ \sigma &= \frac{(s'\varrho' - r'\sigma')\sin\alpha + \sigma'\cos\alpha}{\cos\alpha - r'\sin\alpha}, \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (44)$$

$$s\varrho - r\sigma = \frac{(s'\varrho' - r'\sigma')\cos\alpha - \sigma\sin\alpha}{\cos\alpha - r'\sin\alpha}. \quad \ldots\ldots\ldots \quad (45)$$

39. The general equation of the linear complex

$$\mathrm{A}r + \mathrm{B}s + \mathrm{C} + \mathrm{D}\sigma + \mathrm{E}\varrho + \mathrm{F}(s\varrho - r\sigma) = 0 \; . \; . \quad (7)$$

becomes, if the origin is moved to any point (x^0, y^0, z^0) . . . (30),

$$(\mathrm{A} - \mathrm{F}y^0 - \mathrm{E}z^0)r + (\mathrm{B} + \mathrm{F}x^0 - \mathrm{D}z^0)s + (\mathrm{C} + \mathrm{E}x^0 + \mathrm{D}z^0) + \mathrm{D}\sigma' + \mathrm{E}\varrho' + \mathrm{F}(s\varrho - r\sigma) = 0.$$

If

$$\frac{x^0}{\mathrm{D}} = \frac{y^0}{\mathrm{E}} = \frac{z^0}{\mathrm{F}},$$

the primitive equation is not altered. Consequently the complex remains the same if it be moved parallel to itself along a direction indicated by the last equations. We obtain in denoting by ξ, η, ζ, the angles which this direction makes with OX, OY, OZ,

$$\frac{\cos\xi}{\mathrm{D}} = \frac{\cos\eta}{\mathrm{E}} = \frac{\cos\zeta}{\mathrm{F}}. \quad \ldots\ldots\ldots\ldots \quad (46)$$

40. In order to get OZ congruent with a right line OM of the determined direction and passing through O, we may in the first instance turn the system of coordinates

round OZ in its primitive position through an angle α such that ZX in its new position contains OM. Accordingly we obtain

$$\cos\alpha=\frac{\cos\xi}{\sin\zeta},$$

whence

$$\tan^2\alpha=\frac{1-\cos^2\zeta-\cos^2\xi}{\cos^2\xi}=\frac{\cos^2\eta}{\cos^2\xi}=\frac{E^2}{D^2}.$$

By making use of the formulæ (34), the equation of the complex (7) becomes

$$\begin{aligned}&(A\cos\alpha+B\sin\alpha)r'-(A\sin\alpha-B\cos\alpha)s'\\&+(E\cos\alpha+D\sin\alpha)\varrho'-(E\sin\alpha-D\cos\alpha)\sigma'+C+F(s'\varrho'-r'\sigma')=0,\end{aligned}$$

and may be written thus,

$$A'r+B's+C'+D'\sigma+F'(s\varrho-r\sigma)=0, \quad \ldots\ldots\ldots \quad (47)$$

in omitting the accents of the new coordinates and in putting

$$\left.\begin{aligned}&E\cos\alpha+D\sin\alpha=0,\\&A'=(AD-BE)\frac{\cos\alpha}{D},\quad B'=(AD+BE)\frac{\cos\alpha}{D},\\&D'=(D^2+E^2)\frac{\cos\alpha}{D},\quad C'=C,\quad F'=F.\end{aligned}\right\} \quad \ldots\ldots \quad (48)$$

41. In order to give within ZX to OZ the required direction along OM, the formulæ (44) are to be used after having replaced α by ζ. Accordingly the equation (47) is transposed into the following one,

$$\begin{aligned}&A'(\sin\zeta+r'\cos\zeta)-B's'+C'(\cos\zeta-r'\sin\zeta)\\&+D'\big((s'\varrho'-r'\sigma')\sin\zeta+\sigma'\cos\zeta\big)+F'\big((s'\varrho'-r'\sigma')\cos\zeta-\sigma'\sin\zeta\big)=0,\end{aligned}$$

and may be written thus,

$$A''r+B''s+C''+F''(s\varrho-r\sigma)=0, \quad \ldots\ldots\ldots \quad (48^*)$$

on omitting the accents of the coordinates and putting

$$\left.\begin{aligned}&D'\cos\zeta=F'\sin\zeta,\\&A''=(A'F'-C'D')\frac{\cos\zeta}{F'},\\&B''=-B',\\&C''=(A'F'+A'D')\frac{\cos\zeta}{F'},\\&F''=(D'^2+F'^2)\frac{\cos\zeta}{F'}.\end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (49)$$

42. Finally, the origin may be moved within XY to a point the coordinates of which are x^0 and y^0. Accordingly the equation of the complex, on replacing ϱ and σ by $\varrho+x^0$ and $\sigma+y^0$, becomes

$$(A''-F''y^0)r+(B''+F''x^0)s+C''+F''(s\varrho-r\sigma)=0,$$

and by putting

$$y^0=\frac{A''}{F''},\quad x^0=-\frac{B''}{F''} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (50)$$

is reduced to

$$(s\varrho-r\sigma)=-\frac{C''}{F''}=k. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (51)$$

43. By successive substitution we obtain

$$\begin{aligned} k&=-\frac{C''}{F''} \\ &=-\frac{C'F'+A'D'}{D'^2+F'^2} \\ &=-\frac{CF+(AD-BE)(D^2+E^2)\frac{\cos^2\alpha}{D^2}}{(D^2+E^2)^2\frac{\cos^2\alpha}{D^2}+F^2}; \end{aligned}$$

and finally, on observing that

$$\cos\alpha^2=\frac{D^2}{D^2+E^2},$$

the symmetrical expression

$$k=-\frac{AD-BE+CF}{D^2+E^2+F^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (52)$$

In order to replace OZ and OX by each other, we may make use of the formulæ (41) and (42) on putting $\alpha=\frac{1}{2}\pi$. By means of the last of these formulæ the equation of the linear complex (51) is immediately transformed into the following one,

$$\sigma=kr, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (53)$$

the constant k being the same as before.

Again, on interchanging OY and OX, we get

$$\varrho=ks. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (54)$$

44. If k become equal to zero the complex is of a peculiar description, all its rays meet a fixed line. If the complex be represented by the general equation

$$Ar+Bs+C+D\sigma+E\varrho+F(s\varrho-r\sigma)=0, \quad . \quad . \quad . \quad (7)$$

this peculiar case is indicated by the following condition,

$$AD-BE+CF=0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (55)$$

45. By eliminating from the general equation of the complex σ, ϱ and $(s\varrho-r\sigma)$ by means of the equations

$$\begin{aligned} x&=rz+\varrho, \\ y&=sz+\sigma, \\ sx-ry&=s\varrho-r\sigma. \end{aligned}$$

we get

$$(A+Fy-Ez)r+(B+Fx-Dz)s+(C+Dy+Ex)=0.$$

If there exist a point (x, y, z) where all rays of the complex meet, this point will be determined by means of the following three equations,

$$\left.\begin{aligned} \mathrm{A}-\mathrm{F}y-\mathrm{E}z&=0,\\ \mathrm{B}+\mathrm{F}x-\mathrm{D}z&=0,\\ \mathrm{C}+\mathrm{E}x+\mathrm{D}y&=0. \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (56)$$

These three equations can subsist simultaneously only in the case where (55) is satisfied.

If this condition be satisfied, the locus of points, where all rays of the complex meet, is a right line, the projections of which are represented by the last equations (56).

46. Such rays as belong to both linear complexes,

$$\left.\begin{aligned} \Omega&\equiv\mathrm{A}r+\mathrm{B}s+\mathrm{C}+\mathrm{D}\sigma+\mathrm{E}\varrho+\mathrm{F}(s\varrho-r\sigma)=0,\\ \Omega'&\equiv\mathrm{A}'r+\mathrm{B}'s+\mathrm{C}'+\mathrm{D}'\sigma+\mathrm{E}'\varrho+\mathrm{F}'(s\varrho-r\sigma)=0, \end{aligned}\right\} \quad \ldots\ldots \quad (57)$$

constitute a *linear congruency of rays represented by the system of the two equations.* In order to determine the congruency each of the two complexes,

$$\Omega=0, \qquad \Omega'=0$$

may be replaced by any other represented by

$$\Omega+\mu\Omega'=0, \quad \ldots\ldots\ldots\ldots \quad (58)$$

where arbitrary values are given to the coefficient μ.

In each of the two complexes by means of which the congruency is determined, there is a plane corresponding to each point of space which contains all rays starting from that point. Both planes corresponding to the same point intersect each other along a single ray, belonging to both complexes, *i. e.* to the congruency. *With regard to the congruency one ray corresponds to a given point of space.* The planes corresponding to the same point, in all complexes, represented by (58) meet along a fixed line, the corresponding ray of the congruency.

Conversely, there is in each of the complexes (58) a point corresponding to a given plane in which all rays confined within the plane meet. By means of two such complexes we get, within the given plane, two points; the right line joining the two points is the only ray of the plane common to both complexes, and therefore belonging to the congruency. We call it *the ray of the congruency corresponding to the given plane.*

To each point, as well as to each plane, corresponds only one ray. There are not any two rays of the congruency intersecting one onother, or, in other terms, confined within the same plane.

47. Suppose that AB is any given right line, and A′B′, A″B″ its two conjugate with regard to the complexes Ω, Ω'. Let C be any point of AB. Each ray starting from C, if confined within the plane A′B′C belongs to Ω, if confined within A″B″C to Ω'. Therefore the intersection of the two planes A′B′C, A″B″C, *i. e.* the right line starting from C and meeting both conjugate, is the ray of the congruency which corresponds to the point C. If C move along AB, all rays of the congruency obtained in that way are the

rays of one generation of a *hyperboloid*, while the given right line AB and its two conjugate A′B′, A″B″ are rays of its other generation. In replacing Ω and Ω' by other complexes arbitrarily taken among the complexes (58), the conjugate will be replaced by others, all intersected by the rays of the congruency starting from AB. Hence

The right lines conjugate to a given one, with regard to all complexes intersecting one another along a linear congruency, belong to one generation of a hyperboloid, while the right lines of its second generation are rays of the congruency meeting the given line.

48. If a point move along a given right line of space, according to the last number, its corresponding ray generally describes a hyperboloid. We may say that the same hyperboloid is described by the ray which corresponds to a plane passing through the given right line and turning round it. If the ray be the same in both cases, the point where it meets the given line AB is a point of the surface, and the plane confining both AB and the ray, the tangent plane in that point.

49. The hyperboloid generated by a ray of a linear congruency, the corresponding point of which moves along AB, varies if this line turn round one of its points C. All the new hyperboloids contain the ray which corresponds to C, but there is no other ray common to any two of them. If AB describe a plane, by turning round C through an angle π, there will be one ray of a hyperboloid passing through any point of space. A linear congruency therefore may be generated by a variable hyperboloid turning round one of its rays.

In an analogous way, a linear complex may be generated by a revolving variable congruency.

50. While in each of the two complexes Ω and Ω' there is a fixed line—the axis of the complex around which its rays are symmetrically distributed—there is in a linear congruency a characteristic section parallel to both axes of the complexes, and a characteristic direction perpendicular to it.

The characteristic section, if conducted through the origin O, may be represented by the equation

$$ax+by+cz=0.$$

The two right lines starting from O and parallel to the two axes of the complexes are represented by the double equations,

$$\frac{x}{\mathrm{D}}=\frac{y}{\mathrm{E}}=\frac{z}{\mathrm{F}},$$

$$\frac{x}{\mathrm{D}'}=\frac{y}{\mathrm{E}'}=\frac{z}{\mathrm{F}'}.$$

These lines being confined within the section, we get in order to determine the constants of its equation,

$$a\mathrm{D}+b\mathrm{E}+c\mathrm{F}=0,$$

$$a\mathrm{D}'+b\mathrm{E}'+c\mathrm{F}'=0,$$

whence

$$(\mathrm{D}'\mathrm{E}-\mathrm{E}'\mathrm{D})b+(\mathrm{D}'\mathrm{F}-\mathrm{F}'\mathrm{D})c=0,$$

$$(\mathrm{D}'\mathrm{E}-\mathrm{E}'\mathrm{D})a-(\mathrm{E}'\mathrm{F}-\mathrm{F}'\mathrm{E})c=0.$$

Accordingly the equation of the section becomes

$$(E'F-F'E)x-(D'F-F'D)y+(D'E-E'D)z=0, \quad \dots\dots \quad (59)$$

and the double equation of the right line perpendicular to it,

$$\frac{x}{E'F-F'E}=\frac{-y}{D'F-F'D}=\frac{z}{D'E-E'D}. \quad \dots\dots\dots\dots \quad (60)$$

51. By giving to OZ the characteristic direction, the two complexes (57) will be represented by linear equations of the form

$$\left.\begin{aligned} \Omega &\equiv Ar+Bs+C+D\sigma+E\varrho=0, \\ \Omega' &\equiv A'r+B's+C'+D'\sigma+E'\varrho=0, \end{aligned}\right\} \quad \dots\dots\dots\dots \quad (61)$$

the origin and the direction of OX and OY, perpendicular to OZ, remaining arbitrary.

Again, OZ may be moved parallel to itself, and accordingly ϱ and σ replaced by $(\varrho+x^0)$ and $(\sigma+y^0)$, x^0 and y^0 being the coordinates of the new origin. If especially

$$C+Dy^0+Ex^0=0,$$

$$C'+D'y^0+E'x^0=0,$$

whence

$$x^0=-\frac{C'D-D'C}{D'E-E'D},$$

$$y^0=\frac{C'E-E'C}{D'E-E'D};$$

by the mere disappearance of C and C' the equations of the two complexes become

$$\left.\begin{aligned} \Omega &\equiv Ar+Bs+D\sigma+E\varrho=0, \\ \Omega' &\equiv A'r+B's+D'\sigma+E'\varrho=0. \end{aligned}\right\} \quad \dots\dots\dots\dots \quad (62)$$

OZ in its new position is a completely determined right line, which may be called the *axis of the congruency*. It is easily seen that it intersects at right angles the two axes of rotation of the complexes Ω and Ω', and consequently the axes of all complexes represented by (58).

52. The planes corresponding in the two complexes (62) to a given point (x', y', z') are represented by

$$\left.\begin{aligned} (A-Ez')x+(B-Dz')y+(Ex'+Dy')z&=Ax'+By', \\ (A'-E'z')x+(B'-D'z')y+(E'x'+D'y')z&=A'x'+B'y'. \end{aligned}\right\} \quad \dots\dots \quad (63)$$

In order to express that both corresponding planes are the same, we obtain the following relations,

$$\left.\begin{aligned} &(A-Ez'):(B-Dz'):(Ex'+Fy'):(Ax'+By')= \\ &(A'-E'z'):(B'-D'z'):(E'x'+F'y'):(A'x'+B'y'). \end{aligned}\right\} \quad \dots\dots \quad (64)$$

Since both planes pass through the given point, any two equations, hence derived, are sufficient in order to determine the locus of points having, in both complexes, the same corresponding plane. From any two of the following six equations where the accents are omitted, the remaining four may be derived:

$$(D'E-E'D)z^2-[(B'E-E'B)-(A'D-D'A)]z-(A'B-B'A)=0, \quad . \quad . \qquad (65)$$

$$(B'D-D'B)y^2+[(B'E-E'B)+(A'D-D'A)]xy+(A'E-E'A)x^2=0, \quad . \quad . \qquad (66)$$

$$(A'D-D'A)y+(A'E-E'A)x+(D'E-E'D)yz=0, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (67)$$

$$(B'D-D'B)y-(B'E-E'B)x-(D'E-E'D)xz=0, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (68)$$

$$(A'B-B'A)y+(A'E-E'A)xz-(B'E-E'B)yz=0, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (69)$$

$$(A'B-B'A)x-(A'D-D'A)xz+(B'D-D'B)xz=0^*. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (70)$$

53. According to the first two equations (65), (66), the locus in question is a system of two right lines both intersecting OZ. These lines are confined within two planes parallel to XY and determined by (65); their direction within these planes is given by (66). We shall call them the "*directrices*," and the characteristic section parallel to both and equidistant from them, *the central plane of the linear congruency*. Both "directrices" intersect at right angles the axis of the congruency, as the axes of all complexes do.

54. We may distinguish two general classes of linear congruencies; either both directrices are *real* or both *imaginary*. In a particular case the two directrices are congruent. Finally, one of the two directrices may pass at an infinite distance.

55. If the directrices are real, and the plane XY be conducted through one of them, the following condition,

$$A'B-B'A=0, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (71)$$

is derived from (65). In order to determine within XY the direction of that directrix, we get from (67), by putting $z=0$,

$$(A'D-D'A)y+(A'E-E'A)x=0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (72)$$

There is among the infinite number of complexes containing the congruency, which are represented by

$$\Omega+\mu\Omega'=0,$$

one of a particular description. It is obtained if, starting from (62), we put

$$\mu=-\frac{A}{A'}=-\frac{B}{B'};$$

whence

$$(A'D-D'A)\sigma+(A'E-E'A)\varrho=0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (73)$$

All rays of that complex, and therefore all rays of the congruency, meet within XY a fixed right line, represented by (72), on replacing ϱ and σ by x and y. This line therefore is the axis of that complex, and one of the two directrices of the congruency. In the same way it may be proved that likewise all rays of the congruency meet the other directrix. Hence

All rays of a congruency meet its two directrices.

---

[*] We may observe that any equation which, like those above, is homogeneous with regard to $(A'B-B'A)$, $A'C-C'A)$... will not be altered if the complexes Ω and Ω' are replaced by any of the complexes $(\Omega+\mu\Omega')$.

Accordingly, both directrices being real and known, we may immediately draw through any given point the only corresponding ray of the congruency.

56. In that peculiar class of congruencies indicated by the condition

$$D'E-E'D=0, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (74)$$

one of the two directrices passes at an infinite distance. By putting simultaneously

$$A'B-B'A=0,$$

we get, in order to represent the only remaining directrix, now confined within XY, the same equation as before (72). But among the complexes,

$$\Omega+\mu\Omega'=0,$$

there is, besides the complex (73), the axis of which is the directrix, another complex, represented by

$$D\Omega'-D'\Omega\equiv(A'D-D'A)r+(B'D-D'B)s=0,$$

the rays of which are parallel to a given plane. Its equation may be transformed into

$$Ar+Bs=0; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (75)$$

accordingly the equation of the plane becomes

$$Ax+By=0.$$

Hence in this peculiar case

All rays of the linear congruency meet the only directrix, and are parallel to a given plane.

57. From the last considerations we conclude that among the complexes intersecting each other along a linear congruency, and represented by

$$\Omega+\mu\Omega'=0, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (76)$$

there are in the general case two, of a peculiar description, all the rays of which meet their axes. These axes, the directrices of the congruency, are two conjugate right lines with regard to each of the complexes (76).

Generally there is only one ray of the congruency passing through a given point, as there is only one ray confined within a given plane. But each of the two directrices may be considered as the locus of points, from which start an infinite number of rays, constituting a plane which passes through the other directrix. It may be likewise regarded as enveloped by planes, confining each an infinite number of rays, which converge towards a point of the other directrix.

58. We may represent *any two* complexes Ω, Ω' in any position whatever by equations depending only upon the position of their axes and their constants. Let Δ be the shortest distance of the two axes from each other, and ϑ the angle between their directions.

Suppose that OZ intersects at right angles the axes of both complexes. Let OX be the axis of the first complex Ω, k its constant, OX perpendicular to XZ. The equation of the complex will be

$$\sigma=kr.$$

If the axis OY be turned round O till, in its new position OY', the angle Y'OX becoming ϑ, the plane ZOY' passes through the axis of the second complex, the last equation, by putting

$$\sigma=\sigma' \sin\vartheta,$$
$$r=r'+s'\cos\vartheta,$$

assumes the following form,

$$\sigma'\sin\vartheta=kr'+ks'\cos\vartheta.$$

The axis of the second complex Ω' meets OZ in a point O', O'O being Δ. O' may be regarded as the origin of new coordinates, OY and OZ being replaced by O'Y'' congruent with the axis of Ω', and by O'X'' perpendicular to ZY''; then the second complex Ω' will be represented by the equation

$$\varrho''=k's'',$$

ϱ'' and s'' being the new ray-coordinates and k' the constant of the complex. In order to make O'X'' parallel to OX', it is to be turned round O' till, in its new position O'X''', the angle Y'''O'X'' becomes ϑ. Accordingly, by putting

$$\varrho''=\varrho'''\sin\vartheta,$$
$$s''=r'''\cos\vartheta+s''',$$

the equation of the complex is transformed into the following,

$$\varrho'''\sin\vartheta=k'r'''\cos\vartheta+k's'''.$$

Finally, by displacing the origin O' into O, ϱ''' becomes $\varrho^{\text{IV}}+\Delta r'''$, whence

$$\varrho'''\sin\vartheta=(k'\cos\vartheta+\Delta\sin\vartheta)r'''+k's'.$$

On omitting the accents, both complexes Ω and Ω', referred to the same axes of coordinates OZ, OY', OX, the two last of which include an angle ϑ, are represented by the following equations,

$$\left.\begin{aligned}\sigma\sin\vartheta&=kr+k\cos\vartheta.s,\\ \varrho\sin\vartheta&=(k'\cos\vartheta+\Delta\sin\vartheta)r+k's.\end{aligned}\right\}\quad\dots\dots\quad(77)$$

59. In order to determine the directrices of the congruency represented by the system of the last equations (77), the equations (65) and (66) may be transformed by putting

$$\begin{array}{llll}A=k, & B=k\cos\vartheta, & D=-\sin\vartheta, & E=0,\\ A'=k'\cos\vartheta+\Delta\sin\vartheta, & B'=k', & D'=0, & E'=-\sin\vartheta\end{array}$$

into those following,

$$0=(z\sin\vartheta)^2-[(k+k')\cos\vartheta+\Delta\sin\vartheta]z\sin\vartheta+(kk'\sin^2\vartheta-\Delta k\sin\vartheta\cos\vartheta),\ .\quad(78)$$

$$0=\left(\frac{y}{x}\right)^2-\frac{(k'-k)\cos\vartheta-\Delta\sin\vartheta}{k'}\cdot\frac{y}{x}-\frac{k}{k'}.\quad\dots\dots\dots\quad(79)$$

On denoting the roots of these equations by $z'\sin\vartheta$, $z''\sin\vartheta$, and $\left(\frac{y}{x}\right)'$, $\left(\frac{y}{x}\right)''$, we obtain

$$\begin{aligned}
z'+z'' &= \frac{(k-k')\cos\vartheta+\Delta\sin\vartheta}{\sin\vartheta},\\
(z'-z'')^2 &= \frac{4kk'+[(k-k')\cos\vartheta-\Delta\sin\vartheta]^2}{\sin^2\vartheta},\\
\left(\frac{y}{x}\right)'+\left(\frac{y}{x}\right)'' &= \frac{(k+k')\cos\vartheta-\Delta\sin\vartheta}{k'},\\
\left(\left(\frac{y}{x}\right)'-\left(\frac{y}{x}\right)''\right)^2 &= \frac{4kk'+[(k-k')\cos\vartheta-\Delta\sin\vartheta]^2}{k'^2}.
\end{aligned}$$

The roots of both equations are simultaneously either real, or imaginary, or congruent. In the last case we have

$$(k-k')\cos\vartheta-\Delta\sin\vartheta=2\sqrt{-kk'},$$

whence

$$\left(\frac{y}{x}\right)'=\left(\frac{y}{x}\right)''=\sqrt{-\frac{k}{k'}}.$$

The central plane of the congruency is represented by

$$z=\frac{(k-k')\cos\vartheta-\Delta\sin\vartheta}{2\sin\vartheta}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (80)$$

In two peculiar cases this equation becomes

$$z=\tfrac{1}{2}\Delta,$$

either if

$$\vartheta=\tfrac{1}{2}\pi,$$

or, whatever may be ϑ, if

$$k=k'.$$

Hence the axes of any two complexes selected among those intersecting each other along a given congruency are at equal distances from its central plane if their directions are perpendicular to each other, or if the constants of both complexes are the same.

60. Without entering into a more detailed discussion of the last results we may finally treat the inverse problem: a congruency being given by means of its two directrices, to determine the complexes passing through it. On the supposition of rectangular coordinates, the two directrices may be represented by the following systems of equations,

$$\begin{aligned}
y-ax&=0, \qquad z=\theta,\\
y+ax&=0, \qquad z=-\theta.
\end{aligned}$$

These directrices are the axes of two complexes of a peculiar description, ranging among the infinite number of complexes which intersect each other along the congruency. The two complexes, if moved parallel to themselves till their axes fall within XY, are represented by the equations

$$\begin{aligned}
\sigma-a\varrho&=0,\\
\sigma+a\varrho&=0,
\end{aligned}$$

whence, in order to represent them in their primitive position, the following equations are derived,

$$\sigma - a\varrho + bs - bar = 0,$$

$$\sigma + a\varrho - bs - bar = 0.$$

By adding the two equations, after having multiplied the second by an undetermined coefficient μ, the following equation results,

$$(1+\mu)\sigma - (1-\mu)a\varrho + (1-\mu)bs - (1+\mu)bar = 0,$$

which, on putting

$$\frac{1-\mu}{1+\mu} = \lambda,$$

becomes

$$\sigma - \lambda a\varrho + \lambda bs - bar = 0 \,. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (81)$$

By varying λ all complexes intersecting each other along the congruency are represented by this equation. Their axes are parallel to XY and meet OZ. According to (19) and (52) we may immediately derive the direction of the axes and their constants. The following way of proceeding leads us to the same results, giving besides the position in space of their axes.

By turning OX and OY round OZ through an angle ω, by means of the formula (34), in which α is to be replaced by ω, the last equation is transformed into the following one,

$$(\cos\omega + \lambda a \sin\omega)\sigma' + (\sin\omega - \lambda a \cos\omega)\varrho' + (\lambda\cos\omega + a\sin\omega)bs' + (\lambda\sin\omega - a\cos\omega)br' = 0,$$

whence, by putting

$$\tan\omega = \lambda a, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (82)$$

we obtain

$$(1+\tan^2\omega)\sigma' + (\lambda\tan\omega - a)br' + (\lambda + a\tan\omega)bs' = 0.$$

Finally, by displacing the system of coordinates parallel to itself in such a way that the origin moves along OZ through z^0, we get

$$(1+\tan^2\omega)\sigma' + (\lambda\tan\omega - a)br' + (\lambda + a\tan\omega)bs' - (1+\tan^2\omega)z^0 s' = 0,$$

whence, by putting

$$z^0 = \frac{\lambda + a\tan\omega}{1+\tan^2\omega}\cdot b, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (83)$$

there results

$$\sigma' = -\frac{\lambda\tan\omega - a}{1+\tan^2\omega}\cdot br' = kr'. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (84)$$

The values of $\tan\omega$, z^0, and k remain real if both directrices become imaginary. In this case, XY always remaining the central plane of the congruency and OZ its axis, a, b, and μ are to be replaced by $a\sqrt{-1}$, $b\sqrt{-1}$, $\mu\sqrt{-1}$. If a be real, we may put

$$a = \tan\alpha,$$

2α being the angle between the directions of the two directrices, bisected by XZ. Accordingly we get

$$\lambda = \frac{\tan\omega}{\tan\alpha}, \qquad (85)$$

$$\left.\begin{aligned} z^0 &= \theta\frac{1+\tan^2\alpha}{\tan\alpha}\cdot\frac{\tan\omega}{1+\tan^2\omega} \\ &= \theta\frac{\sin\omega\cos\omega}{\sin\alpha\cos\alpha} \\ &= \theta\frac{\sin 2\omega}{\sin 2\alpha}, \end{aligned}\right\} \qquad (86)$$

$$\left.\begin{aligned} k &= \theta\frac{\tan^2\alpha-\tan^2\omega}{\tan\alpha(1+\tan^2\omega)} \\ &= \theta\frac{\sin^2\alpha\cos^2\omega-\sin^2\omega\cos^2\alpha}{\sin\alpha\cos\alpha} \\ &= \theta\frac{\sin(\alpha+\omega)\sin(\alpha-\omega)}{\sin\alpha\,\cos\alpha}. \end{aligned}\right\} \qquad (87)$$

The expression of z^0 shows that the axis within the central plane is directed along one of the two right lines bisecting, within this plane, the angle between the directions of the two directrices. These two right lines, having a peculiar relation to the congruency, may be called its *second* and *third* axis. The three axes, perpendicular to each other, meet in the *centre* of the congruency.

In order to express the angle ω by means of z^0, we get the following equation,

$$\sin 2\omega = \frac{z^0}{\theta}\sin 2\alpha,$$

indicating two directions perpendicular to each other, and corresponding to any value of z^0.

61. By replacing in the expression

$$z^0 = \frac{\theta}{\sin\alpha\cos\alpha}\cdot\frac{\tan\omega}{1+\tan^2\omega}$$

$\tan\omega$ by $\frac{y}{x}$, we obtain on omitting the accent of z^0,

$$z(y^2+x^2) = \frac{\theta}{\sin\alpha\cos\alpha}\cdot xy. \qquad (88)$$

The axes of all complexes constituting the congruency are confined within the surface represented by that equation. But this equation remaining unaltered if the axes OX and OY are replaced by one another, it is evident that the same surface contained the axes of two different series of complexes; one of the two series constituting the given congruency, while the other constitutes a strange one, obtained by turning the given congruency round its axis through a right angle.

62. In representing any three linear complexes by

$$\left.\begin{array}{l} \Omega \equiv \mathrm{A}r + \mathrm{B}s + \mathrm{C} + \mathrm{D}\sigma + \mathrm{E}\varrho + \mathrm{F}\,(s\varrho - r\sigma) = 0, \\ \Omega' \equiv \mathrm{A}'r + \mathrm{B}'s + \mathrm{C}' + \mathrm{D}'\sigma + \mathrm{E}'\varrho + \mathrm{F}'\,(s\varrho - r\sigma) = 0, \\ \Omega'' \equiv \mathrm{A}''r + \mathrm{B}s'' + \mathrm{C}'' + \mathrm{D}''\sigma + \mathrm{E}''\varrho + \mathrm{F}''(s\varrho - r\sigma) = 0, \end{array}\right\} \quad . \quad . \quad . \quad . \quad . \quad (89)$$

the system of these three equations represents a *linear configuration of rays*. The complexes may be replaced by any three selected among those represented by

$$\Omega + \mu\Omega' + \nu\Omega'' = 0$$

on giving to μ and ν any values whatever. By combining the three complexes Ω, Ω', Ω'' we get three congruencies, and accordingly three couples of directrices. Each ray of the configuration, belonging simultaneously to the three congruencies, meets both directrices of each couple. Hence in the general case the configuration is a *hyperboloid*; *its rays constitute one of its generations, while the directrices of all congruencies passing through it are right lines of its other generation.* Any three directrices are sufficient in order to determine the hyperboloid.

63. Let P and P′, Q and Q′, R and R′ be the three couples of directrices, each couple determining a central plane. The three central planes Π, K, P meet in one point C, which shall be called the *centre* of the configuration. The segment of any ray of a congruency bounded by both directrices being bisected by the central plane, the three right lines drawn through the centre C of the configuration to the three couples of directrices are bisected in the centre; they may be called *diameters* of the configuration.

Let, for instance, π and π' be the extremities of that diameter, $\pi \mathrm{C} \pi'$, which meets both directrices P and P′. The ray of the congruency (Ω, Ω') passing through π is parallel to P′, the ray passing through π' parallel to P. Both planes p and p', drawn through P and P′ parallel to the central plane Π, each confining two right lines (one directrix and the ray parallel to the other) which belong to the two generations of the hyperboloid, touch that configuration, and the point where both right lines in each plane meet is the point of contact.

Draw through the six directrices P and P′, Q and Q′, R and R′ six planes p and p', q and q', r and r' parallel to the central planes Π, K, P. The six planes thus obtained constitute a paralellopiped circumscribed to the configuration, the three diameters of which join each the points of contact within two opposite planes. The axes of the three corresponding congruencies (Ω, Ω'), (Ω, Ω'), (Ω', Ω'') are equal to the distance of the three couples of opposite planes; their centres are easily found.

64. The hyperboloid thus obtained is not changed if the complexes Ω, Ω', Ω'' be replaced by any three others taken among the complexes

$$\Omega + \mu\Omega' + \nu\Omega'' = 0,$$

but the three congruencies vary, and their directrices and the three diameters of the hyperboloid. The directrices may be either real or imaginary; accordingly the three diameters either intersect the hyperboloid or do not meet it. In the intermediate case, where both congruencies are congruent, the corresponding diameter falls within the asymptotic cone of the surface.

65. Conversely, starting from the hyperboloid and any three of its diameters, we may revert to the three corresponding congruencies and the series of complexes by means of which these congruencies are determined. If especially the three diameters are the axes of the hyperboloid, the axes of the three congruencies meet in the same point, the centre of the surface, and are directed along its axes.

There is a double way of reverting from a given hyperboloid to the congruencies, and further on to the complexes. The right lines constituting each of its two generations may be considered as its rays, while the right lines of its other generation will be found to be the directrices of the congruencies passing through the surface.

66. It might be desirable to support in the analytical way the geometrical results explained in the last numbers. For that purpose we may select in order to determine the configuration, three complexes of that peculiar description where all rays meet the axis. Accordingly the axes of the three complexes Ω, Ω′, Ω″ are three of the six directrices, P, Q, R for instance, confined within the planes *p*, *q*, *r*. In assuming these planes as planes of coordinates XY, XZ, YZ, the three complexes, constituting the configuration, are represented by equations of the following form,

$$\left.\begin{array}{l}\Omega\ \equiv \mathrm{C}\quad +\mathrm{D}\sigma\ +\mathrm{E}\varrho=0,\\ \Omega'\equiv \mathrm{B}'s\ +\mathrm{D}'\sigma+\mathrm{F}'(s\varrho-r\sigma)=0,\\ \Omega''\equiv \mathrm{A}''r+\mathrm{E}''\varrho+\mathrm{F}''(s\varrho-r\sigma)=0.\end{array}\right\}\quad \ldots\ldots\quad (90)$$

In order to represent by means of a single equation between x, y, z a configuration determined by means of three equations between ray-coordinates, these coordinates are to be eliminated by means of the following two equations,

$$x=rz+\varrho,$$
$$y=sz+\sigma,$$

to which the third derived one,

$$sx-ry=s\varrho-r\sigma,$$

may be added. In our case we may at first eliminate $s\varrho-r\sigma$, whence

$$(\mathrm{B}'+\mathrm{F}x')s-\mathrm{F}'yr+\mathrm{D}'\sigma=0,$$
$$(\mathrm{A}''-\mathrm{F}''y)r+\mathrm{F}''xs+\mathrm{E}''\varrho=0,$$

and after that ϱ and σ,

$$\mathrm{E}zr+\mathrm{D}zs=\mathrm{C}+\mathrm{D}y+\mathrm{E}x,$$
$$(\mathrm{B}'+\mathrm{F}'x-\mathrm{D}'z)s-\mathrm{F}'yr+\mathrm{D}'y=0,$$
$$(\mathrm{A}''-\mathrm{F}''y-\mathrm{E}''z)r+\mathrm{F}''xs+\mathrm{E}''x=0.$$

Finally, by putting the values of r and s taken from the last two equations into the first one, we obtain

$$\begin{aligned}&\{(B'+F'x-D'z)E''-F''D'y\}Exz\\+&\{(A''-F''y-E''z)D'-E''F'x\}Dyz\\+&\{(A''-F''y-E''z)(B'+F'x-D'z)+F'F''xy\}(C+Dy+Ex)=0,\end{aligned}$$

which, by the disappearance of terms of the third order, becomes

$$\left.\begin{aligned}&A''B'C+A''(B'E+CF')x+B'(A''D-CF'')y-C(A''D'+E''B')z\\&\qquad+A''F'Ex^2-B'F''Dy^2+CE''D'z^2\\&+(A''F'D-B'F''E)xy-(A''D'E+CE''F)xy\\&\qquad+(CF''D'-B'E''D)yz=0.\end{aligned}\right\} \quad . \quad . \quad (91)$$

After dividing by $A''B'C$ and replacing

$$-\frac{E}{C},\ -\frac{D}{C},\ \frac{D'}{B'},\ -\frac{F'}{B'},\ \frac{E''}{A''},\ \frac{F''}{A''}$$

by ξ, η, ζ', ξ', ζ'', η'', the last equation assumes the following symmetrical form,

$$\left.\begin{aligned}&1-(\xi+\xi')x-(\eta+\eta'')y-(\zeta'+\zeta'')z\\&\qquad+\xi\xi'x^2+\eta\eta''y^2+\zeta'\zeta''z^2\\&+(\xi'\eta+\xi\eta'')xy+(\xi'\zeta''+\xi\zeta')xz+(\eta\zeta'+\eta''\zeta')yz=0.\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (92)$$

In order to represent the configuration this equation replaces the three equations (90), which may be written thus,

$$\left.\begin{aligned}&\eta\sigma+\xi\varrho-1=0,\\&\zeta'\sigma-\xi'(s\varrho-r\sigma)-1=0,\\&\zeta''\varrho-\eta''(s\varrho-r\sigma)+1=0.\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (93)$$

It shows that the configuration is a hyperboloid touching the three planes XY, XZ, YZ. The rays within these planes are represented by

$$\left.\begin{aligned}z&=0, &\quad \xi x+\eta y&=1,\\y&=0, &\quad \xi'x+\zeta'z&=1,\\x&=0, &\quad \eta''y+\zeta''z&=1,\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (94)$$

the directrices within them by

$$\left.\begin{aligned}z&=0, &\quad \xi'x+\eta''y&=1,\\y&=0, &\quad \xi x+\zeta''z&=1,\\x&=0, &\quad \eta y+\zeta''z&=1.\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (95)$$

The points of contact, being within each plane the intersection of the ray and the directrix, are easily obtained.

The rays within the three planes of coordinates which form one edge of a circumscribed parallelopiped meet the directrices within the planes forming the opposite edge.

II.—*On Complexes of Luminous Rays within Biaxal Crystals.*

1. A single ray of light when meeting the surface of a doubly refracting crystal is divided into two rays determined by means of their four coordinates, r, s, ϱ, σ. All incident rays constituting a configuration, especially all rays starting from a luminous point and forming a conical surface, constitute within the crystal a new configuration, represented by the system of three equations between ray-coordinates. All incident rays constituting a congruency, emanating, for instance, in all directions from a luminous point, constitute within the crystal, after refraction, another congruency. Finally, a complex of incident rays, all rays, for instance, emanating in all directions from every point of a luminous curve, constitute within the crystal another complex of refracted rays. The congruency of refracted rays is represented by two, the complex by a single equation between ray-coordinates.

2. But before entering into the discussions indicated by the foregoing remarks, a short digression on double refraction might be desirable.

A biaxal crystal being cut along any plane whatever, we may suppose that this plane is congruent with xy, and that the point where an incident ray meets it is the origin of coordinates O. Let

$$x=pz,\ y=qz \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

be the equations of the incident ray, whence

$$\frac{x}{p}=\frac{y}{q}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

the equation of the plane of incidence. In the moment of incidence the front of the corresponding elementary wave, perpendicular to the ray, will be represented by

$$z+qy+px=0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

After the front of the wave has moved in air through the unit of distance, its equation becomes

$$z+qy+px=w \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

on putting

$$1+p^2+q^2=w^2.$$

At this moment the front of the wave intersects xy along a right line, which we may denote by RR, the equation of which is

$$qy+px=w. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

If the optical density of the surrounding medium increases, the value of w decreases in the same ratio.

3. Around the point O, where the incident ray meets the section of the crystal, let the wave-surface be described as it is at that moment when the front of the elementary wave intersects xy along RR. The position of the axes of elasticity of the crystallized medium being known with regard to the axes of coordinates, the equation of the wave-surface only depends upon three constants a, b, c, which are to be referred to the same

unit as w. If both systems of axes are congruent, the wave-surface is represented by the well-known equation

$$(a^2x^2+b^2y^2+c^2z^2)(x^2+y^2+z^2)-[a^2(b^2+c^2)x^2+b^2(a^2+c^2)y^2+c^2(a^2+b^2)z^2]+a^2b^2c^2=0, \quad . \ (6)$$

which, for simplicity, may be written thus,

$$\Omega=0.$$

4. The wave-surface is intimately connected with three ellipsoids, the equations of which are

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1, \quad \ldots \ldots \ldots \quad (7)$$

$$a^2x^2+b^2y^2+c^2z^2=1, \quad \ldots \ldots \ldots \quad (8)$$

$$\frac{x^2}{bc}+\frac{y^2}{ac}+\frac{z^2}{ab}=1. \quad \ldots \ldots \ldots \quad (9)$$

By means of the *first* and the *second* ellipsoid the wave-surface may be obtained most easily. The third ellipsoid has been introduced by myself on account of the following remarkable property. With regard to this ellipsoid the wave-surface is its own polar surface, *i. e.* the polar plane of any point of the surface touches it in another point, and *vice versâ*, the pole of any plane tangent to the surface is one of its points.

The wave-surface and the three ellipsoids depend upon the same constants. When the crystal turns around the point of incidence O, both the surface and the three ellipsoids simultaneously turn with it. In the new position their equations involve three new constants, indicating the position of the axes of elasticity with regard to the axes of coordinates. Now the wave-surface may be represented by

$$\Omega'=0,$$

and the third ellipsoid in the corresponding position by

$$Ax^2+Bxy+Cy^2+2Dxz+2Eyz+Fz^2-1\equiv E=0. \quad \ldots \quad (10)$$

From the six constants of this equation, which may be regarded as known, you may derive the six constants of the wave-surface by determining both the direction and the length of the axes of the third ellipsoid.

Within the plane xy, supposed to be any section whatever of the crystal, OX and OY may be directed along the axes of the ellipse along which this plane is intersected by the third ellipsoid. Accordingly the constant B disappears from the last equation. Besides, if OZ be directed along that diameter of the ellipsoid which is conjugate to the plane xy, and cease therefore, in the general case, to be perpendicular to it, both constants D and E likewise disappear.

5. According to HUYGHENS's principle, we obtain both rays into which an incident ray is divided, when entering the crystal, by the following general construction. Construct the two planes passing through the trace RR and tangent to the wave-surface described within the crystal around the point of incidence O. Let H and H′ be the

points of contact within these planes. The two right lines OH, OH′ drawn through the point of incidence O and the two points of contact H, H′ will be the refracted rays.

By means of the theorem referred to in the last number I have replaced this construction by the following one, much easier to manage. Construct with regard to the third auxiliary ellipsoid E the polar line of the trace RR. This polar line, which may be denoted by SS, meets the wave-surface within the crystal in the two points H and H′, OH and OH′ being, as before, the two refracted rays.

The plane HOH′, containing both refracted rays OH, OH′, may be called *the plane of refraction.* There are, generally speaking, four tangent planes passing through RR, as there are four points where the wave-surface is intersected by SS. We get therefore four rays, all confined within the plane of refraction, but two of them, not entering the crystal, are foreign to the question.

6. The plane of refraction may be constructed solely by means of the third ellipsoid E. The details of this construction depend upon the well-known different modes of determining the polar line SS. On proceeding in this way we meet some remarkable corollaries concerning double refraction*.

7. The poles of all planes passing through the trace RR, represented by

$$qy+px=w \quad . \quad . \quad . \quad (5),$$

are points of SS. All right lines passing through the point of incidence O and these poles fall within the plane of refraction confining SS. These right lines may likewise be regarded as diameters of the ellipsoid E conjugate to diametral planes passing through the trace along which the surface of the crystal, *i. e.* the plane xy, is intersected by the wave-front in its primitive position, the trace being parallel to RR and represented by

$$qy+px=0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

Hence

The plane of refraction is that diametral plane of the ellipsoid E, *the conjugate diameter of which is perpendicular to the plane of incidence in* O.

---

* In concluding a former paper, "Discussion de la forme générale des ondes lumineuses" (Crelle's Journal, No. xix. pp. 1 & 91, Mai 1838), I gave the following construction:—

"Construisez, par rapport à l'ellipsoide directeur, la ligne droite polaire (SS) de celle qui est perpendiculaire au plan d'incidence en O′. Elle coupera la surface de l'onde, decrite autour du point O, en deux points. Les deux lignes droites qui vont du point O aboutir à ces points seront les deux rayons réfractés; tandis que les deux plans, qui, contenant la perpendiculaire en O′ (RR), passent par ces deux mêmes points seront les fronts des deux ondes planes correspondantes. Enfin il a été démontré, dans ce qui précède, que les deux plans de vibration sont ceux qu'on obtient en conduisant par les rayons lumineux (réfractés) des plans perpendiculaires aux fronts des ondes correspondantes."

At the present occasion I resume the discussion, announced by myself twenty-six years ago, of a part of this construction. More recently, in the eighteenth Leçon of his valuable work, 'Théorie mathématique de l'Elasticité' (1852), M. LAMÉ reproduces the curious relation between the wave-surface and the third ellipsoid. He presents in the following Leçon a remarkable theorem, "which is one of those immediately derived from this relation." [8]

Accordingly the plane of refraction, conjugate to (6), is represented by the equation

$$\frac{dE}{dx}q=\frac{dE}{dy}p, \quad \ldots \ldots \ldots \quad (12)$$

which may be expanded into the following one,

$$(Ax+By+Dz)q=(Bx+Cy+Ez)p, \quad \ldots \ldots \quad (13)$$

or

$$(Aq-Bp)x+(Bq-Cp)y+(Dq-Ep)z=0^{*}. \quad \ldots \ldots \quad (14)$$

8. These equations remain unaltered if p and q vary in such a way that the ratio $\frac{p}{q}$ remains the same, *i. e.* if the angle of incidence vary while the plane of incidence remains the same. The same equations do not contain w, the value of which depends upon the density of the surrounding medium. Hence

All rays of light confined within the same plane of incidence, after being divided into two by double refraction, are confined again within the same plane—the plane of refraction. This plane remains the same if the surrounding medium be changed.

9. The plane xy, i. e. the surface of the crystal, containing the trace (11), its conjugate diameter, the equations of which are

$$\frac{dE}{dx}=0, \quad \frac{dE}{dy}=0, \quad \ldots \ldots \ldots \quad (15)$$

or

$$\left.\begin{array}{l} Ax+By+Dz=0, \\ Bx+Cy+Ez=0, \end{array}\right\} \quad \ldots \ldots \ldots \quad (16)$$

is confined within the plane of refraction, whatever may be the incident ray. The same may be proved analytically by observing that (12) is satisfied by means of the two equations (15). Hence

A ray of light of any direction whatever meeting the surface of a biaxal crystal in a fixed point is so refracted that the plane containing both refracted rays passes through a fixed right line (15).

---

* On representing any one of both refracted rays by the equations

$$x=rz, \quad y=sz,$$

the last equation, written thus,

$$(Aq-Bp)r+(Bq-Cp)s+(Dq-Ep)=0, \quad \ldots \ldots \ldots \quad (1)$$

indicates a relation between the direction of the incident ray, determined by the constants p and q, and the direction of the refracted one, determined by r and s.

This equation will not be altered if the incident ray, moved parallel to itself, meet the section of the crystal in any point

$$x=\rho, \quad y=\sigma.$$

If r and s be regarded as variable, ρ and σ being constant, the equation (1) represents the plane of refraction corresponding to the incident ray

$$x=pz+\rho, \quad y=qz+\sigma,$$

and containing both refracted rays.

If without the crystal the plane of incidence turns round the perpendicular to the section, within the crystal the plane of refraction simultaneously turns round the diameter of the third ellipsoid conjugate to the section.

10. In order to construct the plane of refraction, we want to know another diameter conjugate to any plane passing through the trace (11). In selecting among these planes the wave-front itself in its primitive position, the plane of refraction will be obtained by drawing a plane through both diameters conjugate to the section of the crystal and the primitive wave-front.

The wave-front in its primitive position is represented by

$$px+qy+z=0,$$

its conjugate diameter by the equations

$$\left.\begin{aligned}\frac{d\mathrm{E}}{dx}&=p\cdot\frac{d\mathrm{E}}{dz},\\ \frac{d\mathrm{E}}{dy}&=q\cdot\frac{d\mathrm{E}}{dz},\end{aligned}\right\} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (17)$$

which, if expanded, become

$$\left.\begin{aligned}\mathrm{A}x+\mathrm{B}y+\mathrm{D}z&=p(\mathrm{D}x+\mathrm{E}y+\mathrm{F}z),\\ \mathrm{B}x+\mathrm{C}y+\mathrm{E}z&=q(\mathrm{D}x+\mathrm{E}y+\mathrm{F}z).\end{aligned}\right\} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (18)$$

In order to prove in the analytical way that the diameter conjugate to the primitive wave-front falls within the plane of refraction, it is sufficient to observe that, by eliminating $\frac{d\mathrm{E}}{dz}$ between the two equations (17), the equation of the plane of refraction (12) is obtained.

11. If a ray of light meet the surface of a crystal in a given point, the third ellipsoid remains invariably the same as long as the position of the crystal is not altered. Therefore the diameter conjugate to the wave-front remaining likewise the same, whatever may be the section of the crystal passing through the point of incidence, the plane of refraction always passes through that fixed diameter. Again, if the incident ray, displaced parallel to itself, meet the surface of the crystal in a new point, this new point of incidence becomes the centre of the third ellipsoid, likewise displaced parallel to itself. The diameter conjugate to the primitive wave-front, always passing through the point of incidence, retains the same direction. We may finally observe that the surface of the crystal, if a curved one, may be replaced for any incident ray by the plane tangent to it in the point of incidence.

If a ray of light meet a biaxal crystal in a given point, whatever may be the surface bounding the crystal and containing that point, the plane of refraction passes through a fixed right line.

If a system of parallel rays meet the surface of a biaxal crystal, each ray of which after double refraction is divided into two, there is within the crystal a fixed direction, not depending upon the shape of the surface, so that the directions of both refracted rays *into which any incident ray is divided, and that fixed direction, are confined within the same plane.*

12. By putting

$$Dq = Ep,$$

the equation of the plane of refraction becomes

$$(Aq - Bp)x + (Bq - Cp)y = 0,$$

which, after eliminating p and q, may be written thus,

$$(AE - DB)x + (BE - DC)y = 0. \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (19)$$

In this case the plane of refraction is perpendicular to xy and passes through OZ. The plane of incidence perpendicular to xy, or its trace within this plane, is represented by

$$Dy = Ex. \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (20)$$

It is easily seen that this trace is perpendicular to the trace of that diametral plane which, with regard to the ellipsoid E, is conjugate to OZ. Indeed this plane is represented by

$$\frac{dE}{dz} \equiv Dx + Ey + Fz = 0,$$

and its trace within xy by

$$Dx + Ey = 0.$$

Each ray within the plane of incidence (20) is divided by double refraction into two, both confined within the same *vertical* plane of refraction. That is especially the case with regard to the ray incident at right angles; the corresponding plane of refraction, represented by (19), contains the incident ray and both the refracted rays.

13. Besides the vertical ray, there is in each plane of incidence one ray confined with both refracted rays within the same plane. After eliminating p and q between the general equations of the planes of incidence and of refraction,

$$qx = py,$$
$$(Ax + By + Dz)q = (Bx + Cy + Ez)p,$$

the following equation is obtained,

$$B(y^2 - x^2) + (A - C)xy + (Dy - Ex)z = 0, \quad . \; . \; . \; . \; . \; . \; . \quad (21)$$

representing a cone of the second degree, the locus of incident rays which are confined within their corresponding planes of refraction. This cone passes through the vertical OZ, and intersects xy within two right lines perpendicular to each other. These lines are congruent with the two axes of the ellipse

$$Ax^2 + 2Bx + Cy^2 = 1, \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (22)$$

along which the plane xy is intersected by the ellipsoid E. (That is instantly seen by putting B=0 [4].) Hence both rays, grazing the surface of the crystal along the axes of the ellipse (22), are confined with both corresponding refracted rays within the same plane.

If especially the crystal be cut in such a way that xy become a *circular* section of the ellipsoid E, each ray grazing the surface of the crystal will be contained within the corresponding plane of refraction. This plane therefore is easily obtained by means of the trace of the plane of incidence and the diameter OZ′ of the ellipsoid E conjugate to its circular section xy.

14. In the preceding numbers the plane of refraction has been determined without determining SS confined within it. This right line, passing through the infinitely distant pole of xy, is parallel to the diameter OZ′ conjugate to xy and represented by the equations (16), which by eliminating successively y and x may be replaced by the following ones,

$$\left.\begin{array}{l}(B^2-AC)x+(BE-CD)z=0,\\ (B^2-AC)y+(BD-AE)z=0.\end{array}\right\} \quad \dots\dots\dots\dots \quad (23)$$

The direction of SS being known, any one of its points, *i. e.* the pole of any plane passing through RR, will be sufficient to construct it. If the plane be parallel to the diameter just determined, its pole will fall within the plane xy, and may be also regarded as the pole of RR, with regard to the ellipse (22) along which this plane is intersected by E. The trace RR being represented by

$$qy+px=w,$$

where

$$w^2=1+p^2+q^2,$$

the two lines, the equations of which are

$$(Ax+By)\frac{w}{p}=1,$$

$$(Bx+Cy)\frac{w}{q}=1,$$

will meet in the pole mentioned. Hence, on denoting its coordinates by x^0 and y^0,

$$\left.\begin{array}{l}x^0=\dfrac{Bq-Cp}{B^2-AC}\cdot\dfrac{1}{w},\\[2ex] y^0=\dfrac{Bp-Aq}{B^2-AC}\cdot\dfrac{1}{w}.\end{array}\right\} \quad \dots\dots\dots\dots \quad (24)$$

Finally, the equations of SS thus obtained are

$$\frac{x-x^0}{CD-BE}=\frac{y-y^0}{AE-BD}=\frac{z}{B^2-AC}. \quad \dots\dots\dots \quad (25)$$

In order to complete the construction of the two refracted rays, the points (M, M′) in which SS meets the wave-surface Ω within the crystal are to be joined with O by means of two right lines OM and OM′.

15. If rays of every direction meet the crystal in O, the corresponding wave-fronts in that moment when, within the crystal, the wave-surface Ω is formed, will envelope a sphere,

$$x^2+y^2+z^2=1,$$

the radius of which is equal to unity. The locus of poles of the wave-fronts, if taken with regard to the ellipsoid E, is a new ellipsoid, which, referred to axes of coordinates directed along the axes of all auxiliary ellipsoids, is represented by the equation

$$\frac{x^2}{b^2c^2}+\frac{y^2}{a^2c^2}+\frac{z^2}{a^2b^2}=1,$$

or

$$a^2x^2+b^2y^2+c^2z^2=a^2b^2c^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

Its axes are obtained by multiplying the axes of the *second* auxiliary ellipsoid (8), to which it is similar, by *abc*.

16. The new fourth auxiliary ellipsoid (26) is fitted to connect the constructions of the refracted rays if, the section of the crystal remaining the same, the direction of the incident rays vary. Indeed a right line (MM′) drawn through any point Y of the fourth ellipsoid (26) parallel to OZ′, *i. e.* to the diameter conjugate to *xy* with regard to the third ellipsoid E, meets the wave-surface Ω, within the crystal, in two points M and M′. OM and OM′ will be the two refracted rays corresponding to that incident ray which is perpendicular to the plane conjugate to OY.

17. After this digression we resume our subject.

Let *xy* be the section of a biaxal crystal and OZ perpendicular to it. Let a ray of any direction starting from any point of OZ meet the section of the crystal in a point the coordinates of which are

$$x-\varrho, \qquad y=\sigma.$$

Let

$$\left.\begin{aligned} x&=pz+\varrho,\\ y&=qz+\sigma \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

be the equations of the incident ray. In order to express that this ray meets OZ we obtain the following relation,

$$\frac{p}{q}=\frac{\varrho}{\sigma}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

Let

$$\left.\begin{aligned} x&=rz+\varrho,\\ y&=sz+\sigma \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (29)$$

be the equations of any one of the two corresponding refracted rays. Let us finally suppose that, without the crystal, *z* is negative, within it, positive. Accordingly in the equations of the incident ray, positive values of *z*, in the equations of the refracted rays, negative ones are to be rejected.

Again, let

$$\Omega=0$$

be the general equation of the wave-surface, and

$$\mathrm{E}\equiv \mathrm{A}x^2+2\mathrm{B}xy+\mathrm{C}y^2+2\mathrm{D}xz+2\mathrm{E}yz+\mathrm{F}z^2-1=0$$

the equation of the third auxiliary ellipsoid; the position of both being determined by the position of the crystal with regard to the axes of coordinates.

18. According to the footnote of [7], we have between the four constants p, q, r, s, upon which the direction of the incident and the refracted ray depends, the following relation,

$$(Aq-Bp)r+(Bq-Cp)s+(Dq-Ep)=0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

By means of (28) this equation may be transformed into the following one,

$$(A\sigma-B\varrho)r+(B\sigma-C\varrho)s+(D\sigma-E\varrho)=0, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

and then represents a *complex of refracted rays.* As no supposition is made regarding the position of the luminous point on OZ, the corresponding incident rays may start in every direction from all its points. They constitute therefore a complex of rays emanating from OZ, perpendicular to the section of the crystal, and considered as a luminous right line. This complex of incident rays, after entering the crystal, passes into the complex of double refracted rays represented by the last equation.

19. By admitting that OX and OY, within the section of the crystal, were directed along the axes of the ellipse, along which xy is intersected by the ellipsoid E, the constant B disappears from the equation of the complex, which then may be written thus,

$$(Ar+D)\sigma=(Cs+E)\varrho. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (32)$$

We have hitherto supposed OZ to be perpendicular to xy, and will continue to do so for incident rays without the crystal; but for the refracted rays entering it (the axes OX, OY, perpendicular to each other, remaining the same) the direction of OZ may be changed by replacing it by the diameter OZ′ of the ellipsoid E, conjugate to xy. Then the constants D and E likewise disappear, and the equation of the complex assumes the most simple form,

$$Ar\sigma=Cs\varrho.$$

20. On denoting by a_0 and b_0 the two semiaxesof the ellipse along which xy is intersected by the ellipsoid E, we get

$$A=\frac{1}{a_0^2}, \quad B=\frac{1}{b_0^2}.$$

We may suppose, too, that a_0 falling within OX, is greater than b_0 falling within OY, whence the square of the excentricity of the ellipse e_0^2 becomes $\frac{a_0^2-b_0^2}{a_0^2}$.

After having introduced the new constants, the last equation may be written in the following ways,

$$\frac{\sigma}{s}=\frac{a_0^2}{b_0^2}\cdot\frac{\varrho}{r}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (33)$$

$$\frac{r\sigma-s\varrho}{r}=e_0^2.\sigma, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (34)$$

$$\frac{s\varrho-r\sigma}{s}=-\frac{a_0^2-b_0^2}{b_0^2}.\varrho. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (35)$$

Besides, on observing that $\frac{\sigma}{\varrho}=\frac{q}{p}$,

$$\frac{s}{r}=\frac{b_0^2}{a_0^2}.\frac{q}{p} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (36)$$

In order to get a geometrical interpretation of these equations, let any refracted ray of the complex be projected in the ordinary way on the three planes of coordinates XY, XZ and YZ; each axis of coordinates will be met by two of the three projections. The intercepts on OZ′ are $\frac{\sigma}{s}$ and $\frac{\varrho}{r}$; on OY, σ and $\frac{r\sigma-s\varrho}{r}$; on OX, ϱ and $\frac{s\varrho-r\sigma}{s}$. Hence

With regard to all rays of the complex, the two intercepts on each axis of coordinates are in the same ratio.

For OZ′, *i. e.* for the diameter of the ellipsoid E conjugate to the section of the crystal, this ratio is the ratio of the squares of the axes of the ellipse within this plane. For OY, *i. e.* for the shorter axis of this ellipse, it is equal to the square of its excentricity; for OX the greater axis equal to $\left(-e_0^2 . \frac{a_0^2}{b_0^2}\right)$.

Finally, if any incident ray, without, be projected on the section xy of the crystal along OZ, *i. e.* perpendicularly, and one of the two corresponding refracted rays, within the crystal, along OZ′, the projections thus obtained are the traces of the planes of incidence and of refraction, $\frac{q}{r}$ and $\frac{s}{r}$ indicating the trigonometrical tangents of the angles, between the two traces and the greater axis of the ellipse within the section xy. *The ratio of the tangents is equal to the ratio of the squares of the axes of the ellipse.*

21. In order to get a general idea of the distribution of the refracted rays constituting the complex, we may determine first the cone formed by rays passing through any given point within the crystal. If M be this point and x_0, y_0, z'_0 its coordinates, the equations

$$\left.\begin{array}{l} x_0=rz'_0+\varrho, \\ y_0=sz'_0+\sigma, \end{array}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (37)$$

are to be combined with the equation of the complex, which, on putting $\frac{b_0}{a_0}=\beta$, may be written thus,

$$s\varrho=\beta^2 r\sigma. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (38)$$

By eliminating ϱ and σ, we get

$$x_0 s-\beta^2 y_0 r=(1-\beta^2)z'_0 rs. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (39)$$

This equation shows that the locus of rays of the complex which pass through the point M is a cone of the second degree. Its equation in ordinary coordinates x, y, z' (z' being referred to OZ′) is

$$x_0(y-y_0)(z'-z'_0)-\beta^2 y_0(x-x_0)(z'-z'_0)=(1-\beta^2)z'_0(x-x_0)(y-y_0), \quad . \quad . \quad . \quad (40)$$

From this equation we immediately derive that, whatever may be the position of M within the crystal, the cone always contains three rays parallel to OX, OY, OZ′, as well as a fourth ray passing through the origin O. Besides, the cone depends upon the only constant β, the ratio of the two axes of the ellipse, here represented by

$$\frac{x^2}{a_0^2}+\frac{y^2}{b_0^2}=1, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (41)$$

along which xy is intersected by the third auxiliary ellipsoid E.

The equation (39), only depending upon the ratio of the constants x_0, y_0, z_0, shows that the cone in question of double refracted rays *is not at all altered if its centre moves along a right line passing through the origin* O.

22. In the peculiar case where M lies within the section of the crystal xy all corresponding incident rays likewise meet in that same point, constituting the plane of incidence passing through OZ, and represented by

$$y'x=x'y.$$

Here the cone of refracted rays degenerates into a system of two planes, which after putting $z'_0=0$, are represented by

$$\left.\begin{aligned} z'&=0, \\ x_0(y-y_0)&=\beta^2y_0(x-x_0). \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (42)$$

The second of these equations represents the plane of refraction corresponding to the plane of incidence*.

23. If M fall within one of both the other planes of coordinates XZ and YZ, the cone of double refracted rays likewise degenerates into two planes.

24. Either by putting $z'=0$ in (40), or, after having eliminated r and s between the three equations (37) and (38), by replacing the remaining variables ϱ and σ by x and y, we obtain

$$y_0x-\beta^2x^0y=(1-\beta^2)xy. \quad \ldots\ldots\ldots\ldots \quad (43)$$

This equation represents, within xy, the trace of the cone of refracted rays which meet in M. It is an equilateral hyperbola, having its asymptotes parallel to OX and OY, and passing through the projection of M. The coordinates of its centre are

$$y=\frac{y_0}{1-\beta^2}\ x=-\frac{\beta^2x_0}{1-\beta^2},$$

whence

$$\frac{y}{x}=-\frac{1}{\beta^2}\frac{y_0}{x_0}.$$

As the equation (43) does not involve the constant z'_0, we conclude that

The cone of double refracted rays continually changes if its centre be moved along a right line parallel to OZ′, *but its trace within the section of the crystal always remains the same hyperbola.*

25. Secondly, we may determine the curve enveloped by refracted rays confined within any given plane. If the plane be

$$tx+uy+vz+w=0,$$

---

[*] In the present researches, the auxiliary ellipsoid E, which may be considered as described round any point of the section of the crystal, as well as the wave-surface itself, has no other signification than to indicate by its constants the molecular constitution of the crystal so far as the transmission of luminous vibrations is concerned. Our equations only containing the ratio of these constants, the ellipsoid E and its elliptical trace (41) may be supposed here to have any dimensions whatever.

The last equation (42) represents the plane of refraction as it represents its trace within xy. It likewise represents, if the point M falls within the circumference of the ellipse (41), the normal to that curve in the point M. Hence is derived an elegant construction of the plane of refraction.

If within xy round any point of incidence as centre the ellipse (41) be described, the traces of the planes, both of incidence and of refraction, are such two diameters of that ellipse, the second of which is parallel to the normal to it at the point where the first intersects it.

the equation of this curve will result from the combination of the equation of the complex

$$s\varrho=\beta^2 r\sigma \quad . \quad . \quad . \quad . \quad . \quad (38)$$

with the two equations

$$tr+us+v=0,$$

$$t\varrho+u\sigma+w=0,$$

expressing that a ray (r, s, ϱ, σ) falls within that plane. By eliminating r and ϱ, we obtain

$$ws-\beta^2 v\sigma+(1-\beta^2)us\sigma=0, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (44)$$

$\frac{1}{\sigma}$ and $\left(-\frac{s}{\sigma}\right)$ being the coordinates of the projection, within xy', of the refracted ray. The projection envelopes an hyperbola; so does the ray itself within the given plane. The last equation (44) does not contain t, and therefore will not be altered if the given plane turns round its trace within YZ′, represented by

$$uy+vz'+w=0.. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (45)$$

Hence it follows that the projections of all refracted rays which meet that trace are tangents to the same hyperbola (44), the asymptotes of which are parallel to OY and OZ′, and which especially is touched by the trace itself, with regard to which

$$\sigma=-\frac{w}{u}, \quad \frac{\sigma}{s}=-\frac{w}{v}.$$

The refracted rays themselves are tangents to a hyperbolic cylinder having as base the hyperbola (44) and OX as axis.

26. In order to particularize, let us, in the first instance, suppose that the trace (45) is parallel to OZ′ and intersects OY in any point Q, OQ being equal to $\left(-\frac{w}{u}\right)$. Then v being equal to zero, the equation (44) becomes

$$\left(w+(1-\beta^2)u\right)s=0,$$

indicating that the hyperbola of the general case degenerates into two points, falling within OY, one at an infinite distance, while the distance of the other (Q′) from O is

$$OQ'=\sigma=-\frac{1}{1-\beta^2}\frac{w}{u}=\frac{1}{1-\beta^2}OQ. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (46)$$

Accordingly the hyperbolic cylinder degenerates into two right lines, met by all refracted rays. One of the two lines within the plane xy along which the crystal is cut is parallel to OX, and intersects OY in Q′, the other is infinitely distant. Hence all rays within a plane intersecting xz' along a trace (QZ'_0) parallel to OZ′ are divided into two sets. The rays of one set being parallel to the plane xy may be here omitted. The rays of the other set meet in a fixed point of that same plane along which the crystal is cut. If the plane turns round its trace QZ'_0 the fixed point moves, within xy, parallel to OX, describing a right line $Q'X_0$. Each ray meeting both right lines QZ'_0 and $Q'X_0$ is a ray of the complex.

27. If, in the second instance, the trace (45) is parallel to OY and intersects OZ′ in R, OR being equal to $\left(-\frac{w}{v}\right)$, the equation (44) becomes

$$ws=\beta^2 v\sigma,$$

representing a point of OZ′, the distance of which from O is

$$\mathrm{OR}'=-\frac{\sigma}{s}=-\frac{1}{\beta^2}\frac{w}{v}=\frac{1}{\beta^2}\,\mathrm{OR}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (47)$$

The hyperbolic cylinder therefore degenerates into a right line (RX_0) within xz' parallel to OX and passing through R′. Hence

All refracted rays of the complex confined within a plane intersecting yz' along a trace (RY_0) parallel to OY converge into a fixed point of the plane xz'. If the plane turns round its trace, that point describes, within xz', a right line RX_0 parallel to OX. Each ray meeting both lines RY_0 and $\mathrm{R'X}_0$ is a ray of the complex.

28. The axes of coordinates OX and OY may be interchanged by writing a_0 instead of b_0, and reciprocally. Then we get analogous results if, instead of traces within YZ′, we consider traces within XZ′. Especially we may immediately conclude from the last equation written thus,

$$b_0^2\,.\,\mathrm{OR}'=a_0^2\,.\,\mathrm{OR}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (48)$$

that the relation between the two right lines $\mathrm{R'X}_0$ and RY_0 is a mutual one.

29. All rays intersecting two fixed right lines constitute a *linear congruency*, the fixed right lines being its directrices (Sect. I., 55). Consequently *the complex of refracted rays may be generated in three different ways by a variable linear congruency.* In each case the two directrices of the congruency move parallel to any two of the three axes of coordinates OX, OY, OZ′, intersecting the third axis in two points, the distances of which from O are in a given ratio.

30. Hitherto we have supposed that the plane xy is any section whatever of the crystal. Let us now, in particularizing again, admit that the crystal is cut along one of the two circular sections of the third auxiliary ellipsoid E, then represented by

$$\mathrm{A}(x^2+y^2)+\mathrm{F}z^2=1;$$

β being equal to unity, the equation of the complex becomes

$$r\sigma=s\varrho. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (49)$$

In this peculiar case therefore all rays of the complex meet the diameter OZ′, conjugate with regard to E to its circular section xy. Hence *all refracted rays of the complex intersect* OZ′ *as all corresponding incident rays start from* OZ.

Both the diameter of the third auxiliary ellipsoid E perpendicular to its circular section xy, and its diameter conjugate to that section, fall within a principal section of the ellipsoid containing its greatest and least axis, and consequently also its two optic axes. The rectangular axes of coordinates OX and OY may, without changing the equation of the complex, turn round O within the section xy. If one of them, OX for instance, become the vertical projection of OZ', the plane xz' is a *principal plane* of the ellipsoid E, containing the two optic axes, and OY the mean axis of the ellipsoid E.

31. If the plane xy is a principal section of the third auxiliary ellipsoid E (and therefore of all auxiliary ellipsoids), the axis OZ', becoming perpendicular to xy, is congruent with OZ. Then the equation of the ellipsoid E, referred to rectangular coordinates, becomes

$$\frac{x^2}{bc}+\frac{y^2}{ac}+\frac{z^2}{ab}=1,$$

and may be written thus,

$$ax^2+by^2+cz^2=abc.$$

Hence the equation of the complex is

$$ar\sigma=bs\varrho. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (50)$$

If the crystal be turned round OY through an angle α, we get, after replacing x and z by

$$x\cos\alpha-z\sin\alpha,$$

$$x\sin\alpha+z\cos\alpha,$$

the following equation of the ellipsoid E,

$$(a\cos^2\alpha+c\sin^2\alpha)x^2+by^2-2(a-c)\sin\alpha\cos\alpha\,.\,xz+(a\sin^2\alpha+c\cos^2\alpha)z^2=abc. \quad . \quad (51)$$

The axes of the elliptic trace within xy being always directed along OY and OX, the equation of the complex assumes the form of the equation (32), which, after putting $E=0$ and

$$A:C:D=(a\cos^2\alpha-c\sin^2\alpha):b:-(a-c)\sin\alpha\cos\alpha,$$

passes into the following one,

$$(a\cos^2\alpha-c\sin^2\alpha)r\sigma-bs\varrho-(a-c)\sin\alpha\cos\alpha\,.\,\sigma=0. \quad . \quad . \quad . \quad . \quad (52)$$

32. The equations (51) and (52) of the last number belong to the case in which one of the three axes of elasticity, OY, falls within the section of the crystal. The two remaining axes of elasticity are confined within the plane XZ, where one of them, corresponding to C, makes with OZ an angle α, this angle being counted towards OX.

The two equations may be regarded as representing the *general* case of *uniaxal crystals* cut along any plane whatever. Indeed let OC be the single optic axis making with the normal to the section xy of the crystal any angle α. Draw through OC the plane xz perpendicular to xy, and OY perpendicular to that plane. The rectangular system of coordinates being thus determined, the equations (51) and (52), after having replaced c by a, will belong to uniaxal crystals.

33. If the optic axis of an uniaxal crystal falls within the section xy, the equation of the complex, on putting $\alpha=\frac{1}{2}\pi$, becomes

$$cr\sigma=as\varrho.$$

In the case of uniaxal crystals, each plane passing through the optic axis may be regarded as a principal section of the ellipsoid E. Therefore the equation of the complex assumes the form of the equation (50); the form of the two equations being the same as in the general case, where the direction of the third axis is oblique to xy.

If in the case of uniaxal crystals the circular section of E is congruent with the section xy of the crystal, we get in order to represent the complex of double refracted rays, on putting $\alpha=0$, the following equation,

$$rs=s\varrho,$$

indicating that the plane of refraction is congruent with the plane of incidence, or, in other terms, that both the ordinary and the extraordinary ray into which any incident ray, starting from OZ, is divided by double refraction, likewise meet OZ.

34. The preceding fragmentary researches on double refraction—only calculated to present a new and curious instance of a complex—may be concluded by a last remark.

All the results we have hitherto obtained, especially the determination of the complex of double refracted rays, only depend, 1st, upon the direction of the diameter of the ellipsoid E conjugate to the section of the crystal; 2ndly, upon the ratio of the axes of the elliptical trace along which the same ellipsoid meets that section. Here, therefore, the third auxiliary ellipsoid E,

$$ax^2+by^2+cz^2=abc,$$

may be replaced by the following one,

$$ax^2+by^2+cz^2=1,$$

which is similar to it. It is immediately seen that, along the different directions, the reciprocal values of optical elasticity within the crystal are indicated by the *radii vectores* of the new ellipsoid, as the squares of these values are represented by the *radii vectores* of the second auxiliary ellipsoid,

$$a^2x^2+b^2y^2+c^2z^2=1.$$

ADDITIONAL NOTE.

Received December 11, 1865.

I. *Coordinates of a right line.*

1. A right line, if considered as an axis round which a plane revolves, is determined by any two positions of the revolving plane; analytically, by means of two groups of plane-coordinates. If considered as a geometrical locus, described by a point, it is determined by any two positions of the moving point; analytically, by means of two groups of point-coordinates.

Let the plane- and point-coordinates

$$\frac{t}{w},\ \frac{u}{w},\ \frac{v}{w},\qquad \frac{x}{\varpi},\ \frac{y}{\varpi},\ \frac{z}{\varpi}$$

be such that

$$tx+uy+vz+w\varpi=0, \quad \ldots\ldots\ldots\ldots \quad (1)$$

which equation, if geometrically interpreted, indicates that each point $\left(\frac{x}{\varpi}, \frac{y}{\varpi}, \frac{z}{\varpi}\right)$ falls within each plane $\left(\frac{t}{w}, \frac{u}{w}, \frac{v}{w}\right)$, or, which is the same, that each plane $\left(\frac{t}{w}, \frac{u}{w}, \frac{v}{w}\right)$ passes through each point $\left(\frac{x}{\varpi}, \frac{y}{\varpi}, \frac{z}{\varpi}\right)$. I called such coordinates "associated plane- and point-coordinates"*, and here we shall make use of that denomination. By two couples of associated either plane- or point-coordinates,

$$\frac{t}{w}, \frac{u}{w}, \frac{v}{w}, \qquad \frac{t'}{w'}, \frac{u'}{w'}, \frac{v'}{w'},$$

$$\frac{x}{\varpi}, \frac{y}{\varpi}, \frac{z}{\varpi}, \qquad \frac{x'}{\varpi'}, \frac{y'}{\varpi'}, \frac{z'}{\varpi'},$$

the *same* right line is determined.

We may employ homogeneous instead of ordinary equations†; accordingly each group of three coordinates is replaced by a group of four:

$$t, u, v, w, \qquad t', u', v', w',$$

$$x, y, z, \varpi, \qquad x', y', z', \varpi'.$$

2. Both planes (t, u, v, w) and (t', u', v', w'), represented in point-coordinates by the equations

$$tx+uy+vz+w\varpi=0,$$

$$t'x+u'y+v'z+w'\varpi=0,$$

are arbitrarily chosen amongst those passing through the right line, and may be replaced by any two others, the equations of which have the form

$$(t+\mu t')x+(u+\mu u')y+(v+\mu v')z+(w+\mu w')\varpi=0,$$

where μ denotes any arbitrary coefficient. But the position of the right line with regard to the axes of coordinates OX, OY, OZ is not characteristically connected with such a plane, except in the case where the plane itself has a peculiar relation to the axes. There are four such cases: the plane may either pass through the origin, or project the right line on the three planes of coordinates. Accordingly, in putting

$$w+\mu w'=0, \quad v+\mu v'=0, \quad u+\mu u'=0, \quad t+\mu t'=0,$$

the last equation successively becomes

$$\left.\begin{aligned}
(tw'-t'w)x+(uw'-u'w)y+(vw'-v'w)z&=0,\\
(tv'-t'v)x+(uv'-u'v)y-(vw'-v'w)\varpi&=0,\\
(tu'-t'u)x-(uv'-u'v)z-(uw'-u'w)\varpi&=0,\\
-(tu'-t'u)y-(tv'-t'v)z-(tw'-t'w)\varpi&=0.
\end{aligned}\right\} \quad \ldots\ldots \quad (2)$$

---

* Geometrie des Raumes, No. 5.

† I first introduced homogeneous equations into analytical geometry, CRELLE's Journal, v. p. 1, 1830.

Any two of the four planes represented by these equations are sufficient to fix the position of the right line. They contain five constants, which by division may be reduced to four, the necessary number upon which the line depends. Besides the five constants in the two equations we meet a sixth one in both remaining equations. But the right line being determined by the former five, the sixth ought to be a function of them. The equation of condition, connecting the six constants, may, for instance, be obtained by adding the three last equations, after having multiplied the first of them by $-(tu'-t'u)$, the second by $(tv'-t'v)$, and the third by $-(uv'-u'v)$. Thus we obtain

$$(tu'-t'u)(vw'-v'w)-(tv'-t'v)(uw'-u'w)+(uv'-u'v)(tw'-t'w)=0. \quad . \quad . \quad . \quad (3)$$

The following six constants, taken with an arbitrary sign,

$$\pm(uv'-u'v), \quad \pm(tv'-t'v), \quad \pm(tu'-t'u), \quad \pm(tw'-t'w), \quad \pm(uw'-u'w), \quad \pm(vw'-v'w),$$

may be regarded *as the six coordinates of the right line.*

3. In quite a similar manner, when in order to fix the position of the right line we replace the two planes by the two points (x, y, z, ϖ) and (x', y', z', ϖ'), we get the following equations in plane coordinates,

$$\left.\begin{aligned} (x\varpi'-x'\varpi)t+(y\varpi'-y'\varpi)u+(z\varpi'-z'\varpi)v&=0,\\ (xz'-x'z)t+(yz'-y'z)u-(z\varpi'-z'\varpi)w&=0,\\ (xy'-x'y)t-(yz'-y'z)v-(y\varpi'-y'\varpi)w&=0,\\ -(xy'-x'y)u-(xz'-x'z)v-(x\varpi'-x'\varpi)w&=0, \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

representing four points, the first of which is at an infinite distance on the right line of which the position is to be determined, while the three others are the points in which that line meets the three planes of coordinates. Accordingly we may likewise regard the six constants of the last four equations, taken with an arbitrary sign,

$$\pm(x\varpi'-x'\varpi), \quad \pm(y\varpi'-y'\varpi), \quad \pm(z\varpi'-z'\varpi), \quad \pm(yz'-y'z), \quad \pm(xz'-x'z), \quad \pm(xy'-x'y),$$

as *the six coordinates of the right line.* These six coordinates are connected by the following equation of condition:

$$(xy'-x'y)(z\varpi'-z'\varpi)-(xz'-x'z)(y\varpi'-y'\varpi)+(yz'-y'z)(x\varpi'-x'\varpi)=0. \quad . \quad . \quad (5)$$

4. In denoting the distance of the right line from the origin of coordinates by δ, the angles with it makes with the three axes OX, OY, OZ by α, β, γ, and the angles which the normal to the plane passing through it and the origin makes with the same axes by λ, μ, ν, the following relations are obtained:

$$\begin{array}{lcccccccccccc} \text{I.} & & (uv'-u'v) & : & -(tv'-t'v) & : & (tu'-t'u) & : & (tw'-t'w) & : & (uw'-u'w) & : & (vw'-v'w) \\ \text{II.} & = & (x\varpi'-x'\varpi) & : & (y\varpi'-y'\varpi) & : & (z\varpi'-z'\varpi) & : & (yz'-y'z) & : & -(xz'-x'z) & : & (xy'-x'y) \\ \text{III.} & = & \cos\alpha & : & \cos\beta & : & \cos\gamma & : & \delta\cos\lambda & : & \delta\cos\mu & : & \delta\cos\nu. \end{array}$$

5. Hence we conclude that

$$\cos\alpha, \quad \cos\beta, \quad \cos\gamma, \quad \delta\cos\lambda, \quad \delta\cos\mu, \quad \delta\cos\nu$$

may likewise be regarded as line-coordinates. Here the equation of condition between the six coordinates becomes

$$\cos\alpha\,\cos\lambda+\cos\beta\cos\mu+\cos\gamma\,\cos\nu=0,$$

which, added to the two following ones,

$$\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1,$$

$$\cos^2\lambda+\cos^2\mu+\cos^2\nu=1,$$

reduces to four the number of constants upon which the position of the line depends.

6. The two sets of ratios I. and II. retain the same generality after putting $w=w'=\pm1$, $\varpi=\varpi'=\pm1$. If we suppose, again, that both planes and both points, by which the line is determined, are coincident, we get, choosing the under signs, two new sets of equal ratios,

$$\text{IV.}\quad =(udv-vdu):-(tdv-vdt):(tdu-udt):\quad dt\quad:\quad du\quad:\quad dv$$

$$\text{V.}\quad =\quad dx\quad:\quad dy\quad.\quad dz\quad:(ydz-zdy):-(xdz-zdx):(xdy-ydx).$$

Thus we obtain two systems of differential coordinates, dx, dy, dz indicating the direction of the line, dt, du, dv the direction of the normal to the plane passing through it and the origin of coordinates. We may regard x, y, z, t, u, v as functions of time.

7. We can represent the direction of a *force* by the right line, and its intensity by the distance of the two points by which the position of the line is fixed. In denominating the projections of the force on OX, OY, OZ by X, Y, Z, and the projections of its moment with regard to the origin on YZ, XZ, XY by L, M, N, we obtain the following new set of equal ratios:

$$\text{VI.}\quad =\mathrm{X}:\mathrm{Y}:\mathrm{Z}:\mathrm{L}:\mathrm{M}:\mathrm{N}.$$

Therefore X, Y, Z, L, M, N may also be considered as six line-coordinates. The equation of condition between them becomes

$$\mathrm{XL}+\mathrm{YM}+\mathrm{ZN}=0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

8. The six coordinates of each system range into two groups of three, to each coordinate of one group corresponds one of the other. By exchanging the three axes of coordinates, the three couples of corresponding coordinates are exchanged, both groups remaining the same.

We may, in order to pass from the six coordinates of a right line to its five absolute coordinates, divide any five of them by the sixth. Here we meet two cases, in dividing either by a coordinate of the first or the second group.

9. Let us divide the first two and the three last terms of the ratios I. by the third $(tu'-t'u)$. In putting

$$\frac{uv'-u'v}{tu'-t'u}=r,\quad -\frac{tv'-t'v}{tu'-t'u}=s,\quad \frac{tw'-t'w}{tu'-t'u}=-\sigma,\quad \frac{uw'-u'w}{tu'-t'u}=\varrho,\quad \frac{vw'-v'w}{tu'-t'u}=\eta,$$

where, according to the equation of condition (3),

$$\eta=r\sigma-s\varrho,$$

r, s, $(-\sigma)$, ϱ, and η will be the *five* absolute coordinates of the right line. The last two of the four equations (2), representing the planes projecting the right line on the planes XZ and YZ, as well as the projections themselves, may now be written thus,

$$x = rz + \varrho,$$
$$y = sz + \sigma,$$

r and s being the trigonometrical tangents of the angles made by the two projections with the axis OZ, ϱ and σ the segments intercepted by them on the axes OX and OY.

Again, let us divide the first five terms of the set of ratios II. by the sixth $(xy'-x'y)$. In putting

$$\frac{x\varpi'-x'\varpi}{xy'-x'y} = -\varkappa, \qquad \frac{y\varpi'-y'\varpi}{xy'-x'y} = \pi, \qquad \frac{z\varpi'-z'\varpi}{xy'-x'y} = \zeta,$$
$$\frac{yz'-y'z}{xy'-x'y} = p, \qquad -\frac{xz'-x'z}{xy'-x'y} = q,$$

where, according to the equation of condition (5),

$$\zeta = p\varkappa - q\pi,$$

p, q, $(-\varkappa)$, π, and ζ will be the *five* new coordinates. We meet four of them in the last two of the four equations (4), representing the two points where the planes XZ and YZ are intersected by the right line. These equations assume the following form,

$$t = pv + \pi w,$$
$$u = qv + \varkappa w,$$

and may, in denoting the coordinates of the points within their planes by x_y, z_y, and y_x, z_x, be written thus,

$$x_y t + z_y v + w = 0,$$
$$y_x u + z_x v + w = 0;$$

whence

$$p = -\frac{z_y}{x_y}, \quad \pi = -\frac{1}{x_y}, \quad q = -\frac{z_x}{y_x}, \quad \varkappa = -\frac{1}{y_x}.$$

We may add to the former six sets of equal ratios the two following:

$$\text{VII.} \quad = r : s : \quad 1 \quad : (-\sigma) : \varrho : \eta (\equiv r\sigma - s\varrho).$$
$$\text{VIII.} \quad = -\varkappa : \pi : \zeta(\equiv p\varkappa - q\pi) : \quad p \quad : q : \quad 1.$$

10. We have thus obtained eight different systems of line-coordinates, the coordinates being the six terms of each of the eight sets of equal ratios I. to VIII. In changing the position of the origin and the direction of the axes of coordinates, the coordinates of each system are changed. But I do not here transcribe the formulæ of transformation of line-coordinates, observing only that these formulæ may be immediately transferred from one system to any other.

II. *Complexes. Congruencies. Surfaces generated by a moving right line. Developable surfaces and curves of double curvature.*

11. A homogeneous equation between any six line-coordinates is said to represent the *complex* of those lines the coordinates of which verify that equation. According to the identity of ratios I. to VIII., the following equations,

$$\begin{aligned}
&\mathrm{F}[(uv'-u'v),\ -(tv'-t'v),\ (tu'-t'u),\ (tw'-t'w),\ (uw'-u'w),\ (vw'-v'w)]=0,\\
&\mathrm{F}[(x\varpi'-x'\varpi),\ (y\varpi'-y'\varpi),\ (z\varpi'-z'\varpi),\ (yz'-y'z),\ -(xz'-x'z),\ (xy'-x'y)]=0,\\
&\mathrm{F}[\cos\alpha,\ \cos\beta,\ \cos\gamma,\ \delta\cos\lambda,\ \delta\cos\mu,\ \delta\cos\nu]=0,\\
&\mathrm{F}[(udv-vdu),\ -(tdv-vdt),\ (tdu-udt),\ dt,\ du,\ dv]=0,\\
&\mathrm{F}[dx,\ dy,\ dz,\ (ydz-zdy),\ -(xdz-zdx),\ (xdy-ydx)]=0,\\
&\mathrm{F}[\mathrm{X},\ \mathrm{Y},\ \mathrm{Z},\ \mathrm{L},\ \mathrm{M},\ \mathrm{N}]=0,\\
&\mathrm{F}[r,\ s,\ 1,\ (-\sigma),\ \varrho,\ \eta]=0,\\
&\mathrm{F}[(-\varkappa),\ \pi,\ \zeta,\ p,\ q,\ 1]=0,
\end{aligned}$$

represent the same complex; F being supposed to indicate always the same homogeneous function of the different groups of line-coordinates. The *complex* is said to be of the *n*th *degree*, and represented by Ω_n if its equations are of that degree.

12. Starting from the first equation,

$$\Omega_n\equiv\mathrm{F}[(uv'-u'v),\ -(tv'-t'v),\ (tu'-t'u),\ (tw'-t'w),\ (uw'-u'w),\ (vw'-v'w)]=0, \quad . \quad (1)$$

t, u, v, w and t', u', v', w' are to be referred to any two planes passing through any line of the *complex.* Let one of the two planes (t', u', v', w') be any given one. Then the last equation, in regarding t', u', v', w' as constant and t, u, v, w as variable, represents within the given plane a *curve* enveloped by tangent-planes (t, u, v, w). The lines of the *complex*, confined within the plane, also envelope the same curve, the class of which is the same as the degree of the *complex*. Hence

A complex Ω_n of the n*th degree being given, in each plane traversing space there is a curve of the* n*th class enveloped by lines of the complex.*

The equations of such curves fully agree with the general equation of the *complex* itself. We have only to consider in this equation t', u', v', w' as constant in referring them to the given plane, while t, u, v, w are regarded as variable plane-coordinates.

If $n=1$, the curve in each plane is replaced by a point; each line within the plane passing through that point belongs to the linear complex.

If $n=2$, the curves enveloped are conics, which may degenerate into systems of two real or imaginary points.

13. If, in the second equation of the same *complex*,

$$\lambda\Omega_n\equiv\mathrm{F}[(x-x'),\ (y-y'),\ (z-z'),\ (yz'-y'z),\ -(xz'-x'z),\ (xy'-x'y)]=0, \quad . \quad (2)$$

where we put $\varpi'=\varpi=1$, and λ denotes a constant, x', y', z' are referred to any given point in space and therefore regarded as constant, while x, y, z are the variable coordinates of the points of any line of the *complex*, that equation represents a *cone* of the nth order, the geometrical locus of lines of the complex passing through the given point. Hence

A complex of the n*th degree being given, each point of space is the centre of a cone of the* n*th order into which lines of the complex converge.*

In linear complexes the lines meeting in a given point constitute a plane. If $n=2$, the cones are of the second order, and may degenerate into two real or imaginary planes.

14. The right lines constituting a complex Ω_n may be distributed either within planes traversing space, or according to points into which they converge. We hitherto considered Ω_n as a *complex of right lines*, the number of which is ∞^3. We may as well regard it either as a complex of curves, or as a complex of cones, the number both of curves and cones being ∞^2. Therefore we may say that

$$\Omega_n=0$$

represents at the same time as well in each plane a curve of the n*th class as cones of the* n*th order having each point of space as centre.*

The curve in a plane revolving round a given line, or moving parallel to itself, generates a surface. The cone the centre of which describes a given right line envelopes a surface. The number of surfaces both generated by the curve and enveloped by cones is ∞. There is one of each kind of surfaces corresponding to any given line, all surfaces will be exhausted if that line turns in all directions round any of its points. Accordingly we may likewise consider Ω_n as a complex of surfaces, either described by curves or enveloped by cones.

15. In denoting by μ any constant coefficient,

$$\Omega_n+\mu\Omega_m=0 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (3)$$

represents an infinite number of complexes. The lines congruent in any two of them belong simultaneously to all. All these congruent lines constitute a *congruency* (Ω_n, Ω_m), which we say is represented by the equations of the two complexes.

Each plane traversing space confines a curve of each of the two complexes, the mn tangents common to both curves belong to the *congruency*. All curves within the same plane belonging to the different complexes (3) which pass through the congruency, touch the same mn of its lines. Again, each point is the centre of a cone belonging to the different complexes (3). All such cones meet along the same mn lines, likewise belonging to the *congruency*. Therefore in a congruency (Ω_n, Ω_m) *there are* mn *lines confined within each plane as there are* mn *lines passing through each point.* The number of lines constituting a congruency is ∞^2.

If $m=1$, there are in each plane n lines of the *congruency* (Ω_n, Ω_1) passing through the same point, as n of its lines converging into each point fall within the same plane; plane and point corresponding to each other.

16. In denoting by μ and ν any two constant coefficients,

$$\Omega \equiv \Omega' + \mu\Omega'' + \nu\Omega''' = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

represents an infinite number (∞^2) of complexes. All these complexes meet along the lines which simultaneously belong to any three of them, especially to

$$\Omega' = 0, \quad \Omega'' = 0, \quad \Omega''' = 0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

By means of these equations the position of such a line is determined, after having arbitrarily assumed the value of one of the four constants upon which the line depends; in other terms, three of these four constants are functions of the fourth, varying each by an infinitely small quantity if this one does. Hence we conclude that a line the coordinates of which verify the three equations (5), generates a *surface* in passing successively into all its positions. *This surface* (Ω', Ω'', Ω''') *is said to be represented by the system of the three equations* (5).

17. Any point of space being given, there are three cones described by lines which belong to the three complexes (5) and pass through the given point. Generally the three cones (11) do not intersect along the same line. In certain positions only of the point they do. In this case their common intersection belongs to the *surface* (Ω', Ω'', Ω'''), and therefore the point itself also.

Put

$$\left.\begin{aligned} \lambda'\,\Omega' &\equiv F'\,[(x-x'), (y-y'), (z-z'), (yz'-y'z), -(xz'-x'z), (xy'-x'y)]=0, \\ \lambda''\,\Omega'' &\equiv F''\,[(x-x'), (y-y'), (z-z'), (yz'-y'z), -(xz'-x'z), (xy'-x'y)]=0, \\ \lambda'''\Omega''' &\equiv F'''[(x-x'), (y-y'), (z-z'), (yz'-y'z), -(xz'-x'z), (xy'-x'y)]=0. \end{aligned}\right\} \quad . \quad (6)$$

If x', y', z' are referred to any arbitrary point, and x, y, z regarded as variable, these equations represent the three cones, $(x'y'z')$ being their common centre, and their generating lines belonging to the three complexes (5). Without changing the conditions of mutual intersection, the three cones may be moved parallel to themselves till the origin of coordinates becomes their common centre. After that displacement their equations are transformed into the following ones:

$$\left.\begin{aligned} &F'\,[x, y, z, (yz'-y'z), -(xz'-x'z), (xy'-x'y)]=0, \\ &F''\,[x, y, z, (yz'-y'z), -(xz'-x'z), (xy'-x'y)]=0, \\ &F'''[x, y, z, (yz'-y'z), -(xz'-x'z), (xy'-x'y)]=0. \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (7)$$

These equations being homogeneous with regard to (x, y, z), will, in the general case, not be simultaneously verified by the three variables. In order to express that they subsist simultaneously, we obtain, after having eliminated x, y, z,

$$\varphi(x', y', z') = 0, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

φ indicating a function which involves the primitive constants of the three complexes (5). This function might be rendered homogeneous by introducing ϖ'. This equation, in regarding the coordinates as variable, represents in ordinary point-coordinates the *surface* which in line-coordinates is represented by the system of the three equations (5).

18. Likewise there are in each plane traversing space three curves enveloped by lines of the three complexes Ω', Ω'', Ω'''. In the general case these curves have no common tangent. In certain positions of the plane they have, and then the common tangent belongs to the *surface* (Ω', Ω'', Ω'''). Reciprocally, within a plane passing through any generating line of the *surface*, the curves enveloped by the lines of any complex Ω touch the generating line, and continue to do so if the plane revolves round it. The plane in each of its positions is a *tangent-plane* of the *surface*.

Put

$$\left.\begin{aligned}\Omega' &\equiv \mathrm{F}'\,[(uv'-u'v),\ -(tv'-t'v),\ (tu'-t'u),\ (t-t'),\ (u-u'),\ (v-v')]=0,\\ \Omega'' &\equiv \mathrm{F}''[(uv'-u'v),\ -(tv'-t'v),\ (tu'-t'u),\ (t-t'),\ (u-u'),\ (v-v')]=0,\\ \Omega''' &\equiv \mathrm{F}'''[(uv'-u'v),\ -(tv'-t'v),\ (tu'-t'u),\ (t-t'),\ (u-u'),\ (v-v')]=0.\end{aligned}\right\} \quad . \quad (9)$$

In regarding t, u, v as variable plane-coordinates, and referring t', u', v' to the traversing plane, these equations represent, within that plane, the three curves enveloped by lines of the three complexes Ω', Ω'', Ω'''. On this account they may be reduced to equations between two variables only, and therefore will not, in the general case, be verified by any values of the three variables reduced to two. By eliminating the variables between the last three equations, an equation,

$$\psi(t',\ u',\ v')=0, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

will be obtained, which, if t', u', v' are regarded as variable, represents in plane-coordinates the *surface* (Ω', Ω'', Ω''').

19. In order to derive the equations (9) from the equations (6) (both systems of equations representing the same *surface*), we may first pass from (6) to the three new equations,

$$\begin{aligned}&\mathrm{F}'\,[(yz'-y'z),\ -(xz'-x'z),\ (xy'-x'y),\ (x-x'),\ (y-y'),\ (z-z')]=0,\\ &\mathrm{F}''[(yz'-y'z),\ -(xz'-x'z),\ (xy'-x'y),\ (x-x'),\ (y-y'),\ (z-z')]=0,\\ &\mathrm{F}'''[(yz'-y'z),\ -(xz'-x'z),\ (xy'-x'y),\ (x-x'),\ (y-y'),\ (z-z')]=0,\end{aligned}$$

and then replace x, y, z, x', y', z' by t, u, v, t', u', v'. The last equations are likewise obtained by merely exchanging amongst themselves the constant coefficients in each of the three equations (6). The way of exchanging is obvious. Hence, in considering that the equation (10) is derived exactly by the same algebraical operations from (9) as (8) from (7), we may conclude that (10) may be derived from (8) by a mere exchange of constants and a substitution of plane- for point-coordinates.

20. In a congruency (Ω_n, Ω_m) there are mn lines meeting in a given point. Two, three, four of these lines may coincide. In this case the cones of both complexes Ω_n and Ω_m, the common centre of which is the given point, are tangent one to another, or osculate each other along the double or multiple line. In order to get the analy-

tical expression of these new conditions, we may, as we did before, replace both cones by such as have the origin as centre. In putting

$$\frac{x}{z}=p, \quad \frac{y}{z}=q,$$

the equations of these new cones may be written thus (No. 17),

$$\left.\begin{aligned} f(p, q, x', y', z')=0,\\ f'(p, q, x', y', z')=0,\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

f and f' representing two functions of the variables p and q, by means of which the lines constituting the two cones are determined, x', y', z' being the coordinates of the given point. If two of the *mn* intersecting lines of the two cones are coincident along any right line (p, q), we get for the determination of that line, besides the two equations (11), the following new one,

$$\frac{df}{dq} : \frac{df}{dp} = \frac{df'}{dq} : \frac{df'}{dp},$$

which, if expanded, likewise assumes the form

$$f''(p, q, x', y', z')=0, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

f'' indicating a new function. By eliminating p and q between the three equations (11) and (12), we get an equation of the form

$$\psi(x', y', z')=0, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

representing, if x', y', z' be regarded as variable, a *developable surface*, the locus of those points through which *double lines of the congruency* pass, or, in other terms, the locus of the double lines themselves.

In supposing that *three* intersecting lines of the two cones (11) fall within the same line (p, q), the following new equation of condition is obtained

$$\frac{\frac{d^2f}{dp^2}\left(\frac{df}{dq}\right)^2-2\frac{d^2f}{dpdq}\cdot\frac{df}{dq}\cdot\frac{df}{dp}+\frac{d^2f}{dq^2}\left(\frac{df}{dp}\right)^2}{\frac{d^2f'}{dp^2}\left(\frac{df'}{dq}\right)^2-2\frac{d^2f'}{dpdq}\,\frac{df'}{dq}\cdot\frac{df'}{dp}+\frac{d^2f'}{dq^2}\left(\frac{df'}{dp}\right)^2}=\frac{\frac{df}{dp}}{\frac{df'}{dp}}=\frac{\frac{df}{dq}}{\frac{df'}{dq}},$$

which again may be expanded into an equation of the form

$$f'''(p, q, x', y', z')=0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

This equation, combined with the three former equations (11) and (12), furnishes a new equation of condition,

$$\psi(x', y', z')=0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

The system of the two equations (13) and (15) gives, as locus of points through which *triple* lines of the congruency pass, a curve of *double curvature.*

In pursuing the same course a new equation of the same form as (13) and (15) is obtained, which, combined with these, indicates that there is a certain number of points into which quadruple lines of the congruency converge.

In congruencies of a peculiar description only we meet quintuple lines.

21. In quite the same manner we may determine the position of planes within which two, three, four of the mn lines of the congruency (Ω_n, Ω_m) coincide. In that case both curves within the plane, enveloped by lines of the complexes Ω_n and Ω_m, touch or osculate one another on a common tangent.

In operating on the first two equations (9) as we did on the first two equations (6), we get, in order to represent in plane-coordinates the locus enveloped by planes confining a double line of the congruency, the following equation,

$$\psi(t, u, v)=0, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

which, as the remarks of No. 19 here likewise hold, is derived by a mere exchange of constants from (10). Each plane passing through a double line being an enveloping tangent plane of the represented surface, this surface degenerates into a *curve of double curvature.*

Another equation may be derived from (15) in the same way. Let it be

$$\psi'(t, u, v)=0, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

the system of the two equations (16) and (17) representing a *developable surface*, the tangent planes of which confine the triple lines of the congruency. Finally, there are certain tangent planes of the *developable surface* which confine the quadruple lines of the congruency. These planes, as well as the points of the curve of double curvature through which the quadruple lines pass, are determined by associated plane- and point-coordinates, both being functions of the constants of the congruency, and are obtained one from another by the above-mentioned exchange of these constants.

22. The double lines of a congruency constitute a *surface*, degenerated into a *developable* one, as they envelope a surface, degenerated into a *curve of double curvature.* The *developable surface* is represented in point-coordinates by a single equation (13), in plane-coordinates by the system of two equations (16) and (17). *The curve of double curvature* is represented in plane-coordinates by a single equation (16), in point-coordinates by the system of two equations (13) and (15). The tangent-planes of the *surface*, confining triple lines of the congruency, osculate the curve; the points of the *curve*, through which these triple lines pass, are osculating points of the *surface*, in which three consecutive tangent planes meet. The *curve*, in certain points where the tangent is an osculating one, is osculated by a plane in four points. Through such a point pass four consecutive tangent planes of the *surface*, the common intersection of which is a line of inflexion of the developable *surface.* The quadruple lines of the congruency pass through such points, and are confined within such planes*.

---

* In two remarkable papers "On a New Analytical Representation of Curves in Space," published in the third and fifth volume of the Quarterly Journal of Mathematics, Professor CAYLEY employed before me, in order to represent cones, the six coordinates of a right line, depending upon any two of its points. Having lately

III. *On a new System of Coordinates.*

23. We have hitherto determined the position of a right line in space in making use of the ordinary system of three axes OX, OY, OZ intersecting each other. The new question is whether we may substitute for this system another, by means of which we are enabled to fix immediately the position of a right line without recurring to points and planes.

In the ordinary system of coordinates, (1) the position of a point is determined by means of three planes parallel to the planes of coordinates and meeting in that point, (2) the position of a plane by a linear equation between the three coordinates of a point, regarded as variable; both point and plane depending upon three constants.

In an analogous way a right line is determined by the intersection of four linear complexes. Such a linear complex depends upon the position of its axis and a constant (paper presented, No. 29). A right line, regarded as the direction of a *force*, belongs to the complex, if the moment of rotation of the force with regard to the axis, divided by its projection on the axis, be equal to the constant. Accordingly any four axes in space being given, the position of a right line is fixed by means of four constants, obtained by dividing the four moments of rotation with regard to the four axes by the four corresponding projections on the same axes.

The four axes of the complexes constitute the new system of coordinates; the four constants are the four coordinates of the given right line. The right line intersecting the four axes is the origin of coordinates, its four coordinates being equal to zero.

In the new system of coordinates a right line is determined in the most general way by its four coordinates; but an equation between the four coordinates is not in a general way sufficient to represent a linear complex, depending as it does on five constants.

We may *ad libitum* increase the number of coordinates of a right line.

24. Let P, Q, R, S, T, U . . be the axes of any number of complexes, and p, q, r, s, t, u . . the corresponding coordinates of a given right line (according to the last number). Let

$$\begin{gathered}\Omega_p \equiv \Xi_p - p = 0, \quad \Omega_q \equiv \Xi_q - q = 0, \quad \Omega_r \equiv \Xi_r - r = 0, \\ \Omega_s \equiv \Xi_s - s = 0, \quad \Omega_t \equiv \Xi_t - t = 0, \quad \Omega_u \equiv \Xi_u - u = 0 \dots\end{gathered}$$

be the equations of the complexes. In order to express that the complexes meet along the same line, the following equations of condition are obtained,

$$\left.\begin{aligned}\Omega_t &\equiv \varkappa\,\Omega_p + \lambda\,\Omega_q + \mu\,\Omega_r + \nu\,\Omega_s, \\ \Omega_u &\equiv \varkappa'\Omega_p + \lambda'\Omega_q + \mu'\Omega_r + \nu'\Omega_s,\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$
$$. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad .$$

---

only seen the papers, I hasten to mention it now. But, besides the coincidence referred to, the leading views of Professor CAYLEY's paper and mine have nothing in common. On this occasion I may state that the principles upon which my paper is based were advanced by me, nearly twenty years ago (Geometry of Space, No. 258), but this had entirely escaped from my memory when I recurred to Geometry some time since.

where we may suppose that P, Q, R, S are the former four axes of coordinates; $\varkappa$, $\varkappa'$, λ, λ', μ, μ', ν, ν' indicate any constant coefficients.

In putting the coordinates p, q, r, s, t, u.. equal to zero, the general equations of the complexes become

$$\Xi_p=0, \quad \Xi_q=0, \quad \Xi_r=0, \quad \Xi_s=0, \quad \Xi_t=0, \quad \Xi_u=0.$$

These new equations represent complexes of a peculiar kind, the lines of which intersect their axes; they may be said to represent the axes themselves.

In order to satisfy the equation (18), we put

$$\left.\begin{aligned} \Xi_t &\equiv \varkappa\,\Xi_p+\lambda\,\Xi_q+\mu\,\Xi_r+\nu\,\Xi_s, \\ \Xi_u &\equiv \varkappa'\Xi_p+\lambda'\Xi_q+\mu'\Xi_r+\nu'\Xi_s, \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

$$. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad .$$

whence

$$\left.\begin{aligned} t &=\varkappa p +\lambda q +\mu r +\nu s, \\ u &=\varkappa' p+\lambda' q+\mu' r+\nu' s. \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

The equations (19) require that the *origin* met by the axes P, Q, R, S be likewise met by the new axes T, U...

Therefore p, q, r, s, t, u.. may be regarded as coordinates of the right line along which all complexes meet; the axes of the complexes intersecting the same right line being the axes of coordinates. A right line being completely determined by the first four coordinates, those remaining depend upon them by linear equations (20).

The system of four axes of coordinates depends upon 16, of five axes upon 19, of six upon 22 constants.

Having thus established a system of coordinates which, independently of points and planes, fixes the position of a right line in space, we are enabled, by regarding right lines as elements of space, to reconstruct the whole geometry without recurring to the ordinary system. Here we are guided by analogy. As far as I may judge, the task is a most grateful but at the same time a long and laborious one.

IV. *Geometry of Forces.*

25. In recapitulating the contents of the first three paragraphs of this note, new considerations have been suggested to me, which seem calculated, while greatly increasing again this kind of inquiry, to put the key-stone to it. Hitherto, when I borrowed technical terms from mechanical science, the only intention was to simplify the expression. But *force* may be regarded as a merely geometrical notion, and there is only one step more to be taken in order to arrive at a "*Geometry of Forces*," as there is a geometry based on the notion of right lines.

Forces depend upon five independent constants, four of which indicate their position, while the fifth indicates their intensity. We may call these constants the *five coordinates of the forces.*

In order to fix the direction of a *force*, we may employ line-coordinates and choose the following,

$$X,\ Y,\ Z,\ L,\ M,\ N,$$

indicating the projections of the *force* on the three axes of coordinates OX, OY, OZ, and its three moments of rotation with regard to these axes. Between them the following equation of condition holds good,

$$XL+YM+ZN=0$$

(see No. 7). The quotients obtained by dividing any five of them by the sixth are the absolute values of coordinates. From these quotients the intensity of the force has disappeared.

The *same six constants*, reduced by the last equation to five independent ones, *may be regarded as the absolute values of the coordinates of the force.* Instead of homogeneous equations between them, if regarded as variable, representing complexes of lines (of directions of the forces), we now get ordinary equations between the same variables representing *complexes of forces.*

The extension of all former developments thus indicated immediately occurs to us. A single instance may be referred to here. *Forces* constituting a linear complex are such passing in all directions through each point of space as have their intensity equal to the segments taken on their directions from the point to a certain plane corresponding to it. Forces common to two linear complexes and passing through a given point are confined within the same plane, the distance from the points where their directions meet a given line within the plane being their intensity. Forces, the coordinates of which verify simultaneously three linear equations, are distributed through space in such a manner that there is one force of a given intensity passing through each point of space.

The general contents of this note (except § IV.) were in a verbal communication presented by me at the last Birmingham Meeting of the British Association. As they concern the principles on which the original paper is based, giving to them a symmetry and a generality I was not before aware of, I thought it necessary to add the note to that paper. At the same time I also endeavoured to give an idea of the great fertility of the method developed. But as I am now preparing a volume for publication on this subject, I do not think it suitable to enter here into any details. The work will embrace the theory of the general equation of the second degree between line-coordinates, requiring no means of discussion but those employed by me in the case of equations of the same degree between point- or plane-coordinates. The complex of lines represented by such an equation may be regarded likewise as a complex of *curves of the second class*, one of which is confined in each plane, or as a complex of *cones of the second order*, each point of space being the centre of such a cone. In reducing the number of constants upon which the complex depends from 19 to 9, we pass in particularizing step by step from the general complex to a surface of the second order and class, determined by its tangents.

I intend resuming the consideration of the mechanical part of this note. Then a last generalization will occur to us, the equation of condition, hitherto admitted between the six coordinates x, y, z, L, M, N, being removed.

CONTENTS.

I. *On Linear Complexes of Right Lines.*

Preliminary explanations.—Point-coordinates. Equations between them representing surfaces by means of their points. Plane coordinates. Equations between them representing surfaces enveloped by planes, 1. Double definition of right lines, either by means of their points or by means of traversing planes. Rays. Axes. The two projections of a ray within two planes of coordinates depend upon four linear constants, which may be regarded as ray-coordinates, r, s, ϱ, σ and t, u, v_x, v_y. The two points in which two planes of coordinates are intersected by an axis, depend upon four linear constants which are its coordinates, x, y, z_t, z_u and p, q, ϖ, $\varkappa$, 2–5. Complexes of rays or axes represented by one equation between their four coordinates. .Congruent lines of two complexes constitute a congruency, of three complexes a configuration (*surface gauche*). In a complex every point is the vertex of a cone, every plane contains an enveloped cone. In a congruency there is a certain number of right lines passing through a given point, and confined within a given plane, 6, 7. A configuration of rays represented by three linear equations, either between r, s, ϱ, σ or t, u, v_x, v_y, is a paraboloid, or a hyperboloid, 8. A configuration of axes represented by three equations, either between p, q, ϖ, $\varkappa$ or x, y, z_t, z_u, is either a hyperboloid or a paraboloid, 9. In a congruency of rays or axes represented by two linear equations, there is one ray and one axis passing through a given point and confined within a given plane, 10. Construction, by means of two fixed points, of the rays of a congruency represented by two linear equations between t, u, v_x, v_y, 11. Construction, by means of two planes, of the axes of a congruency represented by two linear equations between x, y, z_t, z_u, 12.

Linear complexes of rays.—In a complex represented by a linear equation between r, s, ϱ, σ, all rays traversing a given point constitute a plane; all rays confined within a given plane meet in the same point. Points and planes corresponding to each other, 13–15. A new variable $(s\varrho-r\sigma)$ introduced. The general equation of a *linear complex* is $Ar+Bs+C+D\sigma+E\varrho+F(s\varrho-r\sigma)=0$. Equation of a plane corresponding to a given point, of a point corresponding to a given plane, 16–19. Conjugate right lines. Each ray intersecting any two conjugate lines is a ray of the complex. A ray of the complex may be regarded as two congruent conjugate lines. Principle of polar reciprocity applied, 20. Construction of the plane corresponding to a given point, of the point corresponding to a given plane, 21, 22. Geometrical determination of the constant of the general equation of the complex. There is a characteristic direction given by the double equation $\frac{x}{D}=\frac{y}{E}=\frac{z}{F}$. If that direction falls within xy, the term $(s\varrho-r\sigma)$ disappears and the general equation becomes linear. If any plane perpendicular to it is taken as one of the three planes of rectangular coordinates, and the corresponding point within it as origin, the general equation assumes one of the forms, $s=k\varrho$, $r=k\sigma$, $s\varrho-r\sigma=k$. A linear complex may, without being altered, turn round a fixed line, and move along it parallel to itself, 23–29. Geometrical interpretation of the last equations, 30. Points and planes corresponding to one another with regard to the complex $s\varrho-r\sigma=k$. Geometrical interpretations, 31, 32. Generalization, 33. Conjugate lines with regard to the complex $s\varrho-r\sigma=k$, 34. A linear complex depends upon five constants, four of which give the position of its axis, 35. Formulæ of the transformation of ray-coordinates corresponding to any displacement of the axes of coordinates, 36–38. Analytical determination of the axis of a complex, represented by the general equation. Determination of k, 39–43. In the peculiar case in which k is equal to zero, all rays meet the axis of the complex, 44. Rays passing through the same point, 45.

Linear congruencies of rays.—A linear congruency, along which an infinite number of complexes intersect each other, is represented by the equations of any two of them. Through a given point of space only one ray passes, corresponding to that point, as there is only one ray confined within a given plane, 46. There is in each complex passing through the congruency one line conjugate to a given right line: all these lines belong to one generation of a hyperboloid, the second generation of which contains rays of the congruency. Generation of a linear congruency by a variable hyperboloid, 47–49. Characteristic section of a congruency to which the axes of all passing complexes are parallel. The axis of the congruency is a fixed right line, perpendicular to that section on which the axes of all complexes meet at right angles, 50, 51. The locus of points having in all complexes the same corresponding plane is a system of two right lines, the directrices of the congruency. Central plane parallel to both directrices and equidistant from them. The directrices may be real or imaginary, 52–54. In the first case there are amongst the complexes two of a peculiar description [44] having both directrices as axes. All rays of the congruency meet both its directrices, 55. The peculiar case in which one of the two directrices is infinitely distant, 56. Each of any two complexes being given by means of its constant k and the position of its axis, to determine both directrices of the congruency, 57–59. A congruency being given by means of its two directrices, to determine the constants and the axes of the complexes passing through it. Centre of the congruency. The two secondary axes within the central plane, 60. Locus of the axes of all complexes meeting along the same congruency, 61.

Linear configurations of rays represented by the equations of three linear complexes.

An infinite number of congruencies meet along a linear configuration. Generally it is a hyperboloid. Its rays constitute one of its generations, while the directrices of all congruencies constitute the other, 62. The central planes of all congruencies meet in the same point: the centre of the configuration. Its diameters meet both directrices of the different congruencies, 63. A configuration is determined by means of three complexes, or by means of three congruencies, obtained by combining them two by two. Three couples of planes drawn through both directrices of each of the congruencies parallel to its central plane constitute a parallelopiped circumscribed to the hyperboloid. Each ray intersects all six directrices. The ray within each of the six planes is parallel to the directrix within the opposite plane; the point in which it meets the directrix within the same plane is the point of contact. Three diameters determined by both points of contact within the three couples of opposite planes. Imaginary diameters correspond to imaginary directrices, asymptotes to congruent directrices, 64. A hyperboloid being given, we may return to the congruencies and complexes which constitute it, 65. The equations of the configurations transformed into an equation between x, y, z, 66.

II. *On Complexes of Luminous Rays within Biaxal Crystals.*

Complexes of doubly refracted rays corresponding to complexes of incident rays, 1.

Digression on double refraction. HUYGHENS's principle. The author's construction presented, 1838. Auxiliary ellipsoids. The ellipsoid E, with regard to which the wave-surface is its own polar surface. The plane of refraction, containing both refracted rays, passes through SS, the polar line of RR, along which the surface of the crystal is intersected by the front of the incident elementary wave at that moment when, within the crystal, the wave-surface is formed, 2–6. The plane of refraction is congruent with the diametral plane of E, the conjugate diameter of which is perpendicular to the plane of incidence in O, 7. All rays incident within the same plane are, after double refraction, confined again within the same plane, 8. While the plane of incidence turns round the vertical in O, the corresponding plane of refraction turns round that diameter of E, the conjugate diametral plane of which is the surface of the crystal, 9, 10. Whatever may be the plane or curved surface met by an incident ray in any given point O, all corresponding planes of refraction pass through a fixed right line, 11. Peculiar cases of complexes. The plane of refraction perpendicular to the surface of the crystal. The incident and the two refracted rays confined within the same plane. A circular section of E falling within the surface of the crystal, 12, 13. Analytical determination of SS, 14. A fourth auxiliary ellipsoid, 15, 16.

Complex of doubly refracted rays determined by means of E. Its equation depending upon the constants of E, 17, 18. By taking as axes of coordinates three conjugate diameters of E, two of which, falling within the surface of the crystal, are perpendicular to each other, the general equation of the complex becomes $r\sigma = ks\varrho$, the constant k being the ratio of the squares of the two rectangular diameters, 19. Geometrical interpretation, 20. Refracted rays of the complex passing through a given point constitute a cone of the second order. The cone remains the same if the given point moves along a right line, passing through O, 21. Peculiar cases, 22–24. A hyperbola enveloped by the doubly refracted rays within any given plane. Its determination, 25. Peculiar cases. Geometrical interpretations, 26–28. The complex generated in three different ways by a variable linear congruency, 29. Peculiar case of a complex, the crystal being cut along a circular section of E. All doubly refracted rays meet that diameter of E the conjugate plane of which is the circular section, 30. Peculiar case in which the surface of the crystal is a principal section, 31. Case of uniaxal crystals, 32, 33. The ellipsoid E replaced by a new ellipsoid, the *radii vectores* of which indicate the reciprocal values of optical elasticity, 34.

Additional Note.

Coordinates of a right line, 1–10. Complexes. Congruencies. Surfaces generated by a moving right line. Developable surfaces and curves of double curvature, 11–22. A new system of coordinates, 23, 24. Geometry of forces, 25.

INDEX

TO THE

PHILOSOPHICAL TRANSACTIONS

FOR THE YEAR 1865.

A.

Antedon rosaceus, embryogeny of, 513 (see THOMSON).
Armour-plated ships, magnetic character of, 263 (see EVANS).
Atlantic, principal currents of the, 239 (see FORCHHAMMER).
Atomic heat and atomic weight, relations between, 177 (see KOPP).

B.

BAILEY (J. W.), his researches on Foraminifera, 423 (see PARKER).
BAKERIAN LECTURE, 605 (see ROSCOE).
BEALE (L. S.). New Observations upon the Minute Anatomy of the Papillæ of the Frog's Tongue, 443.—Conclusions, 456; description of the plates, 457.
BINNEY (E. W.). A Description of some Fossil Plants, showing Structure, found in the Lower Coal-seams of Lancashire and Yorkshire, 579.—Concluding remarks, 596; description of the plates, 599.
Blood, influence of physical and chemical effects upon, 687 (see HARLEY).

C.

CAYLEY (A.). On the Sextactic Points of a Plane Curve, 545.—Application to a cubic, 556; Appendix, 569.
Cerebral hemispheres of the Marsupialia and Monotremata, 633 (see FLOWER).
Chemical action of total daylight, 605 (see ROSCOE).
Comatula rosacea, embryogeny of, 513 (see THOMSON).
Compass, effect of ship's magnetism on the, 263 (see EVANS).
Complexes, linear, of right lines, 725 (see PLÜCKER).
Condensers, electric, theory of, 493 (see MAXWELL).
Cubic curve, sextactic points of, 556 (see CAYLEY).

D.

Double refraction, 760 (see Plücker).

E.

Echidna Hystrix, 671 (see Owen).

Electromagnetic field, dynamical theory of, 459 (see Maxwell).

Elements, nature of the chemical, 199 (see Kopp).

Evans (F. J.) and **Smith** (A.). On the Magnetic Character of the Armour-plated Ships of the Royal Navy, and on the Effect on the Compass of particular arrangements of Iron in a Ship, 263.—Deduction of the formulæ employed, 267; physical representation of the results, 271; numerical values of the coefficients, 278; tables of coefficients, 291; effect on the compass of particular masses of soft iron in a ship, 304.

F.

Flower (W. H.). On the Commissures of the Cerebral Hemispheres of the Marsupialia and Monotremata as compared with those of the Placental Mammals, 633.—Description of the plates, 650.

Foraminifera from the Atlantic Ocean, &c., 325 (see Parker).

Forchhammer (G.). On the Composition of Sea-water in the different parts of the Ocean, 203.—Elements which occur in the water of the ocean, 204; on the quantitative analysis of sea-water, 214; distribution of the salts in different parts of the sea, 219; general results of the preceding investigation, 226; principal currents of the Atlantic, 239; chemical decomposition in sea-water, 242; tables, 246.

Fossil plants found in coal-seams, 579 (see Binney).

Frog's tongue, minute anatomy of the papillæ of, 443 (see Beale).

G.

Gases, spectra of, 1 (see Plücker).

Geometry, on a new, of space, 725 (see Plücker).

Glyptodon, osteology of, 31 (see Huxley).

H.

Harley (G.). On the Influence of Physical and Chemical Agents upon Blood; with special reference to the mutual action of Blood and the Respiratory Gases, 687.—Influence of physical agents, 689; of chemical agents—animal products, 699; of vegetable products, 704; of anæsthetics, 713; of mineral substances, 718.

Hittorf (J. W.) and **Plücker** (J.). On the Spectra of Ignited Gases and Vapours, &c., 1 (see Plücker).

Hoplophoridæ, 31 (see Huxley).

Huxley (T. H.). On the Osteology of the genus *Glyptodon,* 31.—Part I. History of the discovery and determination of the remains of the Hoplophoridæ, 31. Part II. Description of the skeleton of *Glyptodon clavipes,* 43; of the skull, 43; of the vertebral column, 58; description of the plates, 69.

I.

Induction, electromagnetic, 466 (see Maxwell).

J.

Jones (T. R.) and Parker (W. K.). Foraminifera from the Atlantic Ocean, 325 (see Parker).

K.

Kopp (H.). Investigations of the Specific Heat of Solid Bodies, 71.—Historical introduction, 71; description of a method of determining the specific heat of solid bodies, 83; determination of the specific heat of individual solid substances, 103; table of the substances whose specific heat has been experimentally determined, 167; on the relations between atomic heat and atomic weight or composition, 177; considerations on the nature of the chemical elements, 199.

L.

Light, electromagnetic theory of, 497 (see Maxwell).

M.

Magnetic character of ships, 263 (see Evans).
Marsupialia, cerebral characters of the, 633 (see Flower).
Maxwell (J. C.). A Dynamical Theory of the Electromagnetic Field, 459.—Introduction, 459; on electromagnetic induction, 466; general equations of the electromagnetic field, 480; mechanical actions in the field, 488; theory of condensers, 493; electromagnetic theory of light, 497; calculation of the coefficients of electromagnetic induction, 506.
Meteorological registration of the chemical action of total daylight, 605 (see Roscoe).
Monotremata, cerebral characters of the, 633 (see Flower).

N.

Nitrogen, different spectra of, 6 (see Plücker).

O.

Owen (R.). On the Marsupial Pouches, Mammary Glands, and Mammary Fœtus of the *Echidna Hystrix*, 671.—Description of the plates, 685.

P.

Parker (W. K.) and Jones (T. R.). On some Foraminifera from the North Atlantic and Arctic Oceans, including Davis Straits and Baffin's Bay, 325. (For Contents, see p. 325.)

PLÜCKER (J.) and HITTORF (J. W.). On the Spectra of Ignited Gases and Vapours, with especial regard to the different Spectra of the same elementary gaseous substance, 1.—Different spectra of nitrogen, 6; spectra of the first and second orders, 13; spectra of sulphur and various other elements, 13; explanation of the plates, 26.

PLÜCKER (J.). On a New Geometry of Space, 725.—On linear complexes of right lines, 725; on complexes of luminous rays within biaxal crystals, 760. Additional note, 774. (For Contents, see p. 788.)

Poisons, influence of various, upon blood, 687 (see HARLEY).

POURTALES (F. L.), his researches on Foraminifera, 429 (see PARKER).

R.

ROSCOE (H. E.). THE BAKERIAN LECTURE. On a Method of Meteorological Registration of the Chemical Action of Total Daylight, 605.—Tables of results, 624.

S.

Sea-water, composition of, in different parts of the ocean, 203 (see FORCHHAMMER).

Sextactic points of a plane curve, 545 (see CAYLEY); 653 (see SPOTTISWOODE).

Sigillaria, 579 (see BINNEY).

SMITH (A.) and EVANS (F. J.). On the Magnetic Character of the Armour-plated Ships of the Royal Navy, &c., 263 (see EVANS).

Specific heat of solid bodies, 71 (see KOPP).

Spectra, different, of the same elementary substance, 1 (see PLÜCKER).

SPOTTISWOODE (W.). On the Sextactic Points of a Plane Curve, 653.

T.

THOMSON (WYVILLE). On the Embryogeny of *Antedon rosaceus,* Linck (*Comatula rosacea* of Lamarck), 513.—Explanation of plates, 542.

V.

Velocity of light, deduction of, from electromagnetic data, 499 (see MAXWELL).

W.

Wave-surface in biaxal crystals, 761 (see PLÜCKER).

LONDON:
PRINTED BY TAYLOR AND FRANCIS, RED LION COURT, FLEET STREET.

PRESENTS

RECEIVED BY

THE ROYAL SOCIETY,

WITH THE

NAMES OF THE DONORS.

From November 1864 to June 1865.

PRESENTS.	DONORS.
ACADEMIES and SOCIETIES.	
Albany:—	
Transactions of the Society for the Promotion of Useful Arts in the State of New York. Vol. IV. Part 2. 8vo. *Albany* 1819.	The Institute.
Transactions of the Albany Institute. Vol. IV. 8vo. *Albany* 1858–64.	———
Basel:—Verhandlungen der Naturforschenden Gesellschaft. Theil IV. Heft 1. 8vo. *Basel* 1864.	The Society.
Berlin:—	
Abhandlungen der Königlichen Akademie der Wissenschaften, aus dem Jahre 1863. 4to. *Berlin* 1864.	The Academy.
Monatsbericht. January and February, and June to December, 1864. 8vo. *Berlin.*	———
Die Fortschritte der Physik im Jahre 1862, dargestellt von der physikalischen Gesellschaft. Jahrgang XVIII. 8vo. *Berlin* 1864.	The Society.
Berliner Astronomisches Jahrbuch für 1867. 8vo. *Berlin* 1864.	The Observatory.
Bern:—	
Mittheilungen der Naturforschenden Gesellschaft, aus dem Jahre 1863. Nr. 531–552. 8vo. *Bern* 1863.	The Society.
Programm der Berner Kantonsschule für das Jahr 1864. 8vo. *Bern* 1864.	———
Verzeichniss der Vorlesungen welche vom 15. Oct. 1863–31. März 1864, 15. April–15. Aug. 1864, 15. Oct. 1864–31. März 1865 an der Hochschule in Bern gehalten werden sollen. 4to. *Bern.*	———
Verzeichniss der Behörden, Lehrer und Studirenden der Berner Hochschule, 1863–64. 8vo. *Bern.*	———
Beitrag zur Casuistik der Beckengeschwülste in geburtshülflicher Beziehung, von J. U. Kürsteiner. 8vo. *Zürich* 1863.	———

PRESENTS.	DONORS.
ACADEMIES and SOCIETIES (*continued*).	
Bern :—	
Beiträge zur Lehre von den Luxationen und Fracturen der obersten Halswirbel, von C. R. Wagner. 8vo. *Zürich* 1863.	The Society.
Einige Versuche über Strychnin-Vergiftung als gerichtlich-toxikologischer Beitrag zu den Strychnin-Vergiftungen, von J. Meyer. 8vo. *Bern* 1864.	————
Ueber angeborne Stenose der Arteria pulmonalis, von C. Stölker. 8vo. *Bern* 1864.	————
Berwick :—Berwickshire Naturalists' Club. Proceedings. Vol. V. No. 2. 8vo. 1865.	The Club.
Birmingham :—Institution of Mechanical Engineers. Proceedings, 5th November 1863; 28th January, 5th May, 2nd August 1864 (2 parts). 8vo.	The Institution.
Bologna :—	
Memorie della Accademia delle Scienze dell' Istituto. Tomo 12. Serie 2, tomo 1 & 2. 4to. *Bologna* 1861–62.	The Institute.
Rendiconto delle Sessioni 1860–61, 1861–62, 1862–63. 8vo. *Bologna.*	————
Bordeaux :—Mémoires de la Société des Sciences Physiques et Naturelles. Tome III. cahier 1. 8vo. *Paris* 1864.	The Society.
Boston, U.S. :—	
Society of Natural History. Boston Journal of Natural History. Vol. VII. No. 4. 8vo. *Boston* 1863.	The Society.
Breslau :—	
Abhandlungen der Schlesischen Gesellschaft für vaterländische Cultur. Abth. für Naturwissenschaften und Medicin. 1862, Heft 3. Phil.-Hist. Abth. 1864, Heft 1. Einundvierzigster Jahresbericht. 8vo. *Breslau* 1864.	The Society.
Bruxelles :—	
Mémoires de l'Académie Royale des Sciences, des Lettres et des Beaux-Arts. Tome XXXIV. 4to. *Bruxelles* 1864.	The Academy.
Mémoires Couronnés et Mémoires des Savants Étrangers. Tome XXXI. 4to. *Bruxelles* 1863.	————
Bulletins. 2e série. Tomes XV., XVI., XVII., XVIII., & XIX. Nos. 1–4. 8vo. *Bruxelles* 1863–65.	————
Mémoires Couronnés et autres Mémoires. Collection in-8vo. Tomes XV. & XVI. 8vo. *Bruxelles* 1863–64.	————
Annuaire 1864 & 1865. 8vo. *Bruxelles* 1864–65.	————
Bulletin de l'Académie Royale de Médecine. Tome VII. Nos. 3–6, 8–11; Tome VIII. Nos. 1–2. 8vo. *Bruxelles* 1864–65.	The Academy.
Calcutta :—	
Asiatic Society of Bengal. Journal, 1864, Nos. 1–5 and Supplementary No. 8vo. *Calcutta* 1864.	The Society.
Memoirs of the Geological Survey of India. Vol. III. Part 2, Vol. IV. Part 2. 8vo. *Calcutta* 1864.	The Geological Museum.
———————————— Palæontologia Indica: III. 2–5. The Fossil Cephalopoda. 4to. *Calcutta* 1864.	————
Memorandum on the results of a cursory examination of the Salt-range in the Punjab and of parts of Bunnoo and Kohat. 8vo. *Calcutta* 1864.	————
Annual Report of the Geological Survey of India and of the Museum of Geology. 8vo. *Calcutta* 1864.	————

Presents.	Donors.
ACADEMIES and SOCIETIES (*continued*).	
Cambridge (Mass.):—	
Harvard College. Annual Reports of the President and Treasurer, 1862–63. 8vo. *Cambridge* 1864.	The College.
Report of the Committee of the Overseers appointed to visit the Observatory. 8vo. *Boston* 1864.	———
Catalogus Universitatis Harvardianæ 1863. 8vo.	———
Catalogue of the Officers and Students, 1863–64. 8vo. *Cambridge* 1863.	———
Address delivered before the Alumni. 8vo. *Cambridge* 1863.	———
Addresses at the Inauguration of Thomas Hill, D.D., as President. 8vo. *Cambridge* 1863.	———
Catania:—Atti dell' Accademia Gioenia di Scienze Naturali. Serie seconda. Tomo I.–XVI. & XIX. 4to. *Catania* 1844–64.	The Academy.
Cherbourg:—Mémoires de la Société Impériale des Sciences Naturelles. Tomes IX. & X. 8vo. *Paris* 1863–64.	The Society.
Christiania:—	
Forhandlingar i Videnskabs-Selskabet. Aar 1863. 8vo. *Christiania* 1864.	The University.
Nyt Magazin for Naturvidenskaberne. Bind XII. Heft 4, Bind XIII. Hefte 1–3. 8vo. *Christiania* 1863–64.	———
Meteorologische Beobachtungen; aufgezeichnet auf Christiania's Observatorium. Lief. 3 & 4. 4to. *Christiania* 1864.	———
Det Kongelige Norske Frederiks Universitets Aarsberetning for Aaret 1862. 8vo. *Christiania*.	———
Index Scholarum in Universitate Regia Fredericiana Jan. & Aug. 1864. 4to. *Christiania*.	———
Om Sneebræen Folgefon, af S. A. Sexe. 4to. *Christiania* 1864.	———
Om de Geologiske Forhold paa Kystrœkningen af Nordre Bergenhus Amt, af M. Irgens og Th. Hiortdahl. 4to. *Christiania* 1864.	———
Chur:—Verhandlungen der Schweizerischen naturforschenden Gesellschaft bei ihrer Versammlung zu Samaden Aug. 1863. 47 Versammlung. 8vo. *Chur*.	The Society.
Columbus:—Ohio State Board of Agriculture. Seventeenth Annual Report. 8vo. *Columbus* 1863.	The Board.
Copenhagen:—Oversigt over det Kgl. danske Videnskabernes Selskabs Forhandlinger, 1862 & 1863. 8vo. *Kjöbenhavn*.	The Society.
Devonshire Association for the Advancement of Science, Literature, and Art. Reports and Transactions. Parts 1–3. 8vo. *London* 1863–64.	The Association.
Dijon:—Mémoires de l'Académie Impériale des Sciences, Arts et Belles-Lettres. Série 2. Tome XI. Année 1863. 8vo. *Dijon* 1864.	The Academy.
Dresden:—Nova Acta Academiæ Cæsareæ Leopoldino-Carolinæ Germanicæ Naturæ Curiosorum. Tomus XXXI. 4to. *Dresdæ* 1864.	The Academy.
Dublin:—	
Geological Society. Journal. Vol. X. Part 2. 8vo. *Dublin* 1864.	The Society.
Natural History Society. Proceedings. Vols. I., II. Parts 2 & 3; Vol. III. Parts 1 & 2; Vol. IV. Parts 1 & 2. 8vo. *Dublin* 1849–65.	The Society.
Royal Dublin Society. Journal. Nos. 31, 32 & 33. 8vo. *Dublin* 1864–65.	The Society.
Royal Irish Academy. Transactions. Vol. XXIV. Antiquities, Part 2. 4to. *Dublin* 1864.	The Academy.

Presents.	Donors.
ACADEMIES and SOCIETIES (*continued*).	
Dudley:—Dudley and Midland Geological and Scientific Society and Field Club. Transactions. Nos. 1–5. 8vo. *Dudley* 1862–65.	The Society.
Edinburgh:—	
Royal Scottish Society of Arts. Transactions. Vol. VI. Part 4. 8vo. *Edinburgh* 1864.	The Society.
Royal Society. Transactions. Vol. XXIII. Part 3. 4to. *Edinburgh* 1864.	The Society.
———— Proceedings. Session 1863–64. 8vo. *Edinburgh* 1864.	————
Frankfurt a. M.:—	
Abhandlungen herausgegeben von der Senckenbergischen naturforschenden Gesellschaft. Band V. Heft 2. 4to. *Frankfurt a. M.* 1864.	The Society.
Der Zoologische Garten, Organ der zoologischen Gesellschaft. Jahrgang I., II., III., IV., & V. Nr. 2–12. 8vo. *Frankfurt a. M.* 1860–64.	The Society.
Freiburg im Breisgau:—Berichte über die Verhandlungen der naturforschenden Gesellschaft. Band III. Heft 2. 8vo. *Freiburg* 1864.	The Society.
Geneva:—Mémoires de la Société de Physique et d'Histoire Naturelle. Tome XVII. Partie 2. 4to. *Genève* 1864.	The Society.
Görlitz:—	
Abhandlungen der naturforschenden Gesellschaft. Band I.–XII. (wanting B. III. Heft 1, and B. VII.). 8vo. *Görlitz* 1827–65.	The Society.
Die Regenverhältnisse Deutschlands, Abdruck aus den Abhandlungen, Band VII. Heft 1. 8vo. *Görlitz* 1865.	————
Göttingen:—Nachrichten von der K. Gesellschaft der Wissenschaften und der Georg-August-Universität, aus dem Jahre 1864. 12mo. *Göttingen* 1865.	The Society.
Haarlem:—Natuurkundige Verhandelingen van de Hollandsche Maatschappij der Wetenschappen. Tweede Verzameling. Deel XVIII., XIX., & XXI. Stuck 1. 4to. *Haarlem* 1863–64.	The Society.
Habana:—Observatorio Magnético y Meteorológico del Real Colegio de Belen de la Compañia de Jesus. Resumen de las Observaciones. Nov., Dec. 1863; Jan.–Dec. 1864; Jan.–March 1865. 8vo.	The Observatory.
Halle:—Zeitschrift für die gesammten Naturwissenschaften, herausgegeben von dem naturw. Vereine für Sachsen und Thüringen in Halle, redigirt von C. Giebel und M. Siewert. Jahrg. 1864. Band XXIII. 8vo. *Berlin* 1864.	The Society.
Hobart Town:—Report of the Royal Society of Tasmania for the year 1863. 8vo. *Hobart Town* 1864.	The Society.
Jena:—Jenaische Zeitschrift für Medicin und Naturwissenschaft, herausgegeben von der medicinisch-naturwissenschaftlichen Gesellschaft zu Jena. Band I. Hefte 2–3. 8vo. *Leipzig* 1864.	The Society.
Kazan:—Imperial Russian University. Outchonia Zapiski (Scientific Papers), 1862. 4 Parts. 8vo. *Kazan* 1862–63.	The University.
Kiel:—Schriften der Universität, aus dem Jahre 1863. Band X. 4to. *Kiel* 1864.	The University.
Kolozsvártt:—Az Erdélyi Muzeum-Egylet Évkönyvei. Kötet III. Füzet 1. 4to. *Kolozsvártt* 1864.	The Museum.
Königsberg:—Schriften der königlichen physikalisch-ökonomischen Gesellschaft. Jahrgang 1863. Abth. 1 & 2. 4to. *Königsberg* 1863.	The Society.
Lausanne:—Bulletin de la Société Vaudoise des Sciences Naturelles. Tome VII. No. 50; tome VIII. Nos. 51 & 52. 8vo. *Lausanne* 1863–65.	The Society.

Presents.	Donors.
ACADEMIES and SOCIETIES (*continued*).	
Leeds:—	
Geological and Polytechnic Society of the West Riding of Yorkshire. Proceedings, December 6, 1839; 1840–42, pp. 1–442 (5 parts); 1863–64. 8vo. *Leeds* 1839–64.	The Society.
Philosophical and Literary Society. Annual Reports for 1861-62 & 1863–64. 8vo. *Leeds* 1862–64.	The Society.
On the Early History of Leeds in Yorkshire. A Lecture by T. Wright. 8vo. *Leeds* 1864.	———
Leipzig:—	
Berichte über die Verhandlungen der königlich-sächsischen Gesellschaft der Wissenschaften. Math.-Phys. Classe, 1863, 1 & 2; Phil.-Hist. Classe, 1863, 1–3; 1864, 1. 8vo. *Leipzig* 1863–64.	The Society.
Darlegung der theoretischen Berechnung der in den Mondtafeln angewandten Störungen, von P. A. Hansen. Zweite Abhandlung. 8vo. *Leipzig* 1864.	———
Elektrodynamische Maassbestimmungen insbesondere über elektrische Schwingungen, von W. Weber. 8vo. *Leipzig* 1864.	———
Lisbon:—	
Memorias da Academia Real das Sciencias. Classe de Sciencias Mathematicas, Physicas e Naturaes: nova serie, Tomo III. parte 1. Classe de Sciencias Moraes, Politicas e Bellas Lettras: nova serie, Tomo III. parte 1. 4to. *Lisbôa* 1863.	The Academy.
Annaes do Observatorio do Infante D. Luiz. Vol. I. 1856 a 1863; Vol. II. 1863–64, Nos. 1–3. fol. *Lisbôa* 1864.	The Observatory.
Relatorio do Serviço do Observatorio do Infante D. Luiz, 1863–64. 8vo. *Lisbôa* 1864.	———
Liverpool:—	
Historic Society of Lancashire and Cheshire. Transactions. New Series. Vol. III. 8vo. *Liverpool* 1863.	The Society.
Literary and Philosophical Society. Proceedings, 1863–64. No. 18. 8vo. *London* 1864.	The Society.
London:—	
Anthropological Society. Anthropological Review. Nos. 4–8. 8vo. *London* 1864–65.	The Society.
British Association. Report of the Thirty-third Meeting, held at Newcastle-upon-Tyne, 1863. 8vo. *London* 1864.	The Association.
——————— Index to Reports and Transactions from 1831 to 1860 inclusive. 8vo. *London* 1864.	———
British Horological Institute. The Horological Journal. Vol. VI. Nos. 71 & 72; Vol. VII. Nos. 73–82. 8vo. *London* 1864–65.	The Institute.
British Meteorological Society. Proceedings. Vol. II. Nos. 13–19. 8vo. *London* 1864–65.	The Society.
——————— Catalogue of Books in the Library, 1864. List of Members, 1864. 8vo.	———
Chemical Society. Journal. July to December 1864; January to April 1865. 8vo. *London*.	The Society.
Entomological Society. Transactions. Third series. Vol. II. Parts 1–4. Vol. III. Part 1. 8vo. *London* 1864–65.	The Society.

PRESENTS.	DONORS.
ACADEMIES and SOCIETIES (*continued*).	
London :—	
Geological Society. Quarterly Journal. Nos. 79–82. Vol. XX. Parts 3 & 4; Vol. XXI. Parts 1 & 2. 8vo. *London* 1864–65.	The Society.
Geological Survey of Great Britain, Memoirs. Catalogue of the Collection of Fossils in the Museum of Practical Geology. By T. H. Huxley and R. Etheridge. 8vo. *London* 1865.	The Geological Survey Office.
——— Catalogue of the Mineral Collections. By W. W. Smyth, T. Reeks, and F. W. Rudler. 8vo. *London* 1864.	———
——— Descriptive Catalogue of the Rock Specimens. 8vo. *London* 1862.	———
——— Descriptive Catalogue of the Geological, Mining, and Metallurgical Models. 8vo. *London* 1865.	———
——— Catalogue of the published Maps, Sections, Memoirs, and other publications. 8vo. *London* 1865.	———
——— Iron Ores of Great Britain. Parts 3 & 4. 8vo. *London* 1861–62.	———
——— Mineral Statistics of the United Kingdom for 1861, 1862, and 1863. 8vo. *London* 1862–64.	———
——— Memoirs illustrative of the Geological Map, Sheets 4, 7, 10, 12, 13, 45, 53 N.E., 71 N.E., 80 N.E., 82 S.E., 88 S.W., 89 S.W., 89 S.E., Scotland Sheets 32 & 34. 8vo. *London* 1861–64.	———
——— Figures and Descriptions illustrative of British Organic Remains. Decades 10 & 11. 4to. *London* 1861–64.	———
Institution of Civil Engineers. Minutes of Proceedings, Session 1861–62. Vol. XXI. 8vo. *London* 1862. General Index, Vols. I. to XX. 8vo. *London* 1865.	The Institution.
Linnean Society. Transactions. Vol. XXIV. Part 3. 4to. *London* 1864.	The Society.
——— Journal of Proceedings. Vol. VIII. Zoology, Nos. 29 & 30; Botany, Nos. 29–32, & Vol. IX. Nos. 33 & 34. 8vo. *London* 1864–65.	———
——— List, 1864. 8vo.	———
Pathological Society. Transactions. Vol. XV. 8vo. *London* 1864.	The Society.
——— A General Index to the first fifteen volumes of the Transactions, compiled by T. Holmes. 8vo. *London* 1864.	———
Photographic Society. The Photographic Journal. Nos. 147–158. 8vo. *London* 1864–65.	The Society.
Royal Agricultural Society. Journal. Vol. XXV. Part 2. Second Series, Vol. I. Part 1. 8vo. *London* 1864–65.	The Society.
Royal Asiatic Society. Journal. New Series. Vol. I. Part 1. 8vo. *London* 1864.	The Society.
Royal Astronomical Society. Memoirs. Vol. XXXII. 4to. *London* 1864.	The Society.
——— Monthly Notices. Vol. XXIV. No. 9; Vol. XXV. Nos. 1–7. 8vo. *London* 1864–65.	———
Royal Geographical Society. Journal. Vols. XXXIII. & XXXIV. 8vo. *London* 1863–64.	The Society.
——— Proceedings. Vol. VIII. Nos. 4–6; Vol. IX. Nos. 1 & 2. 8vo. *London* 1864–65.	———
Royal Horticultural Society. Proceedings. Vol. IV. Nos. 10–12; Vol. V. Nos. 1–6. 8vo. *London* 1864–65.	The Society.

Presents.	Donors.
ACADEMIES and SOCIETIES (*continued*).	
London:—	
Royal Institute of British Architects. Sessional Papers, 1864–65. Part I. Nos. 1–4; Part 2. Nos. 1–6; Part 3. Nos. 1–3 & 5. 4to. *London* 1865.	The Institute.
Royal Institution. Proceedings. Vol. IV. Parts 3 & 4. 8vo. *London* 1864.	The Institution.
Royal Medical and Chirurgical Society. Medico-Chirurgical Transactions. Vol. XLVII. 8vo. *London* 1864.	The Society.
———— Proceedings. Vol. IV. Nos. 5 & 6; Vol. V. No. 1. 8vo. *London* 1864–65.	————
———— Index to the Catalogue of the Library. 8vo. *London* 1860.	————
Royal Society of Literature. Transactions. Second series. Vol. VIII. Part 1. 8vo. *London* 1864.	The Society.
———— Annual Report; the President's Address; List of Members, 1864. 8vo. *London.*	————
Royal United Service Institution. Journal. Vol. VIII. Nos. 31–33, and Appendix. 8vo. *London* 1864–65.	The Institution.
Society for the Suppression of Mendicity. Forty-sixth Report. 8vo. *London* 1864.	The Society.
Society of Antiquaries. Proceedings. Second series. Vol. II. Nos. 4 & 5. 8vo. *London* 1863.	The Society.
Zoological Society. Transactions. Vol. V. Parts 1–3. 4vo. *London* 1862–64.	The Society.
———— Proceedings of the Scientific Meetings, for 1863. 3 parts. 8vo. *London.*	————
Luxembourg:—Société des Sciences Naturelles. Tome VII. 8vo. *Luxembourg* 1864.	The Society.
Madrid:—Memorias de la Real Academia de Ciencias. 1ª serie: Ciencias Exactas. Tomo I. parte 2. 2ª serie: Ciencias Fisicas. Tomo I. parte 3; Tomo II. parte 1. 8vo. *Madrid* 1863–64.	The Academy.
Mantua:—Accademia di Scienze, Belle Lettere ed Arti. Anno 1864. 8vo. *Mantova.*	The Academy.
Mauritius:—Meteorological Society. Proceedings and Transactions. Vol. VI. 8vo. *Mauritius* 1864.	The Society.
Milan:—	
Memorie del Reale Istituto Lombardo di Scienze, Lettere ed Arti. Vol. IX. fasc. 2 & 4. 4to. *Milano* 1862–63.	The Institute.
Atti. Vol. III. fasc. 58, 15 & 16. 4to. *Milano* 1862–63.	————
Anuario, 1864. 8vo. *Milano.*	————
Rendiconti. Classe di Scienze Matematiche e Naturali: Vol. I. fasc. 4–7. Classe di Lettere e Scienze Morali e Politiche: Vol. I. fasc. 3–7. 8vo. *Milano,* 1864.	————
Atti della fondazione scientifica Cagnola dalla sua istituzione in poi. Vol. III. 8vo. *Milano* 1862.	The Society.
Atti della Società Italiana di Scienze Naturali. Vol. V. fasc. 4 & 5; Vol. VI. fasc. 1–3. 8vo. *Milano* 1863–64.	The Society.
Modena:—Memorie della Società Italiana delle Scienze. Serie seconda. Tomo I. 4to. *Modena* 1862.	The Society.

PRESENTS.	DONORS.
ACADEMIES and SOCIETIES (*continued*).	
Montpellier:—Mémoires de l'Académie des Sciences et Lettres. Section des Sciences: Tome V. Section de Médecine: Tome III. fasc. 4 & 5; Tome IV. fasc. 1. Section des Lettres: Tome III. fasc. 3 & 4. 4to. *Montpellier* 1861–63.	The Academy.
Montreal:—The Canadian Naturalist and Geologist. New Series. Vol. I. Nos. 1–6. 8vo. *Montreal* 1864.	The Natural History Society.
Moscow:—Bulletin de la Société Impériale des Naturalistes. Année 1863, Nos. 1–4; 1864, No. 1. 8vo. *Moscou* 1863–64.	The Society.
Munich:—	
Sitzungsberichte der königl. bayer. Akademie der Wissenschaften, 1864. I. Hefte 3–5; II. Hefte 1–4. 8vo. *München* 1864.	The Academy.
Gelehrte Anzeigen. Bände I.–V. 4to. *München* 1835–37.	———
Ueber den Begriff der bürgerlichen Gesellschaft. Vortrag von W. H. Riehl. 4to. *München* 1864.	———
König Maximilian II. und die Wissenschaft. Rede von J. v. Döllinger. 8vo. *München* 1864.	———
Chinesische Texte zu Dr. J. H. Plath's Abhandlung: Abtheilung II. Der Cultus der alten Chinesen. 4to. *München* 1864.	———
Die Stellung Venedigs in der Weltgeschichte: Rede von G. M. Thomas. 4to. *München* 1864.	———
Naples:—	
Società Reale. Atti dell' Accademia delle Scienze Fisiche e Matematiche. Tomo I., Anno III. fasc. 1–6. 4to. *Napoli* 1863–64.	The Society.
——— Rendiconto: Anno II. fasc. 4–12; Anno III. fasc. 1 & 2, 1863–64. Rendiconto delle tornate e dei lavori dell' Accademia di Scienze Morali e Politiche: Anno II. Aprile–Dicembre 1863; Anno III. Maggio–Dicembre 1864; Anno IV. Gennaio 1865. Rendiconto della Reale Accademia di Archeologia, Lettere e Belle Arti: Anno 1863; Anno 1864, Gennaio–Giugno. 4to. *Napoli* 1863–65. Annuario 1865.	———
Neuchâtel:—Bulletin de la Société des Sciences Naturelles. Tome VI. troisième cahier. 8vo. *Neuchâtel* 1864.	The Society.
Newcastle-upon-Tyne:—Tyneside Naturalists' Field Club. Transactions. Vol. VI. Part 2. 8vo. *Newcastle* 1864.	The Club.
Paris:—	
Comptes Rendus Hebdomadaires des Séances de l'Académie des Sciences. June to December 1864, January to June 1865. Tome LIX. Table de Matières. 4to. *Paris* 1864–65.	The Institute.
Connaissance des Temps pour l'an 1866, publié par le Bureau des Longitudes. 8vo. *Paris* 1864.	The Bureau.
Annuaire pour l'an 1865. 12mo. *Paris*.	———
Dépôt de la Marine. Annales Hydrographiques. 2^e–4^e trimestre de 1863; 1^r trimestre de 1864. 8vo. *Paris*.	The Dépôt de la Marine.
——— Annuaire des Marées des Côtes de France pour 1865. 12mo. *Paris* 1863.	———
——— Instructions Nautiques sur les côtes est de la Chine, &c., traduites sur la dernière édition du China Pilot, par M. de Vautré. 8vo. *Paris* 1863.	———

PRESENTS.	DONORS.
ACADEMIES and SOCIETIES (*continued*).	
Paris:—	
Dépôt de la Marine. Pilote de l'île Guernesey, traduit par M. Massias. 8vo. *Paris* 1864.	The Dépôt de la Marine.
——— Instructions Nautiques sur les côtes de la Patagonie, par Paul Martin. 8vo. *Paris* 1863.	———
——— Pilote de l'île Vancouver, par G. H. Richards, traduit par H. Périgot. 8vo. *Paris* 1863.	———
——— Formule générale pour trouver la Latitude et la Longitude, par L. Pagel. 8vo. *Paris* 1863.	———
——— Instructions Nautiques sur la Mer Baltique et le Golfe de Finlande, par A. Le Gras. Tome I. 8vo. *Paris* 1864.	———
——— Les côtes du Brésil, Description et Instructions Nautiques, par Ernest Mouchez. II^e section. 8vo. *Paris* 1864.	———
——— Maps and Charts.	———
Annales des Mines. 6^e série. 2^e–6^e liv. de 1864; 1^e liv. de 1865. 8vo. *Paris*.	L'École des Mines.
Annuaire de l'Institut des Provinces, des Sociétés Savantes et des Congrès Scientifiques. 2^e série. Vol. VI. 8vo. *Paris* 1864.	The Institute.
Bulletin International de l'Observatoire Impérial. June 11 to December 31, 1864; January 1 to June 14, 1865. folio. *Paris*.	The Observatory.
Comptes Rendus des Séances et Mémoires de la Société de Biologie. 3^e série. Tome V. Année 1863. 8vo. *Paris* 1864.	The Society.
Bulletin de la Société d'Encouragement pour l'Industrie Nationale. 2^e série. Tome XI. January to December 1864; Tome XII. January to April 1865. 4to. *Paris*.	The Society.
Bulletin de la Société de Géographie. 5^e série. Tome VII. 8vo. *Paris* 1864.	The Society.
Bulletin de la Société Géologique. 2^e série. Tome XXII. feuilles 1–7. 8vo. *Paris* 1864.	The Society.
Penzance:—Royal Geological Society of Cornwall. Forty-sixth and Forty-seventh Annual Reports. 8vo. *Penzance* 1860.	The Society.
Philadelphia:—	
Academy of Natural Sciences. Proceedings. 1864. Nos. 1–5. 8vo. *Philadelphia*.	The Academy.
Franklin Institute. Journal. Nos. 461–472. 8vo. *Philadelphia* 1864–65.	The Institute.
——— Charter and By-Laws. 8vo. *Philadelphia* 1864.	———
Rome:—	
Atti dell' Accademia Pontificia de' Nuovi Lincei. Anno XVII. Sess. 1–7. 4to. *Roma* 1864.	The Academy.
Bullettino Meteorologico dell' Osservatorio del Collegio Romano, compilato dal P. Angelo Secchi. Vol. II. No. 12; Vol. III. Nos. 5, 6, 8, 10–13; Vol. IV. Nos. 1, 2 & 4. 4to. *Roma* 1863–65.	The Observatory.
Memorie dell' Osservatorio del Collegio Romano. Nuova serie. Vol. II. 4to. *Roma* 1863.	The College.
Rouen:—Congrès Scientifique de France. 32^e session. Rouen. 31 Juillet 1865. 4to. *Rouen*.	The Congress.
St. Petersburg:—	
Mémoires de l'Académie Impériale des Sciences. Tome V. Nos. 1–9; Tome VI. Nos. 1–12. 4to. *St. Pétersbourg* 1862–63.	The Academy.

Presents.	Donors.
ACADEMIES and SOCIETIES (*continued*).	
St. Petersburg:—	
Mémoires de l'Académie Impériale des Sciences. Bulletin. Tome V. Nos. 3–8; Tome VI. Nos. 1–5; Tome 7. Nos. 1 & 2. 4to. *St. Pétersbourg* 1863.	The Academy.
Annales de l'Observatoire Physique Central de Russie, par A. T. Kupffer. Années 1860 & 1861. 4to. *St. Pétersbourg* 1863.	The Administration of Mines.
Comptes Rendus Annuels. Années 1861, 1862 & 1863. 4to. *St. Pétersbourg* 1862–64.	———
Considérations sur la prévision des tempêtes et spécialement sur celles du 1 au 4 Décembre 1863. 4to. *St. Pétersbourg* 1864.	———
Horæ Societatis Entomologicæ Rossicæ. Fasciculus 1. 8vo. *Petropoli* 1861.	The Linnean Society.
Stockholm:—	
Kongliga Svenska Vetenskaps-Akademiens Handlingar. Ny Följd. Bandet IV. Häftet 2. 4to. *Stockholm* 1862.	The Academy.
Öfversigt af . . . Förhandlingar. Årgången XX. 8vo. *Stockholm* 1864.	———
Meteorologiska Jakttagelser i Sverige. Bandet IV. 4to. *Stockholm* 1862.	———
Toronto:—	
The Canadian Journal of Industry, Science, and Art. New series. Nos. 52–56. 8vo. *Toronto* 1864–65.	The Canadian Institute.
Abstracts of Meteorological Observations made at the Magnetical Observatory, 1854–59, 1860–62. 2 vols. 4to. *Toronto* 1864.	The Observatory.
Toulouse:—Mémoires de l'Académie Impériale des Sciences, Inscriptions et Belles-Lettres. 6^{e} série. Tome II. 8vo. *Toulouse* 1864.	The Academy.
Upsala:—	
Universitets Arskrift, 1863. 8vo. *Upsala.*	The University.
Nova Acta Regiæ Societatis Scientiarum Upsaliensis. Seriei tertiæ. Vol. V. fasc. 1. 4to. *Upsaliæ* 1864.	The Society.
Refractors-Beobachtungen der K. Universitäts-Sternwarte in Upsala vom Feb. 1862 bis Jan. 1864. 8vo. *Upsala* 1864.	The Observatory.
Utrecht:—Meteorologische Waarnemingen in Nederland en zijne Bezittingen, door het Koninklijk Nederlandsch Meteorologisch Instituut, 1863. (2 copies.) 4to. *Utrecht* 1864.	The Institute.
Venice:—	
Atti dell' Ateneo Veneto. Serie 2. Vol. I. puntata 1. 8vo. *Venezia* 1864.	The Athenæum.
Memorie dell' I. R. Istituto Veneto di Scienze, Lettere ed Arti. Vol. II. parte 1^{a} e 2^{a}. 4to. *Venezia* 1862–64.	The Institute.
Atti. Serie terza. Tomo IX. disp. 1–10; Tomo X. disp. 1–4. 8vo. *Venezia* 1863–65.	———
Vienna:—	
Denkschriften der kaiserlichen Akademie der Wissenschaften. Math.-Naturw. Classe: Bände XXII. & XXIII. Phil.-Hist. Classe: Band XIII. 4to. *Wien* 1864.	The Academy.
Sitzungsberichte. Math.-Nat. Classe. Band XLVII. Abth. 2. Heft 5; Band XLVIII. Abth. 1. Hefte 1–5, Abth. 2. Hefte 1–5; Band XLIX. Abth. 2. Hefte 1–5; Band L. Abth. 1. Hefte 1. 8vo. *Wien* 1863–64.	The Academy.
——— Phil.-Hist. Classe. Band XLII. Hefte 1–3; Band XLIII. Hefte 1 & 2; Band XLIV. Hefte 1–3; Band XLV. Hefte 1–3; Band XLVI. Hefte 1–3. 8vo. *Wien* 1863–64.	———

Presents.	Donors.
ACADEMIES and SOCIETIES (*continued*).	
Vienna:—	
Anzeiger. Math.-Naturw. Classe. 1. Jahrgang, 1864, Nr. 14–28; 1865, Nr. 1–14. 8vo. *Wien* 1864–65.	The Academy.
Almanach, Jahrgang 13 & 14. 1863, 1864. 8vo. *Wien.*	———
Tabulæ Codicum Manu Scriptorum præter Græcos et Orientales in Bibliotheca Palatina Vindobonensi asservatorum: edidit Academia Cæsarea Vindobonensis. Vol. I. 8vo. *Vindobonæ* 1864.	———
Mittheilungen der k.-k. geographischen Gesellschaft. VII. Jahrgang, 1863. 8vo. *Wien.*	The Society.
Jahrbuch der k.-k. geologischen Reichsanstalt. B. XIII. Nr. 4; B. XIV. Nr. 2, 3 & 4. 8vo. *Wien* 1863–64.	The Institute.
Bericht über die Erhebungen der Wasser-Versorgungs-Commission des Gemeinderathes der Stadt Wien. I. Text 4to. II. Atlas in case. *Wien* 1864.	Sir Charles Lyell, Bart., F.R.S.
Washington:—	
Report of the Commissioner of Patents for the Year 1861. Arts and Manufactures. 2 vols. 8vo. *Washington* 1863.	The Patent-Office, Washington.
Smithsonian Contributions to Knowledge. Vol. XIII. 4to. *Washington* 1864.	The Smithsonian Institution.
Smithsonian Miscellaneous Collections. Vol. V. 8vo. *Washington* 1864.	———
Würzburg:—	
Physikalisch-medicinische Gesellschaft. Würzburger naturwissenschaftliche Zeitschrift. Band IV. Hefte 2 & 3; Band V. Hefte 1–4. 8vo. *Würzburg* 1863–64.	The Society.
Würzburger medicinische Zeitschrift. Band V. Hefte 2–6. 8vo. *Würzburg* 1864.	———
Zürich:—	
Vierteljahrsschrift der naturforschenden Gesellschaft. Jahrgang 7. Hefte 3 & 4; Jahrg. 8. Hefte 1–4. 8vo. *Zürich* 1862–63.	The Society.
An die Zürcherische Jugend auf die Jahre 1863, 1864. 4to *Zürich.*	———
Nouveaux Mémoires de la Société Helvétique des Sciences Naturelles. Band XX. 4to. *Zürich* 1864.	The Society.
ADOLPH (W.) The Simplicity of the Creation, or the Astronomical Monument of the Blessed Virgin, a New Theory of the Solar System. 8vo. *London* 1864.	The Author.
AIRY (G. B., F.R.S.) Astronomical, Magnetical, and Meteorological Observations made at the Royal Observatory, Greenwich, in 1862. 4to. *London* 1864.	The Lords Commissioners of the Admiralty.
——— Seven-Year Catalogue of 2022 Stars, deduced from Observations extending from 1854 to 1860. 4to. *London* 1862.	———
——— Plan of the Buildings and Grounds of the Royal Observatory, Greenwich. 4to. *London* 1863.	———
——— Determination of the Longitude of Valentia in Ireland, by Galvanic Signals, in 1862. 4to. *London.*	———
AKIN (Dr. C. K.) On the Transmutation of Spectral Rays. Part 1. (Excerpt from Brit. Assoc. Rep.) 8vo. *London* 1863.	The Author.
——— On the Origin of Electricity. (Excerpt from Trans. Camb. Phil. Soc.) 4to. *Cambridge* 1865.	———

PRESENTS.	DONORS.
ALFONSO X. DE CASTILLA. Libros del Saber de Astronomia, copilados, anotados y comentados por Manuel Rico y Sinobas. 2 vols. fol. *Madrid* 1863.	The Royal Academy of Madrid.
ANDREW (J. A.) Address to the Legislature of Massachusetts. 8vo. *Boston* 1864.	The Legislature.
ANONYMOUS:—	
Account of several New Inventions and Improvements now necessary for England. *London* 1691.—The New Invention of Mill'd Lead. *London* 1691. In 1 vol. 12mo.	H. Dircks, Esq.
American Ephemeris and Nautical Almanac for 1865. 8vo. *Washington* 1863.	The Secretary of the United States Navy.
Army Lists of the Roundheads and Cavaliers, containing the Names of the Officers in the Royal and Parliamentary Armies of 1642. Edited by Edward Peacock. 4to. *London* 1863.	H. Dircks, Esq.
Army Medical Department. Statistical, Sanitary, and Medical Reports for 1862. 8vo. *London* 1864.	The Director-General.
Aspects of Nature, and their Teachings. 8vo. 1864.	The Author.
Calendar of the M'Gill College and University, Montreal. 8vo. *Montreal* 1864.	The University.
Catalogue of Additions to the Manuscripts in the British Museum in the years 1846–47. 8vo. *London* 1864.	The British Museum.
Catalogue of Autograph Letters forming part of the Collection of a Member of the Incorporated Law Society. 4to. *London* 1862.	H. Dircks, Esq.
Catalogue of the Library of H.R.H. the Princess Elizabeth, Landgravine of Hesse Homburg. 8vo. *London* 1863.	———
Catalogue of the Melbourne Public Library. 8vo. *Melbourne* 1861.	The Library.
Certain Necessary Directions, as well for the Cure of the Plague as for preventing the Infection. 4to. *London* 1665.	H. Dircks, Esq.
Companion (A) to Ragland Castle. 4to. *Monmouth* 1833.	———
Constitutions of the Musæum Minervæ. 4to. *London* 1636.	———
Des eröfneten Ritter Platz anderer Theil. 8vo. *Hamburg* 1704.	———
Description d'un Appareil destiné a reproduire les Aurores Polaires et les phénomènes qui les accompagnent. 8vo. *Paris.*	The Author.
Descriptive Catalogue of the Pathological Specimens in the Museum of the Royal College of Surgeons. Supplement II. 4to. *London* 1864.	The College.
Descriptive Inventory of James Cox's Museum. 4to. *London* 1774.	H. Dircks, Esq.
Discourse of the Necessity of encouraging Mechanick Industry. 4to. *London* 1690.	———
English Life Table. Tables of Lifetimes, Annuities, and Premiums, with an Introduction by W. Farr, M.D., F.R.S. 8vo. *London* 1864.	The Registrar-General.
Entomologist's Annual for 1865. 12mo. *London* 1865.	H. T. Stainton, Esq.
First Biennial Report of the Progress of the Geological Survey of Michigan. 8vo. *Lansing* 1861.	A. Winchell, Esq.
General Description of Sir John Soane's Museum. 12mo. *London.*	The Trustees.
Humane Industry; or a History of most Manual Arts. 8vo. *London* 1661.	H. Dircks, Esq.
Introductory Report of the Commissioner of Patents for 1863. 8vo. *Washington.*	The Commissioner.
London University Calendar, 1865. 8vo.	The University.
London's Dreadful Visitation, or a Collection of all the Bills of Mortality for this present year. 4to. *London* 1665.	H. Dircks, Esq.

PRESENTS.	DONORS.
ANONYMOUS (*continued*).	
Magnum in Parvo, or the Practice of Geometry, by R. P. 8vo. *London* 1671.	H. Dircks, Esq.
Mercury; or the Secret and Swift Messenger. 8vo. *London* 1641.	———
New Touch-stone for Gold and Silver Wares. 8vo. *London* 1679.	———
Notice sur les Observations Météorologiques faites dans les Pays Bas et dans ses possessions aux Indes Orientales et Occidentales. 8vo. *Utrecht* 1858.	The Meteorological Institute, Utrecht.
Observations Météorologiques faites à Nijné-Taguilsk, année 1863. 8vo. *Paris* 1864.	The Prince Demidoff.
Pleasant and Compendious History of the first Inventers and Instituters of the most famous Arts, Misteries, Laws, Customs, and Manners in the Whole World. 12mo. *London* 1686.	H. Dircks, Esq.
Reise der österreichischen Fregatte Novara um die Erde. Geologischer Theil. Band I. 4to. *Wien* 1864.	Dr. F. von Hochstetter.
——— Statistisch-commercieller Theil, von K. von Scherzer. Band I. 4to. *Wien* 1864.	Dr. von Scherzer.
Report of the Superintendent of the Coast Survey, showing the Progress of the Survey during 1861. 4to. *Washington* 1862.	The Survey.
Salomons Pest-House, or Towre Royall, newly re-edified and prepared to preserve Londoners from the Plague. 4to. *London* 1630.	H. Dircks, Esq.
Shutting (The) up of Infected Houses, as it is practised in England, soberly debated. 4to. *London* 1665.	———
South (The) Sea Bubble, and the numerous fraudulent projects to which it gave rise in 1720. 12mo. *London* 1825.	———
Statistique de la France.—Prix et Salaires a diverses époques. Deuxième série. Tome XII. 4to. *Strasbourg* 1863.	Unknown.
Sveriges Geologiska Undersökning pa offentlig bekostnad utförd, under ledning af A. Erdmann. Parts 6–13. 8vo. *Stockholm* 1863–64.	The Geological Board of Sweden.
Third Report of the Commission appointed to inquire into the best mode of distributing the Sewage of Towns. 8vo. *London* 1865.	J. B. Lawes, F.R.S.
Transactions and Reports of the Special Committee on Iron, between 21st January 1861 and March 1862. 1862–63. 3 vols. folio. Photographs to accompany the Reports, in 6 vols. oblong.	The Lords Commissioners of the Admiralty.
Treasure of Evonymus, conteyninge the Wonderfull hid Secretes of Nature, touchinge the most apte formes to prepare and destyl Medicines, for the conservation of helth, translated by P. Morwing. 4to. *London* 1559.	H. Dircks, Esq.
True and Faithful Account of the several Informations exhibited to the Committee appointed by the Parliament to inquire into the late dreadful burning of the City of London. 4to. 1667.	———
Twenty-eighth Annual Report of the Council of the Art-Union of London. 8vo. *London* 1864.	The Art-Union.
Weekly Returns of Births and Deaths in the City of Dublin. Vol. I. 1864, Nos. 1–52 (wanting No. 29); Vol. II. 1865, Nos. 1–23. Quarterly Returns of Births and Deaths, 1864, Nos. 1 & 2. 8vo. *Dublin* 1864–65.	The Registrar-General, Ireland.
Witty Apophthegms, delivered at several times and upon several occasions by King James, King Charls, the Marquess of Worcester, Lord Bacon, and Sir Thomas Moor. 12mo. *London* 1669.	H. Dircks, Esq.
ARGYLL (Duke of, F.R.S.) Address to the Royal Society of Edinburgh, 5th December, 1864. 8vo. *Edinburgh* 1864.	The Author.

PRESENTS.	DONORS.
BABBAGE (C., F.R.S.) Passages from the Life of a Philosopher. 8vo. *London* 1864.	The Author.
BACHE (A. D.) Records and Results of a Magnetic Survey of Pennsylvania and Parts of adjacent States. (Excerpt from Smithsonian Contrib.) 4to. *Washington* 1863.	The Author.
BACON (Francis) The Wisedome of the Ancients, done into English by Sir A. Gorges. 12mo. *London* 1619.	H. Dircks, Esq.
BAKER (T.) The Steam-Engine, or the Powers of Flame: an Original Poem. 8vo. *London* 1857.	————
BARBER (J. T.) A Tour through South Wales and Monmouthshire. 8vo. *London* 1803.	————
BATE (J.) The Mysteries of Nature and Art, in foure severall parts, the second edition. (The Booke of Extravagants.) 4to. *London* 1635.	————
BAYLY (T.) Herba Parietis, or the Wall-Flower: a History which is partly true, partly romantick, morally divine. folio. *London* 1650.	————
———— The Golden Apophthegms of his Royall Majesty King Charles I. and Henry Marq. of Worcester. 4to. *London* 1660.	————
BEATTIE (W.) Raglan Castle. 8vo. *London*.	————
BECHI (E.) I Soffioni Boraciferi di Triavale. Memoria letta alla Reale Accademia dei Georgofili. 8vo. 1863.	The Author.
BELAVENETZ (Captain) Blank Forms for the Computation of the Deviations and Magnetic Observations on Board of Ship. 8vo. *St. Pétersbourg* 1864.	The Author.
———— On the Magnetic Condition of the Iron-clad 'Pervenetz.' 8vo. *St. Pétersbourg* 1865.	————
BELLET (L.) et ROUVRE (C. de) Notice sur le nouveau système de Locomotive Électro-Magnétique. 8vo. *Paris* 1864.	Mons. L. Bellet.
BENSON (L. S.) Argumentum ad Hominem. Geometrical Disquisitions. 8vo. *London* 1864.	The Author.
BESSONI (J.) Il Theatro de gl' Instrumenti & Machine. fol. *Lione* 1582.	H. Dircks, Esq.
[BIRCH (T.)] An Inquiry into the share which K. Charles I. had in the Transactions of the Earl of Glamorgan, for bringing over a body of Irish Rebels to assist that King in 1645 and 1646. 8vo. *London* 1747. Second edition, 8vo. *London* 1756.	————
BOATE (G.) Irelands Naturall History, now published by S. Hartlib. 8vo. *London* 1652.	————
BÖHM (J. G.) und ALLÉ (M.) Magnetische und meteorologische Beobachtungen zu Prag. Jahrgang 25. 4to. *Prag* 1865.	The Observatory.
BOND (H.) The Longitude Found, or a Treatise shewing an easie and speedy way to find the Longitude. 4to. *London* 1676.	H. Dircks, Esq.
[BOSWELL (Rev. J.)] The Case of the Royal Martyr considered with candour. 2 vols. 8vo. *London* 1758.	————
BOUDIN (J. C. M.) Études Anthropologiques: Considérations sur le culte et les pratiques religieuses de divers peuples anciens et modernes: Culte de Phallus; Culte du Serpent. 8vo. *Paris* 1864.	G. Witt, F.R.S.
BOUTON (L.) Plantes Médicinales de Maurice. 8vo. *Port Louis* 1864.	The Author.
BRAYLEY (E. W., F.R.S.) Physical Constitution of the Sun. No. 2. 12mo. *London* 1865.	The Author.

PRESENTS.	DONORS.
BRODIE (Sir B. C., F.R.S.) Works, with an Autobiography, collected and arranged by Charles Hawkins. 3 vols. 8vo. *London* 1865.	The Editor.
BROWN (J.) The Description and Use of a Joynt-Rule. 12mo. *London* 1661.	H. Dircks, Esq.
———— The Triangular Quadrant. 12mo. *London* 1662.	————
BURT (T. S., F.R.S.) Miscellaneous Papers on Scientific Subjects. Vol. III. Part 2. 8vo. *London* 1865.	The Author.
CALVERT (F.) Contributions towards the Ancient Geography of the Troad: On the site of Gergis. 8vo.	The Author.
CAPRA (A.) La Nuova Architettura Famigliare. 4to. *Bologna* 1678.	H. Dircks, Esq.
CAUTLEY (Colonel Sir Proby T., F.R.S.) Ganges Canal: a Valedictory Note to Sir Arthur Cotton. 8vo. *London* 1864.	The Author.
CECCHI (F.) e ANTONELLI (G.) Il Barometro Areometrico a Bilancia della Loggia dell' Orgagna in Firenze. 8vo. *Pisa* 1863.	The Authors.
CERVO (F.) Legge per la Religione in Italia, Progetto. 8vo. *Napoli* 1864.	The Author.
CHALLIS (Rev. J., F.R.S.) Astronomical Observations made at the Observatory of Cambridge. Vol. XX. 4to. *Cambridge* 1864.	The Syndicate of the Observatory.
CHEVREUL (M. E., For. Mem. R.S.) Recherches Chimiques sur la Teinture: douzième à quatorzième mémoires. 4to. *Paris* 1863. (Two copies.)	The Author.
COLDEN (C.) The Principles of Action in Matter, the Gravitation of Bodies and the Motions of the Planets explained from those Principles. 8vo. *London* 1751.	H. Dircks, Esq.
CORVISART (L.) Collection de mémoires sur une fonction méconnue du Pancréas, la Digestion des Aliments Azotés. 8vo. *Paris* 1857–1863.	The Author.
CRÜGER (H.) On the Meteorology of Trinidad. 8vo. *London* 1864.	The Scientific Association of Trinidad.
DAUBENY (C., F.R.S.) Essay on the Trees and Shrubs of the Ancients. 8vo. *Oxford* 1865.	The Author.
DAVIS (J. B.) The Neanderthal Skull: its peculiar conformation explained anatomically. 8vo. *London* 1864.	The Author.
———— Sur les Déformations Plastiques du Crâne. 8vo. *Paris*.	————
DAVY (Dr. John, F.R.S.) On some of the more important Diseases of the Army, with Contributions to Pathology. 8vo. *London* 1862.	The Author.
———— Physiological Researches. 8vo. *London* 1863.	————
———— Letters to the Editors of the Philosophical Magazine, in reply to charges made by C. Babbage, Esq., against the late Sir Humphry Davy, when President of the Royal Society. (Excerpt from Phil. Mag.) 8vo. *London* 1864–65.	————
———— The same. (2 copies.)	Sir James South, F.R.S.
DEAN (J.) The Gray Substance of the Medulla Oblongata and Trapezium. Photographs. (Excerpt from Smithsonian Contributions.) 4to. *Washington* 1864.	The Author.
DE LA RIVE (A.) Quelques considérations sur les Courants Électriques Terrestres. 8vo. *Genève* 1865.	The Author.
DESARGUES. Œuvres, réunies et analysées par M. Poudra. 2 vols. 8vo. *Paris* 1864.	The Editor.
DIGGES (L.) A Prognostication everlastinge of righte good effecte, corrected and augmented by T. Digges. 4to. *London* 1576.	H. Dircks, Esq.

PRESENTS.	DONORS.
DILLON (E. W.) Physiological Researches relative to the functions of the Cephalic Nerves. 8vo. *Berne* 1864.	The Author.
DIRCKS (H.) A Biographical Memoir of Samuel Hartlib, Milton's familiar friend, with Bibliographical Notices of Works published by him, and a reprint of his pamphlet entitled "An Invention of Engines of Motion." 8vo. *London* 1865.	The Author.
——— The Life, Times, and Scientific Labours of the second Marquis of Worcester, to which is added a reprint of his Century of Inventions, 1663, with a Commentary thereon. 8vo. *London* 1865.	———
DOVE (H. W., For. Mem. R.S.) Darstellung der Wärmeerscheinungen durch fünftägige Mittel. Theil II. 4to. *Berlin* 1863.	The Author.
——— Die Witterungserscheinungen des nördlichen Deutschlands im Zeitraum von 1858–1863. (Preussische Statistik, VI.) 4to. *Berlin* 1864.	———
DOYNE (W. T.) Report upon the Plains and Rivers of Canterbury, New Zealand. folio. *Christchurch* 1864.	The Author.
DUHAMEL (J. M. C.) Des Méthodes dans les Sciences de Raisonnement. 8vo. *Paris* 1865.	The Author.
DUTOIT (Eug.) Die Ovariotomie in England, Deutschland und Frankreich. 8vo. *Würzburg* 1864.	The Author.
[DYMOCK (Cressy)] An Invention of Engines of Motion lately brought to perfection. 4to. *London* 1651.	H. Dircks, Esq.
EDWARDS (G. N.) and WILLETT (A.) Statistical Tables of the Patients under treatment in the Wards of St. Bartholomew's Hospital during 1864. 8vo. *London* 1865.	The Hospital.
ELGAR (J.) Hemerologium, or Diary for Nineteen Years: MS., 1750. J. Newton, Tabulæ Mathematicæ, 1654, &c. In 1 vol. 12mo.	H. Dircks, Esq.
ÉLIE DE BEAUMONT (L., For. Mem. R.S.) Tableau des Données Numériques qui fixent 159 cercles du Réseau Pentagonal. 4to. *Paris* 1863.	The Author.
——— Tableau des Données Numériques qui fixent les 362 points principaux du Réseau Pentagonal. 4to. *Paris* 1864.	———
ELLIS (H. S.) On the Oyster (*Ostrea edulis*) in the West of England. 8vo. *London* 1865.	The Author.
ESCHER (P.) Elementare Theorie der Differenzen Briggischer und trigonometrischer Logarithmen. 4to. *Wien* 1864.	The Author.
EUDES-DESLONGCHAMPS (E.) Études Critiques sur les Brachiopodes Nouveaux ou peu connus. Fasc. 3. 8vo. *Caen* 1863.	The Author.
——— Documents sur la Géologie de la Nouvelle-Calédonie. 8vo. *Caen* 1864.	———
——— Études sur les Étages Jurassiques Inférieurs de la Normandie. 4to. *Paris* 1864.	———
——— Recherches sur l'Organisation du Manteau chez les Brachiopodes Articulés et principalement sur les spicules calcaires contenus dans son intérieur. 4to. *Paris* 1864.	———
EVANS (F. J., F.R.S.) and SMITH (A., F.R.S.) Ueber die Deviationen des Compasses. Nach dem Englischen, von Dr. F. Schaub. 8vo. *Wien* 1864.	The Translator.

PRESENTS.	DONORS.
EVEREST (Sir G., F.R.S.) Tables for the use of the Great Trigonometrical Survey. MS. 4to.	The Author.
EWBANK (T.) A Descriptive and Historical Account of Hydraulic and other Machines for raising Water, Ancient and Modern. 8vo. *New York* 1854.	H. Dircks, Esq.
FALCONER (J.) Cryptomenysis Patefacta: or the Art of secret Information disclosed without a Key. 8vo. *London* 1685.	———
FIGUIER (L.) Exposition et Histoire des principales Découvertes Scientifiques modernes. 4 vols. 8vo. *Paris* 1862.	———
FLOWER (W. H., F.R.S.) Annual Report of the Conservator of the Royal College of Surgeons to the Museum Committee, 1865. 8vo. *London.*	The College.
FOSBROKE (Rev. T. D.) A Picturesque and Topographical Account of Raglan Castle. 8vo. *Monmouth* 1831.	H. Dircks, Esq.
FOX (F. and C.) Report respecting the past and present state of Brislington House, near Bristol, a private Asylum for the Insane. 8vo. *Bristol.*	The Authors.
FRENCH (J.) The Art of Distillation. Second edition. 4to. *London* 1653.	H. Dircks, Esq.
FRITZSCHE (J.) Ueber den Pollen. 4to. *St. Petersburg* 1837.	The Author.
——— A Series of Thirty Tracts on Chemistry &c. 8vo. *St. Petersburg* 1839–64.	———
GAFFAREL (J.) Unheard-of Curiosities, concerning the Talismanical Sculpture of the Persians, the Horoscope of the Patriarkes, and the Reading of the Stars. Englished by E. Chilmead. 8vo. *London* 1650.	H. Dircks, Esq.
GALILEO (Galilei) Opere; prima edizione completa condotta sugli autentici Manoscritti Palatini. 15 volumes in 16, and Supplemento. 8vo. *Firenze* 1842–1856.	The Museum of Physical Science and Natural History, Florence.
GARRIGOU (F.) Lettre à M. le professeur N. Joly. 8vo. *Toulouse* 1862.	The Author.
——— L'Homme Fossile des Cavernes de Lombrive et de Lherm (Ariége). 8vo. *Toulouse* 1862.	———
——— Étude Chimique et Médicale des Eaux Sulfureuses d'Aix (Ariége). 8vo. *Paris* 1862.	———
GIANELLI (G. L.) La Vaccinazione e le sue leggi in Italia, Memoria. 4to. *Milano* 1864.	The Author.
GLOCKER (E. F.) Zwei Karten zur geognostischen Beschreibung der Preuss. Oberlausitz. 4to. 1857.	The Naturforschenden Gesellschaft, Görlitz.
GOSAIN BUNWALIDAS, surnamed VALI. Goolzare Hall, or the Garden of Divine Ecstacy. Published by Burjorjee Sorabjee Ashburner, from the Original Persian Work. 8vo. *Bombay* 1862.	The Editor.
GOYA (F.) Los Proverbios: Coleccion de diez y ocho laminas. oblong. *Madrid* 1864.	The Academy of San Fernando.
GRANT (R., F.R.S.) Report of the Professor of Astronomy in the University of Glasgow. 8vo. *Glasgow* 1865.	The University.
GRAY (J. E., F.R.S.) A Second Letter to the Earl of Ellesmere, on the Management of the Library of Printed Books in the British Museum. 8vo. *London* 1849.	The Author.
——— Handbook of British Water-Weeds, or Algæ. 8vo. *London* 1864.	———
——— Address to the Zoological and Botanical Section of the British Association, Bath, Sept. 15, 1864. 8vo. *London* 1864.	———

PRESENTS.	DONORS.
GROTIUS (H.) Syntagma Arateorum: Opus Poeticæ et Astronomiæ studiosis utilissimum. 4to. *Lugd. Bat.* 1600.	H. Dircks, Esq.
GUEST (E., F.R.S.) The Invasion of Britain by Julius Cæsar: a Memoir. 8vo. *London* 1864.	The Author.
HAIDINGER (W., For. Mem. R.S.) A Series of Thirty-three Papers on Meteoric Subjects. 8vo. *Vienna* 1859–63.	The Author.
HAILES (J. D.) The Hailesian System of Astronomy. 8vo. *Cambridge* 1865.	The Author.
HALLIWELL (J. O., F.R.S.) Rara Mathematica, or a Collection of Treatises on the Mathematics. 8vo. *London* 1839.	H. Dircks, Esq.
HAMEL (J.) England and Russia, comprising the Voyages of J. Tradescant the Elder, Sir H. Willoughby, R. Chancellor Nelson, and others to the White Sea. Translated by J. S. Leigh. 8vo. *London* 1854.	———
HANNON. The Periplus of Hannon, King of the Karchedonians, concerning the Lybian parts of the Earth beyond the Pillars of Herakles. 4to. *London* 1864.	Dr. Simonides.
HARTLIB (S.) A Discours of Husbandrie used in Brabant and Flanders. 4to. *London* 1605 [1650].	H. Dircks, Esq.
——— His Legacie, or an Enlargement of the Discourse of Husbandry used in Brabant and Flaunders. 4to. *London* 1651.	———
——— His Legacy of Husbandry. 4to. *London* 1655.	———
——— An Essay for Advancement of Husbandry-Learning. 4to. *London* 1651.	———
——— The Reformed Husband-man. 4to. *London* 1651.	———
——— A Designe for Plentie, by an Universall planting of Fruit-Trees. 4to. *London.*	———
HARTNUP (J) Report of the Astronomer to the Marine Committee, Mersey Docks and Harbour Board. 8vo. *Liverpool* 1864.	The Committee.
HEATH (C.) Historical and Descriptive Accounts of the ancient and present state of Ragland Castle. 8vo. *Monmouth* 1825.	H. Dircks, Esq.
HELMHOLTZ (H., For. Mem. R.S.) Die Lehre von den Tonempfindungen als physiologische Grundlage für die Theorie der Musik: zweite Ausgabe. 8vo. *Braunschweig* 1865.	The Author.
HÉRICOURT (Comte Achmet d') Annuaire des Sociétés Savantes de la France et de l'Étranger. 2 vols. in 1. 8vo. *Paris* 1863–64.	H. Dircks, Esq.
——— ——— Tome II. 8vo. *Paris* 1864.	The Author.
HERMITE (—) Sur les Théorèmes de M. Kronecker relatifs aux formes quadratiques. 4to. *Paris* 1862.	The Author.
——— Sur un nouveau Développement en série des Fonctions. 4to. *Paris* 1864.	———
——— Remarque sur le développement de cos am *x*. 4to. *Paris* 1864.	———
HERO Alexandrinus. Spiritali . . . ridotti in lingua volgare da Alessandro Giorgi. 4to. *Urbino* 1592.	H. Dircks, Esq.
——— Gli Artificiosi e Curiosi Moti Spiritali, tradotti da G. B. Aleotti. 4to. *Bologna* 1647.	———
——— De gli Automati overo Machine se moventi libri due, tradotti da B. Baldi. 4to. *Venetia* 1589.	———

PRESENTS.	DONORS.
HERO Alexandrinus. Buch von Lufft- und Wasser-Künsten, aus dem Lateinischen ins Teutsche gebracht durch A. Carionem. 4to. *Franckfurt* 1688.	H. Dircks, Esq.
HERSCHEL (A. S.) On the Detonating Meteor of December 5, 1863. 8vo. *Manchester* 1864.	The Author.
——— and GREG (R. P.) General Radiant-Points of Shooting Stars. (Excerpt from Proc. Brit. Meteor. Soc.) 8vo. *London* 1865.	The Authors.
HODY (Baron de) Godefroid de Bouillon à Boulogne-sur-Mer, à Bruxelles, et à Jérusalem. 8vo. *Bruxelles* 1863.	The Author.
HOEK (M.) Recherches Astronomiques de l'Observatoire d'Utrecht. Liv. II. 4to. *La Haye* 1864.	The Observatory.
——— et OUDEMANS (A. C.) Recherches sur la quantité d'éther contenue dans les liquides. 4to. *La Haye* 1864.	———
——— Sur les Contractions dans les Mélanges de Liquides. 4to. *La Haye* 1864.	The Authors.
HORATIIS (C. de) Nuovi Elementi della Scienza Acustico-Musicale applicabili alla Scienza delle Arti. 8vo. *Napoli* 1865.	The Author.
HÖRNES (Dr. M.) Die fossilen Mollusken des Tertiär-Beckens von Wien. Band II. Nr. 5 & 6. Bivalven. 4to. *Wien* 1865.	The Geological Institute, Vienna.
HUGHES (D. E.) Expériences sur la Forme et la Nature des Électro-Aimants. 8vo. *Paris* 1864.	The Author.
HUGUENY (F.) Recherches expérimentales sur la Dureté des corps et spécialement sur celle des Métaux. 8vo. *Paris* 1865.	The Author, by R. Mallet, F.R.S.
——— Recherches sur la composition chimique et les propriétés qu'on doit exiger des Eaux Potables. 8vo. *Paris* 1865.	———
HUME (A.) Of the Orthographie and Congruitie of the Britan Tongue. Edited by H. B. Wheatley. 8vo. *London* 1865.	The Editor.
ICERY (M. E.) Mémoire sur le Pou à Poche Blanche. 8vo. *Maurice* 1864.	The Author.
JACKSON (S.) An Authentick Account of the Weights, Measures, Exchanges, Customs, Duties, Port Charges, &c., and correct Batty Tables, made use of and paid at the several ports of the East Indies. 4to. *London* 1764.	Dr. Sharpey, Sec. R.S.
JEFFREYS (J. G., F.R.S.) The Upper Tertiary Fossils at Uddevalla in Sweden: 1863. Report of the Committee appointed for exploring the Coasts of Shetland by means of the Dredge. (Excerpt from Report Brit. Assoc.) 8vo. *London* 1863.	The Author.
JOLY (N.) Conférence Publique sur l'Hétérogénie ou Génération Spontanée, 28 Juin 1864. 8vo. *Paris* 1864.	The Author.
JOUGLEZ DE LIGNE (—) La Rade de Dunkerque. 8vo. *Paris* 1864.	The Author.
JOURNALS:—	
Cosmos. June to December 1864; January to June 1865. 8vo. *Paris.*	The Editor.
Giornale per l'Abolizione della Pena di Morte; diretto da Pietro Ellero. 9–11. 8vo. *Bologna* 1864–65.	The Editor.
Les Mondes. June to December 1864; January to June 1865. 8vo. *Paris.*	The Editor.
Notes and Queries. June to December 1864; January to June 1865. 4to. *London.*	The Editor.
The American Journal of Science and Arts. Vol. XXXVIII. Nos. 112–114; Vol. XXXIX. Nos. 115–117. 8vo. *New Haven* 1864–65.	The Editors.

Presents.	Donors.
JOURNALS (*continued*).	
The Athenæum. June to December 1864; January to May 1865. 4to. *London.*	The Editor.
The Builder. June to December 1864; January to June 1865. folio. *London.*	The Editor.
The Chemical News. June to December 1864; January to June 1865. 4to. *London.*	The Editor.
The Educational Times. Vol. XVII. Nos. 40–47; Vol. XVIII. Nos. 49–51. 4to. *London* 1864–65.	The Editor.
The Intellectual Observer. July to September 1864; May 1865. 8vo. *London.*	The Editor.
The London Review. July to December 1864; January to June 1865. folio. *London.*	The Editor.
The Mining and Smelting Magazine. Vol. V. No. 30; Vol. VI. Nos. 31–36; Vol. VII. Nos. 37 & 38. 8vo. *London* 1864–65.	The Editor.
The Philosophical Magazine. July to December 1864; January to June 1865. 8vo. *London.*	W. Francis, Esq.
The Quarterly Journal of Science. Nos. 2–6. 8vo. *London* 1864–65.	The Editors.
The Reader. June to December 1864; January to June 1865. folio. *London.*	The Editor.
KÖLLIKER (A., For. Mem. R.S.) Kurzer Bericht über einige im Herbst 1864 an der Westküste von Schottland angestellte vergleichend-anatomische Untersuchungen. 8vo. *Würzburg* 1864.	The Author.
——— Icones Histiologicæ oder Atlas der vergleichenden Gewebelehre. Abth. I. 4to. *Leipzig* 1864.	———
KOPS (J.) en HARTSEN (F. A.) Flora Batava, Afbeelding en Beschrijving van Nederlandsche Gewassen. Afl. 187-191. 4to. *Amsterdam.*	H. M. the King of the Netherlands.
LA HIRE (P. de) Traité de Mécanique. 8vo. *Paris* 1695.	H. Dircks, Esq.
LAMONT (Dr. J., For. Mem. R.S.) Annalen der königlichen Sternwarte bei München. Bände XIII. & XIV. 8vo. *München* 1864–65.	The Observatory.
LANCIA DI BROLO (F.) Statistica dei Sordomuti di Sicilia nel 1863. 8vo. *Palermo* 1864.	The Author.
LA NOUË (Lord de) Politicke and Militarie Discourses, translated by E. A. 4to. *London* 1587.	H. Dircks, Esq.
LAWES (J. B., F.R.S.) On the Chemistry of the Feeding of Animals for the production of Meat and Manure. 8vo. *Dublin* 1864.	The Author.
——— and GILBERT (J. H., F.R.S.) Report of Experiments on the Growth of Wheat for twenty years in succession on the same land. 8vo. *London* 1864.	The Authors.
——— Further Report of Experiments with different Manures on Permanent Meadow Land. 8vo. *London* 1863.	———
LEIBNITII de Expeditione Ægyptiaca Scripta Omnia, adjecta præfatione historico-critica: edidit Onno Klopp. 8vo. *Hanoveræ* 1864.	The Editor.
LEVI (Leone) Introductory Lecture on International Maritime Law. 8vo. *London* 1865.	The Author.
LEVI (M. R.) La Patologia Cellulare considerata nei suoi fondamenti e nelle sue applicazioni. 8vo. *Venezia* 1863.	The Author.
LEYBOURN (W.) Pleasure and Profit, consisting of Recreations of divers kinds. fol. *London* 1694.	H. Dircks, Esq.

PRESENTS.	DONORS.
LILLY (W.) The Book of Knowledge, by Erra Pater. A rich Cabinet with Variety of Inventions, by J. White. *London* 1684. 2 vols. in 1. 8vo.	H. Dircks, Esq.
LITTROW (C. von) Annalen der k.-k. Sternwarte in Wien. Dritter Folge zwölfter Band. Jahrgang 1862. 8vo. *Wien* 1863.	The Observatory.
——— und WEISS (E.) Meteorologische Beobachtungen an der k.-k. Sternwarte in Wien von 1775 bis 1855. Band IV. 8vo. *Wien* 1863.	———
LODGE (E.) Portraits of the Earls of Worcester, 1544–1667. 8vo.	H. Dircks, Esq.
LOWE (E. J.) Our Native Ferns. Part XX. 8vo. *London* 1865.	The Author.
LUBBOCK (Sir J., F.R.S.) Prehistoric Times, as illustrated by Ancient Remains and the Manners and Customs of Modern Savages. 8vo. *London* 1865.	The Author.
LUCAS (C.) A Treatise named Lucarsolace, divided into fower Bookes. 4to. *London* 1590.	H. Dircks, Esq.
MACKENZIE (W.) Entoptics.—Letter from Dr. Jago. 8vo. *Glasgow* 1864.	The Author.
MAGRINI (Luigi) Sulla Importanza dei Cimelj Scientifici e dei Manoscritti di Alessandro Volta. 8vo. *Milano* 1864.	The Istituto Lombardo.
MAILLY (E.) Essai sur les Institutions Scientifiques de la Grande Bretagne et de l'Irlande. V. 8vo. *Bruxelles* 1865.	The Author.
MAIN (Rev. R., F.R.S.) Astronomical and Meteorological Observations made at the Radcliffe Observatory in the year 1862. Vol. XXII. 8vo. *Oxford* 1865.	The Radcliffe Trustees.
MAREY (E. J.) Physiologie Médicale de la Circulation du Sang. 8vo. *Paris* 1863.	The Author, by Dr. Humphry.
M[ARKHAM] (G.) The English Husbandman. 4to. *London* 1613.	H. Dircks, Esq.
——— The Souldiers Exercise, in three bookes. 4to. *London* 1639.	———
MARTINS (C.) Tableau Physique du Sahara Oriental de la province de Constantine, Souvenirs d'un Voyage exécuté pendant l'hiver de 1863 dans l'Oued-Rir et dans l'Oued-Souf. 8vo. *Paris* 1864.	The Author.
——— Deux Ascensions Scientifiques au Mont-Blanc, leurs résultats immediats pour la Météorologie, la Physique du Globe et les Sciences Naturelles. 8vo. *Paris* 1865.	———
MAUMENÉ (E.) Théorie Géuérale de l'exercice de l'Affinité. 8vo. *Paris* 1864.	The Author.
MAUPERTUIS (De) Lettre sur le Progrès des Sciences. 8vo. 1752.	H. Dircks, Esq.
MEEHAN (Rev. C. P.) The Confederation of Kilkenny. 16mo. *Dublin* 1862.	———
MELSENS (—.) Deuxième Note sur les Matières Albuminoïdes. 8vo. *Bruxelles* 1864.	The Author.
MERET (L. E.) De l'Instinct et de l'Intelligence des Animaux. 8vo. *Paris* 1864.	The Author.
MERRIFIELD (C. W., F.R.S.) Address delivered at the opening of the Royal School of Naval Architecture and Marine Engineering at South Kensington, Nov. 1, 1864. 8vo. *London* 1864.	The Author.
MILLER (W. A., Treas. R.S.) Elements of Chemistry. Part 2. Inorganic Chemistry. Third edition. 8vo. *London* 1864.	The Author.
MILNER (Rev. T.) The Gallery of Geography. Parts 10–19. 8vo. *London* 1864.	Messrs. Chambers.
MILTON (John) Of Education. To Master S. Hartlib. 4to.	H. Dircks, Esq.

PRESENTS.	DONORS.
MIQUEL (F. A. G.) Annales Musei Botanici Lugduno-Batavi. Tom. I. fasc. 4–10. fol. *Amst.* 1863–64.	The Museum.
MISCELLANEOUS:—	
Collection of Oriental Manuscripts in the Zend, Pehlvi, and Persian Languages. Persian Dictionary, by Abdool Ruhim, 2 vols. 4to. Mahabharat, Persian translation, fol. Amrat Kand, Persian translation from Sanscrit, 4to. Hitopodesh, Persian translation from Sanscrit, 4to. Sárōd, Persian translation, 4to. The Gita, Persian translation from Sanscrit, 4to. Vendidad, original text, fol. Vendidad, with Pehlvi and Persian translations, fol. Shah Namah, 2 vols. fol. Commentary on Shah Namah, 4to. Jog bishust, translated from Sanscrit, 4to. Laksmi Narayan, translated from Sanscrit, fol. Gulshan-i-raz, 4to. Commentary on Gulshan-i-raz, 8vo. Kirān-ūl-Sá dain. 8vo. Sālūkh.(4), fol. & 8vo. Khordah Avasta, with Persian translation, 4to. The Divān of Áfiz, 8vo. Lexicon to Desātir, 4to. Arabic Syntax, 4to. Pehlvi Vocabulary, with Persian definitions, 8vo. Pehlvi Vocabulary, fol. Avasta, original text with Pehlvi translation, fol. Traditions of Zartusht, in Pehlvi, 4to. 1. History of the Peshdadyan; 2. Catechism in Zoroastrian Ritual; 3. Pehlvi Vocabulary; 4. Translation of Khorda Avasta; 5. Zend Vocabulary to No. 4. 4to. Letters by Harkaran, 4to. Letters by Mirza Mahomed Tahir, Minister to the King of Persia, 8vo. Letters by Sadyaki, 8vo. Letters by Hasni, 8vo. Letters by Berehmun, fol. Ethics, by Ali Shah of Bokhara, 4to. History of Framarz, son of Rūstam, 8vo. History of Ranjit Singh, fol. Biography of Zoroaster, by Behram, 4to. Rhetoric, by Khajah Mā'ruff, fol. Moral Precepts by Jāmir, fol. Morals by Hūsan ul Vaiz of Kāshag'h, 4to. Arjūn Bikhad, translation, fol. Gulzāri hāl, 4to. Fables in imitation of the Gulistan, 8vo. Allegorical Poem founded on the Kūrān, 8vo. The Lover and the Beloved, an Allegorical Poem, 8vo. Prayers and Praises to the Sun, &c., by Behram, 8vo. 1. Birds; 2. Diwān of Ūhmed, fol. Diwān of Saib, 8vo. Diwān of Sanāi, fol. Diwān of Wali. fol. The Mirror of Philosophy, fol. Collection of Essays on Meditation, fol. The Treasury of Knowledge. On the Connexion between the Soul and the Body, 4to. Mahomedanism and Zoroastrianism, an Essay, 8vo. A Discourse on the Zoroastrian Religion, 8vo. 1. Lunar Sect contrasted with the Zoroastrian; 2. Dictates of Jāmāsp on the Zoroastrian ritual, 4to. A Guide to the Study of the Kūrān, 8vo. The Religion of the Hindus compared with that of the Persians, 4to. On the Unity of God and his Manifestations, 8vo. On the Knowledge of God, fol. A Poem on Ali, 8vo. Kabalistic Philosophy, 8vo. Astronomy (2), 4to. Alchemy and Astronomy, 4to. 1. Astrology; 2. Incantations, believed to be cures for certain diseases, in the Zend languages, 4to. On the Constellations, 4to. On Intercalation, 4to. Geography, 4to. On Morals, 4to. On Ethics, 8vo. On Diseases and their treatment (3), 8vo and fol. Letters, 8vo. History of Kings and Prophets of Persia, fol. Doctrine of the Sheeahs, with Book of Martyrs, fol. Catechism, 4to. Moral Tales, 4to. Science of Rhetoric, Prosody and Rhyme, fol. Poem by Shiraz, fol. 1. On the Stars; 2. A Fable of Mouse and Cat; 3. Laws of the Stars; 4. Essay on Meditation, fol. 1. Poems, translated from the Sanscrit;	Burjorjee Sorabjee Ashburner, Esq.

PRESENTS.	DONORS.
MISCELLANEOUS (*continued*).	
2. History of the Turks; 3. Salukh; 4. Memoir of Chundreban, fol. 1. Translation of Jāwidān, from Pehlvi into Persian; 2. A medical work; 3. History of Astronomy. fol. 1. Prayers, poetical; 2. Theological Ethics; 3. Prayers, prose, 8vo. Arabic and Persian Dictionary, 4to. Pehlvi Dictionary, with Persian definitions and Commentary, 4to. A Liturgical Work, 8vo. Also a Collection of Thirty-six Volumes of Persian Manuscripts on paper, 8vo and fol., with titles untranslated. One MS. written on Palm leaves.	Burjorjee Sorabjee Ashburner, Esq.
Descriptive Diagram of a Seismometer invented by Lieut.-Colonel Ramstedt of Helsingfors, and named by him Telegraph of the Accidental Movements of the Earth. *Stockholm.*	The Author.
Photograph of the Moon (40 in. x 31 in.), by Warren De la Rue, F.R.S. Framed and glazed.	W. De la Rue, F.R.S.
Photo-lithographic Impressions of Traces produced simultaneously by the self-recording Magnetographs at Kew and Lisbon. 4to.	The Kew Observatory.
Portrait of Edward Somerset, Sixth Earl and Second Marquis of Worcester. Proof engraving by W. Faithorne. Framed and glazed. Funeral Certificate of the Marquess of Worcester, copied from the Original in the Heralds' Colledge, with the Arms emblazoned. Framed and glazed.	H. Dircks, Esq.
MOHN (H.) Bewölkung in Christiania. 8vo.	The Author.
———— Den Magnetiske Declination i Christiania udledet af Observationer 1842–1862. 8vo. 1863.	————
MOON (G. W.) The Dean's English: a Criticism on the Dean of Canterbury's Essay on the Queen's English. 8vo. *London* 1864.	The Author.
MOORE (Sir Jonas) Contemplationes Geometricæ, in two Treatises. 8vo. *London* 1660.	H. Dircks, Esq.
MORGAN (S.) Horologiographia Optica. Dialing Universall and perticuler. 4to. *London* 1652.	————
MUELLER (F., F.R.S.) Fragmenta Phytographiæ Australiæ. Vol. IV. 8vo. *Melbourne* 1863–64.	The Author.
———— The Plants indigenous to the Colony of Victoria. Lithograms. 8vo. *Melbourne* 1864–65.	————
———— The Vegetation of the Chatham Islands. 8vo. *Melbourne* 1864.	————
MURCHISON (Sir R. I., F.R.S.) Address at the Anniversary Meeting of the Royal Geographical Society, 23rd May 1864. 8vo. *London.*	The Author.
MYDORGE (C.) Examen du livre des Recreations Mathematiques et de ses Problemes en Geometrie, Mechanique, Optique et Catoptrique. 8vo. *Paris* 1638.	H. Dircks, Esq.
NAMIAS (G.) Della Infezione Biliosa del Sargue (Colemia) Discorsi. 8vo. *Venezia* 1863.	The Author.
NEPAIR (J.) A Description of the admirable Table of Logarithmes, translated by Edward Wright. 12mo. *London* 1618.	J. Robson, Esq.
NEWTON (H. A.) On November Star Showers. 8vo. *New Haven* 1864.	The Author.
———— Abstract of a Memoir on Shooting Stars. (Excerpt from Amer. Journ. Sci.). 8vo. *New Haven* 1865.	————

Presents.	Donors.
ORESME (N.) Traictie de la première invention des Monnoies, textes français et latin; et Traité de la Monnoie de Copernic, texte latin et traduction française, publiés et annotés par L. Wolowski. 8vo. *Paris* 1864.	The Editor.
ORMOND (Marquesse of)'s Proclamation concerning the Peace concluded with the Irish Rebells. 4to. *London* 1649.	H. Dircks, Esq.
PAGE (T.) Report upon the System of Weaving by compressed Air in the Pneumatic Loom. 4to. *Cambridge* 1864.	The Author.
PAGLIARDINI (Tito) Essays on the Analogy of Languages. Second Essay: The International Alphabet, or a Plea for Phonetic Spelling. 8vo. *London* 1864.	The Author.
PANCIROLLUS (Guido) Rerum Memorabilium sive Deperditarum partes prior et posterior, ab H. Salmuth. 4to. *Francofurti* 1631.	H. Dircks, Esq.
——— The History of many memorable things lost, which were in use among the Ancients. 2 vols. in 1. 8vo. *London* 1715.	———
PAPILLON (D.) A Practicall Abstract of the Arts of Fortification and Assailing. 4to. *London* 1645.	———
PARLATORE (F.) Studi Organografici sui fiori e sui frutti delle Conifere. 4to. *Firenze* 1864.	The Author.
——— Intorno due Dissertazioni Botaniche di Michelangelo Poggioli Considerazioni. 8vo. *Roma* 1864.	———
PARRY (C. C.) Physiographical Sketch of a portion of the Rocky Mountain Range. 8vo. 1862.	The Author.
PERIGAL (H.) Revolution and Rotation. 8vo. *London* 1864.	The Author.
PETTY (Sir W.) A Declaration concerning the newly invented Art of Double Writing. 4to. *London* 1648.	H. Dircks, Esq.
PHIPSON (T. L.) The Use of Salt in Agriculture. Prize Essays. 8vo. *Liverpool* 1863.	The Author.
——— The Utilization of Minute Life, being practical studies on Insects, Crustacea, Mollusca, Worms, Polypes, Infusoria and Sponges. 8vo. *London* 1864.	———
——— On Vanadium Ochre, and other sources of Vanadic Acid. (Excerpt from Chem. Soc. Journ.) 8vo. *London.*	———
——— Le Préparateur-Photographe ou Traité de Chimie à l'usage des Photographes. 12mo. *Paris* 1864.	———
——— Note on Vegetable Ivory. 8vo.	———
——— Description of a new Apparatus for producing Sulphuretted Hydrogen. 8vo.	———
PICTET (F. J.) Note sur la succession des Mollusques Gastéropodes pendant l'époque crétacée dans la région des Alpes Suisses et du Jura. 8vo. *Genève* 1864.	The Author.
PLANTAMOUR (E.) Résumé Météorologique de l'Année 1862–1863 pour Genève et le Grand St. Bernard. 8vo. *Genève* 1863–64.	The Author.
——— et HIRSCH (A.) Détermination Télégraphique de la Différence de Longitude entre les Observatoires de Genève et de Neuchâtel. 4to. *Genève* 1864.	The Authors.
PLAT (Sir H.) The Jewell House of Art and Nature, conteining divers rare and profitable Inventions. 4to. *London* 1594. Another edition, 4to. *London* 1653.	H. Dircks, Esq.

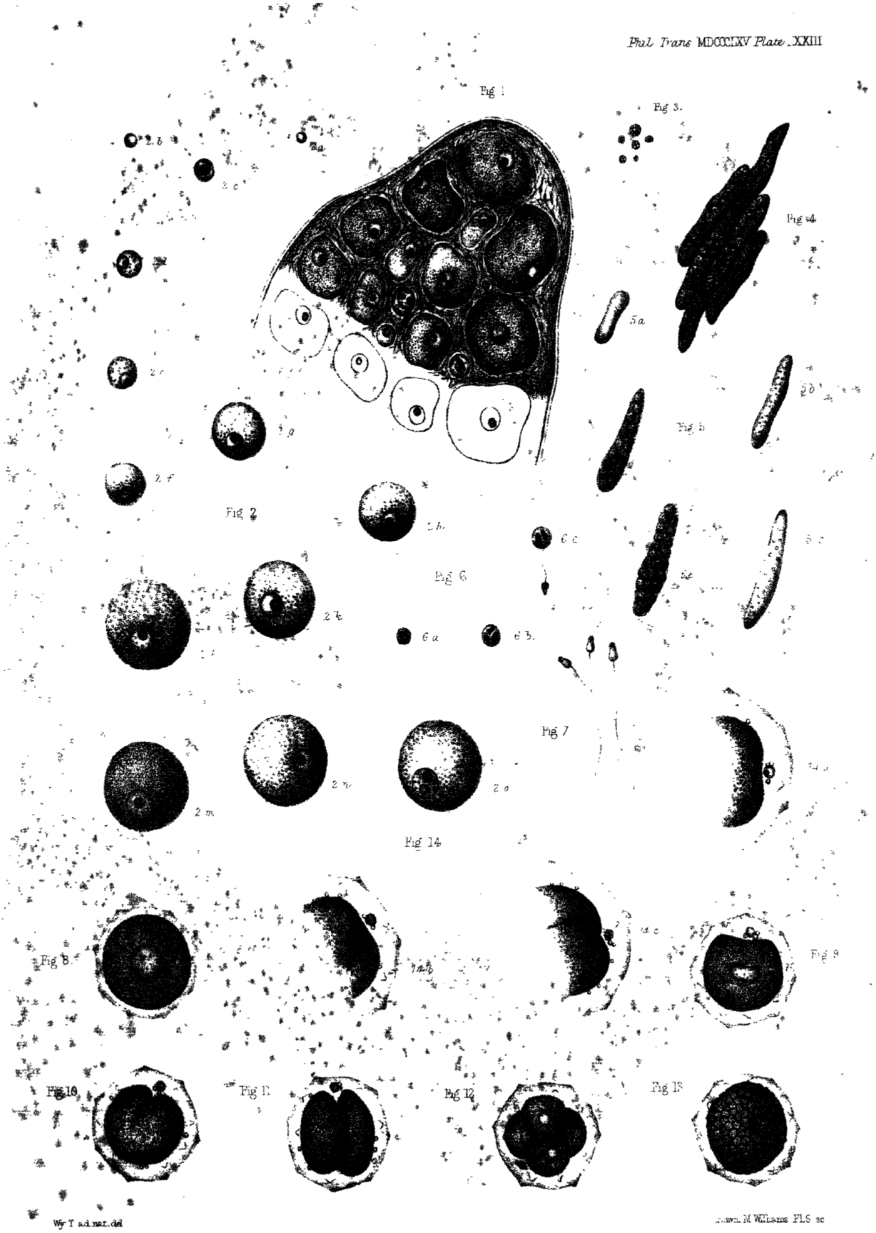
Presents.	Donors.
PLAT (Sir H.) The Garden of Eden, or a description of all Flowers and Fruits now growing in England. 12mo. *London* 1675.	H. Dircks, Esq.
PLATEAU (J.) Sur un problème curieux de Magnétisme. 4to. *Bruxelles* 1864.	The Author.
PLAYFAIR (Lyon, F.R.S.) On the Food of Man in relation to his useful Work. 8vo. *Edinburgh* 1865.	The Author.
POGGIOLI (Giuseppe) De Amplitudine Doctrinæ Botanicæ qua præstitit Fridericus Cæsius M. A. Poggioli in Archigymnasio Romano Doctoris decurialis botanices Commentatio. 8vo. *Romæ* 1865.	The Author.
PONCELET (J. V., For. Mem. R.S.) Applications d'Analyse et de Géométrie. Tome II. 8vo. *Paris* 1864.	The Author.
PORTA (J. B.) Natural Magick in twenty Books. fol. *London* 1658.	H. Dircks, Esq.
POUDRA (—.) Histoire de la Perspective Ancienne et Moderne. 8vo. *Paris* 1864.	The Author.
——— Des Réseaux. 8vo. *Paris* 1865.	———
——— Mémoire sur les Trigones, Tétragones, Hexagones. 8vo. *Paris* 1865.	———
——— Théorie Générale des Faisceaux et des Involutions avec les Applications aux tracés des Courbes des differents ordres. 8vo. *Paris* 1865.	———
PURGOTTI (S.) Intorno alcuni scritti inediti di Michelangelo Poggioli, Nota. 8vo. *Perugia* 1864.	The Author.
QUETELET (A., For. Mem. R.S.) Annales de l'Observatoire Royal de Bruxelles. Tome XVI. 4to. *Bruxelles* 1864.	The Observatory.
——— Annuaire de l'Observatoire Royal, 1864. 34^{e} année. 8vo. *Bruxelles* 1863.	———
——— Observations des Phénomènes Périodiques. 4to. *Bruxelles.*	———
——— Statisque et Astronomie. Sur le Cinquième Congrès de Statistique tenu à Berlin Sept. 1863. Physique du Globe. Phénomènes Périodiques. Notice sur la périodicité des Étoiles Filantes du mois de Novembre. Sur les Étoiles Filantes et leurs lieux d'apparition. Étoiles Filantes de la période du 10 Août 1863. (Excerpts from Bull. Acad. Roy. de Brux.) 8vo. *Bruxelles.*	———
QUETELET (E.) Sur le Mouvement Propre de quelques Étoiles. (Excerpt from Mém. Acad. Roy. Brux.) 4to. *Bruxelles* 1863.	The Author.
RADAU (R.) Sur la Formule Barométrique. 8vo. *Paris* 1864.	Mons. d'Abbadie.
——— Sur un Problème d'Analyse proposé par Lagrange. 8vo. *Paris*	The Author, by Mons. d'Abbadie.
RADCLIFFE (C. B.) Lectures on Epilepsy, Pain, Paralysis, and certain other Disorders of the Nervous System. 8vo. *London* 1864.	The Author.
RAMELLI (Agostino). Le Diverse et Artificiose Machine. fol. *Parigi* 1588.	H. Dircks, Esq.
RANKINE (W. J. M., F.R.S.) Papers on the Action of Waves upon a ship's keel, Isochronous-rolling ships, the Uneasy Rolling of Ships, and the Computation of the probable Engine-power and Speed of proposed Ships. 4to. *London* 1864.	The Author.
RATHBORNE (A.) The Surveyor, in four Books. fol. *London* 1616.	H. Dircks, Esq.

PRESENTS.	DONORS.
RICRAFT (J.) A Survey of Englands Champions and Truths faithfull Patriots. 8vo. *London* 1647, rpr.	H. Dircks, Esq.
RINUCCINI (Gio. Batista) Arcivescovo di Fermo. Nunziatura in Irlanda, negli anni 1645 a 1649, publicata per cura di G. Aiazza. 8vo. *Firenze* 1844.	———
ROBINSON (J.) On the present position of the Agricultural Labourer. 8vo. *London* 1864.	The Author.
SAFFORD (T. H.) On the Right Ascension of the Pole Star. 8vo. *Camb. Mass.* 1864.	Harvard Observatory.
SALTER (J. W.) and BLANFORD (H. F.) Palæontology of Niti in the Northern Himalaya, being descriptions and figures of the Palæozoic and Secondary Fossils collected by Col. R. Strachey. 8vo. *Calcutta* 1865.	Colonel Strachey, F.R.S.
SANNA SOLARO (J. M.) Mémoire sur le premier bassin de Dinothérium découvert dans le département de la Haute-Garonne. 8vo. *Toulouse* 1864.	The Author.
——— Nouvelle Théorie de la Grêle. 8vo. *Paris* 1863.	———
SANTAREM (Visconde de) Quadro Elementar das Relaçoes Politicas e Diplomaticas de Portugal com as diversas potencias do Mundo. Tomo IX. 8vo. *Lisboa* 1864.	The Lisbon Academy.
SARGANT (W. L.) On certain Results and Defects of the Reports of the Registrar General. 8vo. *London* 1864.	The Author.
——— Inconsistencies of the English Census of 1861. (Excerpt from Statistical Journal.) 8vo. *London* 1865.	———
SCACCHI (A.) Della Polisimmetria dei Cristalli. 4to. *Napoli* 1863.	The Royal Society, Naples.
SCHERZER (Dr. K.) Narrative of the Circumnavigation of the Globe by the Austrian Frigate Novara in 1857–59. 3 vols. 8vo. *London* 1861.	The Author.
SCHMIDT (A.) Hämatologische Studien. 8vo. *Dorpat* 1865.	The Author.
SCHOLEFIELD (J.) Twenty-four Hours under the Commonwealth: a Drama. 8vo. *London* 1863.	The Author.
SCHOLZ (B.) Anfangsgründe der Physik, als Vorbereitung zum Studium der Chemie. 8vo. *Wien* 1837.	H. Dircks, Esq.
SCHOPPERUS (H.) Πανοπλια omnium illiberalium Mechanicarum aut Sedentariarum Artium genera continens. 8vo. *Francofurti* 1568.	———
SCHOTTUS (G.) Magiæ Universalis Naturæ et Artis partes III. et IV. 4to. *Bambergæ* 1677.	———
SCHWENTER (D.) Deliciæ Physico-Mathematicæ oder mathemat. und philosophische Erquickstunden. 3 vols. 4to. *Nürnberg*, 1636–92.	———
SEGUIN, Aîné. Mémoire sur l'Origine et la Propagation de la Force. 8vo. *Paris* 1857.	The Author.
SELLON (E.) Annotations on the Sacred Writings of the Hindus. 8vo. *London* 1865.	The Author.
SMITH (Dr. E., F.R.S.) Address to the Subsection of Physiology of the British Association, Bath, Sept. 15, 1864. 8vo. *London.*	The Author.
SMITH (James) The Question, Are there any Commensurable relations between a Circle and other Geometrical Figures? answered. *London* 1860. Letter to the President and Vice-Presidents of the British Association, Liverpool, 1862. A Hole in Smith's Circle, by a Cantab. *London* 1859. (In 1 vol.)	H. Dircks, Esq.
SMITH (R. Angus, F.R.S.). Report on the Air of Mines and Confined Places. fol. *London* 1864.	The Author.

PRESENTS.	DONORS.
SMYTH (Admiral, F.R.S.) Sidereal Chromatics, being a reprint, with additions, from the "Bedford Cycle of Celestial Objects" and its "Hartwell Continuation" on the Colours of Multiple Stars. 8vo. *London* 1864.	Admiral Smyth and Dr. Lee, F.R.S.
SMYTH (C. P., F.R.S.) Astronomical Observations made at the Royal Observatory, Edinburgh. Vol. XII. 4to. *Edinburgh* 1863.	The Observatory.
———— Report read by the Astronomer Royal for Scotland to the Board of Visitors of the Royal Observatory, Edinburgh, 4th November 1864. 4to.	————
SORBIERE (S.) A Voyage to England, done into English. 8vo. *London* 1709.	H. Dircks, Esq.
SPRIGGE (J.) Anglia Rediviva, England's Recovery, being the History of the Army under Sir Thomas Fairfax. fol. *London* 1647.	————
STAINTON (H. T.) The Natural History of the Tineina. Vol. IX. 8vo. *London* 1865.	The Author.
STEWECHIUS (G.) Veteres de Re Militari scriptores. 8vo. *Lugd. Bat.* 1592.	H. Dircks, Esq.
STURLEVANT (S.), ROVENZON (J.), and DUDLEY (Dud). On the Manufacture of Iron. 4to. *London* 1612–1665, rpr.	————
STURM (L. C.) Kurtzer Begriff der gesambten Mathesis, bestehend in V. Theilen. 8vo. *Frankfurt an der Oder* 1707.	————
SYMONS (G. J.) British Rainfall. On the Distribution of Rain over the British Isles during 1860, 1861, 1862, 1863, 1864. 8vo. *London* 1862–65.	The Author.
TARTAGLIA (N.) Opere. 4to. *Venetia* 1606.	H. Dircks, Esq.
TATNALL (R.) An Antidote against the sinfull palpitation of the Heart or Fear of Death. 4to. *London* 1665.	————
TEYLER (J.) Architectura Militaris. 4to. 1553.	————
TODHUNTER (I., F.R.S.) A History of the Mathematical Theory of Probability from the time of Pascal to that of Laplace. 8vo. *Cambridge* 1865.	The Author.
TOYNBEE (J., F.R.S.) Beneficence in Disease: being an Introducytory Address delivered at the opening of St. Mary's Hospital Medical School, October 1, 1864. 12mo. *London* 1865.	The Author.
TREADWELL (D.) On the Construction of Hooped Cannon. 8vo. *Boston* 1864.	The Author.
TRITHEMIUS (J.) Libri Polygraphiæ VI. 8vo. *Argentinæ* 1600.	H. Dircks, Esq.
TURNOR (H.) Astra Castra: Experiments and Adventures in the Atmosphere [Prospectus]. 4to. *London* 1864.	The Author.
TYMME (T.) A Dialogue Philosophical wherein Natures secret closet is opened. 4to. *London* 1612.	H. Dircks, Esq.
VEGETIUS RENATUS et alii Scriptores Antiqui de Re Militari. 8vo. *Vesaliæ* 1670.	————
VERNON (G. V.) On the direction of the Wind at Manchester during the years 1849–61. On the Number of Days on which Rain falls annually at London. On the Rainfall at Oldham, by J. Heap. (Excerpts from the Lit. Phil. Soc. Manchester Mem.) 8vo. *London* 1864.	The Author.
———— On Excess or Deficiency of Temperature. On the Temperature of November. (Excerpts from the same.) 8vo. *London* 1865.	————
VICARS (J.) England's Worthies, under whom all the Civill and bloody Warres since anno 1642 to anno 1647 are related. 8vo. *London* 1647, rpr. 1845.	H. Dircks, Esq.

PRESENTS.	DONORS.
WAKEMANN (T.) Antiquarian Excursions in the Neighbourhood of Monmouth. 8vo. *Monmouth* 1860.	H. Dircks, Esq.
WATTS'S (H.) Dictionary of Chemistry. Parts 17–26. 8vo. *London* 1864–65.	The Editor.
WESTALL (W.) Six Views of Raglan Castle. folio.	H. Dircks, Esq.
WHARTON (G.) Calendarium Carolinum, or a New Almanack after the old fashion for 1662. 12mo. *London.*	———
WHEATLEY (H. B.) Notes on some English Heterographers. 8vo. *London* 1865.	The Author.
WHICHCORD (J.) On Hydraulic Lifts. 4to. *London* 1864.	The Author.
WIDDOWES (D.) Naturall Philosophy, or a Description of the World, namely, of Angels, of Man, of the Heavens, of the Ayre, of the Earths, of the Water, and of the Creatures in the whole World. 4to. *London* 1621.	H. Dircks, Esq.
——— Naturall Philosophy, the second edition. 4to. *London* 1631.	———
WILCOCKS (Alex.) Thoughts on the influence of Ether in the Solar System, its relations to Zodiacal Light, Comets, the Seasons, and periodical Shooting Stars. 4to. *Philadelphia* 1864.	The Author.
WILLIAMS (J.) Some Account of a Japanese Chronological Work. 8vo. *London* 1863.	The Author.
——— On an Example of Chinese Paper Currency of the Ming Dynasty. 8vo. *London* 1864.	———
WILLIAMSON (George) Observations on the Human Crania contained in the Museum of the Army Medical Department, Fort Pitt, Chatham. 8vo. *Dublin* 1857.	The Author.
——— Military Surgery. 8vo. *London* 1863.	———
WILSON (Erasmus, F.R.S.) An Inquiry into the relative frequency, the duration, and cause of Diseases of the Skin. 8vo. *London* 1864.	The Author.
WINCHELL (A.) On the Saliferous Rocks and Salt Springs of Michigan. 8vo. 1862.	The Author.
——— Descriptions of Fossils from the Marshall and Huron Groups of Michigan. 8vo. 1862.	———
——— Salt Manufacture of the Saginaw Valley, Michigan. 8vo. 1862.	———
——— Description of Elephantine Molars in the Museum of the University. 8vo. *Ann Arbor* 1863.	———
——— Fossils from the Potsdam of Wisconsin and Lake Superior. 8vo. 1863.	———
WINKLER (T. C.) Musée Teyler: Catalogue Systématique de la Collection Paléontologique. Liv. II. 8vo. *Harlem* 1864–65.	The Author.
WOLOWSKI (L.) Les Finances de la Russie. 8vo. *Paris* 1864.	The Author.
——— La Question des Banques. 8vo. *Paris* 1864.	———
WORCESTER (Marquis of) A Century of the names and scantlings of such Inventions as at present I can call to mind to have tried and perfected. 12mo. *London* 1663.	H. Dircks, Esq.
——— A Century, &c., 1663, reprinted, 8vo. *London.*	———
——— A Century, &c., 1663, reprinted, 12mo. *London* 1746.	———
——— A Century, &c. 12mo. *Glasgow* 1767.	———
——— A Century, &c. 12mo. *London* 1786.	———

PRESENTS.	DONORS.
WORCESTER (Marquis of) The Century of Inventions, from the Original MS., with Historical and explanatory Notes by C. F. Partington. 8vo. *London* 1825. The six editions in a morocco case.	H. Dircks, Esq.
YATES (James, F.R.S.) Some Account of a Volume containing portions of Ptolemy's Geography and of the "Geographi Græci Minores." 8vo. *London* 1864.	The Author.
YOUNGE'S (Walter) Diary from 1604 to 1628, edited by George Roberts. 4to. *London* 1848.	H. Dircks, Esq.
ZEISING (H.) Theatrum Machinarum. 5 vols. in 1. 4to. *Leipzick* 1612–14.	———
ZEPHAROVICH (V. Ritter von) Krystallographische Studien über den Idokras. 8vo. *Wien* 1864.	The Author.



Wy T ad nat. del

...awn M Williams FLS sc

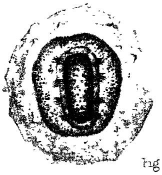


Fig. 4.

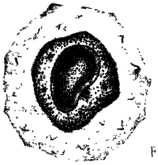


Fig. 3.

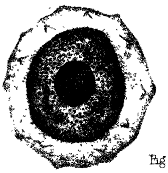


Fig. 2.

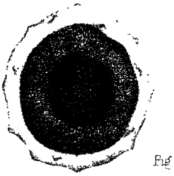
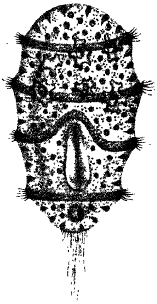


Fig. 1.

Fig. 7.


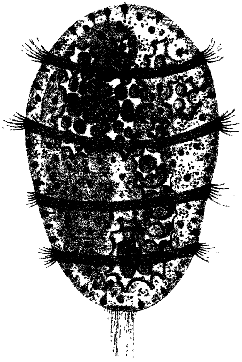


Fig. 6.

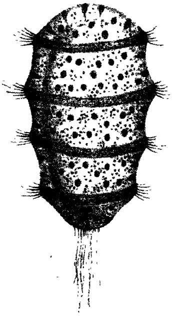


Fig. 5.

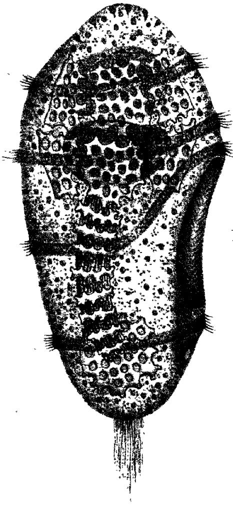
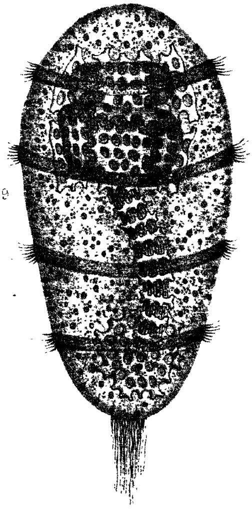
Fig. 8.


Fig. 9.


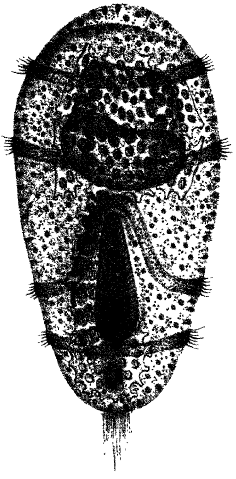


Fig. 10.

W. T. ad nat. del.

Edwn. M. Williams F.L.S. sc.

Phil. Trans. MDCCCLXV *Plate* XXV

Fig. 3

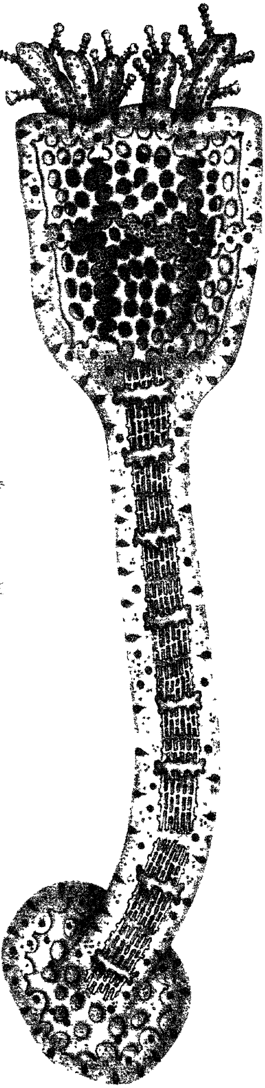


Fig.

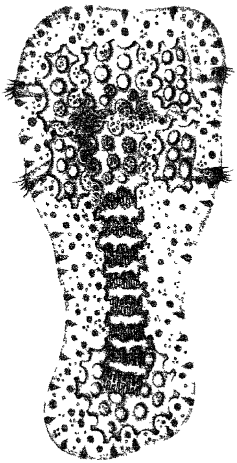
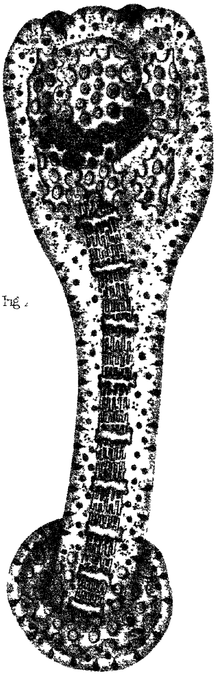
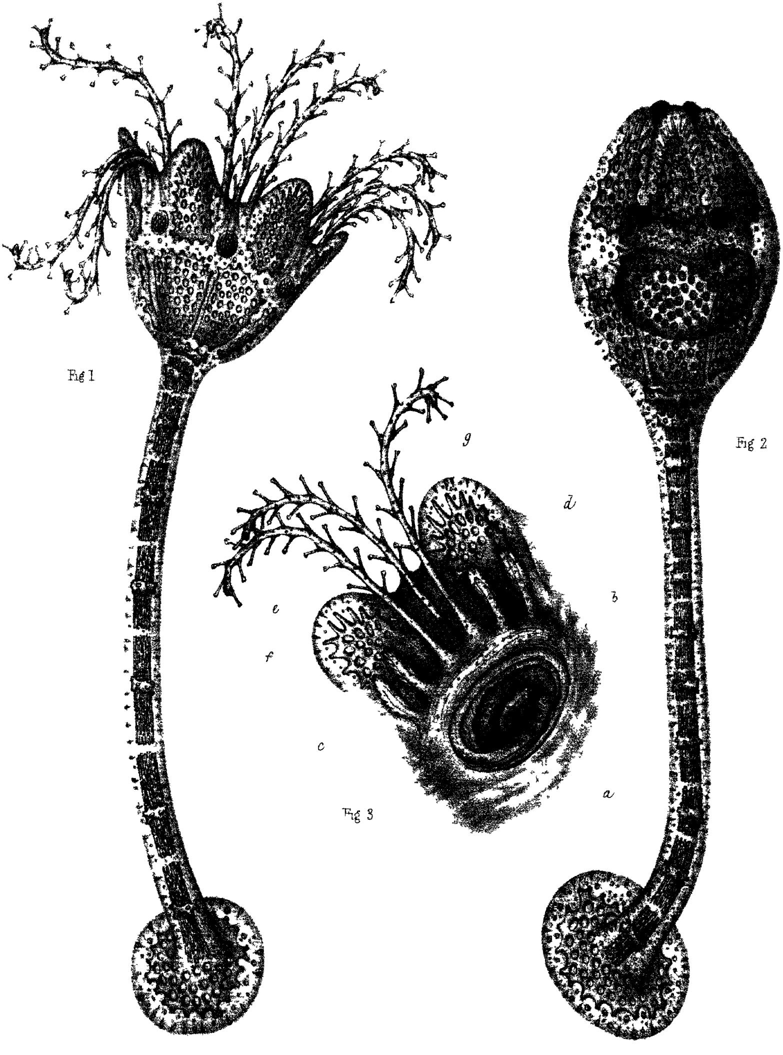


Fig.



W. T. ad nat. del.

Edwin M. Williams F.L.S. &c



Wy T ad nat del.

Edwin M Williams F.L.S. sc

Phil. Trans. MDCCCLXV Plate XXVII.

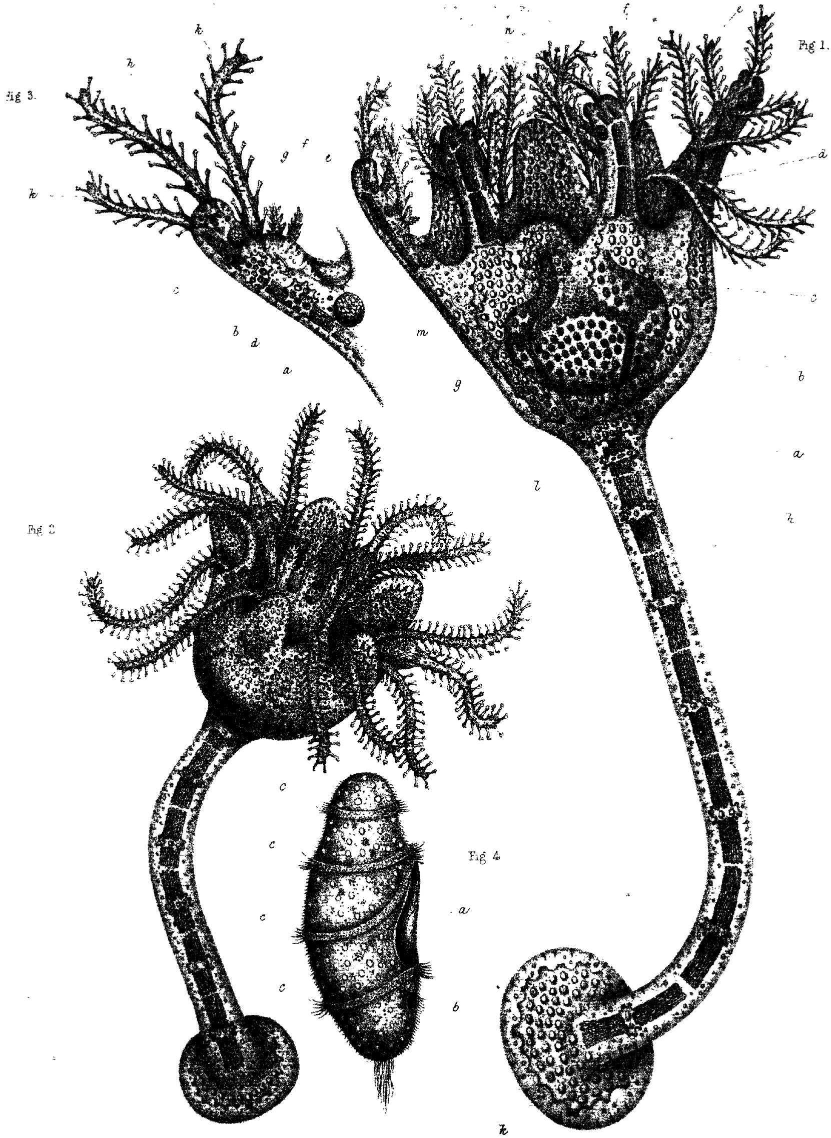


Fig 1.

Fig 2

Fig 3.

Fig 4

W. T. ad nat. del.

Edwin M. Williams F.L.S. sc

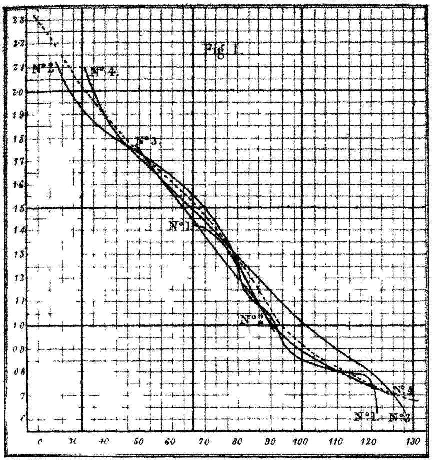


Fig. 1.

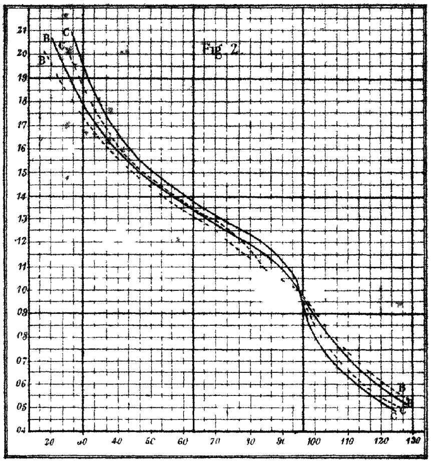


Fig. 2.

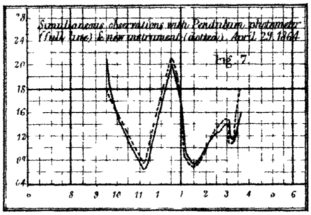

Simultaneous observations with Pendulum photometer (full line) & new instrument (dotted). April 29 1864

Fig. 7.

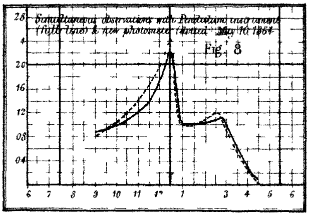
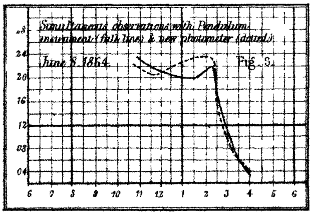
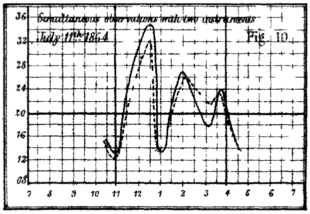

Simultaneous observations with Pendulum instrument (full line) & new photometer (dotted). May 10 1864

Fig. 8.


Simultaneous observations with Pendulum instrument (full line) & new photometer (dotted).

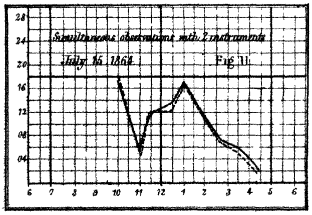
June 8. 1864.

Fig. 9.


Simultaneous observations with two instruments

July 11th 1864

Fig. 10.


Simultaneous observations with 2 instruments

July 15. 1864

Fig. 11.

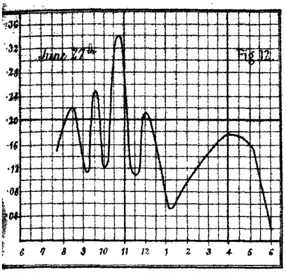


Fig. 12.

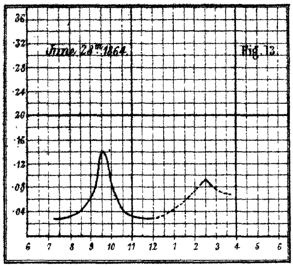


Fig. 13.

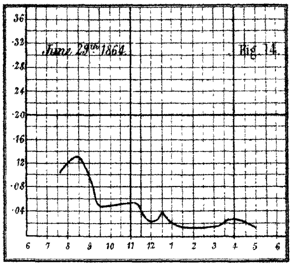


Fig. 14.

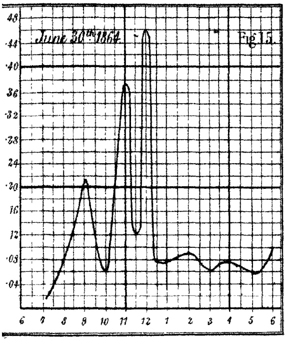


Fig. 15.

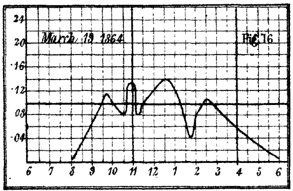


Fig. 16

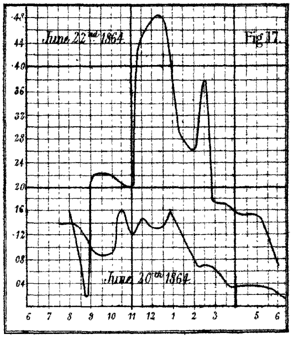


Fig. 17.

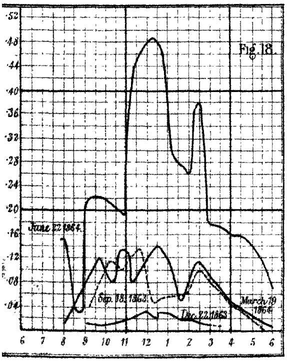


Fig. 18.

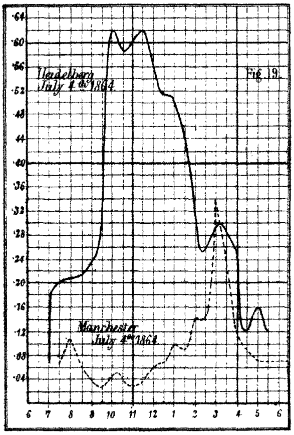


Fig. 19.

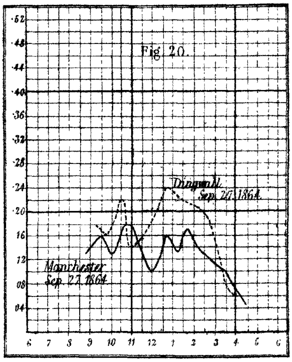
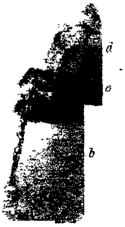


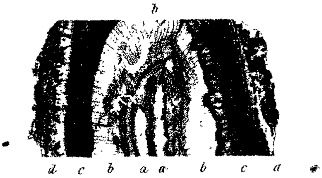
Fig. 20.

Plate A 1

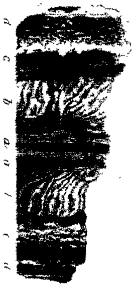
3



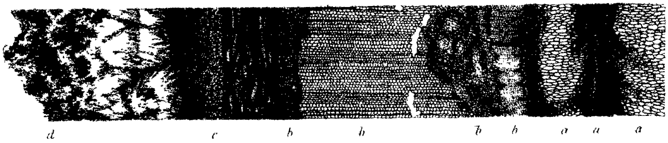
1



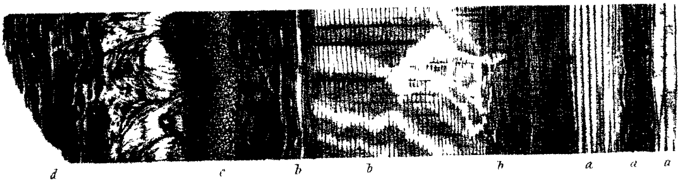
2



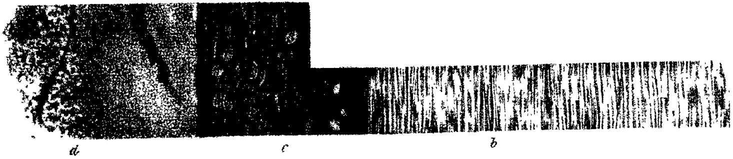
4



5



6

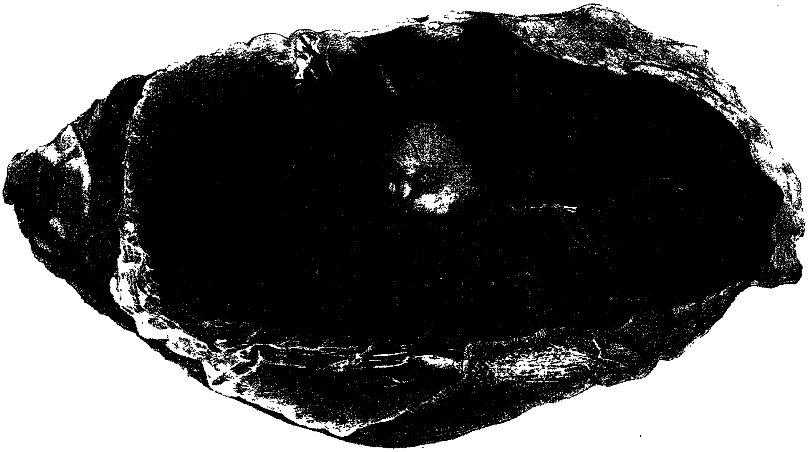


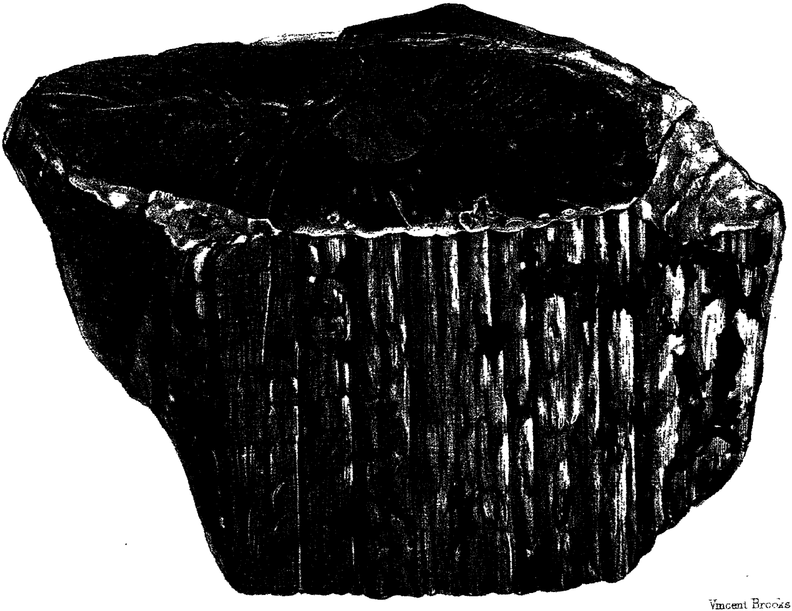
J N Fitch del et lith

Vincent Brooks Imp

Phil. Trans. MDCCCLXV. Pl. XXXI.

Plate A II.



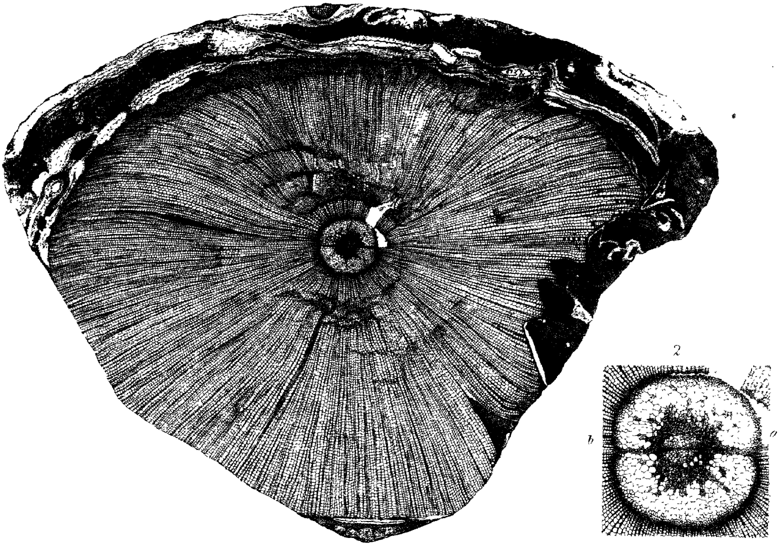


J N Fitch, del et lith

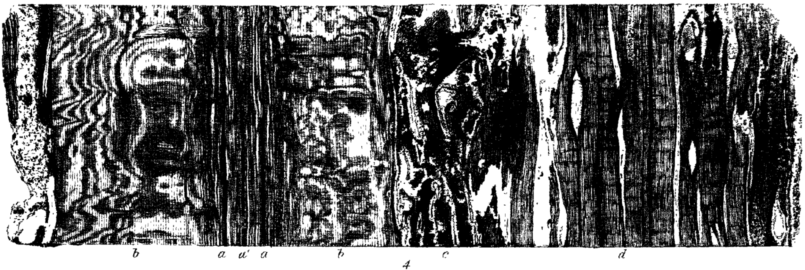
Vincent Brooks, Imp

1

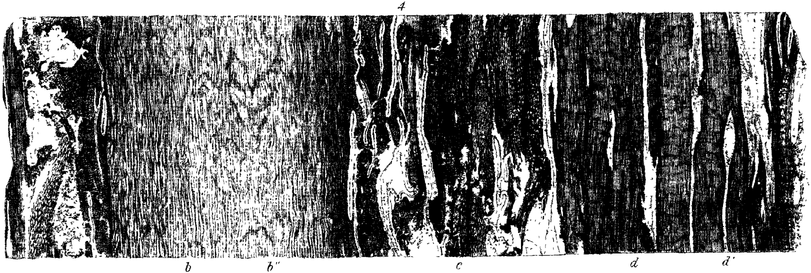
2



3



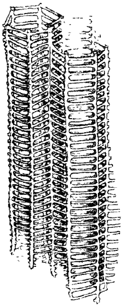
4



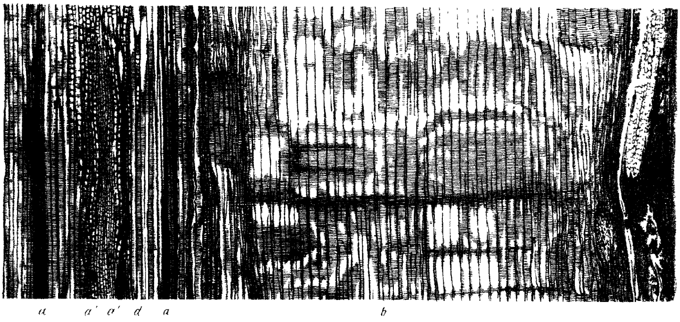
J N Fitch del et lith

Vincent Brooks Imp

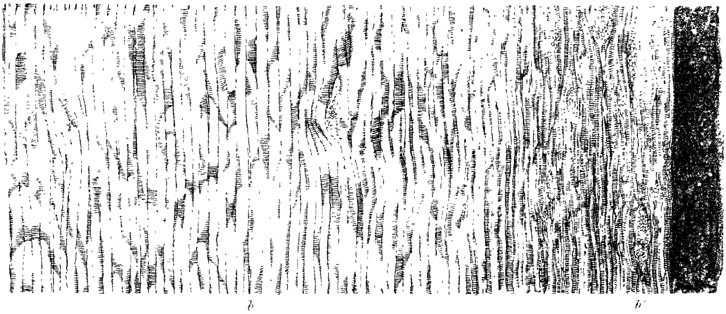
2



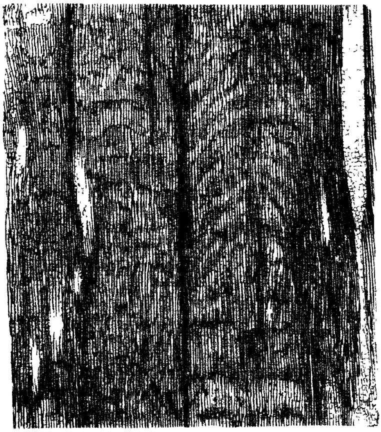
1



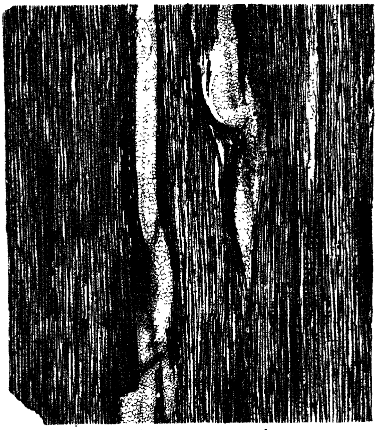
3



4



5

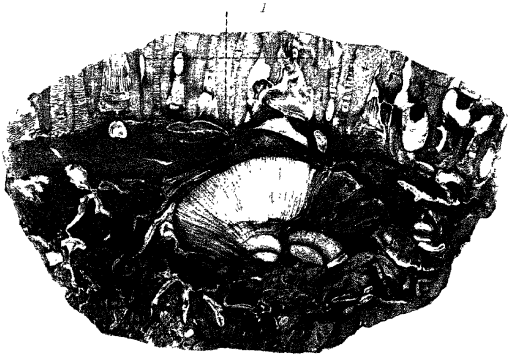


J N Fitch, del et lith

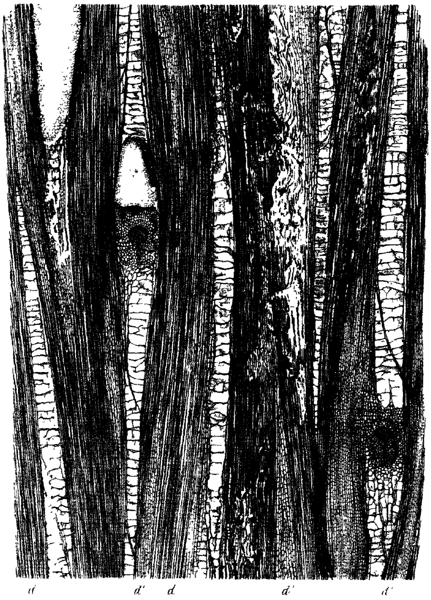
Vincent Brooks, Imp

Plate A V.

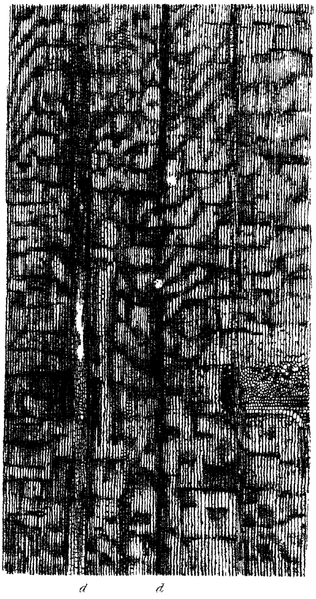
1



2

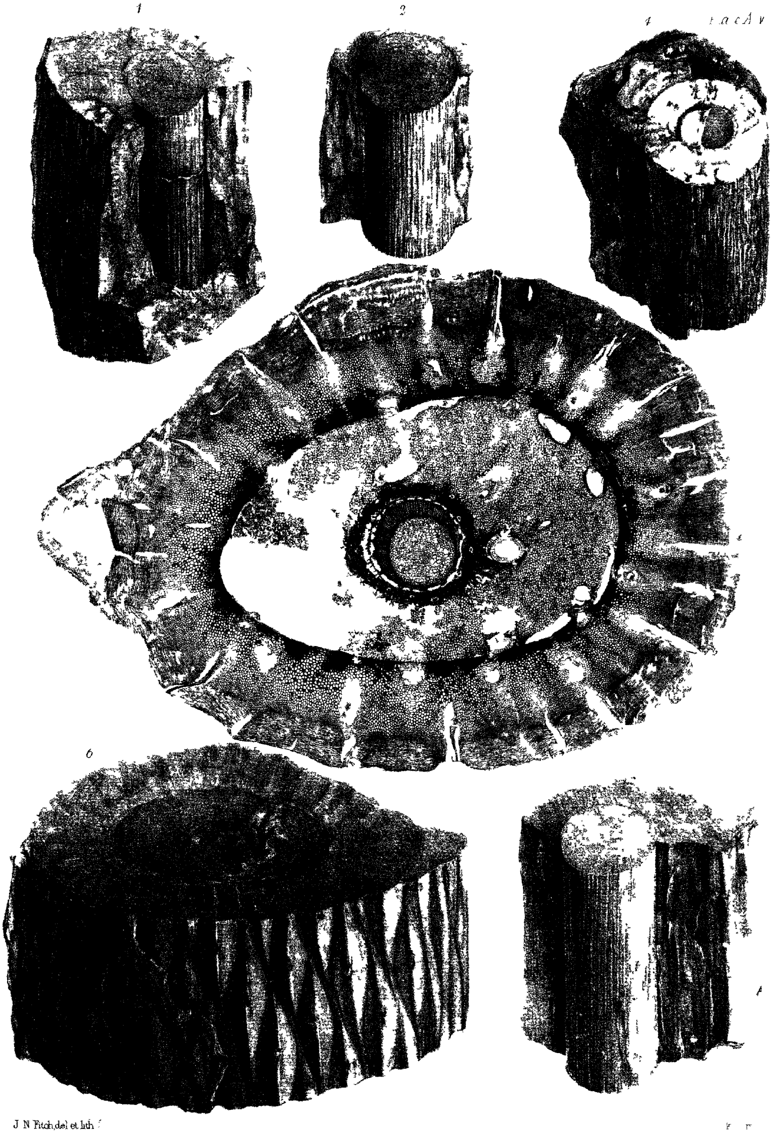


3



J. N. Fitch del. et lith.

Vincent Brooks Imp.



J. N. Fitch, del. et lith.

Fig 1

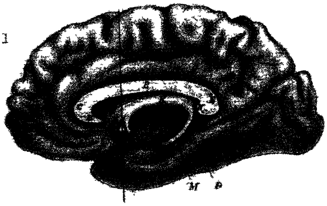


Fig 2

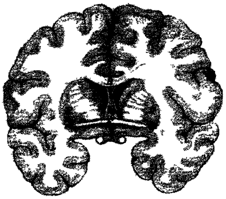


Fig 3

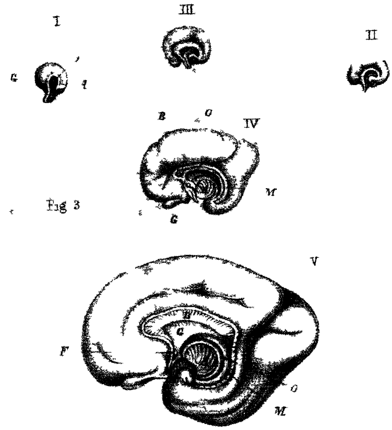


Fig 4

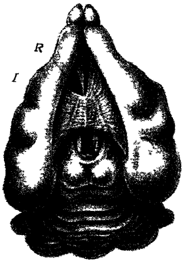


Fig 5

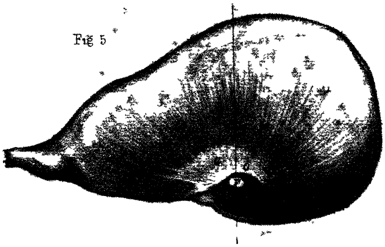
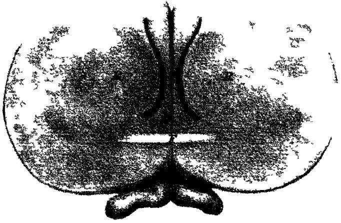


Fig 6



W.H.F. ad nat. del. (except Fig. 3)

Edwin M Williams F.L.S. Sc

Fig 1.

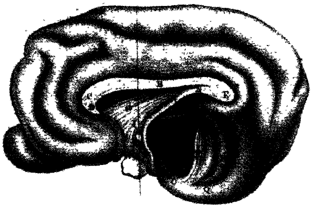


Fig. 2.

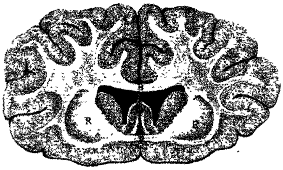


Fig. 3.

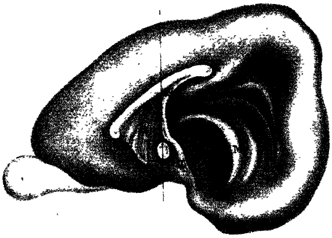


Fig. 4.

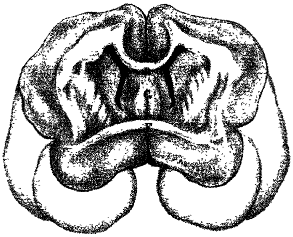


Fig. 5

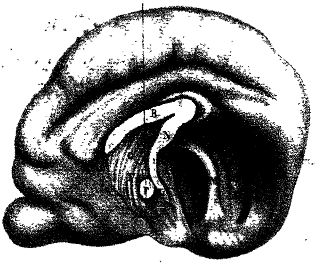


Fig. 6.

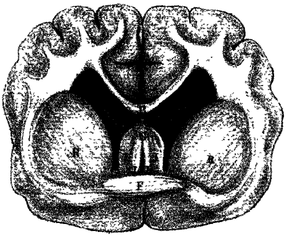


Fig. 7.

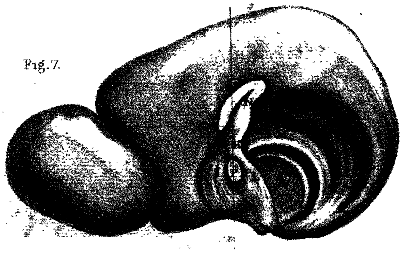
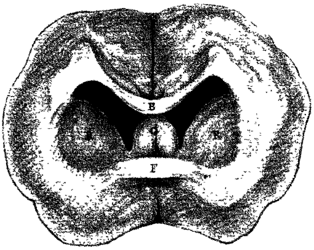


Fig. 8.



W.H.F. ad nat. del.

Edwin M. Williams, F.L.S. Sc.

Phil Trans MDCCCLXV *Plate* XXXVIII.

Fig 1

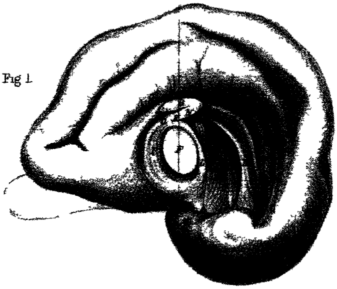


Fig 2

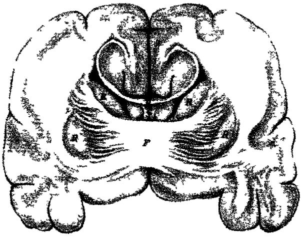


Fig 3

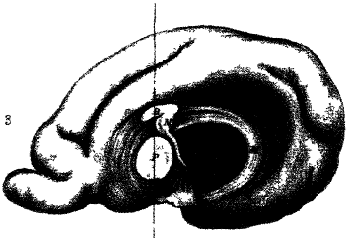


Fig 4

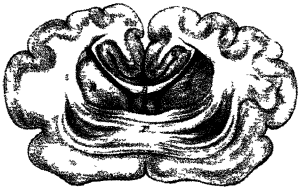


Fig 5

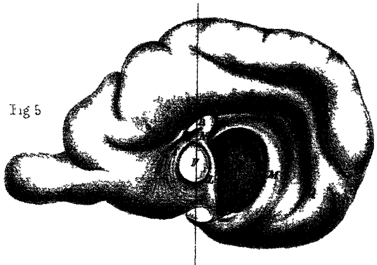


Fig 6

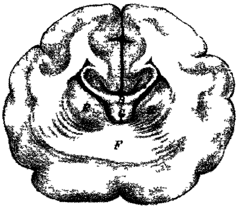


Fig 7

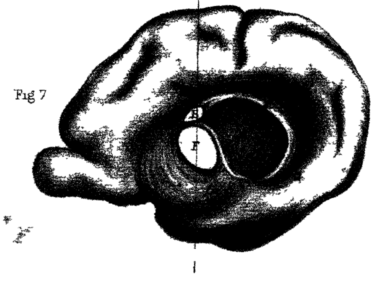
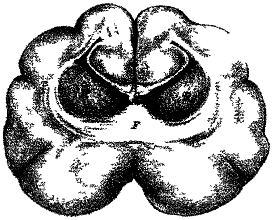
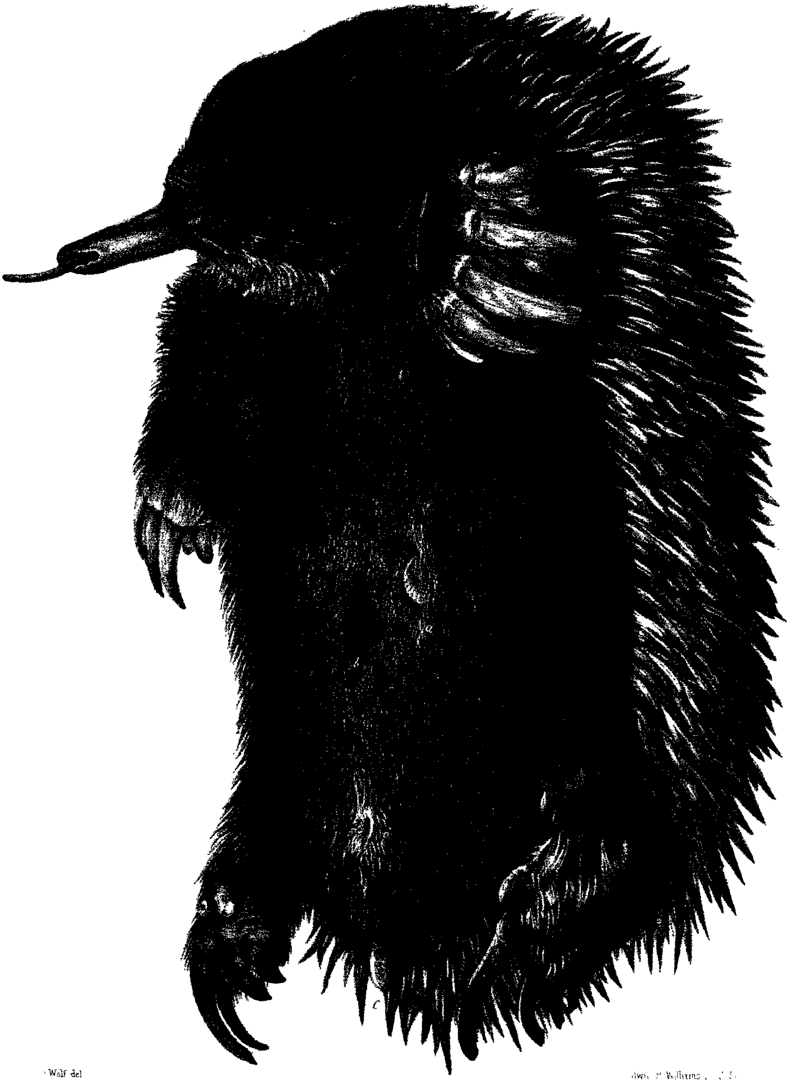


Fig 8



W.H.F ad nat del.

Edwin M Williams F.L.S Sc



Wolf del.

[illegible] Williams [illegible]

Fig 1

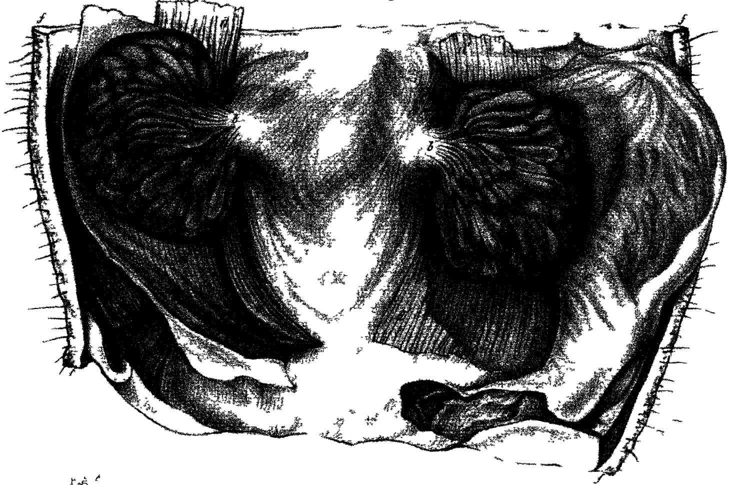


Fig 4

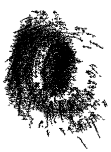


Fig 7

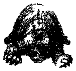


Fig 6

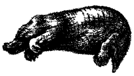


Fig 8

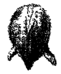


Fig 5

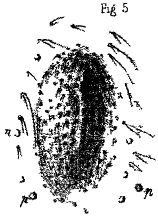


Fig 9

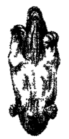


Fig 10

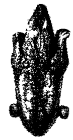


Fig 2

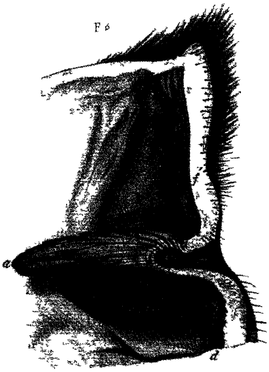


Fig 3

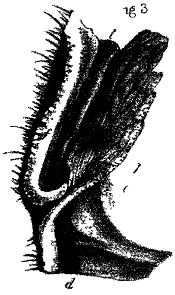


Fig 11

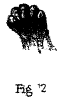


Fig 13

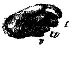


Fig 12

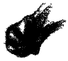
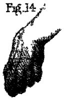


Fig 14



R. Owen F.R.S. del.

Edwin M. Williams F.L.S. Sc.

Fig 1

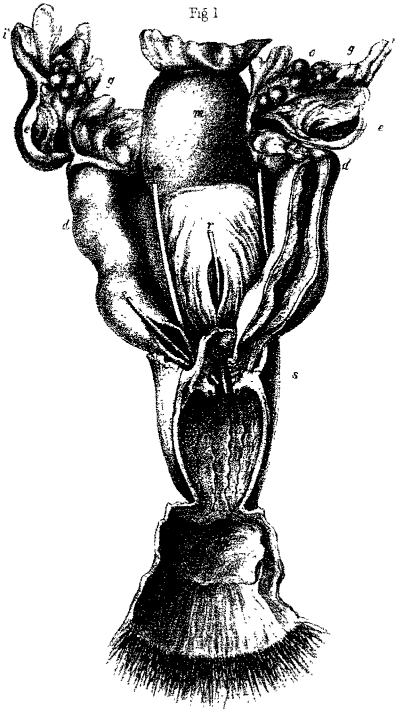


Fig 2

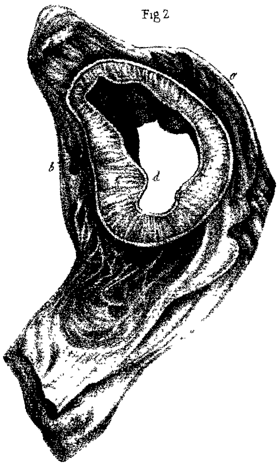


Fig 5

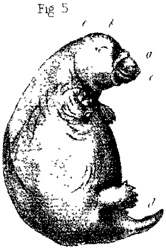


Fig 4

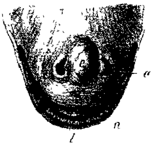
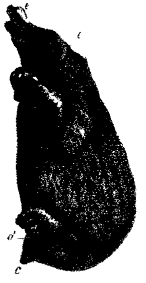


Fig 3



R. Owen, F.R.S. del.

Edwin M. Williams F.L.S. Sc.

I. A. R. I. 75.

IMPERIAL AGRICULTURAL RESEARCH
INSTITUTE LIBRARY
NEW DELHI.

Date of issue.	Date of issue.	Date of issue.